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# Optimizing the service area and trip selection of an electric automated taxi system used for the last mile of train trips 

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#### Abstract

We propose two integer programming models for optimizing an automated taxi (AT) system for last mile of train trips. Model S1: trip reservations are accepted or rejected by the operator according to the profit maximization; model S 2 : any reservation on a selected zone by the model must be satisfied. Models were applied to a case-study. Results indicate that fleet size influences the profitability of the taxi system: a fleet of 40 ATs is optimal in S1 and 60 ATs in S2. Having electric ATs constrains the system for small fleets because ATs will not have time for charging.


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## 1. Introduction

Within the last decade, technology development has accelerated the process of vehicle automation. An automated vehicle (AV), also known as a driverless car and a self-driving car is an advanced type of vehicle which can drive itself on existing roads and can navigate many types of roadways and environmental contexts with reduced direct human input (Fagnant and Kockelman, 2013). SAE International identifies six levels of driving automation from level 0 "no automation" to level 5 "full automation" (SAE International, 2014). Fully AVs are expected to bring significant benefits, such as mitigating traffic congestion, reducing car crashes, improving fuel efficiency and alleviation the negative impacts on environment (Bierstadt et al., 2014). Although further evidence is still needed to assess if those advantages are indeed real.

In recent years, most of the effort has been put into the technology challenges of creating fully AVs. Google has driven more than one million miles on public streets under autonomous mode since 2009 and is planning to make these cars available to the public in 2020 (Google, 2015). In addition to this, many automobile manufactures like General Motor, MercedesBenz, Audi, Nissan, BMW, Renault all expect to sell vehicles which can drive themselves by 2020.

With respect to the impacts of these vehicles on mobility, it has mainly focused on traffic impacts under different driving and infrastructure conditions. Most of these studies use micro-simulation tools or mathematical analysis to estimate the changes in road capacity and congestion under different levels of vehicle automation and cooperation (Bose and Ioannou, 2003; Calvert et al., 2011; van Arem et al., 2006).

Regarding the use of AVs as transit systems, physical trials have been the most relevant method of study so far. Automation in transit is not new, and there are currently many tram and metro lines operating without a driver in cities like Paris or Barcelona. Moreover automation is the basis for the so called Personal Rapid Transit (PRT) system. This is a flexible door-todoor public transport with small capacity vehicles which have their own dedicated infrastructure (Juster and Schonfeld, 2013). There are several examples in the world of its application but probably one of the most well-known is the case of

[^0]London Heathrow Airport, the ULTra (Urban Light Transit). Based on these notions combined with the recent development of vehicle automation there are several projects which are testing, in pilot experiments, the use of automated road transit systems. One of the most notable is the CityMobil2 project (CityMobil2 - Cities demonstrating Automated Road Passenger Transport, n.d.; Csepinszky et al., 2014) in which several field experiments are being run in Europe to test the possibilities of automated bus systems. In 2015, the province of Gelderland in the Netherlands is developing a project named WEpods with two self-driving vehicles, which are used between Ede/Wageningen railway station and Wageningen University \& Research Center (WEpods project DAVI - Dutch Automated Vehicle Initiative, n.d.). These are key experiments to test the technology but also the sensitivity of travellers to its characteristics, such as demand responsiveness or the absence of a driver. Nevertheless little has been done regarding the planning and operation of future transit systems constituted by AVs. Winter et al. (2016) use the case study of the WEpods project in the Netherlands to establish a method to dimension a fleet of automated buses, but these are only shadowing the existing bus line.

The most related research to our topic is looking at the combination between traditional taxis and carsharing (a short term rental of a vehicle that the traveller will drive himself) (Martin et al., 2010; Shaheen et al., 1999): with the advent of automation, the use of AVs in carsharing services may provide a new type of door-to-door service, competing with the traditional taxis or even shared taxi services such as Uber (Martinez et al., 2015) because these new systems will be able to avoid extra human costs associated with both traditional taxis and carsharing. Carsharing systems generate the problem of vehicle imbalance due to the one-way nature of the trips (Correia and Antunes, 2012; Jorge and Correia, 2013; Jorge et al., 2014; Kek et al., 2009) however, in the case of AVs, it is possible to do the relocation with lower costs between different areas because there is no need for a driver.

Fagnant and Kockelman (2014) proposed a simulation method to study the implications of shared ATs. They used an agent-based model for system operations and described the results of a case-study application where AVs were compared to conventional vehicle ownership and use. Their results indicate that each shared AV can replace around 11 conventional vehicles, but adds up to $10 \%$ more travel distance. Using the same technique, the International Transport Forum (ITF) built a model to test the introduction of $100 \%$ autonomous fleets of taxis to satisfy transport demand in a city (International Transport Forum, 2015). Results show that fleets always decrease: with the subway still in operation each AV could remove 9 out of 10 cars in the city if a maximum 5 min waiting time is to be guaranteed, whilst without metro the number stays in 5 vehicles removed per AV.

In this work we do not study the full substitution of traditional transit networks but propose instead to analyze the potential of using automated taxis (ATs) as a last mile connection of train trips. Given parking space availability, a properly functioning road infrastructure and smooth traffic, the use of the private automobile is highly attractive especially at longer distances (Ford, 2012). Moreover in multimodal trips it has been shown that a relatively high disutility is caused by the access and egress modes of transport (Hess, 2009; Hoogendoorn-Lanser et al., 2006). At the same time, to make transport more efficient, concentrating passengers in higher capacity vehicles such as trains leads to cost and pollution savings, hence the use of fully automated electric vehicles to feed these higher capacity systems in a seamless way may be a good solution to bring more people to public transport and improve transport sustainability. The use of AVs for the first/last mile connection has been analyzed before but mainly on a technology perspective (Chong et al., 2011).

In this paper, we present an optimization approach to define the service area of an AT system which satisfies passengers' requests to access or egress a train station, in order to maximize the profit of the AT system. Since AVs can be relocated at a lower cost (no need to hire staff), the model considers the possibility of the vehicles travelling alone as a relocation method. Moreover the system is based on mandatory pre-booking, allowing accepting or rejecting demand according to the profit maximization function. From a methodological point of view the models are based on the ones by Correia and Antunes (2012), hence this paper contributes to the literature by introducing a novel application of these formulations to the case where automated vehicles are used, thus avoiding the high costs that today the traditional carsharing operators have to consider.

A zoning problem is by definition a planning problem, however to select trips is typically an operational problem which should be solved on a daily basis. In this paper we assume that our models are used on a daily basis for trip selection (operational purpose), but by running them with simulated trips for several replications before implementing the system we are able to obtain the zones which should be included in the service area around the station.

The paper is organized as follows. In Section 2 we introduce the mathematical models for two different trip selection schemes. Section 3 applies the models to the case study of the Delft Zuid train station in the city of Delft (the Netherlands). Finally a discussion on the results and main conclusions drawn from the model application are presented in Section 4.

## 2. Mathematical models

In this section, we describe the formulation of two integer programming (IP) models in order to determine the optimal service area and trips to be served by an AT system. The two formulations depend on how trips are selected from the total number of reservations done in one typical day ( 24 h in advance booking).

The first scheme (S1) is called free service. The model works in the assumption that the taxi company can achieve total control over trip selection, by being free to accept or reject requests according to the profit maximization. Waiting time is not applicable for the passengers since the trip is only served exactly at the starting time of the request. The model allocates each AT to a specific trip only if it will bring a higher daily profit. Otherwise, this request will be rejected with no extra penalty,
even though there might be available taxis in the system. Such service scheme is flexible and profitable but will lead to unhappy customers because they may be in a situation in which they have their trips rejected but they know that some ATs are usually available nearby.

The second scheme (S2) is full service, which guarantees that all demand to/from selected zones must be satisfied. It does not mean that all the requests to/from potential zones will be met because zones are to be selected by the model too. Compared to $S 1$, this scheme provides a favourable taxi service which assures that no requests will be missed from the served zones.

We consider in this paper a first-mile/last-mile transport service for accessing and egressing from train stations, which means that we will not consider requests to use the system between two service zones where a service zone is a candidate area of the city for offering the transport service. Moreover, no requests will be considered when trips begin before the service period or when they end after that period. In order to provide a better service, we set up some relocation time before and after the service period and allow the taxis to move between the train station and service zones. At the beginning of the operation time, all vehicles are at the station where parking is free for the company. Similarly, taxis will all come back to the train station at the end of the operation period. This guarantees that the taxi fleet has enough time (normally during night time) to do the necessary full-charging of the battery and any required maintenance. When stopped in a service zone the car must pay for that parking.

Besides accepting or rejecting trips to and from the station, vehicles can travel empty from a service zone to another one to pick up another traveller. There is no need to have special equipment at a service zone because vehicles will only park there.

Before presenting the two models it is important to state all the assumptions which were considered in their formulations. To simplify we treat all origins and destinations of passengers' requests in a same zone as coming or going to the same point: the centroid of the service zone. Plus, we regard taxis as flows, which means that we do not differentiate a specific taxi. This hinders the computation of the specific battery charge that each vehicle has during the day, as modelled in other papers (Correia and Santos, 2014), but it simplifies the problem to be solved. In order to make this system feasible, a central management service is required to compute the best solution and give instructions to ATs. Thus all the requests have to be priorly known before the operation day via a reservation system. We argue that normally train trips are planned in advance according to the timetable. This means that real-time booking is less likely to happen as much as in the Uber (Uber, 2016) system.

### 2.1. Model S1 - free service

Model S1 is the one with free service scheme which allows the system to freely select trips according to the profit maximization. In this model, if there is at least one trip served in a zone, this zone is included in the total service area.

Considering the described system and its assumptions, we formulate the following integer programming problem:
Sets
$\boldsymbol{N}=\{0,1, \ldots, i, \ldots, N\}$ : set of the train station plus the candidate service zones ( $i=0$ represents the train station).
$\boldsymbol{N}^{\prime}=\{1, \ldots, i, \ldots, N\}$ : set of candidate service zones.
$\boldsymbol{T}=\{0,1, \ldots, t, \ldots, T\}$ : set of time instants in the service period. The time between two consecutive time instants is considered to be one time step where the number of time steps in a day is $T$.
$\boldsymbol{T}^{\prime}=\left\{-\delta_{\max }, \ldots,-1,0,1, \ldots, t, \ldots, T, T+1, \ldots, \delta_{\max }\right\}$ : set of time instants in the operation period, including the service period $\{0 \ldots T\}$ and relocation period $\left\{-\delta_{\max } \ldots 0\right\}$ and $\left\{T \ldots \delta_{\max }\right\} . \delta_{\max }$ is the maximum travel time between the train station and any potential zone.

## Decision variables

$x_{i}$ : equals to 1 if the candidate zone $i$ can be served, otherwise $0, \forall i \in \boldsymbol{N}^{\prime}$;
$D_{0 i}^{t, t+\delta_{0 i}}$ : the number of trips satisfied from the train station to service zone $i$ from time instant $t$ to time instant $t+\delta_{0 i}$, $\forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}, t+\delta_{0 i} \leqslant T$;
$D_{i 0}^{t, t+\delta_{i 0}}$ : the number of trips satisfied from service zone $i$ to the train station from time instant $t$ to time instant $t+\delta_{i 0}$, $\forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}, t+\delta_{i 0} \leqslant T$;
$S_{i}^{t, t+1}$ : the number of vehicles stocked at zone $i$ from time instant $t$ to time instant $t+1, \forall i \in \boldsymbol{N}, \forall t \in \boldsymbol{T}^{\prime}, t+1 \leqslant T+\delta_{\max }$;
$U_{i j}^{t, t+\delta_{i j}}$ : the number of taxis travelling from zone $i$ to zone $j$ from time instant $t$ to time instant $t+\delta_{i j}, \forall i, j \in \boldsymbol{N}, i \neq j, \forall t \in \boldsymbol{T}^{\prime}$,
$t+\delta_{i j} \leqslant T+\delta_{\max } ;$
$V_{i}^{t}$ : the number of available vehicles at zone $i$ at time instant $t, \forall i \in \boldsymbol{N}, \forall t \in \boldsymbol{T}^{\prime}$.

## Auxiliary variables

$Z_{0}$ : the number of parking spots at the train station;
$\sigma_{i}$ : the total idle time in time steps that all taxis spend at service zone $i$ in a day, $\forall i \in \boldsymbol{N}^{\prime}$;
$L_{t}$ : average driving distance per vehicle from the beginning of the day until time instant $t, \forall t \in \boldsymbol{T}$;
$\theta$ : share of satisfied demand (percentage).

## Parameters \& input

$F$ : taxi fleet size in the system. The fleet size is an input of the model instead of a decision variable in order to guarantee the model linearity;
$\delta_{i j}$ : the travel time in time steps between zone $i$ and zone $j, \forall i, j \in \boldsymbol{N}, i \neq j$;
$\delta_{\max }$ : the maximum travel time in time steps between train station and any service zone,

$$
\delta_{\max }=\max \left\{\delta_{i j}\right\} \quad i=0, \forall j \in \boldsymbol{N}^{\prime} \text { or } \forall i \in \boldsymbol{N}^{\prime}, j=0
$$

$d_{i j}$ : the travel distance between zone $i$ and zone $j, \forall i, j \in \boldsymbol{N}, i \neq j$;
$Q_{0 i}^{t, t+\delta_{0 i}}$ : the number of passenger requests from the train station to zone $i$ from time instant $t$ to time instant $t+\delta_{0 i}, \forall i \in \boldsymbol{N}^{\prime}$, $\forall t \in \boldsymbol{T}, t+\delta_{0 i} \leqslant T$;
$Q_{i 0}^{t, t+\delta_{i 0}}$ : the number of passenger requests from zone $i$ to the train station from time instant $t$ to time instant $t+\delta_{i 0}, \forall i \in \boldsymbol{N}^{\prime}$, $\forall t \in \boldsymbol{T}, t+\delta_{i 0} \leqslant T$;
$R$ : battery range of a vehicle expressed in driving distance with full battery (km);
$E$ : distance that can be driven with a one time-step charging (km/time step);
$P$ : price rate per driving distance ( $€ / \mathrm{km}$ );
$C_{m 1}$ : vehicle maintenance costs per driving distance ( $€ / \mathrm{km}$ );
$C_{d}$ : depreciation cost per vehicle per day ( $€ /$ day );
$C_{m 2}$ : parking space maintenance cost in the train station per spot per day ( $€$ /spot $\times$ day);
$C_{p}$ : parking price at the service zones per spot per time step ( $€ /$ spot $\times$ time step);
$M$ : large number.
Objective

$$
\begin{equation*}
\operatorname{Max} \Pi=P \cdot\left(\sum_{\substack{i \in \boldsymbol{N}^{\prime}, t \in \boldsymbol{T}, t+\delta_{0 i} \leqslant T}} D_{0 i}^{t, t+\delta_{0 i}} \cdot d_{0 i}+\sum_{\substack{i \in \boldsymbol{N}^{\prime}, t \in \boldsymbol{T}, t+\delta_{i 0} \leq T}} D_{i 0}^{t, t+\delta_{i 0}} \cdot d_{i 0}\right)-C_{m 1} \cdot \sum_{\substack{i, j \in \boldsymbol{N}, i \neq j, t \in \boldsymbol{T}^{\prime} \\ t+\delta_{i j} \leq T+\delta_{\max }}} U_{i j}^{t, t+\delta_{i j}} \cdot d_{i j}-C_{d} \cdot F-C_{m 2} \cdot Z_{0}-C_{p} \cdot \sum_{i \in \boldsymbol{N}^{\prime}} \sigma_{i} \tag{1}
\end{equation*}
$$

Objective function (1) maximizes the total profit ( $\Pi$ ) during a typical day of operations, taking into account the revenues paid by the passengers, vehicle maintenance costs, vehicle depreciation costs, parking space maintenance costs in the train station and parking costs in the service zones. There is no extra cost of rejecting requests in the system. At the same time, we use accepted and rejected requests percentage as an indicator to assess the performance of the AT system.

Constraints

$$
\begin{equation*}
V_{0}^{-\delta_{\max }}=F \tag{2}
\end{equation*}
$$

Constraint (2) describes the initial status of the AT fleet. It imposes that at the beginning of the operation period (before the beginning of service period), all vehicles are stocked at the train station.

$$
\begin{equation*}
V_{i}^{-\delta_{\max }}=0 \quad \forall i \in \boldsymbol{N}^{\prime} \tag{3}
\end{equation*}
$$

Constraints (3) impose that there cannot be any vehicle in the service zones at the beginning of the operation period.

$$
\begin{equation*}
V_{0}^{T+\delta_{\max }}=F \tag{4}
\end{equation*}
$$

Constraint (4) imposes that at the end of the operation period (after the end of service period), all taxis come back to the train station.

$$
\begin{equation*}
V_{i}^{T+\delta_{\max }}=0 \quad \forall i \in \boldsymbol{N}^{\prime} \tag{5}
\end{equation*}
$$

Constraints (5) guarantee that no taxi is available in any service zone at the end of the operation period.

$$
\begin{equation*}
S_{i}^{t, t+1}=S_{i}^{t-1, t}+\sum_{j \in \boldsymbol{N}, i \neq j} U_{j i}^{t-\delta_{j i}, t}-\sum_{\substack{j \in \sum_{N} \neq j, t+\delta_{i j} \leq T+\delta_{\text {max }}}} U_{i j}^{t, t+\delta_{i j}} \quad \forall i \in \boldsymbol{N}, \forall t \in \boldsymbol{T}^{\prime}, t+1 \leqslant T+\delta_{\max } \tag{6}
\end{equation*}
$$

Constraints (6) yield the vehicle stock level at the train station or service zone $i$ from time instant $t$ to time instant $t+1$. The number of stocked taxis for time period from $t$ to $t+1$ equals to the stocked taxis for time period from $t-1$ to $t$ plus the vehicles coming into the train station or service zones minus the vehicles getting out.

$$
\begin{equation*}
V_{i}^{t+1}=V_{i}^{t}-\sum_{\substack{j \in N, i \neq j \\ t+\delta_{i j} \leq T+\delta_{\text {max }}}} U_{i j}^{t, t+\delta_{i j}}+\sum_{j \in \boldsymbol{N}, i \neq j} U_{j i}^{t+1-\delta_{j i}, t+1} \quad \forall i \in \boldsymbol{N}, \forall t \in \boldsymbol{T}^{\prime}, t+1 \leqslant T+\delta_{\max } \tag{7}
\end{equation*}
$$

Constraints (7) are flow conservation constraints which compute the number of available taxis at node $i_{t+1}$ (the train station or service zone $i$ at time instant $t+1$ ) as a function of the number of taxis at time instant $t$ minus the vehicles getting out plus the vehicles coming into the train station or service zones at next time instant.

We use $S_{i}^{t, t+1}$ to describe those taxis who are waiting from one time instant to the next time instant (a whole time step) at the train station or a service zone. During this time, taxis are waiting for the next trip or charging to get electric power (if they are at the train station). $V_{i}^{t}$ is used to represent the instantaneous number of vehicles at a time instant $t$. We assume that the vehicles become available at a time instant $t$ after they arrive at that point and before they go out to a next destination. This can be seen in Fig. 1.

$$
\begin{equation*}
\sigma_{i}=\sum_{\substack{t \in T^{\prime} \\ t+1 \leqslant+\delta_{\max }}} S_{i}^{t, t+1} \quad \forall i \in \boldsymbol{N}^{\prime} \tag{8}
\end{equation*}
$$

Constraints (8) compute the total parking time in time steps for the whole day in each service zone. To be more specific, $S_{i}^{t, t+1}$ is the stock of vehicles, so it corresponds to the number of parking spots occupied for one time step in each service zone $i$ in that period. Summing them up yields the total parking time which will be used to calculate the parking costs in each service zone.

$$
\begin{equation*}
V_{0}^{t} \leqslant Z_{0} \quad \forall t \in \boldsymbol{T}^{\prime} \tag{9}
\end{equation*}
$$

Constraints (9) ensure that the parking capacity in the train station is enough for the number of ATs present there at any time instant.

$$
\begin{equation*}
D_{0 i}^{t, t+\delta_{0 i}} \leqslant Q_{0 i}^{t, t+\delta_{0 i}} \cdot x_{i} \quad \forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}, t+\delta_{0 i} \leqslant T \tag{10}
\end{equation*}
$$

Constraints (10) assure that the satisfied trips between the train station and service zone $i$ which begin at time instant $t$ and finish at time instant $t+\delta_{0 i}$ must be lower than or equal to the passengers' requests on the same OD . And if zone $i$ cannot be served ( $x_{i}=0$ ), the satisfied demand must be zero.

$$
\begin{equation*}
D_{i 0}^{t, t+\delta_{i 0}} \leqslant Q_{i 0}^{t, t+\delta_{i 0}} \cdot x_{i} \quad \forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}, t+\delta_{i 0} \leqslant T \tag{11}
\end{equation*}
$$

Constraints (11) assure that the satisfied trips between service zone $i$ and the train station which begin at time instant $t$ and finish at time instant $t+\delta_{i 0}$ must be lower than or equal to the passengers' requests on the same OD . And if zone $i$ cannot be served ( $x_{i}=0$ ), the satisfied demand must be zero.

$$
\begin{equation*}
\sum_{\substack{j \in \boldsymbol{N}, t \in \boldsymbol{T}^{\prime}, i \neq j, t+\delta_{i j} \leqslant T+\delta_{\max }}} U_{i j}^{t, t+\delta_{i j}}+\sum_{\substack{j \in \boldsymbol{N}, t \in \boldsymbol{T}^{\prime}, i \neq j, t+\delta_{j i} \leqslant T+\delta_{\max }}} U_{j i}^{t, t+\delta_{j i}} \leqslant M \cdot \boldsymbol{x}_{i} \quad \forall i \in \boldsymbol{N}^{\prime} \tag{12}
\end{equation*}
$$

Constraints (12) guarantee that if a zone cannot be served, there are no taxis travelling between this zone and any other zones or the train station.

$$
\begin{equation*}
x_{i} \leqslant \sum_{\substack{t \in T, T \\ t+\delta_{0} \leqslant T}} D_{0 i}^{t, t+\delta_{0 i}}+\sum_{\substack{t \in T, t+\delta_{i 0} \leqslant T}} D_{i 0}^{t, t+\delta_{i 0}} \quad \forall i \in \boldsymbol{N}^{\prime} \tag{13}
\end{equation*}
$$

Constraints (13) assure that if no trip from zone $i$ is satisfied by the ATs then that zone is not selected.

$$
\begin{equation*}
D_{0 i}^{t, t+\delta_{0 i}} \leqslant U_{0 i}^{t, t+\delta_{0 i}} \quad \forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}, t+\delta_{0 i} \leqslant T \tag{14}
\end{equation*}
$$

Constraints (14) impose the condition that the number of vehicles travelling between the train station and service zone $i$ must be greater than or equal to the number of people travelling on that OD (in this model the vehicle capacity is just one seat), since the vehicles can travel without any human inside.


Fig. 1. Vehicle movement in zone $i$.

$$
\begin{equation*}
D_{i 0}^{t, t+\delta_{i 0}} \leqslant U_{i 0}^{t, t+\delta_{i 0}} \quad \forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}, t+\delta_{i 0} \leqslant T \tag{15}
\end{equation*}
$$

Constraints (15) impose the condition that the number of vehicles travelling between service zone $i$ and the train station must be greater than or equal to the number of people travelling on that OD, since the vehicles can travel without any human inside.

$$
\begin{equation*}
\sum_{\substack{j \in \mathbb{j} i \neq j, t+\delta_{i j} \leqslant T+\delta_{\text {max }}}} U_{i j}^{t, t+\delta_{i j}} \leqslant V_{i}^{t} \quad \forall i \in \boldsymbol{N}, \forall t \in \boldsymbol{T}^{\prime} \tag{16}
\end{equation*}
$$

Constraints (16) guarantee that the vehicles leaving a service zone at time instant $t$ is less than or equal to the available vehicles at that service zone $i$.

$$
\begin{equation*}
\theta=\sum_{i \in \boldsymbol{N}^{\prime}}\left(\sum_{\substack{t \in \boldsymbol{T}, t+\delta_{0 i} \leqslant T}} D_{0 i}^{t, t+\delta_{0 i}}+\sum_{\substack{t \in \boldsymbol{T}, t+\delta_{i 0} \leqslant T}} D_{i 0}^{t, t+\delta_{i 0}}\right) / \sum_{i \in \boldsymbol{N}^{\prime}}\left(\sum_{\substack{t \in \boldsymbol{T} \\ t+\delta_{0 i} \leqslant T}} Q_{0 i}^{t, t+\delta_{0 i}}+\sum_{\substack{t \in \boldsymbol{T} \\ t+\delta_{i 0} \leqslant T}} Q_{i 0}^{t, t+\delta_{i 0}}\right) \tag{17}
\end{equation*}
$$

Constraint (17) yields the percentage of satisfied demand.

$$
\begin{equation*}
L_{t}=\sum_{\substack{i \in \boldsymbol{N}, j \in \boldsymbol{N}, i=j, t_{\in} \in \boldsymbol{T}^{\prime}, t_{1}+\delta_{i j} \leqslant t}} U_{i j}^{t_{1}, t_{1}+\delta_{i j}} \cdot d_{i j} / F \quad \forall t \in \boldsymbol{T}^{\prime} \tag{18}
\end{equation*}
$$

Constraints (18) compute the average driving distance per vehicle from the beginning of the operation period to a particular time instant $t$.

$$
\begin{equation*}
L_{t}-R \leqslant \sum_{\substack{t_{1} \in T^{\prime}, t_{1}+1 \leqslant t}} S_{0}^{t_{1}, t_{1}+1} \cdot E / F \quad \forall t \in \boldsymbol{T}^{\prime} \tag{19}
\end{equation*}
$$

Constraints (19) impose the charging condition i.e. when the average driving distance per vehicle exceeds the driving range of a full battery, taxis must stay long enough at the train station to get sufficient power for the remaining average driving distance.

$$
\begin{align*}
& V_{i}^{t} \geqslant 0 \quad \forall i \in \boldsymbol{N}, \forall t \in \mathbf{T}^{\prime}  \tag{20}\\
& S_{i}^{t, t+1} \geqslant 0 \quad \forall i \in \boldsymbol{N}, \forall t \in \mathbf{T}^{\prime}, t+1 \leqslant T+\delta_{\max }  \tag{21}\\
& D_{0 i}^{t, t+\delta_{0 i}} \geqslant 0 \quad \forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}, t+\delta_{0 i} \leqslant T  \tag{22}\\
& D_{i 0}^{t, t+\delta_{i 0}} \geqslant 0 \quad \forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}, t+\delta_{i 0} \leqslant T  \tag{23}\\
& U_{i j}^{t, t+\delta_{i j}} \geqslant 0 \quad \forall i, j \in \boldsymbol{N}, i \neq j, \forall t \in \mathbf{T}^{\prime}, t+\delta_{i j} \leqslant T+\delta_{\max }  \tag{24}\\
& Z_{0} \geqslant 0  \tag{25}\\
& x_{i}=(0,1) \quad \forall i \in \boldsymbol{N}^{\prime} \tag{26}
\end{align*}
$$

Constraints (20)-(26) define the domain for the decision variables.

### 2.2. Model S2 - full service

This model considers that all requested trips to/from served zones must be satisfied. The model is based on the previous by adding the following constraints:

$$
\begin{align*}
& D_{0 i}^{t, t+\delta_{0 i}} \geqslant Q_{0 i}^{t, t+\delta_{0 i}}+M \cdot\left(x_{i}-1\right) \quad \forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}, t+\delta_{0 i} \leqslant T  \tag{27}\\
& D_{i 0}^{t, t+\delta_{i 0}} \geqslant Q_{i 0}^{t, t+\delta_{i 0}}+M \cdot\left(x_{i}-1\right) \quad \forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}, t+\delta_{i 0} \leqslant T \tag{28}
\end{align*}
$$

Constraints (27) and (28) guarantee that when zone $i$ is selected as service zone ( $x_{i}=1$ ), the satisfied trips will be equal to or more than the requests. This means all requests generated to/from zone $i$ are satisfied, because working with constraints (10) and (11) will impose $D_{0 i}^{t, t+\delta_{0 i}}=Q_{0 i}^{t, t+\delta_{0 i}}$ and $D_{i 0}^{t, t+\delta_{i 0}}=Q_{i 0}^{t, t+\delta_{i 0}}$.

### 2.3. Bounding the problem

The search for the optimum set of trips to be satisfied can be accelerated by bounding the problem with extra constraints. They do not change the solution region but tight the bounds on that space by eliminating non-integer solutions of the relaxation process of the traditional branch and bound search method. Thus the gap to find the optimal solution closes faster since certain nodes of the tree will not be explored.

$$
\begin{align*}
& S_{i}^{t, t+1} \leqslant F \cdot x_{i} \quad \forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}^{\prime}, t+1 \leqslant T+\delta_{\max }  \tag{29}\\
& V_{i}^{t} \leqslant F \cdot x_{i} \quad \forall i \in \boldsymbol{N}^{\prime}, \forall t \in \boldsymbol{T}^{\prime} \tag{30}
\end{align*}
$$

These constraints impose that zones can only have taxis there when they are selected as being part of the service area. These constraints are already imposed by the interaction of constraints (6), (7) and (12). In the branch and bound process it may happen that the relaxed variables on the left hand side of constraints (6) and (7) are equivalent to the right hand side, but in reality not all of them may have a value greater than 0 at the same time. Imposing bound constraints (29) and (30) will not allow those solution nodes to be explored in the relaxation process.

### 2.4. A small scale example

We apply model S1 and model S2 to a small scale example to illustrate how they work and the type of results which can be obtained. In this example, we consider passenger requests between 4 potential zones and the train station. There are 8 time steps from time instant 0 to 8 in the service period. As the maximum travel time between any zone and the train station is 3 time steps, we set the operation period from time instant -3 to 11 . Table 1 and Fig. 2 show the passenger requests in the service period. Each line corresponds to one request. The trip duration is based on the travel time between two points which is a set of parameters in this model. For example, there is a request from the train station to zone 4 starting at time instant 0 and the trip duration is 3 time steps.

Table 2 presents the travel time and travel distance between the train station and potential zones. For this small example we consider the following parameters: price rate $P=1 € / \mathrm{km}$, vehicle maintenance cost $C_{m 1}=0.05 € / \mathrm{km}$, vehicle depreciation cost $C_{d}=17 € /$ day, parking space maintenance $C_{m 2}=5 € /$ spot $\times$ day and parking price $C_{p}=0.25 € /$ spot $\times$ time step.

In Fig. 3 we show the graphical results of model $S 1$ for vehicle movements ( $U_{i j}^{t, t+\delta_{i j}}$ ) in the optimal solution and the number of available ATs stocked at each zone and at the train station $\left(V_{i}^{t}\right)$. Beyond the service period, the ATs are allowed to travel between the train station and the service zones. In this case, two taxis do relocations in the first relocation period and another one in the second relocation period. Without this, the system cannot serve trips like $D_{10}^{0,1}$ or getting back to the train station for daily maintenance. Together with the demand in Fig. 2, each solid line represents the accepted request $\left(D_{0 i}^{t, t+\delta_{0 i}}\right.$ or $\left.D_{i 0}^{t, t+\delta_{i 0}}\right)$. The dashed lines are the taxi relocations $\left(U_{i j}^{t, t+\delta_{i j}}-D_{0 j}^{t, t+\delta_{0 j}}\right.$ or $\left.U_{i j}^{t, t+\delta_{i j}}-D_{i 0}^{t, t+\delta_{i 0}}\right)$ which are vehicle movements without passengers. Moreover, a line from Fig. 2 which disappears in Fig. 3, indicates that a request from zone 2 to the train station starting at time instant 3 is rejected in the optimal solution.

Model S2 is also applied to the small scale example, with optimal results shown in Fig. 4. With the service rule imposing that all requests from selected zones must be satisfied, the model decided to serve zone 1, zone 2 and zone 4 (all requests to/ from these zones being satisfied). However, no trips in zone 3 means this zone is apparently not leading to maximizing the profit.

Table 1
Passenger requests list for the small scale example.

| Starting time | Origin | Destination | Number of requests |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 4 | 1 |
| 0 | 0 | 2 | 1 |
| 2 | 0 | 3 | 1 |
| 3 | 0 | 2 | 1 |
| 4 | 0 | 1 | 1 |
| 6 | 0 | 3 | 1 |
| 6 | 0 | 2 | 1 |
| 0 | 1 | 0 | 1 |
| 2 | 3 | 0 | 1 |
| 2 | 4 | 0 | 1 |
| 3 | 2 | 0 | 1 |
| 4 | 4 | 0 | 1 |
| 6 | 1 | 0 | 1 |



Fig. 2. Passenger requests in time-space network for the small scale example.

## 3. Case study

We apply our model to a bigger case study city, the city of Delft, which is located in the Netherlands. The municipality has a total area of $24 \mathrm{~km}^{2}$ and a population of about 100,000 . The city has two train stations but we focus on the one with less demand, Delft Zuid, which is located on a section of the railway line between The Hague and Rotterdam. It counts 142 sprinter trains per day, which provide slower stopping train service not only at significant stations but also at smaller stations between cities (Sprinter SLT_About NS_NS, n.d.). This station is connected to the city area of Delft by just one regional bus line hence its main purpose is local accessibility where the TU Delft campus is one of the most significant trip attractors/generators.

The demand is mainly concentrated in two peak periods: 7:00-9:00 am and 4:00-6:00 pm. The average headway in the two directions is 15 min . Moreover, given that the first train leaves at 5:27 am and the last train at 1:01 am in the next day, we choose 6:00-22:00 as the service period of the AT service to provide accessibility to the train passengers. The time step will be 5 min in order to reduce computational time. Thus for the 16 h service period in a day, there will be 192 time steps considered from time instant 0 to time instant 192. The maximum travel time between the train station and the service zone is 3 time steps hence the model operation period is defined from time instant -3 to time instant 195.

We divided the catchment area of the Delft Zuid station into 48 potential service zones trying to follow a principle of homogeneous land use in each zone (Fig. 5). The average size of each zone is $500 \mathrm{~m} \times 500 \mathrm{~m}$.

For the purpose of applying models S1 and S2 for studying the AT system in Delft Zuid station, we need these data: (1) all requests during an average day; (2) costs of running the system; and (3) driving distance and travel time between the train station and any potential service zone.

In order to determine the demand, we conducted a face to face field survey at the train station on the 2nd and 4th of June, 2015. This survey allowed obtaining a stated preference of the passengers for using the ATs, and also an estimation of the probability of each potential zone being an origin or a destination in these rail connection trips. Regarding the starting time of each request, we estimate that the passenger arriving time at the train station coming from the city is uniformly distributed. Then in order to get the departure time from the service zone, we subtract the determinist taxi travel time in that OD pair. Correspondingly, we assume that the passengers who just finished the railway trip will leave the train station to go to the city area immediately. Consequently there is equal probability of passengers to leave the station at the end of each train headway during a day.

The total number of passengers using Delft Zuid station in a working day is 4668 , a number provided by NS (the passenger railway operator in the Netherlands) for an average day in 2013. From the data we see that $50 \%$ of the passengers are arriving at the station and the other $50 \%$ leaving the station, yielding perfect balance. Each trip was allocated to either using an AT or another transport mode based on the survey results.

Table 2
Travel time and travel distance.

| Zones | Travel time (time step) |  |  |  |  | Travel distance (km) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 |
| 0 | - | 1 | 1 | 2 | 3 | - | 2 | 3 | 4 | 6 |
| 1 | 1 | - | 2 | 1 | 3 | 2 | - | 4 | 2 | 6 |
| 2 | 1 | 2 | - | 1 | 2 | 3 | 4 | - | 2 | 3 |
| 3 | 2 | 1 | 1 | - | 2 | 4 | 2 | 2 | - | 4 |
| 4 | 3 | 3 | 2 | 2 | - | 6 | 6 | 3 | 4 | - |



Fig. 3. Results from model S 1 for the small scale example.


Fig. 4. Results from model S2 for the small scale example.

In order to generate the AT reservations we use the Monte Carlo simulation, where we select an origin, a destination, and a starting time for each trip based on the estimated probabilities brought from the survey. Using a random seed, we get a new set of AT passenger requests each time when a replication is generated.

Travel distance $d_{i j}$ and travel time $\delta_{i j}$ between any potential service zones and the train station were calculated using the web mapping service Google Maps. These two inputs are computed between each centroid of a zone, where travel times will be considered deterministic which means they will not change throughout the day as mentioned before.

The cost values considered are as follows:

- $P$ (price rate): $1 € / \mathrm{km}$.
- $C_{m 1}$ (vehicle maintenance cost): $0.05 € / \mathrm{km}$.
- $C_{d}$ (depreciation cost): $17 € /$ day.
- $C_{m 2}$ (parking space maintenance): $5 € /$ spot $\times$ day.
- $C_{p}$ (parking price in service zones): $0.25 € /$ spot $\times$ time step.

We use as reference the price rate of Uber in Amsterdam and Rotterdam in the Netherlands (Uber, 2016). The fare is $1.4 € / \mathrm{km}$ together with $2 €$ as base fare. ATs do not need drivers which makes the operating cost lower. Besides, a lower fare will attract more passengers to use this brand new transport mode.

We chose Renault Twizy for our one seat capacity vehicle. The battery range of this electric vehicle is 80 km and normally the charging time is 4 h (Discover Renault Twizy, n.d.). Based on this and the assumption that the battery reached by


Fig. 5. Potential service zone location.
charging is uniform, the charging efficiency is $1.67 \mathrm{~km} /$ time step in this case. Therefore, the charging parameters considered are as follows:

- $R$ (battery range): 80 km .
- $E$ (distance that can be driven with a one time-step charging): $1.67 \mathrm{~km} /$ time step.


## 4. Experiments and results

Several experiments were done with the S1 and S2 trip selection schemes. The optimization models were run in an i7 processor @2.10 GHz, 8.00 GB RAM computer under a Windows 764 -bit operation system by Xpress, an optimization tool which uses advanced branch-and-cut algorithms for solving integer programming problems (FICO, 2014).

The passenger request generation is done, as referred, through Monte Carlo simulation. The detailed information of each trip including if it is going to be taken by an AT, its origin, its destination and the starting time are randomly generated thus we have decided to run 10 replications namely 10 sets of passenger requests. The final results of each model run are presented as an average value for each indicator.

The average total number of reservation requests for the whole day with the 10 replications was 2061 . These were associated to 44 potential zones, which indicate that 4 zones do not have any requests.

### 4.1. Fleet size variation

Four scenarios were built with a fleet of $20,40,60$ and 80 ATs for models S1 and S2. The computation time of model S1 is less than 3 min for one replication and a bigger fleet size leads to faster running time. For model S 2 the running time of 20 vehicles is about 16 min and for 40 vehicles it is 9 min . But for 60 and 80 vehicles one replication run is less than 1 min . The results obtained for scheme S1 are presented in Table 3.

For model S1 which is free level of service, in a scenario with 20 taxis, only $60.2 \%$ of the potential requests are satisfied (1242 out of 2061) generating a maximum daily profit of $2365.3 €$. The three indicators of taxi movement show that vehicles in this scenario have a higher usage: each taxi drives an average of $204.8 \mathrm{~km}, 9.4 \mathrm{~h}$, serving 62 trips in a day. From the scenario with 40 taxis in the system, we can see that the satisfied requests percentage increases to $94.9 \%$. Having more ATs not only increases the number of served trips, but also improves the daily profit to $3391.3 €$, which is the highest value among the four scenarios in model S1. When the number of ATs goes to 60 , the percentage of accepted requests reaches $99.9 \%$. Only 2 passenger requests are rejected. In this scenario, the taxi usage is much lower than the first one, with 129.4 km average driving and 34 trips served by each vehicle. With regards to the fleet of 80 taxis, it is easy to find that the optimization of

Table 3
Optimization results for model S1 for fleet size variation.

| Total <br> requests | Fleet <br> size | Profit <br> $(€ /$ day $)$ | Total service <br> zones | Requests satisfied <br> (trips) | Requests <br> satisfied $(\%)$ | Driving distance/ <br> Veh. $(\mathrm{km})$ | Driving time/ <br> Veh. (h) | Satisfied <br> trips/Veh. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2061 | 20 | 2365.3 | 43 | 1242 | 60.2 | 204.8 | 6.4 |  |
|  | 40 | 3391.3 | 44 | 1955 | 94.9 | 185.0 | 62 |  |
|  | 60 | 3128.0 | 44 | 2059 | 99.9 | 129.4 | 49 |  |
|  | 80 | 2691.2 | 44 | 2061 | 100.0 | 97.1 | 4.3 | 4.7 |

Table 4
Optimization results for model S 2 for fleet size variation.

| Total <br> requests | Fleet <br> size | Profit <br> $(€ /$ day $)$ | Total service <br> zones | Requests satisfied <br> $($ trips $)$ | Requests <br> satisfied $(\%)$ | Driving distance/ <br> Veh. $(\mathrm{km})$ | Driving time/ <br> Veh. (h) | Satisfied <br> trips/Veh. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2061 | 20 | 1645.8 | 21 | 913 | 44.3 | 188.2 | 4.3 |  |
|  | 40 | 2716.3 | 35 | 1648 | 80.0 | 157.6 | 4.4 |  |
|  | 60 | 3078.2 | 43 | 2034 | 98.7 | 128.0 | 41 |  |
|  | 80 | 2691.2 | 44 | 2061 | 100.0 | 97.1 | 6.2 | 4.7 |

service zone location and trip selection covers all the demand and all the zones having passenger requests. Although there are more requests satisfied by the system in this scenario, which denotes a higher level of service, the daily profit is lower than the best scenario, even lower than the third scenario because of the increase of depreciation cost associated to a bigger fleet size.

The number of served zones for the first scenario is 43 and for the remaining is always 44 . Hence there are always 43 zones being chosen as service zones regardless of the number of trips served from each zone. When the ATs are insufficient for the requests (fleet size 20), the model indeed closes one zone because the system should not serve any request from it.

Using the same demand, we applied the trip selection scheme S2. Results are presented in Table 4.
For the scenario with a fleet of 20 ATs , only 21 zones and 913 requests are served which is significantly less than the same scenario in model S 1 . When increasing the fleet of vehicles in the system, the service area expands and the satisfied trips increase. It is improved by using 40 vehicles as ATs resulting in 35 zones on average for which all trips are satisfied. But this is still less than the same scenario in model S1. Similarly, this change can also be seen in the third scenario. When 80 taxis are available, the satisfied demand reaches $100 \%$, with all potential zones which have requests being served. In this model, the most profitable scenario is the one with 60 ATs. A possible reason is that 40 ATs can serve more trips but they cannot capture all the requests from these zones, which makes these vehicles redundant to the system and leads to less profit for the taxi company.

The results for the service area optimization are shown in Fig. 6. For trip selection scheme $S 1$, the model selects the potential trips to serve, therefore, the zones with satisfied trips, even if just one, are selected to comprise the service area. There are 44 zones with requests out of the 48 potential ones. When increasing the fleet size from 20 to 40 , more trips from each zone are served by AVs and some zones are actually fully served.

For model S2, the filling colour level in each zone indicates the number of replications when this zone was selected. In the scenario with 20 AVs , there are 4 zones which are never selected to be part of the service area. Results for the other areas are very diverse. For example, the zone on the west side is served in 4 replications and in the other 6 replications the model decided to reject all trips to/from that zone. This shows that there is an important influence of the trip stochasticity. In the scenario with 40 vehicles, all zones have more replications in which they are selected. Moreover, there are 7 zones which are chosen in every replication, which indicates that these should always be part of the system.

In order to verify if 10 replications are sufficient to remove the noise of the simulation results, we compute the coefficient of variation (CV) of each indicator for 10 runs. Results are shown in Table 5 . We can see that almost all indicators have a relatively low level of CV, which means that under the requests variability over 10 simulations the model results are stable. The maximum value of CV is the one with 20 ATs in model S2. This may be because the ATs are insufficient to serve all the requests.

### 4.2. Electric vs conventional vehicles

In the models, constraints (18) and (19) are used to compute the average driving distance and guarantee that on average the charging time is enough to power all the vehicles. Here we do a comparison between conventional ATs and electric ATs to test the impact of different technologies on the profit of the system. Though we assume that in the future the technology of electric vehicles will be mature which makes the vehicle maintenance costs the same as in conventional vehicles. Therefore in this computing the parameter $C_{m 1}$ (vehicle maintenance cost) remains the same.

As the scenario with 20 taxis has the most efficient vehicle usage and the scenario with 40 ATs has the best daily profit, we choose 20 and 40 as fleet size to run the model without constraints (18) and (19) for both trip selection schemes.


Fig. 6. Optimization results for service zone location.

Table 5
Coefficient of variation of each indicator for models S1 and S2.

| Fleet size | S1 |  |  | S2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Profit ( $€ /$ day $)$ | Requests satisfied (Trips) | Total service zones | Profit ( $€ /$ day) | Requests satisfied (Trips) | Total service zones |
| 20 | 0.97\% | 1.08\% | 2.89\% | 3.93\% | 4.22\% | 10.92\% |
| 40 | 1.79\% | 1.41\% | 1.71\% | 3.76\% | 3.76\% | 6.14\% |
| 60 | 2.25\% | 1.56\% | 1.71\% | 2.42\% | 1.97\% | 3.53\% |
| 80 | 2.82\% | 1.74\% | 1.71\% | 2.82\% | 1.74\% | 1.71\% |

Table 6 shows the optimal results of using electric and conventional taxis. In model S1, when there are 20 taxis available in the system, the conventional ATs can serve more trips than the electric ATs. The different results prove that constraints (18) and (19) constrain the feasible region and influence the optimal solution. This also shows the fact that conventional ATs can serve trips continuously and electric vehicles cannot play the same role on the system performance. Regarding the 40 taxis fleet, there is no difference between the two optimization results thus in this case the stocking time at the train station is long enough on average for the vehicles to get enough power for the trips.

Model S2 has a different outcome for this comparison. It is easy to see that the optimal results are the same between the scenario with electric taxis and with conventional taxis when the fleet size is 20 and 40 . This shows that constraints (27) and (28) have more significant impact on the feasible region than constraints (18) and (19) therefore taking out the constraints about electric vehicle does not influence the optimal solution of this model. From a practical point of view, these results show that the ATs have enough time to get charged when they have to serve all the requests in a service area.

### 4.3. Automated taxis vs human driven taxis

Automated taxis can drive themselves between any two zones when they are empty. But if the taxis are not automated, the company needs to hire staff to drive conventional taxis for vehicle relocation like in any carsharing system (Correia and Antunes, 2012; Jorge et al., 2014). In this section, we do a comparison between automated taxis and human driven taxis on the daily profit thus exploring the benefits of using ATs.

Table 6
Optimization results for electric ATs and conventional ATs for models S1 and S2.

| Model | Vehicle type | Fleet <br> size | Profit <br> $(€ /$ day $)$ | Requests satisfied <br> $(\%)$ | Model | Vehicle type | Fleet <br> size | Profit <br> $(€ /$ day $)$ | Requests satisfied <br> $(\%)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| S1 | Electric | 20 | 2365.3 | 60.20 | S2 | Electric | 20 | 1645.8 |  |
|  | Conventional |  | 3071.0 | 76.60 |  | Conventional | 44.30 |  |  |
|  | Electric | 40 | 3391.3 | 94.90 | Electric | 40 | 1645.8 | 44.30 |  |
|  | Conventional |  | 3391.3 | 94.90 | Conventional | 2716.3 | 80.00 |  |  |

Table 7
Optimization results for automated relocation and human driven relocation for scheme S1 and S2.

| S1 |  |  |  |  | S2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Fleet <br> size | Relocation | Profit <br> ( $€$ /day) | Requests satisfied (trips) | Requests <br> satisfied (\%) | Fleet <br> size | Relocation | Profit <br> ( $€ /$ day) | Requests satisfied (trips) | Requests satisfied (\%) |
| 20 | Automatic | 2365.3 | 1242 | 60.2 | 20 | Automatic | 1645.8 | 913 | 44.3 |
|  | Human driven | 1948.4 | 1242 | 60.2 |  | Human Driven | 1150.8 | 908 | 44.1 |
| 40 | Automatic | 3391.3 | 1955 | 94.9 | 40 | Automatic | 2716.3 | 1648 | 80.0 |
|  | Human driven | 2383.7 | 1943 | 94.3 |  | Human <br> Driven | 1864.5 | 1670 | 81.1 |
| 60 | Automatic | 3128.0 | 2059 | 99.9 | 60 | Automatic | 3078.2 | 2034 | 98.7 |
|  | Human driven | 2036.4 | 2042 | 99.1 |  | Human Driven | 1994.4 | 2034 | 98.7 |
| 80 | Automatic | 2691.2 | 2061 | 100.0 | 80 | Automatic | 2691.2 | 2061 | 100.0 |
|  | Human driven | 1598.1 | 2044 | 99.2 |  | Human Driven | 1594.8 | 2061 | 100.0 |

For the objective function (1), staff costs should now be added. The new objective function is hence as follows:

$$
\begin{align*}
\operatorname{Max} \Pi=P & \left(\sum_{\substack{i \in \boldsymbol{N}^{\prime}, t \in \boldsymbol{T}, t+\delta_{0 i} \leqslant T}} D_{0 i}^{t, t+\delta_{0 i}} \cdot d_{0 i}+\sum_{\substack{i \in \boldsymbol{N}^{\prime}, t \in \boldsymbol{T}, t+\delta_{i 0} \leqslant T}} D_{i 0}^{t, t+\delta_{i 0}} \cdot d_{i 0}\right)-C_{m 1} \cdot \sum_{\substack{i, j \in \boldsymbol{N}, i \neq j, t \in \boldsymbol{T}^{\prime} \\
t+\delta_{i j} \leqslant T+\delta_{\max }}} U_{i j}^{t, t+\delta_{i j}} \cdot d_{i j}-C_{d} \cdot F-C_{m 2} \cdot Z_{0}-C_{p} \cdot \sum_{i \in \boldsymbol{N}^{\prime}} \sigma_{i}-C_{h} \\
& \cdot\left(\begin{array}{l}
\left.\sum_{\substack{i, j \in \boldsymbol{N}, i \neq j, t \in \boldsymbol{T}^{\prime} \\
t+\delta_{i j} \leqslant T+\delta_{\max }}} U_{i j}^{t, t+\delta_{i j}} \cdot \delta_{i j}-\sum_{\substack{i \in \boldsymbol{N}^{\prime}, t \in \boldsymbol{T}, t+\delta_{0 i} \leqslant T}} D_{0 i}^{t, t+\delta_{0 i}} \cdot \delta_{0 i}-\sum_{\substack{i \in \boldsymbol{N}^{\prime}, t \in \boldsymbol{T}, t+\delta_{i 0} \leqslant T}} D_{i 0}^{t, t+\delta_{i 0}} \cdot \delta_{i 0}\right)
\end{array},\right. \tag{31}
\end{align*}
$$

where $C_{h}$ is the cost of a driver per time step ( $€$ /time step).
According to the regulations in the Netherlands, the minimum wage for employees is $9 € / \mathrm{h}$, thus in this model we set $C_{h}=0.75 € /$ time step.

Table 7 shows the results from applying the optimization model using the automated driving mode and the human driven mode when relocating taxis between zones and train station.

In model S1, there is no difference between automated driving and human driven relocations regarding the number of served trips when there are 20 taxis in the system. But the daily profit of manually driven cars is lower than in the automation mode which reflects the most significant benefits of using automated taxis. In the scenarios with 40,60 and 80 taxis, the optimal solutions change in relation to the number of satisfied trips. Besides, the cost for the staff, together with less satisfied trips, makes the profit of the taxi company lower than when the vehicles are automatic. This happens because the system decides to reject some trips to avoid more driving costs. Even though accepting these trips will bring revenues, it cannot cover the high relocation cost.

In model S2, the optimal solutions of automated and human driven taxis only change in the scenario with 20 and 40 taxis. In the 60 and 80 vehicles' fleet the satisfied trips remain the same number and the profit is lower due to the driver costs. This indicates that when taxis are sufficient for this case study (scenarios with 60 and 80 ), constraints (27) and (28) in mandatory trip satisfaction have a stronger influence on the feasible region of solutions than adding driver costs.

## 5. Discussion and conclusions

AVs have gained great momentum in the recent decades evolving from concept vehicles to starting to drive in our roads in very visible and challenging pilot studies. Current research puts an emphasis on technology looking at aspects such as
reliability and safety, and also on the interactions between the vehicles and other agents like human beings, conventional vehicles, and the existing network. Despite this rapid change, there are still many open questions regarding their application in the future, especially in what regards their use as public transport.

The contribution of this paper is to provide a mathematical model to plan a last mile reservation based AT system to provide access to train stations. We considered two schemes: free (S1) and full service (S2). In the first the company may select operational zones and the trips which it wants to satisfy and in the second when selecting a zone all trips must be satisfied. The models were both established with the objective to maximize the daily profit of such system by deciding on service zone locations and reservations to be accept. Additionally, we allow the taxis to move without passengers between service zones. This is the potentially most significant benefit of AVs compared to conventional taxis: ATs can move themselves without any driver which reduces operation costs. The models consider the vehicles to be electric thus some constraints were included in order to guarantee that on average taxis are idle enough time for the charging to happen.

The models were applied to the case study of Delft Zuid train station in Delft, the Netherlands. This case study provided interesting insights into the effect of service zone location and trip selection on the profitability of the AT system. From that application we were able to take the following conclusions.

The first conclusion is that zone location and trip selection are able to diminish the negative impact of taxi imbalance hence contributing to the maximization of the profit. When the system is free to choose requests to serve (S1), the daily profit is always higher than the scheme with full service (S2), because this cannot reject inconvenient trips for the system without cancelling a whole operational zone. However model $S 2$ has the advantage of fully covering a zone hence it guarantees level of service at the expense of a lower profit. The second is that fleet size is an important factor on the profitability of the AT system because it influences the service capacity offered to passengers, as well as the total sunk cost of the fleet ownership. This conclusion can be drawn from observing the optimal daily profit of the two models where for 40 taxis under S1 and 60 taxis under S2 the maximum profit is achieved. The third conclusion is that when the model is free to choose trips (S1), having electric ATs constrains the system in accepting more requests for small fleets. For bigger fleets, the time spent at the train station should on average be enough to charge them. But the optimal results do not change using conventional taxis or electric ones in S 2 since full service is more limitative than getting enough power. The fourth conclusion is that using ATs can significantly decrease the relocation costs and improve the daily profit of such taxi company comparing to manpower relocation. In model S 1 , with a small fleet this staff cost does not change the optimal solution but with bigger fleets the optimum solutions point to serve less trips to avoid paying more for drivers, which leads to less revenue for the taxi company. For model S2, with bigger fleets the optimal solution does not change, which indicates that full service has more influence than adding human driver costs in the objective function.

Despite the current version of the optimization models being practice-ready, there are some possible improvements which will make this model more realistic. First, we intend to take into account the negative impact of rejecting travelling requests by generating a penalty in the objective function. Moreover, in this paper we only apply the model under fixed total demand and price, but it would be interesting to assess the effects of varying prices and total demand on the daily profit of the system. Third, this model only considers a service between a train station and an urban area, however there may be synergy effects from combining these trips with inter-zonal trips.

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## References

Bierstadt, J., Gooze, A., Gray, C., Peterman, J., Raykin, L., Walters, J., 2014. Effects of Next - Generation Vehicles on Travel Demand and Highway Capacity. Bose, A., Ioannou, P.A., 2003. Analysis of traffic flow with mixed manual and semiautomated vehicles. IEEE Trans. Intell. Transport. Syst. 4, 173-188. http:// dx.doi.org/10.1109/TITS.2003.821340.

Calvert, S.C., Van Den Broek, T.H.A., Van Noort, M., 2011. Modelling cooperative driving in congestion shockwaves on a freeway network. In: IEEE Conference on Intelligent Transportation Systems, Proceedings. ITSC, pp. 614-619. http://dx.doi.org/10.1109/ITSC.2011.6082837.
Chong, Z.J., Qin, B., Bandyopadhyay, T., Wongpiromsarn, T., Rankin, E.S., Ang, M.H., Frazzoli, E., Rus, D., Hsu, D., Low, K.H., 2011. Autonomous personal vehicle for the first- and last-mile transportation services. In: Proceedings of the 2011 IEEE 5th International Conference on Cybernetics and Intelligent Systems, CIS 2011, pp. 253-260. http://dx.doi.org/10.1109/ICCIS.2011.6070337.
CityMobil2 - Cities demonstrating Automated Road Passenger Transport [WWW Document], n.d. [http://www.citymobil2.eu/en/](http://www.citymobil2.eu/en/).
Correia, G.H. de A., Santos, R.F.G., 2014. Optimizing the use of electric vehicles in a regional car rental fleet. Transport. Res. Rec.: J. Transport. Res. Board 2454 , 76-83. http://dx.doi.org/10.3141/2454-10.
Correia, G.H.D.A., Antunes, A.P., 2012. Optimization approach to depot location and trip selection in one-way carsharing systems. Transport. Res. Part E: Logist. Transport. Rev. 48, 233-247. http://dx.doi.org/10.1016/j.tre.2011.06.003.
Csepinszky, A., Giustianiani, G., Holguin, C., Parent, M., Flamentm, M., Alessandrini, A., 2014. Safe Integration of fully automated road transport systems in urban environments: the basis for the missing legal framework. Transport. Res. Board, 1-17.
Discover Renault Twizy [WWW Document], n.d. [http://www.renault.fr/gamme-renault/vehicules-electriques/twizy/twizy/](http://www.renault.fr/gamme-renault/vehicules-electriques/twizy/twizy/).
Fagnant, D.J., Kockelman, K., 2013. Preparing a nation for autonomous vehicles: opportunities, barriers and policy recommendations for capitalizing on selfdriven vehicles. Transport. Res. Part A: Policy Pract. 77, 167-181. http://dx.doi.org/10.1016/j.tra.2015.04.003.
Fagnant, D.J., Kockelman, K.M., 2014. The travel and environmental implications of shared autonomous vehicles, using agent-based model scenarios. Transport. Res. Part C: Emerg. Technol. 40, 1-13. http://dx.doi.org/10.1016/j.trc.2013.12.001.
FICO, 2014. Getting Started with Xpress Release 7.7.

Ford, H.J., 2012. Shared Autonomous Taxis: Implementing an Efficient Alternative to Automobile Dependency. Princeton University.
Google, 2015. Google Self-Driving Car Project Monthly Report.
Hess, D.B., 2009. Access to public transit and its influence on ridership for older adults in two U.S. cities. J. Transp. Land Use 2, 3-27. http://dx.doi.org/ 10.5198/jtlu.v2i1.11.

Hoogendoorn-Lanser, S., van Nes, R., Hoogendoorn, S.P., Bovy, P.H.L., 2006. Home-activity approach to multi-modal travel choice modelling. Transport. Res. Rec.: J. Transport. Res. Board 1985, 180-187.
International Transport Forum, 2015. Urban Mobility System Upgrade: How shared Self-driving Cars could Change City Traffic.
Jorge, D., Correia, G., 2013. Carsharing systems demand estimation and defined operations: a literature review. Eur. J. Transp. Infrastruct. Res. 13, 201-220.
Jorge, D., Correia, G.H.A., Barnhart, C., 2014. Comparing optimal relocation operations with simulated relocation policies in one-way carsharing systems. IEEE Trans. Intell. Transport. Syst. 15, 1667-1675. http://dx.doi.org/10.1109/TITS.2014.2304358.
Juster, R., Schonfeld, P., 2013. Comparative analysis of personal rapid transit as an urban transportation mode. Transport. Res. Rec.: J. Transport. Res. Board 2350, 128-135. http://dx.doi.org/10.3141/2350-15.
Kek, A.G.H., Cheu, R.L., Meng, Q., Fung, C.H., 2009. A decision support system for vehicle relocation operations in carsharing systems. Transport. Res. Part E: Logist. Transport. Rev. 45, 149-158. http://dx.doi.org/10.1016/j.tre.2008.02.008.
Martin, E., Shaheen, S.A., Lidicker, J., 2010. Impact of carsharing on household vehicle holdings. Transport. Res. Rec.: J. Transport. Res. Board 2143, 150-158. http://dx.doi.org/10.3141/2143-19.
Martinez, L.M., Correia, G.H.A., Viegas, J.M., 2015. An agent-based simulation model to assess the impacts of introducing a shared-taxi system: an application to Lisbon (Portugal). J. Adv. Transport. 49, 475-495. http://dx.doi.org/10.1002/atr.
SAE International, 2014. Summary of SAE International's Level of Driving Automation for On-Road Vehicles.
Shaheen, S., Sperling, D., Wagner, C., 1999. A short history of carsharing in the 90's. J. World Transp. Policy Pract. 5, 16-37. http://dx.doi.org/10.1007/ s11116-007-9132-x.
Sprinter SLT_About NS_NS [WWW Document], n.d. [http://www.ns.nl/en/about-ns/trains-of-ns/sprinter-slt.html](http://www.ns.nl/en/about-ns/trains-of-ns/sprinter-slt.html).
Uber [WWW Document], 2016. [https://www.uber.com/](https://www.uber.com/).
van Arem, B., van Driel, C.J.G., Visser, R., 2006. The impact of cooperative adaptive cruise control on traffic-flow characteristics. IEEE Trans. Intell. Transport. Syst., 429-436 http://dx.doi.org/10.1049/el.
WEpods Project DAVI - Dutch Automated Vehicle Initiative [WWW Document], n.d. [http://davi.connekt.nl/wepods-project/](http://davi.connekt.nl/wepods-project/).
Winter, K., Cats, O., Correia, G.H. de A., van Arem, B., 2016. Designing an automated demand-responsive transport system: fleet size and performance analysis for the case of a campus-train station service. In: Transportation Research Board 95th Annual Meeting.


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