# OPTIMIZING SERVICE LEVEL, PRICE, AND INVENTORY DECISIONS FOR A SUPPLY CHAIN WITH RETAILERS' COMPETITION AND COOPERATION UNDER VMI STRATEGY

# Marzieh Karimi, Hasan Khademi-Zare\*<sup>®</sup>, Yahia Zare-Mehrjerdi and Mohammad Bagher Fakhrzad

Abstract. In a vendor-managed inventory (VMI) system, a manufacturing vendor manages their retailer inventories. Studies on VMI-type supply chains mostly have not considered competition between retailers. There are few works on the price competition; however, to the best of the authors' knowledge, none of the papers formulated a service competition strategy. The service level is one of the competitive factors among competing retailers. Sometimes retailers choose to compete cooperatively instead of competing independently with the manufacturer. The present work investigates service, price, and inventory decisions under retailers' competition and cooperation. Considering the manufacturer and retailers as the leader and followers, respectively, a Stackelberg game model of the problem is developed. The present study proposes a solution algorithm to search the Stackelberg–Nash equilibrium in the retailer cooperation and retailer independence models. The algorithm is numerically demonstrated to explore the impacts of decision parameters. To validate the model, a number of parameters are subjected to sensitivity analyses. It was found that a higher self-service (cross-service) level parameter would lead to higher (lower) profits of the retailer and manufacturer and the total profit in the two models. Retailer cooperation enhances retailer performance; however, manufacturer and system profits decline. Furthermore, when retailers cooperate, they are motivated to offer lower service levels.

Received May 30, 2021. Accepted March 4, 2022.

# 1. INTRODUCTION

Today, in order to enhance the competitiveness of firms and make quick responses to ever-changing customer demands in the market, it is required to optimize the management of supply chain (SC). Integration is a key indicator in the management of SCs. VMI is a new technique in this respect. It is a strategy of inventory management that enables vendors to have access to the sales data of their buyers and allows for managing their inventory levels. VMI empathizes inventory management improvement and the enhancement of the service level with no rise in the distribution cost or inventory level. It is useful to not only vendors but also buyers. Recently, many companies across a variety of industries are increasingly considering VMI as a strategic alternative for lowering costs and improving core skills. Samsung and Sony, for example, recently conducted VMI with Suning to develop a new cooperation model. In light of the VMI, DH Corporation and its distributor experienced

© The authors. Published by EDP Sciences, ROADEF, SMAI 2022

Keywords. Vendor managed inventory, service competition, price competition, Stackelberg, Nash games.

Department of Industrial Engineering, Yazd University, Yazd, Iran.

<sup>\*</sup>Corresponding author: hkhademiz@yazd.ac.ir

an increase in sales and a decrease in inventory levels. VMI helped Point Spring and Driveshaft Company (PSD) increase inventory efficiency. VMI has been popular in the high-tech industry; for example, Dell and HP improved their performance by reducing inventory levels and expenses through VMI [56].

In addition to its application in various industries, VMI has recently become one of the important research topics in supply chain literature. The majority of research in the VMI-type SC literature addresses a single retailer or numerous retailers in the absence of competition between them, avoiding the complexity that may occur as a result of retailers' competition. Despite the fact that commercial competition among independent retailers in a SC is a typical occurrence and is crucial in making optimal SC decisions, there is very little study on VMI systems that consider horizontal competition amongst retailers. Pricing used to be treated as the most important factor of competition [45]. Today, due to environmental and technological dynamics, retailers cannot only reduce their prices in the competition with other retailers and need to adopt more complicated strategies. Few studies focused on the non-price competition factor (advertising) among participating retailers in a VMI system [20,69]. Information technology (IT) has encouraged competing companies to sell comparable products at almost the same prices. Since services have a significant impact on consumer decisions to buy a product, they are considered as a new dimension of competition; in a way, competitors offer higher service levels to obtain a greater share of the market. With several independent retailers in a VMI system, each retailer can compete with other retailers by providing services to customers, resulting in a service competition amongst retailers. As a result, there is service competition between retailers in the VMI-type SC with numerous retailers, in addition to price competition. The impacts of these competitions on the members' operational strategies and profitability are crucial and intriguing issues that need to be addressed. To the best of the authors' knowledge. none of the papers in the VMI-type SC literature formulated pricing and service strategies together for multiple competing retailers. Retailers sometimes choose to engage in co-operational competition rather than individual competition with the manufacturer to maximize their total profit. For instance, a hot-wind retail store may share its inventory with competitors to increase customers. In this case, cooperation is preferred by the retailers [41]. Therefore, the effects of retailers' cooperation on system and player performances in case of service and price competition should also be studied. To fill these research gaps the present work models price and services competition in a VMI system mainly by to respond to the following questions:

- (1) How can modeling of retail service and pricing competition across retailers be modeled, and how can exploring its effects on the strategies and profitability of each VMI-type SC member be identified?
- (2) How does cooperation strategy influence the profitability and optimal strategies of the members?
- (3) How does the intensity of service and price competitions affect the optimal strategies and profitability?

Hence, the present study develops a nonlinear mathematical model to predict service level and dynamic price competitions in a VMI-type SC with a single manufacturer and several rival retailers. Each of the rivals adopts a pricing strategy and a service level under a suitable level of inventory. The present study makes significant contributions to the literature.

- The proposed model has several rival retailers that simulates in addition to the rivals' price competition, the rivals' service competition as an innovation of the present work.
- Two base models are adopted to investigate the problem, including a retail cooperation (RC) model and a retail independence (RI) model.
- The RI and RC models are comparatively analyzed, exploring the effects of crucial parameters on the profits and optimal strategies of the SC members. It demonstrates that retailer profits could be improved by cooperation, suggesting that retailers tend to engage in cooperation, unlike the system and manufacturer.

The remainder of the study is organized as follows: Section 2 reviews the literature; Section 3 describes the base model and proposes assumptions; Section 4 obtains the Stackelberg–Nash equilibrium in the RI and RC models, Section 5 solves the model; Section 6 discusses a numerical example; Section 7 performs sensitivity analyses; Section 8 provides the discussion and, Section 9 concludes the paper.

# 2. LITERATURE REVIEW

This section discusses the main streams of the work, namely, service competition, retailers' competition and cooperation and vendor management inventory strategies.

# 2.1. Service competition

Concerning service strategies, Tsay and Agrawal [54] studied a single-supplier dual-retailer SC where the rival retailers competed for service and price. It was reported that the two rivals preferred competition enhancement in some cases. Tsay and Agrawal [55] investigated cooperation and competition in a SC with a retail channel and a direct channel. Bernstein and Fedregruen [9] introduced a service and price competition-based general stochastic equilibrium inventory framework. Bernstein and Fedregruen [11] analyzed service and price-based coordination mechanisms in SCs of decentralized structures. They incorporated the competition of independent retailers under random demands. Yao et al. [68] explored the effects of information sharing on optimal strategies when value-added services were provided by a retailer. To compare the influences of retail risk sensitivity on channel member strategies between two SCs, Xiao and Yang [62] developed a demand uncertainty-based competitive price-service framework. It was demonstrated that a larger risk sensitivity of a retailer would reduce the optimal service level and retail price. Wu [59] evaluated common service and price strategies in various channels. The service levels could be either sequentially or simultaneously adjusted by the vendors and retailers. Wu [60] studied a bi-tier SC in terms of service and price competition between the reproduction of two manufacturers and new products. The price and service competition levels were found to influence the recycling cost and service investment, particularly for the manufacturers of new products. A bi-level SC model with a single vendor and two rival retailers competing in service level was investigated [21]. Rezapour and Farahani [45] introduced a bi-level competitive SC model with retailer price and service level competition. Ali et al. [2] analyzed a demand disruption-subjected SC model with several rival retailers and price and service strategies. Pi et al. [41] evaluated a demand disruption-subjected two-channel SC model with retailers' cooperation and competition in terms of service and price policies. The vendor provided a product by an online direct channel and two rival retailers. Game theory was adopted to achieve Stackelberg–Nash equilibrium. In this study, service competition under a VMI-type SC will be considered which has been neglected so far.

## 2.2. Retailers' competition and cooperation

Concerning the competition and cooperation of retailers, Bernstein and Federgruen [8] studied the distribution of a single product by a single manufacturer to N retailers in terms of replenishment and price policies based on Bertrand and Cournot competition. Their study was extended by incorporating the competition and cooperation behavior of retailers [14]. Bernstein and Federgon [10] built a demand uncertainty-based SC model with a decentralized structure, a single vendor, and a number of rival retailers. They proposed contracts to enable the SC to have centralized behavior. Cachon [12] examined the inventory decisions of a bi-level single-manufacturer multi-retailer SC model. The retailers could choose to either compete or cooperate. Anderson and Bao [4] analyzed decentralized and centralized SC models in terms of price competition. A SC model with a single supplier and several differential retailers was introduced under coordination and competition, reporting that suppliers would tend to maximize the number of retailers [17]. Numerous studies have been conducted in the inventory [1, 35, 49, 65, 76] and return strategy [13] literature by considering the competition of retailers. Yan and Zhao [64] evaluated the coordination and inventory sharing of retailers within the SC with retailers' independent order quantity determination and collaborative inventory sharing. Shao et al. [48] studied a SC model with a decentralized structure, a single monopolist manufacturer, and several dependent retailers in terms of transshipment price policies. Huang et al. [32] introduced a single-manufacturer dual-retailer SC in terms of price competition and cooperation. Glock and Kim [24] investigated the forward vendor integration approach and multi-retailer competition. In their model the vendor could choose to cooperate with a retailer. The replenishment and pricing policies of single-manufacturer multi-retailer SC models with centralized and decentralized structures were compared [14]. Zhang et al. [77] proposed a single-manufacturer dual-retailer SC

framework based on retailer inventory competition and transshipment. Yu and Huang [69] and Deng *et al.* [20] studied the VMI system based on the competition between retailers in the presence of price and advertising. In contrast to earlier studies, the present work focuses on retailers' competition and cooperation in a VMI-type SC with joint price and service competitions.

## 2.3. Vendor management inventory

Many studies have been conducted on VMI-type SCs. The benefits of adopting this inventory cooperative technique are measured in one area of the literature [7,31,50,57,67]. Another part of the literature examines the best decisions made by SC members when using a VMI contract. For instance, Darwish and Edah [16] studied a VMI-type SC model with several retailers and a single manufacturer. They incorporated deterministic demand into the model. The manufacturer was penalized when an item exceeded the upper inventory limit. This could be considered as a capacity constraint. On the other hand, Kassagri et al. [33] incorporated several retailers and a one manufacturer into a VMI model in order to manage deteriorating product management. They adopted the PSO and GA techniques to identify optimal solutions. Later, a bi-level VMI-type SC with several retailers and products and a single manufacturer was established to optimize joint replenishment by using the GA and TLBO methods in nonlinear model solving [40]. After, Pramudyo and Luong [42] developed a SC of several retailers and one manufacturer based on the VMI strategy and stochastic demand. They sought to obtain minimized total system cost by adjusting the lot sizes of the manufacturer and retailers, replenishment frequency, and retailer cycle time. In the same year, the VMI approach was adopted to coordinate a SC of a single manufacturer and a single retailer under five contracts (*i.e.*, quantity flexibility, buy-back, revenue sharing, quantity discounts, and sale rebate) [47]. Simultaneously, Hariga et al. [28] adopted a VMI consignment approach with a single manufacturer and a single retailer to study integrated environmental and economic variables. Next, Karampour et al. [34] studied inventory profit maximization and transportation carbon emission minimization through a green SC model under VMI strategy. They adopted the MOKA, MORDA, and NSGA-II approaches to solve the problem. While earlier studies did not use a game theory approach, the Stackelberg game is used in some works to describe the competitive relationships between the manufacturer and retailers. This methodology is widely used in the VMI system [3,6,19,20,27,37,43,44,51,57,70,71,73,75]. In VMI models, the manufacturer manages the finished product inventories for all the retailers as the leader dominating the SC, and the retailers are followers.

The type of demand is a significant parameter that distinguishes VMI system model from the others. To facilitate modeling, most of the research in the literature assumed a fixed demand [5, 16, 18, 25, 26, 28, 30, 31, 33, 34, 36, 38, 40, 42, 46, 47, 50, 52, 53, 57, 58, 67, 74]. However, real-life demands are a (linear or nonlinear) function of the price [3, 6, 19, 27, 29, 37, 43, 44, 51, 57, 70, 71, 73, 75]. For example, Yu et al. [70] explored a VMI system with several retailers and a single manufacturer in which each retailer's demand followed the Cobb-Douglas demand function. They adopted the Stackelberg-game theory approach to identify the advantages of the manufacturer's information on the retailers. Later, Almehdawe and Mantin [3] introduced a Stackelberg game model of VMI system with several retailers and one manufacturer in which the demand of each retailer considered a decreasing and convex function due to its retail price They studied two scenarios under the model. The manufacturer serves as the leader in one of the scenarios, while one of the retailers was selected to be the leader in the other scenario. Thereafter, A Stackelberg game model was built to enable a single manufacturer within a VMI system to optimally select retailers whose demand rate is a linear function of price. A combined GA-analytical-dynamic programming model was formulated to solve their bi-level mixed-integer nonlinear model [73]. Next, Taleizadeh et al. [51] introduced a VMI Stackelberg game framework to optimize the prices, production rate, and replenishment cycle and frequency for perishable goods. Concurrently, Rasay et al. [44] studied VMI performance optimization by developing a mixed-integer nonlinear approach. In their study, the demand of each retailer for the product is determined by a constant elasticity demand function. They incorporated revenue-sharing contracts, a centralized framework, and wholesale price contracts into the approach. Afterwards, Giovani et al. [19] evaluated the impacts of collaborative advertising within bilateral

monopolies by using a differential Stackelberg framework. They combined the management of inventory and production with advertising and pricing approaches within a consignment-contracted SC.

Business competition among individual retailers in a SC, on the other hand, is a common occurrence that plays a significant role in making optimal SC decisions; in the above studies, due to the exclusion of complexities, the competition between retailers was not considered. However, there is very little research on VMI systems dealing with competing horizontal entities (retailers). In the competition context, pricing used to be treated as the most important factor [45]. In addition, retailers today compete with more complicated strategies than simply lowering retail prices. Few studies focused on the non-price competition factor (advertising) among participating retailers in a VMI-type SC. For example, Yu and Huang [69] optimized the price and advertising strategies of a tri-level VMI system framework with a single manufacturer and several suppliers and retailers. They incorporated retailer-retailer and retailer-manufacturer competitions into their model. Deng *et al.* [20] studied a demand uncertainty -based multi-retailer VMI-type SC model in terms of advertising and price policies. These VMI studies did not focus on the problem of service competition as a competition factor; however, this non-price competitive parameter has a significant impact on consumers' decision to buy a product [22]. This study seeks to take a step further by considering a VMI system with multiple competing retailers, which will allow for contributing to the existing literature by explaining how the members choose their optimal strategies in the face of service and price competitions between retailers. Table 1 shows some studies on VMI models.

# 3. PROBLEM DESCRIPTION AND FORMULATION

This section describes problem, notations and mathematical framework.

# 3.1. Problem definition

The study context is defined as follows for the problem definition:

- (i) There are a single vendor (manufacturer) and several retailers in the VMI system. At a constrained production capacity, a product is produced by the manufacturer and then was replenished to retailers.
- (ii) The VMI system's manufacturer tends to have the capability of managing retailer inventories. The inventory costs are shared by the manufacturer and retailers. Each of the retailers pays a demand size-based inventory cost of  $\xi_i$  to the manufacturer. Also, the manufacturer pays the remaining inventory cost.
- (iii) The manufacturer adopts a common replenishment cycle policy to diminish inventory costs and levels [20,41,59,72].
- (iv) The retailers sell product in competing markets. The income of a retailer is dependent on the retail price and service level polities of both the retailer and their rivals.
- (v) Retailer product demand depends on the price and service levels of the retailer and the rivals. Linear retailer demand functions were assumed as [2,9,11,15,39,41,45,54,60-63,66]:

$$D_{i}(p,s) = K_{i} - ap_{i} + \sum_{\substack{j=1\\j \neq i}}^{n} bp_{j} + \alpha s_{i} - \sum_{\substack{j=1\\j \neq i}}^{n} \beta s_{j}.$$
(3.1)

Therefore, a retailer has a market demand in the form of a reducing (increasing) function of their price (service level) and an increasing (reducing) function of their rival prices (service levels).

(iv) The Stackelberg–Nash game is illustrated in Figure 1. It consists of two games: a vertical Stackelberg game and a horizontal Nash game. The decision results,  $p_i$  and  $s_i$ , are visible to the other retailers, can be reacted to by them, and can thus influence their profitability. As a result, all the retailers play a Nash game. In addition, a Stackelberg game is used to formulate the manufacturer–retailer leader–follower interaction. The manufacturer's decision  $(w_p, C, y_i)$  is influenced by the decision results  $p_i$  and  $s_i$  of n retailers.

Figure 2 shows the flowchart of the methodology to define the formulation and solution procedure.

1056	M. KARIMI <i>ET AL</i> .																										
	sbohtəm noituloZ				Exact	Exact	Exact	Exact Hvbrid	Methahuristic	Heuristic	Exact	Exact	Exact	Heuristic	Exact	Exact	Exact	Nietnanuristic	Exact	Exact	Heuristic	Exact	Exact	Exact	Exact	Exact	ti retailers); c and multi tralized and
	рисоше			* Exact	т *	ш, ,				щ	Η			¥	, <del>,</del>						• == *		Т	-	Η	*	mul <sup>-</sup> endor (cent
			Purchase	*	*	* 3	 	- *	*	*	*	*			*	, A					~			*		*	s and gle v
	Objective function		Production	*	*	* -	<del>(</del>	* *			*			*	÷	*		*			*	*				*	ndors (sing ); C&
			ransprotention	*	*	* :	÷ *	* *		*		* ÷	* •	*	*					*	*					*	ti ve -MB lized
		Cost	Shortage	*	*	÷	÷ -×	*				* ÷	* ·	*			*	÷			*		*		*	*	(mul); SV.
			Inventory holding	*	*	* -	÷ -×	*	*	*	*	* ÷	* ·	*	* -	* →	÷ *	÷ -*	*	*	*	*	*	*	*	*	/-MR tailer) (dec
			Bring	*	*	* 3	÷ -×	*	*	*	*	* ÷	*		* -	* →	÷ *	÷	*	*	*		*	*	*	*	; MV le ret ; DC
TABLE 1. Some studies on VMI models.			Determinism	C	U	U d	50	50	) D	Ŋ	U	0	D	D	U :		ວເ	50	00	0	) D	D	D	D	D	υ	ilers) sing ailer)
	Cooperation between retailers																										i reta and le ret
	<sup>1</sup> Competition factors		gniaitrevbA			*															*						nd mult facturer nd sing]
	<sup>1</sup> Comp∈ factors		Price			*															*					*	lor al nanu ier a
	<sup>1</sup> Cc fact		Service level																							*	vend gle n suppl
	Competition types		Between retailers	No	$N_{O}$	Nash	No	No No	No	No	No	No	No	No	No S	No	No	No No		No	Nash	No	No	No	No	Nash	(single sin (single sin (single s
			Between manufacturer and retailers	Stackelberg	$\operatorname{Stackelberg}$	$\operatorname{Nash}_{\widetilde{\alpha}}$ .	Stackelberg	Stackelberg	No	No	Stackelberg	Stackelberg	No	Stackelberg	No 	Stackelberg	No	C4 colcolle oue	No	No	Stackelberg	Stackelberg	No	No	No	Stackelberg	ilers); SV-MR (single vendor and multi retailers); MV-MR (multi vendors and multi retailers); t (single supplier, single manufacturer and single retailer); SV-MB (single vendor and multi ailer); SS-SR (single supplier and single retailer); DC (decentralized); C&DC (centralized and
	Structure				DC	DC	C&DC		0 U	C	DC	$C$ $\mathcal{C}$ $C$	C	DC	G G G C	C&DC	с С			00	DC	C&DC	C	U	C	DC	l multi reta SS-SM-SF l single ret
		SM-MR	SM-MR	SM-MR	SM-MK	SM-MR.	SV-MR	SV-MR	SV-MR	SV-MR	MV-MR	SM-MR	SV-SB	SS-SM-SR	SV-MB	ALM-VC	SV-SR	SM-MR.	SV-MR	SV-SR	SV-SB	SM-SR	SS-SR	SM-MR	ufacturer and single buyer); ufacturer and		
			ələitrA	Yu et al. [70]	Yu et al. $[71]$	Yu and Huang [69]	Almahda and Mantin [3]	Braide <i>et al.</i> [0] Yu <i>et al.</i> [73]	Sadeghi and Niaki [46]	Govindan [26]	Taleizadeh $et \ al. \ [51]$	Rasay et al. [44]	Hong et $al.$ [31]		Hemmati <i>et al.</i> [29]	Haji et al. $\begin{bmatrix} 27 \\ 6 \end{bmatrix}$	Taleizadeh <i>et al.</i> [52]	Framuayo and Luong [42]	GIOVAIIII <i>et al</i> . [19] Hariga <i>et a</i> l [98]	Son and Ghosh [50]		Wei et al. $[57]$	Taleizadeh $et al.$ [53]	Giovanni [18]	Ashraf $et [al.] 5$	Current study	Notes. SM-MR (single manufacturer and multi retailers); SV-MR (single vendor and multi retailers); MV-MR (multi vendors and multi retailers); SV-SB (single vendor and single buyer); SS-SM-SR (single supplier, single manufacturer and single retailer); SV-MB (single vendor and multi buyers); SM-SR (single manufacturer and single retailer); SS-SR (single supplier and single retailer); DC (decentralized); C&DC (centralized and

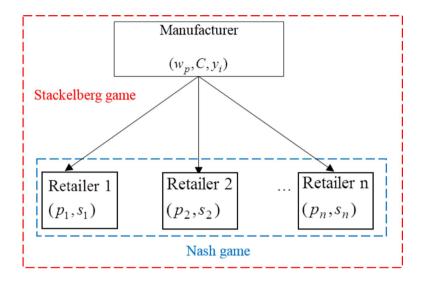


FIGURE 1. Schematic of the problem and Nash-Stackelberg game.

# 3.2. Notations

The notations included:

# Parameters

- $D_i(p,s)$  Demand rate of the retailer i
- $K_i$  Market scale for the retailer i
- *a* Self-price elasticity of demand
- *b* Cross-price elasticity of demand
- $\alpha$  Self-service level effect of demand
- $\beta$  Cross-service level effect of demand
- *Cap* Production rate of the manufacture
- Cm Production cost of the product by the manufacturer (\$/unit)
- $Hr_i$  Holding cost at retailer *i*'s side ( $\frac{1}{\min(\min)}$
- Hm Holding cost at the manufacturer's side (/unit/time)
- $Br_i$  Backorder cost paid by the manufacturer to retailer *i*'s side ( $\frac{1}{\sqrt{1}}$
- $Sr_i$  Fixed order cost of the product for the retailer *i* (\$/order)
- Sm Fixed order cost for the product at the manufacturer's side (\$/order setup)
- $\phi_i$  Transportation cost per unit product shipped from the manufacturer to retailer *i* (\$/unit)
- $\xi_i$  Inventory cost paid to the manufacturer by the retailer *i* (\$/unit/time)
- $\eta$  The service cost factor for the retailer  $\eta > 0, \frac{1}{\eta}$  reflects the service investment efficiency.

# Manufacturer decision variables

- C Common replenishment cycle time for the product
- $y_i$  Fraction of backlogging time for retailer i
- $w_p$  The manufacturer's wholesale price (\$/unit)

# Retailer decision variables

- $p_i$  The sales price of retailer *i* (\$/unit)
- $s_i$  The service level of retailer i

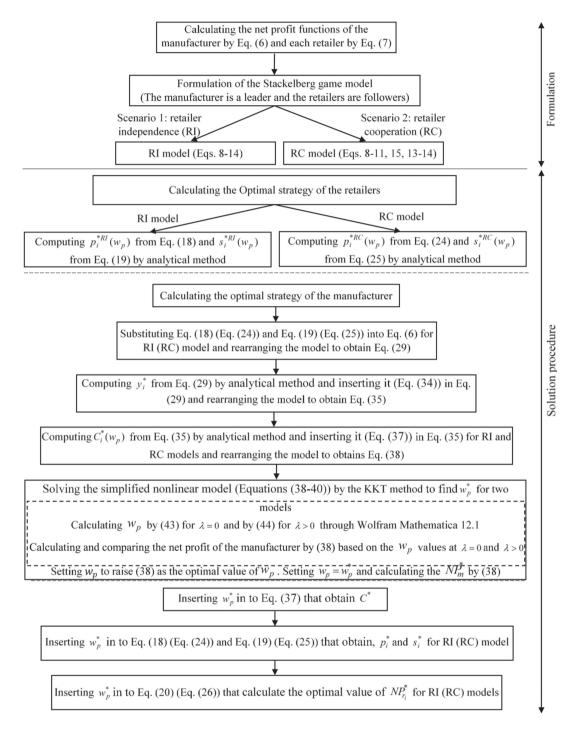


FIGURE 2. Flowchart of research methodology.

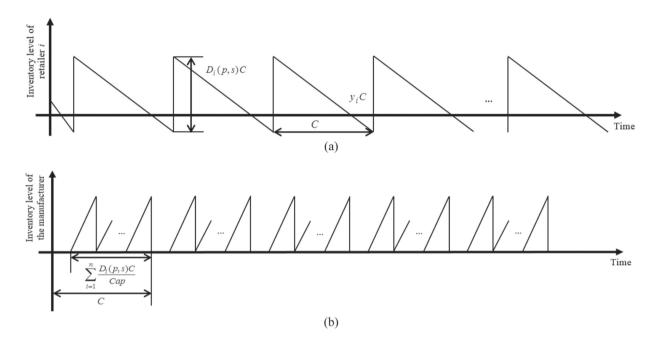


FIGURE 3. Inventory level of (a) retailer i and (b) manufacturer.

# 3.3. Player profit function

As mentioned, the proposed model consists of two players: the manufacturer and the retailers. In this section, the profit function for each player is provided in detail.

## 3.3.1. The manufacturer's net profit

The net manufacturer profit is obtained by subtracting the manufacturer income from the total cost. Also, the manufacturer income refers to the amounts paid by the retailers, including the amounts paid by the retailers for (a) purchasing the product at a wholesale price of  $w_p$  and (b) managing their inventory. Therefore, the manufacturer income is calculated as:

$$TR_m = \sum_{i=1}^n D_i(p, s)(w_p + \xi_i).$$
(3.2)

The costs of the VMI system are classified into direct costs and indirect costs. The direct manufacturer cost involves the transportation and production costs.

$$TDC_m = \sum_{i=1}^n D_i(p,s)(Cm + \varphi_i).$$
(3.3)

Additionally, the indirect costs are those of the SC inventory system and are divided into the inventory system costs of the retailers and the inventory system costs of the manufacturer. Figure 3a depicts the retailer warehouse inventory level. As can be seen, the warehouse inventory cost rate of retailer i (*i.e.*, inventory maintenance and ordering costs) is obtained as:

$$\mathrm{TIC}_{r_i} = \frac{1}{C} \left[ \frac{D_i(p,s)C^2}{2} \left( Hr_i(1-y_i)^2 + Br_i y_i^2 \right) + Sr_i \right].$$
(3.4)

Figure 3b illustrates the manufacturer warehouse inventory level. As can be seen, the manufacturer warehouse inventory cost rate (*i.e.*, inventory maintenance and setup costs) is found as:

$$TIC_{m} = \frac{1}{C} \left[ Hm \sum_{i=1}^{n} \frac{D_{i}(p,s)^{2}C^{2}}{2Cap} + Sm \right].$$
(3.5)

Hence, the net manufacturer profit can be found by:

$$NP_{m}(y_{i}, C, w_{p}) = TR_{m} - \sum_{i=1}^{n} TIC_{r_{i}} - TIC_{m} - TDC_{m} = \sum_{i=1}^{n} D_{i}(p, s)(w_{p} + \xi_{i}) - \frac{1}{C} \left[ \sum_{i=1}^{n} Sr_{i} + Sm \right] - \frac{C}{2} \left[ \sum_{i=1}^{n} D_{i}(p, s) \left( Hr_{i}(1 - y_{i})^{2} + Br_{i}y_{i}^{2} \right) + Hm \frac{\sum_{i=1}^{n} D_{i}(p, s)^{2}}{Cap} \right] - \sum_{i=1}^{n} D_{i}(p, s)(Cm + \phi_{i}).$$
(3.6)

## 3.3.2. Each retailer's net profits

For the service level  $s_i$  of retailer i, the retailer is assumed to have a service cost of  $1/2\eta s_i^2$ . This allows for easily controlling the analysis and ensures the concavity of the profit function on  $s_i$ . In other words, service level improvement would have a reducing return on the service expenditure [2,9,15,21,39,41,54,55,60,62,63,66,68]. The net retailer profit may be derived as:

$$NP_{r_i}(p_i, s_i) = (p_i - w_p - \xi_i)D_i(p, s) - \frac{\eta s_i^2}{2}.$$
(3.7)

Retailers could choose cooperation when retail pricing and providing value-added services. In other words, the retailers that refuse to choose cooperation could perform retail pricing and determine the service level based on profit maximization. This is incorporated in the form of the RI model, in which n retailers independently implement price and service policies at the same time. Those who cooperate could implement pricing and service policies in the form of an integrated system based on total profit maximization. This forms the RC model. Then, the RI and RC models are compared, where pricing and service strategies are sequentially implemented by the leader (*i.e.*, manufacturer) and followers (*i.e.*, retailers).

# 4. Stackelberg game

Based on the net profits of the manufacturer and retailers for retailer cooperation (RC) and retailer independence (RI) scenarios, the present study formulated the Stackelberg game model.

# 4.1. RI model

The mathematical formulation of the RI model is written as: where equation (4.1) represents the manufacturer objective function, equation (4.5) stands for the retailer objective function, equation (4.2)

$$\operatorname{Max} \operatorname{NP}_{m}(y_{i}, C, w_{p}) = \sum_{i=1}^{n} D_{i}(p, s)(w_{p} + \xi_{i}) - \frac{1}{C} \left[ \sum_{i=1}^{n} Sr_{i} + Sm \right] - \frac{C}{2} \left[ \sum_{i=1}^{n} D_{i}(p, s) \left( Hr_{i}(1 - y_{i})^{2} + Br_{i}y_{i}^{2} \right) + Hm \frac{\sum_{i=1}^{n} D_{i}(p, s)^{2}}{Cap} \right] - \sum_{i=1}^{n} D_{i}(p, s)(Cm + \phi_{i}) \quad (4.1)$$

s.t.

$$\sum_{i=1}^{n} D_i(p,s) \le Cap \tag{4.2}$$

OPTIMIZING SERVICE LEVEL, PRICE, AND INVENTORY DECISIONS

$$0 \le y_i \le 1 \tag{4.3}$$

$$C \succ 0, w_p \ge 0 \tag{4.4}$$

Max NP<sub>r<sub>i</sub></sub>(p<sub>i</sub>, s<sub>i</sub>) = (p<sub>i</sub> - w<sub>p</sub> - 
$$\xi_i$$
)D<sub>i</sub>(p, s) -  $\frac{\eta s_i^2}{2}$ 
(4.5)

$$(p_i - w_p - \xi_i) \succ 0 \tag{4.6}$$

$$p_i, s_i \ge 0 \tag{4.7}$$

constrains the production capacity, equation (4.3) the backorder fraction of retailers (0,1), equation (4.6) expresses the current basic retailer condition, and equations (4.4) and (4.7) represent positive variables.

## 4.2. RC model

This subsection formulates the model for the cooperation scenario of n retailers under service provision and retail price alternation. The retailers form an integrated system to determine the joint price and service policies based on total profit NP<sub>r</sub> maximization. All the RC equations are the same as the RI ones, excluding the retailer profit function. Hence, the RC model is derived by using equation (4.8) as a substitute for equation (4.5):

Max NP<sub>r</sub>(p<sub>i</sub>, s<sub>i</sub>) = 
$$\sum_{i=1}^{n} (p_i - w_p - \xi_i) D_i(p, s) - \frac{\eta s_i^2}{2}$$
. (4.8)

# 5. Solution procedure

For equilibrium calculation, the best retailer response function is calculated. Then, the optimal manufacturer decision based on the best retailer reactions is analyzed.

## 5.1. Optimal strategy of the retailers

In this section, the best response functions of retailers calculate by the analytical method.

#### 5.1.1. RI model

When  $p_i \succ w_p + \xi_i$  is not met by a retailer, there will be a retailer loss within the chain in the form of a negative net profit. Such a retailer will be excluded from the system. This is ignored in the discussion below. For the maximization  $(p_i - w_p - \xi_i)D_i(p,s) - \eta \frac{s_i^2}{2} = (p_i - w_p - \xi_i)\left(K_i - ap_i + \sum_{\substack{j=1\\j\neq i}}^n bp_j + \alpha s_i - \sum_{\substack{j=1\\j\neq i}}^n \beta s_j\right) - \eta \frac{s_i^2}{2}$  as the concave function of  $s_i$  and  $p_i$ , the unique optimal service level and retail price based on the wholesale price  $w_p$  are represented as:

Lemma 5.1. For n rival retailers, the equilibrium solution is:

$$\tilde{p}_j^{\text{RI}}(w_p) = \frac{K_j + (\alpha + \beta(1-n))s_j + a(w_p + \xi_j)}{2a + b(1-n)}$$
(5.1)

$$\tilde{s}_j^{\text{RI}}(w_p) = \frac{\alpha(p_j - w_p - \xi_j)}{\eta}.$$
(5.2)

Proof. Based on equation (4.5), the Hessian matrix is  $\begin{pmatrix} -2a & \alpha \\ \alpha & -\eta \end{pmatrix}$  over  $(p_i, s_i)$ . It is negatively definite following from  $\eta \succ \frac{\alpha^2}{2a}$ . The optimal retailer reactions in (5.1) and (5.2) are found by solving the first-order conditions  $\frac{\partial \operatorname{NP}_{r_i}(p_i, s_i, w_p)}{\partial p_i} = 0$  and  $\frac{\partial \operatorname{NP}_{r_i}(p_i, s_i, w_p)}{\partial s_i} = 0$  for  $(p_i, s_i)$ .

M. KARIMI ${\it ET}$   ${\it AL}.$ 

**Theorem 5.2.** The optimal retail price for  $\eta \succ \alpha^2/_{2a}$  would be:

$$p_i^{*\text{RI}}(w_p) = \frac{\eta K_i + (a\eta - \alpha(\alpha + \beta(1-n)))(w_p^* + \xi_i)}{\eta(2a + b(1-n)) - \alpha(\alpha + \beta(1-n))}.$$
(5.3)

Also, the service level is:

$$s_i^{*\mathrm{RI}}(w_p) = \frac{\alpha(K_i - (a + b(1 - n))(w_p^* + \xi_i))}{\eta(2a + b(1 - n)) - \alpha(\alpha + \beta(1 - n))}.$$
(5.4)

*Proof.* Equations (5.1) and (5.2) represent the optimal retail price and service level for retailer *i*, respectively. Equations (5.3) and (5.4) were obtained by solving the resulting equation system of *n* retailers (*i.e.*, the total retailer price and service functions).

The retailer profit is found by the insertion of equations (5.3) and (5.4) in equation (4.5) as:

$$NP_{r_i}^{*RI}(w_p) = \frac{\eta(2a\eta - \alpha^2)(K_i - (a + b(1 - n))(w_p + \xi_i))^2}{2(\eta(2a + b(1 - n)) - \alpha(\alpha + \beta(1 - n)))^2}.$$
(5.5)

Therefore, the optimal retailer demand rate can be represented as:

$$D_i^{*\mathrm{RI}}(w_p) = \frac{a\eta(K_i - (a + b(1 - n))(w_p^* + \xi_i))}{\eta(2a + b(1 - n)) - \alpha(\alpha + \beta(1 - n))}$$
(5.6)

 $\eta \succ \frac{\alpha^2}{2a}$  implies that the service investment cannot be excessively low-cost. This assumption is typically applied in economic studies [54,63]. Here, a unique optimum is ensured by this assumption. The present study assumed that  $\eta \succ \frac{\alpha^2}{2a}$  would be the case in the entire paper.

# 5.1.2. RC model

This subsection provides the results of the retailers' cooperation scenario in service provision and retail price alternation. The retailers function as an integrated system in joint price and service strategy implementation based on total profit NP<sub>r</sub> maximization. As with the RI model, the RC retailer decision model is dependent on  $w_p$  from the manufacturer. For the maximization of  $\sum_{i=1}^{n} (p_i - w_p - \xi_i) D_i(p,s) - \eta \frac{s_i^2}{2} = \sum_{i=1}^{n} (p_i - w_p - \xi_i) \left( K_i - ap_i + \sum_{\substack{j=1 \ j \neq i}}^{n} bp_j + \alpha s_i - \sum_{\substack{j=1 \ j \neq i}}^{n} \beta s_j \right) - \eta \frac{s_i^2}{2}$  as the concave function of  $s_i$  and  $p_i$ , the unique optimal service level and retail price based on the wholesale price of  $w_p$  are represented as:

Lemma 5.3. For n rival retailers, the equilibrium solution is shown as:

$$\tilde{p}_{j}^{\text{RC}}(w_{p},s_{j}) = \frac{K_{j} + (\alpha + \beta(1-n))s_{j} + (a+b(1-n))(w_{p} + \xi_{j})}{2(a+b(1-n))} \qquad \forall j = 1,\dots,n$$
(5.7)

$$\tilde{s}_{j}^{\text{RC}}(w_{p}, p_{j}) = \frac{(\alpha + \beta(1-n))(\tilde{p}_{j} - w_{p} - \xi_{j})}{\eta} \qquad \forall j = 1, \dots, n.$$
(5.8)

Proof. Based on equation (4.8), the Hessian matrix is  $\begin{pmatrix} -2a & \alpha \\ \alpha & -\eta \end{pmatrix}$  over  $(p_i, s_i)$ . It is negatively definite following from  $\eta \succ \frac{\alpha^2}{2a}$ . The optimal retailer reactions in equations (5.7) and (5.8) are found by solving the first-order conditions  $\frac{\partial NP_{r_i}(p_i, s_i, w_p)}{\partial p_i} = 0$  and  $\frac{\partial NP_{r_i}(p_i, s_i, w_p)}{\partial s_i} = 0$  for  $(p_i, s_i)$ .

**Theorem 5.4.** For  $\eta \succ \frac{\alpha^2}{2a}$ , the optimal retail price is obtained as:

$$p_i^{*\text{RC}}(w_p) = \frac{\eta K_i + (\eta(a+b(1-n)) - (\alpha + \beta(1-n))^2)(w_p + \xi_i)}{2\eta(a+b(1-n)) - (\alpha + \beta(1-n))^2}.$$
(5.9)

Also, the service level is:

$$s_i^{*\mathrm{RC}}(w_p) = \frac{(\alpha + \beta(1-n))(K_i - (a+b(1-n))(w_p + \xi_i))}{2\eta(a+b(1-n)) - (\alpha + \beta(1-n))^2}.$$
(5.10)

*Proof.* The optimal retailer price and service level are represented in equations (5.7) and (5.8). Equations (5.9) and (5.10) are derived by solving the resulting equation system of n retailers.

The retailer profit is found by the insertion of equations (5.9) and (5.10) into equation (4.8):

$$NP_{r_i}^{*RC}(w_p) = \frac{\eta(K_i - (a + b(1 - n))(w_p + \xi_i))^2}{2(2\eta(a + b(1 - n)) - (\alpha + \beta(1 - n))^2)}$$
(5.11)

$$NP_r^{*RC}(w_p) = \sum_{I=1}^n \frac{\eta(K_i - (a+b(1-n))(w_p + \xi_i))^2}{2((2\eta(a+b(1-n))) - (\alpha + \beta(1-n))^2)}.$$
(5.12)

Therefore, the optimal retailer demand rate is calculated as:

$$D_i^{*\mathrm{RC}}(w_p) = D_i^*(p_i^*(w_p), s_i^*(w_p)) = \frac{\eta(a+b(1-n))(K_i - (a+b(1-n))(w_p + \xi_i))}{2\eta(a+b(1-n)) - (\alpha+\beta(1-n))^2}.$$
(5.13)

# 5.2. Optimal strategy of the manufacturer

The decisions of the manufacturer involve replenishment cycle C, backlogging fraction  $y_i$ , and wholesale price  $w_p$ . To identify the optimal manufacturer strategy, the RI and RC indexes are not applied since both the RC and RI models have the same objective function and constraints for the manufacturer. For the RI (RC) model, the manufacturer decision model is derived by inserting equation (5.3) (Eq. (5.9)), equation (5.4) (Eq. (5.10)), and equation (5.6) (Eq. (5.13)) in equation (3.6):

$$\operatorname{Max} \operatorname{NP}_{m}(y_{i}, C, w_{p}) = \sum_{i=1}^{n} D_{i}^{*}(w_{p})(w_{p} + \xi_{i}) - \frac{1}{C} \left[ \sum_{i=1}^{n} Sr_{i} + Sm \right] \\ - \frac{C}{2} \left[ \sum_{i=1}^{n} D_{i}^{*}(w_{p}) \left( Hr_{i}(1 - y_{i})^{2} + Br_{i}y_{i}^{2} \right) + Hm \frac{\sum_{i=1}^{n} D_{i}^{*}(w_{p})^{2}}{Cap} \right] \\ - \sum_{i=1}^{n} D_{i}^{*}(w_{p})(Cm + \phi_{i})$$
(5.14)

s.t.

$$\sum_{i=1}^{n} D_i^*(w_p) \le Cap \tag{5.15}$$

$$0 \le y_i \le 1$$
  $\forall i = 1, 2, ..., n \text{ and}$  (5.16)  
 $C \succ 0, w_p \ge 0.$  (5.17)

The second derivative of equation (5.14) with respect to  $y_i$  is given by:

$$\frac{\partial^2 NP_m(y_i, C, w_p)}{\partial y_i^2} = -C \cdot D_i^*(w_p)(Hr_i + Br_i) < 0.$$
(5.18)

Thus, regardless of C and  $w_p$ ,  $NP_m(y_i, C, w_p)$  is a concave function of  $y_i$ . The optimal  $y_i$  is found based on the first derivative  $\frac{\partial NP_m(y_i, C, w_p)}{\partial y_i} = 0$  of equation (5.14) with respect to  $y_i$  as:

$$y_i^* = \frac{Hr_i}{Hr_i + Br_i}.$$
(5.19)

The insertion of equation (5.19) in equation (5.14) gives:

$$NP_{m}(C, w_{p}) = \sum_{i=1}^{n} D_{i}^{*}(w_{p})(w_{p} + \xi_{i} - Cm - \phi_{i}) - \frac{1}{C} \left[ \sum_{i=1}^{n} Sr_{i} + Sm \right] - \frac{C}{2} \left[ \sum_{i=1}^{n} D_{i}^{*}(w_{p}) \left( \frac{Hr_{i}Br_{i}}{Hr_{i} + Br_{i}} \right) + Hm \frac{\sum_{i=1}^{n} D_{i}^{*}(w_{p})^{2}}{Cap} \right].$$
(5.20)

The second derivative of equation (5.20) with respect to C is

$$\frac{\partial^2 \mathrm{NP}_m(C, w_p)}{\partial C^2} = -\frac{2}{C^3} \left( \sum_{i=1}^n Sr_i + Sm \right) < 0.$$
(5.21)

Thus, regardless of  $w_p$ ,  $NP_m(C, w_p)$  represents a concave function of C.

Since  $\frac{\partial \operatorname{NP}_m(C, w_p)}{\partial C} = 0$ , the optimal  $C^*$  is given by

$$C^{*}(w_{p}) = \sqrt{\frac{2(\sum_{i=1}^{n} Sr_{i} + Sm)}{\sum_{i=1}^{n} D_{i}^{*}(w_{p})\left(\frac{Hr_{i}Br_{i}}{Hr_{i} + Br_{i}}\right) + \frac{Hm\sum_{i=1}^{n} D_{i}^{*}(w_{p})^{2}}{Cap}}}.$$
(5.22)

The insertion of equation (5.22) in (5.20) makes the net manufacturer profit a function of  $w_p$  as:

$$NP_{m}(w_{p}) = \sum_{i=1}^{n} D_{i}^{*}(w_{p})(w_{p} + \xi_{i} - Cm - \phi_{i}) - \sqrt{2\left(\sum_{i=1}^{n} Sr_{i} + Sm\right)\left[\sum_{i=1}^{n} D_{i}^{*}(w_{p})\left(\frac{Hr_{i}Br_{i}}{Hr_{i} + Br_{i}}\right) + \frac{Hm \sum_{i=1}^{n} D_{i}^{*}(w_{p})^{2}}{Cap}\right]}$$
(5.23)

$$\sum_{i=1}^{n} D_i^*(w_p) \le Cap \quad \text{and} \tag{5.24}$$

$$w_p \ge 0. \tag{5.25}$$

Based on equation (5.23), the model contains a continuous variable  $w_p$ . Therefore, the optimal model values are calculated using the Kuhn–Tucker condition. The Lagrange multiplier is  $\lambda$ , and the Lagrange function is defined as: /

$$\operatorname{Max} L_m(w_p, \lambda) = \operatorname{NP}_m(w_p) + \lambda \left( Cap - \sum_{i=1}^n D_i^*(w_p) \right).$$
(5.26)

The Kuhn–Tucker condition is shown as:

$$\frac{\partial \mathrm{NP}_m(w_p)}{\partial w_p} - \lambda \frac{\partial \sum_{i=1}^n D_i^*(w_p)}{\partial w_p} = 0$$

$$\lambda \left( Cap - \sum_{i=1}^n D_i^*(w_p) \right) = 0.$$
(5.27)

For  $\lambda = 0$ , there is

$$\frac{\partial \mathrm{NP}_m(w_p)}{\partial w_p} = 0. \tag{5.28}$$

And gives the corresponding critical point  $w_p$ .

For  $\lambda > 0$ , the corresponding critical point  $w_p$  and  $\lambda$  are calculated by:

$$\frac{\partial \mathrm{NP}_{m}(w_{p})}{\partial w_{p}} - \lambda \frac{\partial \sum_{i=1}^{n} D_{i}^{*}(w_{p})}{\partial w_{p}} = 0$$

$$\sum_{i=1}^{n} D_{i}^{*}(w_{p}) = Cap.$$
(5.29)

A comparison of the objective function to the calculated  $w_p$  values, the optimal value of  $w_p$  is found by choosing the one with a larger NP<sub>m</sub>.

As a result, it can be said that the Stackelberg game equilibrium may be derived from solutions meeting the optimality conditions -i.e., equations (5.9) and (5.10) for the RI model, equations (5.3) and (5.4) for the RC model, and equations (5.19), (5.22), and (5.27) for the RC and RI models.

#### 5.3. Computational solution algorithm

A solution algorithm is proposed to drive the Stackelberg game equilibrium in a number of steps, including:

**Step 1.** Calculating optimal  $w_p$ 

- **Step 1.1.** Calculating  $w_p$  by (5.28) for  $\lambda = 0$  and by (5.29) for  $\lambda > 0$  through Wolfram Mathematica 12.1
- Step 1.2. Calculating and comparing the net profit of the manufacturer by (5.23) based on the  $w_p$  values at  $\lambda = 0$  and  $\lambda > 0$
- **Step 1.3.** Setting  $w_p$  to raise (5.23) as the optimal value of  $w_p$ . Setting  $w_p = w_p^*$  and calculating the maximum net manufacturer profit NP<sup>\*</sup><sub>m</sub> by (5.23).
- **Step 2.** Calculating the optimal value of  $y_i^*$  by (5.19) and the optimal value of  $C^*$  by (5.22)
- **Step 3.** Calculating the optimal value of  $p_i^*$  by (5.3) and the optimal value of  $s_i^*$  by (5.4) for the RI model and calculating the optimal value of  $p_i^*$  by (5.9) and the optimal value of  $s_i^*$  by (5.10) for the RC model
- **Step 4.** Calculating the optimal value of  $NP_{r_i}^*$  by inserting the optimal value of  $w_p^*$  in (5.5) and (5.11) to maximize the retailer profit in the RI and RC models, respectively.

To find equilibrium (*i.e.*, the optimal solution), step 3 finds  $p_i^*$  and  $s_i^*$  for RC and RI, step 2 gives  $y_i^*$  and  $C^*$ , step 1 identifies  $w_p^*$ , step 1.3 obtains NP<sup>\*</sup><sub>m</sub>, and step 4 reveals NP<sup>\*</sup><sub>ri</sub>.

# 6. Numerical example

This section examines a numerical example to evaluate the developed model and solution algorithm. The inputs were extracted from the proposed assumptions and earlier studies [24, 41]. For simplification, the same inputs were applied to the rival retailers, as in [20]. The RI and RC inputs included  $n = 2, K_i = 150, a = 2, b = 0.35, \alpha = 0.5, \beta = 0.15, \eta = 0.1, \xi_i = 8, Br_i = 500, Hr_i = 1, Sr_i = 50, \varphi_i = 20$ . The optimal values of retailer and manufacturer decisions in the RC and RI models are reported in Table 2.

# 7. Sensitivity analysis

Sensitivity analysis has been conducted with the model for parameters in three groups: manufacturer-related parameter, retailer-related parameters, and impact of retailer's competition.

#### 7.1. Manufacturer parameters

The effects of the production capacity (Cap) and production cost (Cm) on VMI system performance are investigated in this section.

Policy	Actor										
	Manufa	acturer	Reta	ilers							
	RI	RC	RI	RC							
Selling price $(w_p^*, p_i^*)$	58.881	58.635	87.747	85.937							
Demand rate $(\hat{D}_m^*, \hat{D}_i^*)$	84.556	63.696	42.278	31.848							
Profit $(NP_m^*, NP_{r_i}^*)$	1956.43	1454.30	335.15	386.53							
Service level $(s_i^*)$			105.696	67.557							
Fraction of backlogging $(y_i^*)$			0.002	0.002							
Replenishment cycle $(C^*)$	2.414	2.889									

TABLE 2. Optimal RI/RC manufacturer and retailers decisions.

## 7.1.1. Production capacity

Figure 4 plots the production capacity *versus* the wholesale price, retailer service level, manufacturer profit, retailer profit, and total chain profit in the two models. As can be seen, a rise in the production capacity of the manufacturer reduced the retailer price and wholesale price and raises the retailer service level, manufacturer profit, retailer profit, and total chain profit in the two models.

## 7.1.2. Production cost

Figure 5 depicts production cost *versus* VMI system performance. As can be seen, increased production costs raised the retailer price and wholesale price and diminished the manufacturer profit, manufacturer profit, and total profit in the RI and RC models.

# 7.2. Retailer-related parameters

In this section, the influences of the cross-price elasticity, self-service level, cross-service level, and service cost factor on VMI system performance are evaluated.

## 7.2.1. Cross-price elasticity

Figure 6 shows the impact of the cross-price elasticity (b) on the optimal strategies and performance. According to Figure 6, the retailer price, wholesale price, the retailer service level, manufacturer profit, retailer profit, and total chain profit rise in b, in the RI and RC models. This rise is larger in the RI model as compared to the RC model in Figure 6d, suggesting that a larger degree of retailer-retailer cross-service competition is more desirable in the RI model to not only the manufacturer but also the SC.

#### 7.2.2. Self-service level

Figure 7 plots the self-service level  $\alpha$  versus the performance of the VMI system and optimal strategies. According to Figure 7a, the optimal retail price (wholesale price) rises (declines) as  $\alpha$  rises for both the RI and RC models. Also, it can be inferred from Figure 7b that the optimal retailer service level rises as  $\alpha$  increases in the RI and RC models. Figure 7c implies a retailer profit increase in  $\alpha$ . As a result, a rise in  $\alpha$  is unexpectedly found to increase the manufacturer profit and the total chain profit.

## 7.2.3. Cross-service level effect

Figure 8 plots the cross-service level  $\beta$  versus the performance of the VMI system and optimal strategies. According to Figure 8a, an increase in  $\beta$  reduces (increases) the optimal retail price (wholesale price) strategies. Also, the RC model has smaller optimal retail prices as compared to the RI model. However, the RC model has a larger wholesale price. According to Figure 8b, a rise in  $\beta$  reduces the optimal retail service in the two models. It can also be inferred from Figure 8b that the RI model has higher optimal service levels as compared to the RC model. In other words, retailers' cooperation diminishes retailer competition and encourages the retailers

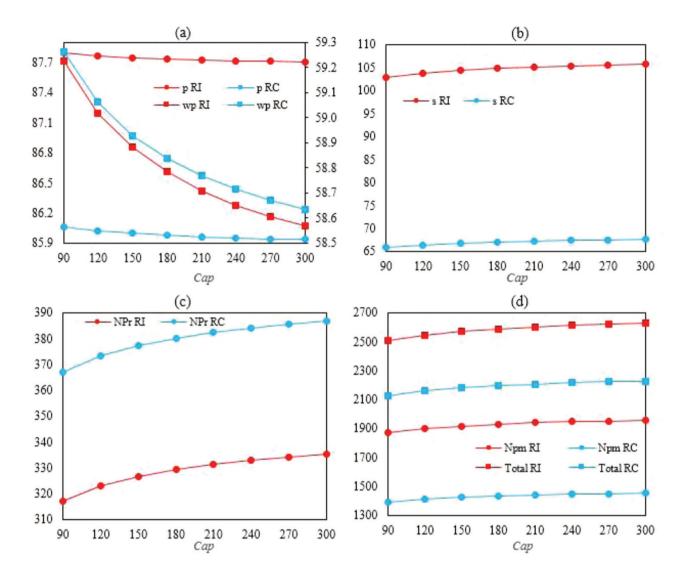


FIGURE 4. The influence of *Cap.* (a) Retail/Wholesale price. (b) Service level. (c) Retailer profit. (d) Manufacturer and total profit.

to offer lower service levels. According to Figure 8c, a rise in  $\beta$  reduced the retailer profits in the two models. This suggests that a lower demand exists at a larger  $\beta$ . It is also inferred from Figure 8c that the RC model has a larger retailer profit than the RI model. As can be seen in Figure 8d, the total chain profit and manufacturer profit decline as  $\beta$  rises. That is, lower  $\beta$  values are more desirable to the SC and manufacturer. The RI model has a greater manufacturer profit and total SC profit than the RC model. Thus, retailers' cooperation is expectedly observed to be harmful to the performance of the system.

# 7.2.4. Service cost factor

Figure 9 depicts the service cost factor  $\eta$  versus the wholesale price, retailer service level, price, manufacturer profit, retailer profit, and total profit in the two models. According to Figure 9a, a rise in  $\eta$  diminishes (raises) the optimal price (wholesale price) strategies. Also, the RC model has smaller optimal retail prices as compared

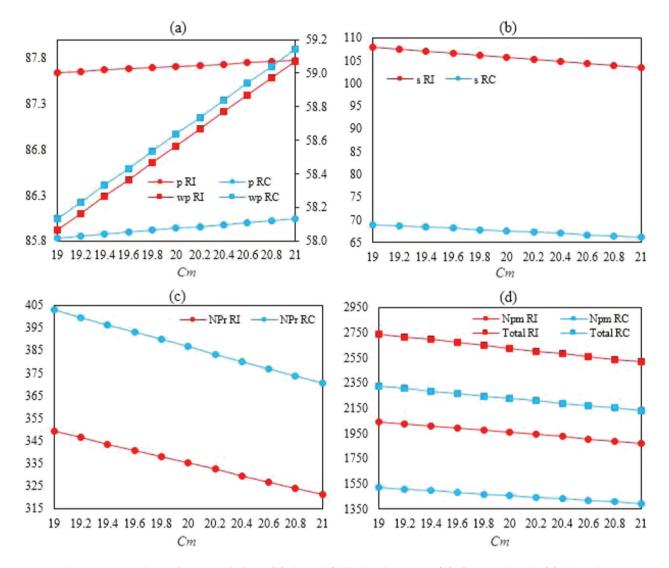


FIGURE 5. The influence of Cm. (a) Retail/Wholesale price. (b) Service level. (c) Retailer profit. (d) Manufacturer and total profit.

to the RI model, while the RC model has larger wholes ale prices. As can be seen in Figure 9b, an increase in  $\eta$  reduces the optimal retail service in the two models.

Also, the RI model has greater optimal service levels as compared to the RC model. In other words, retailers' cooperation would diminish retailer competition and encourage the retailers to offer lower service levels. According to Figure 9c, a rise in  $\eta$  reduces the retailer profit in the two models. This suggests lower demand at a larger  $\eta$ . It can also be inferred from Figure 9c that the RC model has higher retailer profits than the RI model. According to Figure 9d, a rise in  $\eta$  diminishes not only the manufacturer profit but also the total chain profit. That is, the manufacturer and SC prefer a smaller  $\eta$ . The RI model has a larger manufacturer profit and total chain profit than the RC model. As a result, it is unexpectedly concluded that the cooperation of retailers is undesirable for the system's performance.

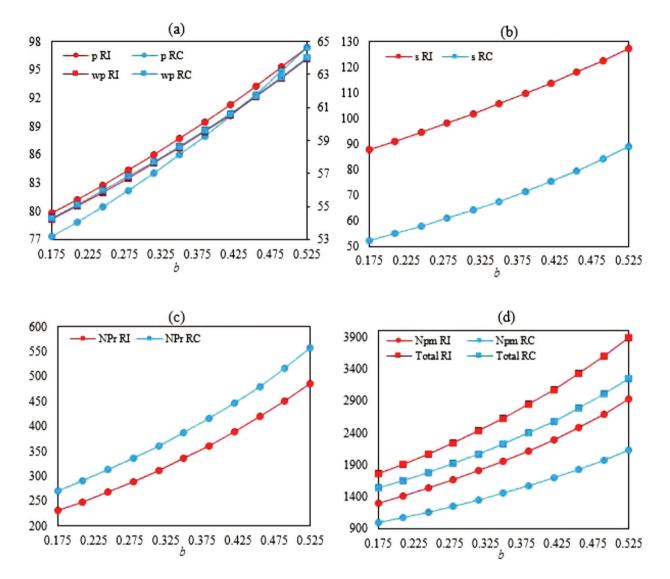


FIGURE 6. The influence of b. (a) Retail/Wholesale price. (b) Service level. (c) Retailer profit. (d) Manufacturer and total profit.

# 7.3. Impacts of retailers' competition

In order to evaluate the effects of retailers' competition on VMI decision-making, the present study adopted cross-price sensitivity and cross-service level sensitivity coefficients  $b_{12} = 0.35, b_{21} = 0.5, \beta_{12} = 0.15, \beta_{21} = 0.3$  to identify the manufacturer and retail profit variations.

Figure 6 reports the cross-price elasticity sensitivity outcomes of symmetric retailer scenarios. Figure 10 plots  $b_{21}$  versus the performance of the VMI system and optimal strategies under retailer asymmetry. According to Figure 10a, a rise in  $b_{21}$  would raise the optimal price strategies. According to Figure 10b, a rise in  $b_{21}$  reduces the optimal service level of retailer 1 and raises that of retailer 2 in the two models. Also, the RI model has greater optimal service levels as compared to the RC model. In other words, retailers' cooperation would diminish retailer competition and encourage the retailers to offer lower service levels. According to Figure 10c,

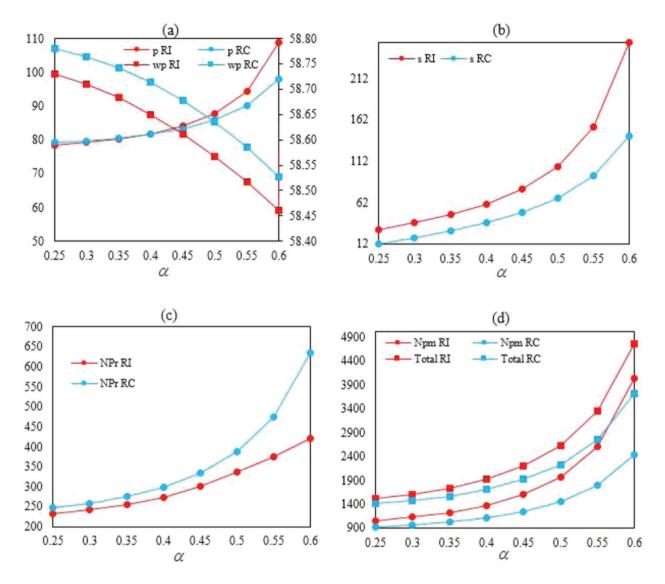


FIGURE 7. The influence of  $\alpha$ . (a) Retail/Wholesale price. (b) Service level. (c) Retailer profit. (d) Manufacturer and total profit.

an increase in  $b_{21}$  reduces the profit of retailer 1 and increases that of retailer 2 in the two models. Hence, it can be said that a rise in  $b_{21}$  diminishes (raises) the demand of retailer 1 (retailer 2). Also, the RC model has a larger retailer profit as compared to the RI model. According to Figure 10d, a rise in  $b_{21}$  increases not only the manufacturer profit but also the total profit. The RI model has a larger manufacturer profit and total profit than the RC model. As a result, the cooperation of retailers is unexpectedly found to have an adverse impact on the system's performance.

Figure 8 reports the cross-service elasticity sensitivity outcomes of symmetric retailer scenarios. Figure 11 plots  $\beta_{21}$  versus the performance of the VMI system and optimal strategies under retailer asymmetry. According to Figure 11a, a rise in  $\beta_{21}$  diminishes the optimal prices. According to Figure 11b, a rise in  $\beta_{21}$  raises (diminishes) the optimal service level of retailer 1 (retailer 2) in the two models. Also, the RI model has higher optimal service

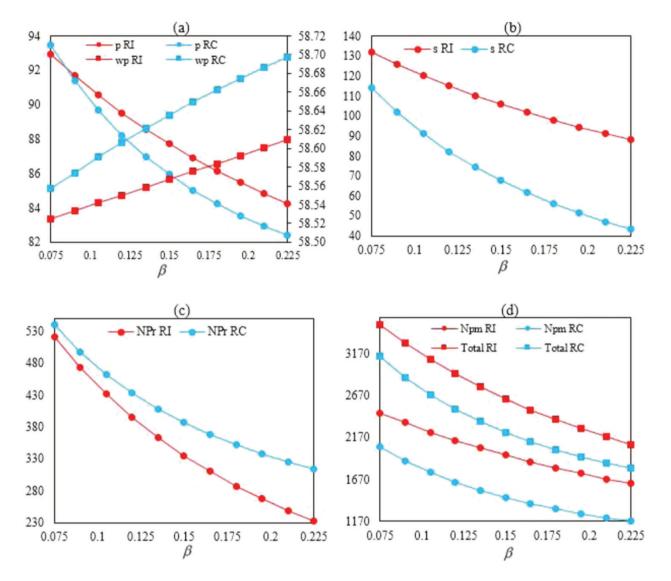


FIGURE 8. The influence of  $\beta$ . (a) Retail/Wholesale price. (b) Service level. (c) Retailer profit. (d) Manufacturer and total profit.

levels than the RC model. In other words, retailer cooperation reduces retailer competition and encouraged the retailers to offer lower service levels. According to Figure 11c, a rise in  $\beta_{21}$  slightly increases (reduces) the profit of retailer 1 (retailer 2) due to increased (reduced) retailer demand, leading to increased (reduced) profits of retailer 1 (retailer 2). It can also be inferred from Figure 11c that the RC model has larger retailer profits than the RI model. According to Figure 11d, an increase in  $\beta_{21}$  reduces not only the manufacturer profit but also the total SC profit. The RI model has higher manufacturer and total profits than the RC model. As a result, it is unexpectedly found that the cooperation of retailers would be harmful to the system's performance.

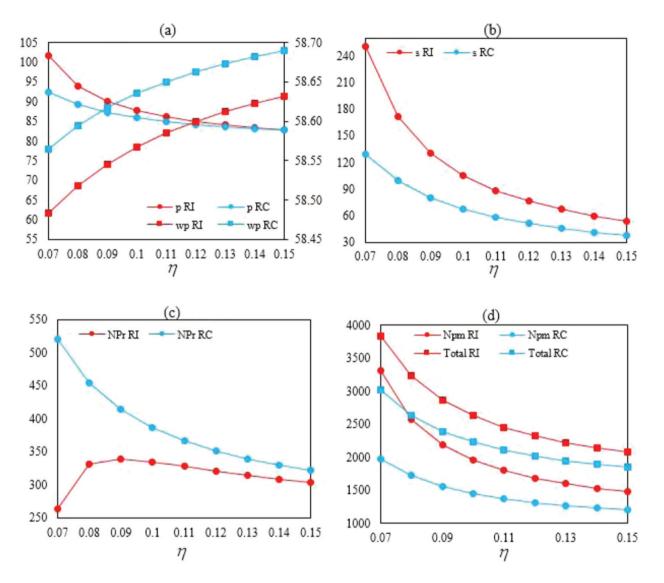


FIGURE 9. The influence of  $\eta$ . (a) Retail/Wholesale price. (b) Service level. (c) Retailer profit. (d) Manufacturer and total profit.

# 8. DISCUSSION

In this section, the results of sensitivity analyses were compared with results to earlier works.

- When the retailers cooperate, the retailer profit increases, but the manufacturer and total profits reduce. This finding is consistent with [41]. Retailers' cooperation reduces the retailer competition and encourages the retailers to offer lower service levels.
- When self-services increase, the demand increases. This enables retailers to increase their price [45] and service level. Moreover, a rise in demand entails a decline in the wholesale price. Due to increased demand, retailer profit [45], manufacturer profit and total profit rise.

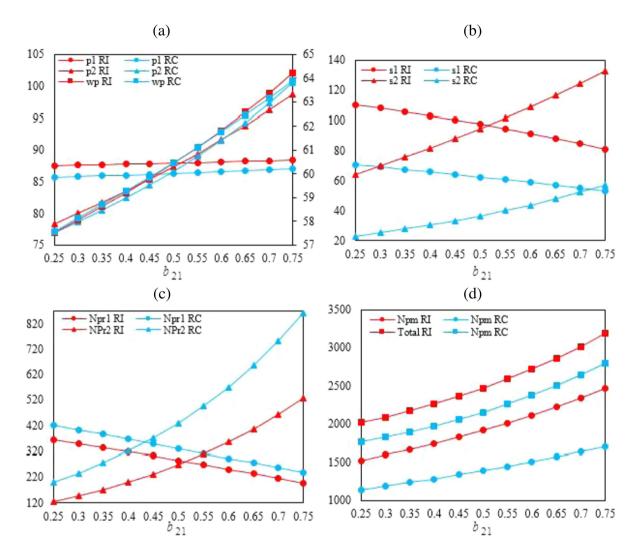


FIGURE 10. The influence of  $b_{21}$ . (a) Retail/Wholesale price. (b) Service level. (c) Retailer profit. (d) Manufacturer and total profit.

- A rise in cross-service level reduces causes the demand, diminishing the retailer price and service level and vice versa. This results in an increased wholesale price. Therefore, the manufacturer profit, retailer profit, and total profit decrease. This is consistent with [21].
- The demand increases when cross-price elasticity increases. The retailer can increase its price [14, 23] and service level due to increased demand, which, in turn, enables the manufacturer to increase the price. Therefore, the manufacturer profit [20], retailer profit [14, 20, 23, 45], and total profit increase.
- When a manufacturer raises the production capacity, retailers lower retail pricing due to lower wholesale prices and enhance service levels to stimulate demand, resulting in higher profits for the retailer, manufacturer, and system which is similar to [66].
- If the production cost increases, the retailer price increases due to the increased wholesale price. However, the retailer needs to decrease the service level to prevent a further rise in the price. This, in turn, diminishes the manufacturer profit, retailer profit, and total profits decreases.

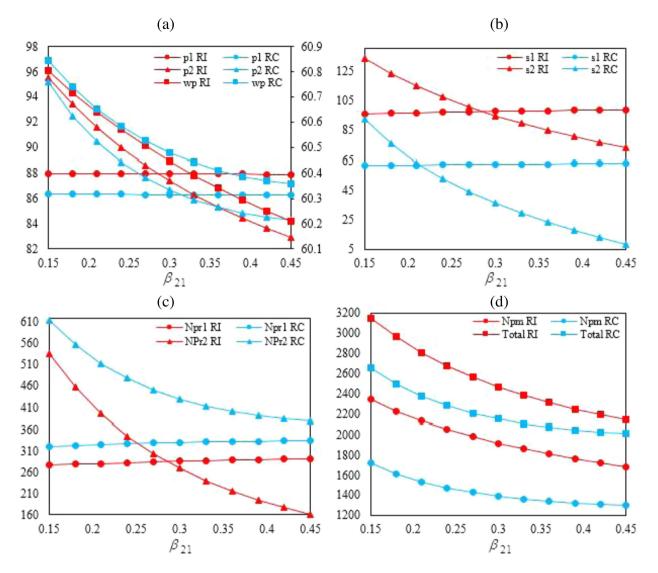


FIGURE 11. The influence of the  $\beta_{21}$ . (a) Retail/Wholesale price. (b) Service level. (c) Retailer profit. (d) Manufacturer and total profit.

# 9. Conclusions

The present study focused on retailers' competition and cooperation in SCs under the VMI approach. Retailers' competition included price competition and service competition. A bi-level mathematical framework was developed to evaluate optimal and inventory strategies in a single-manufacturer multi-retailer system. The retailers distributed the manufacturer's product. The retailer market demand rate was a reducing (increasing) function of the subject retailer's price (service level) and an increasing (reducing) function of the rival retailers' prices (service) levels. For the formulation of the problem, a Stackelberg game model was developed, where the manufacturer function as the leader, while the retailers served as followers. The problem was examined in a retailer independence (RI) model and a retailer cooperation (RC) model. In the former, retailers adopted competition and independent operational strategy determination at the same time. In the latter, however, retailers chose cooperation for total profit maximization. The Nash-equilibrium approach was adopted to identify optimal service and price strategies in the two models. To solve the Stackelberg game model, a computational algorithm was employed to theoretically analyze the optimal response function. Through the mathematical framework and numerical example, the retail service-price interaction was revealed to be dependent on cross-price, cross-service, and self-service sensitivities. Optimal strategies and optimal performance were compared, analyzing the effects of important variables on the optimal profits and strategies of the members in the SC within the RC and RI models. It was found that retailers tend to choose cooperation as it would improve their profits. Retailer cooperation, however, was demonstrated to diminish the manufacturer and total SC profits.

Since the findings of the sensitivity analysis are consistent with theoretical studies, some managerial recommendations can be made as follows:

- (1) Retailers can adopt a cooperation strategy to reduce the competitive position (leadership) of the manufacturer and reduce the competition between them. Thus, in addition to a retail profit rise, they can offer a lower level of service. However, in the cooperational situation of retailers, the manufacturer's profit is lower.
- (2) In order to increase the profits of the manufacturer and retailer, the following strategies can be used:
  - Increasing the production capacity by the manufacturer reduces the wholesale price and retail price. Therefore, the increased demand raises the profits of the retailers and manufacturer.
  - If the cross-price elasticity of demand increases, the demand of the main retail increases; thus it is possible to increase the price and provide more services. At the same time, the wholesale price and, consequently, the manufacturer's profit increase.
  - By increasing the self-service level effect of demand, the retailer price rises. However, due to the increased demand for better service, more profits are earned. Although the manufacturer price decreases very slightly, the manufacturer profit will rise.
- (3) The following strategies should be considered to avoid reduced profits of both the manufacturer and retailer.
  - Avoiding a rise in the manufacturer's production cost since a higher production cost leads to a higher wholesale price, followed by the retail price, and, consequently, lower profits of the members. This strategy is implemented by reducing the material cost or the cost of converting material into a final product.
  - Avoiding an increase in the cross -service level effect of the demand since the main retailer's demand decreases as the competitor's service level increases. This leads to a decrease in the retailer price and profit and ultimately the manufacturer's profit.

The proposed framework had a number of limitations. These limitations could be tackled to extend the model in future studies. For example, the proposed framework incorporated linear, deterministic demand. Stochastic demand may be applied in future works. Also, the present study investigated horizontal retailers' cooperation, while future research could evaluate vertical manufacturer–retailer cooperation where the manufacturer can enjoy the advantages of retail services - e.g., product reputation may be improved by higher retail service levels. The present study was grounded on symmetric data, and future works could examine the decisionmaking policies of SC members in a system with asymmetric data. Finally, the comparison of the results of the two models to the centralized model is also significant.

Acknowledgements. The authors would like to thank the anonymous referees for their helpful suggestions on the work, which have substantially enhanced its presentation.

# References

- [1] E. Adida and V. DeMiguel, Supply chain competition with multiple manufacturers and retailers. Oper. Res. 59 (2011) 156–172.
- [2] S.M. Ali, M.H. Rahman, T.J. Tumpa, A.A.M. Rifat and S.K. Paul, Examining price and service competition among retailers in a supply chain under potential demand disruption. J. Retail. Consum. Serv. 40 (2018) 40–47.
- [3] E. Almehdawe and B. Mantin, Vendor managed inventory with a capacitated manufacturer and multiple retailers: retailer versus manufacturer leadership. Int. J. Prod. Econ. 128 (2010) 292–302.

#### M. KARIMI ET AL.

- [4] E.J. Anderson and Y. Bao, Price competition with integrated and decentralized supply chains. Eur. J. Oper. Res. 200 (2010) 227-234.
- [5] Z. Ashraf, D. Malhotra, P.K. Muhuri and Q.M. Lohani, Interval type-2 fuzzy vendor managed inventory system and its solution with particle swarm optimization. Int. J. Fuzzy Syst. 23 (2021) 2080–2105.
- [6] S. Braide, Z. Cao and X. Zeng, Volume discount pricing strategy in the VMI supply chain with price sensitive demand. J. Oper. Res. Soc. 64 (2013) 833–847.
- [7] H.M. Beheshti, I.J. Clelland and K.V. Harrington, Competitive advantage with vendor managed inventory. J. Promot. Manag. 26 (2020) 836-854.
- [8] F. Bernstein and A. Federgruen, Pricing and replenishment strategies in a distribution system with competing retailers. Oper. Res. 51 (2003) 409–426.
- [9] F. Bernstein and A. Federgruen, A general equilibrium model for industries with price and service competition. Oper. Res. 52 (2004) 868–886.
- [10] F. Bernstein and A. Federgruen, Decentralized supply chains with competing retailers under demand uncertainty. Manag. Sci. 51 (2005) 18–29.
- [11] F. Bernstein and A. Federgruen, Coordination mechanisms for supply chains under price and service competition. Manuf. Serv. Oper. Manag. 9 (2007) 242–262.
- [12] G.P. Cachon, Stock wars: inventory competition in a two-echelon supply chain with multiple retailers. Oper. Res. 49 (2001) 658–674.
- [13] J. Chen and B. Chen, Competing with customer returns policies. Int. J. Prod. Res. 54 (2016) 2093–2107.
- [14] K. Chen and T. Xiao, Pricing and replenishment policies in a supply chain with competing retailers under different retail behaviors. Comput. Ind. Eng. 103 (2017) 145–157.
- [15] B. Dan, G. Xu and C. Liu, Pricing policies in a dual-channel supply chain with retail services. Int. J. Prod. Econ. 139 (2012) 312–320.
- [16] M.A. Darwish and O.M. Odah, Vendor managed inventory model for single-vendor multi-retailer supply chains. Eur. J. Oper. Res. 204 (2010) 473–484.
- [17] A. David and E. Adida, Competition and coordination in a two-channel supply chain. Prod. Oper. Manag. 24 (2015) 1358–1370.
- [18] P. De Giovanni, Smart supply chains with vendor managed inventory, coordination, and environmental performance. Eur. J. Oper. Res. 292 (2021) 515–531.
- [19] P. De Giovanni, S. Karray and G. Martín-Herrán, Vendor management inventory with consignment contracts and the benefits of cooperative advertising. *Eur. J. Oper. Res.* 272 (2019) 465–480.
- [20] H. Deng, Y. Li, Z. Wan and Z. Wan, Partially smoothing and gradient-based algorithm for optimizing the VMI system with competitive retailers under random demands. *Math. Probl. Eng.* 2020 (2020) 3687471.
- [21] B.C. Giri and T. Maiti, Service competition in a supply chain with two retailers under service level sensitive retail price and demand. Int. J. Manag. Sci. Eng. Manag. 9 (2014) 133–146.
- [22] B.C. Giri and B.R. Sarker, Coordinating a two-echelon supply chain under production disruption when retailers compete with price and service level. Oper. Res. 16 (2016) 71–88.
- [23] B.C. Giri, A. Chakraborty and T. Maiti, Quality and pricing decisions in a two-echelon supply chain under multi-manufacturer competition. To appear in: Int. J. Manag. Sci. Eng. Manag. 78 (2015) 1927–1941.
- [24] C.H. Glock and T. Kim, The effect of forward integration on a single-vendor-multi-retailer supply chain under retailer competition. Int. J. Prod. Econ. 164 (2015) 179–192.
- [25] H. Golpîra, E. Najafi, M. Zandieh and S. Sadi-Nezhad, Robust bi-level optimization for green opportunistic supply chain network design problem against uncertainty and environmental risk. Comput. Ind. Eng. 107 (2017) 301–312.
- [26] K. Govindan, The optimal replenishment policy for time-varying stochastic demand under vendor managed inventory. Eur. J. Oper. Res. 242 (2015) 402–423.
- [27] A. Haji, M. Afzalabadi and R. Rasoul, Pricing and inventory decisions in a vendor managed inventory system with revenue sharing contract. Uncertain Supply Chain Manag. 6 (2018) 299–320.
- [28] M. Hariga, S. Babekian and Z. Bahroun, Operational and environmental decisions for a two-stage supply chain under vendor managed consignment inventory partnership. Int. J. Prod. Res. 57 (2019) 3642–3662.
- [29] M. Hemmati, S.M.T. Fatemi Ghomi and M.S. Sajadieh, Vendor managed inventory with consignment stock for supply chain with stock-and price-dependent demand. Int. J. Prod. Res. 55 (2017) 5225–5242.
- [30] V. Hemmelmayr, K.F. Doerner, R.F. Hartl and M.W. Savelsbergh, Vendor managed inventory for environments with stochastic product usage. Eur. J. Oper. Res. 202 (2010) 686–695.
- [31] X. Hong, W. Chunyuan, L. Xu and A. Diabat, Multiple-vendor, multiple-retailer based vendor-managed inventory. Ann. Oper. Res. 238 (2016) 277–297.
- [32] H. Huang, H. Ke and L. Wang, Equilibrium analysis of pricing competition and cooperation in supply chain with one common manufacturer and duopoly retailers. Int. J. Prod. Econ. 178 (2016) 12–21.
- [33] M.A. Kaasgari, D.M. Imani and M. Mahmoodjanloo, Optimizing a vendor managed inventory (VMI) supply chain for perishable products by considering discount: two calibrated meta-heuristic algorithms. Comput. Ind. Eng. 103 (2017) 227–241.
- [34] M.M. Karampour, M. Hajiaghaei-Keshteli, A.M. Fathollahi-Fard and G. Tian, Metaheuristics for a bi-objective green vendor managed inventory problem in a two-echelon supply chain network. To appear in: Sci. Iran. (2020) DOI: 10.24200/sci.2020.53420.3228.

- [35] D. Konur and J. Geunes, Supplier wholesale pricing for a retail chain: implications of centralized vs. decentralized retailing and procurement under quantity competition. Omega 65 (2016) 98–110.
- [36] J.Y. Lee and L. Ren, Vendor-managed inventory in a global environment with exchange rate uncertainty. Int. J. Prod. Econ. 130 (2011) 169–174.
- [37] Y. Li, Z. Wan and J. Liu, Bi-level programming approach to optimal strategy for vendor-managed inventory problems under random demand. ANZIAM J. 59 (2017) 247–270.
- [38] K.P. Lin, P.T. Chang, K.C. Hung and P.F. Pai, A simulation of vendor managed inventory dynamics using fuzzy arithmetic operations with genetic algorithms. *Expert Syst. Appl.* 37 (2010) 2571–2579.
- [39] J.C. Lu, Y.C. Tsao and C. Charoensiriwath, Competition under manufacturer service and retail price. Econ. Model. 28 (2011) 1256–1264.
- [40] S.H.R Pasandideh, S.T. Akhavan Niaki and P. Ahmadi, Vendor managed inventory in the joint replenishment problem of a multi-product single-supplier multiple-retailer supply chain: a teacher-learner-based optimization algorithm. J. Model. Manag. 13 (2018) 156–178.
- [41] Z. Pi, W. Fang and B. Zhang, Service and pricing strategies with competition and cooperation in a dual-channel supply chain with demand disruption. *Comput. Ind. Eng.* **138** (2019) 106130.
- [42] C.S. Pramudyo and H.T. Luong, One vendor and multiple retailers system in vendor managed inventory problem with stochastic demand. Int. J. Ind. Syst. Eng. 31 (2019) 113–136.
- [43] H. Rasay and Y.Z. Mehrjerdi, Modelling, analysing and improving the revenue sharing contract in a one vendor-multi retailer supply chain based on the Stackelberg game theory. J. Manuf. Technol. Manag. 31 (2017) 402–423.
- [44] H. Rasay, Y.Z. Mehrjerdi and M.S.F. Nezhad, Modeling and numerical analysis of revenue sharing contract based on the Stackelberg game theory. Int. J. Supply Oper. Manag. 1 (2015) 439.
- [45] S. Rezapour and R.Z. Farahani, Supply chain network design under oligopolistic price and service level competition with foresight. Comput. Ind. Eng. 72 (2014) 129–142.
- [46] J. Sadeghi and S.T.A. Niaki, Two parameter tuned multi-objective evolutionary algorithms for a bi-objective vendor managed inventory model with trapezoidal fuzzy demand. Appl. Soft Comput. 30 (2015) 567–576.
- [47] A. Sainathan and H. Groenevelt, Vendor managed inventory contracts-coordinating the supply chain while looking from the vendor's perspective. Eur. J. Oper. Res. 272 (2019) 249–260.
- [48] J. Shao, H. Krishnan and S.T. McCormick, Incentives for transshipment in a supply chain with decentralized retailers. Manuf. Serv. Oper. Manag. 13 (2011) 361–372.
- [49] P.P. Shenoy, Competitive inventory models. RAIRO-Oper. Res. 21 (1987) 1-19.
- [50] J.Y. Son and S. Ghosh, Vendor managed inventory with fixed shipping cost allocation. Int. J. Logist. Res. Appl. 23 (2020) 1–23.
- [51] A.A. Taleizadeh, M. Noori-daryan and L.E. Cárdenas-Barrón, Joint optimization of price, replenishment frequency, replenishment cycle and production rate in vendor managed inventory system with deteriorating items. Int. J. Prod. Econ. 159 (2015) 285–295.
- [52] A.A. Taleizadeh, S. Tavakoli, I. Konstantaras and M. Rabbani, A vendor managed inventory on consignment with penalty as a supply chain coordination arrangement. *RAIRO-Oper. Res.* 53 (2019) 1343–1355.
- [53] A.A. Taleizadeh, I. Shokr and F. Joali, Optimizing vendor-managed inventory systems with limited storage capacity and partial backordering under stochastic demand. RAIRO-Oper. Res. 54 (2020) 179–209.
- [54] A.A. Tsay and N. Agrawal, Channel dynamics under price and service competition. Manuf. Serv. Oper. Manag. 2 (2000) 372–391.
- [55] A.A. Tsay and N. Agrawal, Channel conflict and coordination in the e-commerce age. Prod. Oper. Manag. 13 (2004) 93-110.
- [56] J. Tyan and H.M. Wee, Vendor managed inventory: a survey of the Taiwanese grocery industry. J. Purch. Supply Manag. 9 (2003) 11–18.
- [57] Q. Wei, J. Zhang, G. Zhu, R. Dai and Sh. Zhang, Retailer vs. vendor managed inventory with considering stochastic learning effect. J. Oper. Res. Soc. 71 (2020) 628–646.
- [58] D. Weraikat, M.K. Zanjani and N. Lehoux, Improving sustainability in a two-level pharmaceutical supply chain through vendor-managed inventory system. Oper. Res. Health Care. 21 (2019) 44–55.
- [59] D. Wu, Joint pricing-servicing decision and channel strategies in the supply chain. Cent. Eur. J. Oper. Res. 19 (2011) 99–137.
- [60] C.H. Wu, Price and service competition between new and remanufactured products in a two-echelon supply chain. Int. J. Prod. Econ. 140 (2012) 496–507.
- [61] Y. Xia and S.M. Gilbert, Strategic interactions between channel structure and demand enhancing services. Eur. J. Oper. Res. 181 (2007) 252–265.
- [62] T. Xiao and D. Yang, Price and service competition of supply chains with risk-averse retailers under demand uncertainty. Int. J. Prod. Econ. 114 (2008) 187–200.
- [63] T. Xiao and D. Yang, Risk sharing and information revelation mechanism of a one-manufacturer and one-retailer supply chain facing an integrated competitor. Eur. J. Oper. Res. 196 (2009) 1076–1085.
- [64] X. Yan and H. Zhao, Inventory sharing and coordination among n independent retailers. Eur. J. Oper. Res. 243 (2015) 576–587.
- [65] S.L. Yang and Y.W. Zhou, Two-echelon supply chain models: considering duopolistic retailers' different competitive behaviors. Int. J. Prod. Econ. 103 (2006) 104–116.

#### M. KARIMI ${\it ET}$ ${\it AL}.$

- [66] D. Yang, T. Xiao and H. Shen, Pricing, service level and lot size decisions of a supply chain with risk-averse retailers: implications to practitioners. Prod. Plan. Control. 20 (2009) 320–331.
- [67] Y. Yao, P.T. Evers and M.E. Dresner, Supply chain integration in vendor-managed inventory. Decis. Support Syst. 43 (2007) 663–674.
- [68] D.Q. Yao, X. Yue and J. Liu, Vertical cost information sharing in a supply chain with value-adding retailers. Omega 36 (2008) 838–851.
- [69] Y. Yu and G.Q. Huang, Nash game model for optimizing market strategies, configuration of platform products in a vendor managed inventory (VMI) supply chain for a product family. *Eur. J. Oper. Res.* 206 (2010) 361–373.
- [70] Y. Yu, F. Chu and H. Chen, A Stackelberg game and its improvement in a VMI system with a manufacturing vendor. Eur. J. Oper. Res. 192 (2009) 929–948.
- [71] Y. Yu, G.Q. Huang and L. Liang, Stackelberg game-theoretic model for optimizing advertising, pricing and inventory policies in vendor managed inventory (VMI) production supply chains. Comput. Ind. Eng. 57 (2009) 368–382.
- [72] Y. Yu, G.Q. Huang, Z. Hong and X. Zhang, An integrated pricing and deteriorating model and a hybrid algorithm for a VMI (vendor-managed-inventory) supply chain. *IEEE Trans. Autom. Sci. Eng.* 8 (2011) 673–682.
- [73] Y. Yu, Z. Hong, L.L. Zhang, L. Liang and C. Chu, Optimal selection of retailers for a manufacturing vendor in a vendor managed inventory system. Eur. J. Oper. Res. 225 (2013) 273–284.
- [74] Y. Yugang, L. Liang and G.Q. Huang, Leader-follower game in vendor-managed inventory system with limited production capacity considering wholesale and retail prices. Int. J. Logist. Res. Appl. 9 (2006) 335–350.
- [75] Y. Zare Mehrjerdi and M. Akhbari, The Stackelberg game model for vendor-managed inventory systems: a wholesale price versus a two-part tariff contract. Int. J. Supply Oper. 7 (2020) 148–163.
- [76] P. Zhang, Y. He and C.V. Shi, Transshipment and coordination in a two-echelon supply chain. RAIRO-Oper. Res. 51 (2017) 729–747.
- [77] P. Zhang, X. Xu, V. Shi and J. Zhu, Simultaneous inventory competition and transshipment between retailers. Int. J. Prod. Econ. 230 (2020) 107781.

# **Subscribe to Open (S2O)** A fair and sustainable open access model

This journal is currently published in open access under a Subscribe-to-Open model (S2O). S2O is a transformative model that aims to move subscription journals to open access. Open access is the free, immediate, online availability of research articles combined with the rights to use these articles fully in the digital environment. We are thankful to our subscribers and sponsors for making it possible to publish this journal in open access, free of charge for authors.

## Please help to maintain this journal in open access!

Check that your library subscribes to the journal, or make a personal donation to the S2O programme, by contacting subscribers@edpsciences.org

More information, including a list of sponsors and a financial transparency report, available at: https://www.edpsciences.org/en/maths-s2o-programme