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Optimum design of concrete slab using genetic algorithm

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ABSTRACT

The optimum design of concrete slab is solved and investigated using genetic algorithm. In ordinal optimum design of slabs, the constraints are restricted to the stresses in concrete and reinforcing bars. However, in order to obtain durable concrete structures, the constraint for crack width is very important. Adding to this, the variables like spacings of reinforcing bars are usually determined as discrete one. Therefore, the genetic algorithm is useful for this kind of problems.

INTRODUCTION

The optimum design of civil engineering structures usually has many constraints and sometimes two or more objectives. The objective functions or the constraints or both are generally expressed by nonlinear function of design variables. The difficulty lies when the design variables are to be determined as the discrete variables. This is very common if the diameters of reinforcing bar materials are available in finite type and the spacings of reinforcement are specified by discrete round number.

In this case, so called the Integer Programming or Mixed-integer Programming scheme[1),2)] is used to obtain the solution. However, these techniques are generally very hard to apply because of its complexity of algorithms.

In order to overcome this difficulty, the genetic algorithm (GA)has been applied to many optimization problems including civil engineering[4)-13)].

Many studies have been performed and developed by Goldberg[4)-7)] in 1980s, since the GA algorithm is first introduced by Holand in 1970s[3)] till today.
In this paper, the optimum design problem of concrete slab is formulated and solved by the simple genetic algorithm (SGA) scheme[4)].
In ordinal optimal design based upon the working stressed design, the stresses induced in concrete and reinforcing bars are only concerned. However, in order to make durable concrete structures, the crack widths occurring in concrete structures should be restricted to an allowable value. Therefore, in this paper, the constraint for crack widths is included.

SIMPLE GA

The general procedures of evolution process of GA are performed by repeating the operations of (1) initialization, (2) selection, (3) crossover, (4) mutation.
In the SGA[4)], a site is chosen randomly for crossover operations. At this site, the crossover is accomplished by swapping all character of strings. The procedures are simply repeated and the current solution is evolved. After the crossover, the mutation operation is introduced to extend the search domain. The mutation is performed by changing the binary digit 0 of certain locus of some chromosomes to 1 or reversely. The certain locus of chromosomes is selected randomly. After repeating this cycles, the objective function is expected to converges to minimum or maximum value, asymptotically.
The decoded design value of current generation is regarded as the optimum solution when the fitness, usually average fitness, is not improved furthermore.
The SGA algorithm is very simple and it seems to be better to obtain a good solution than obtaining no solution, even if the solution is not exact one.

OPTIMUM DESIGN FORMULATION OF CONCRETE SLAB

(1)Minimum Cover Thickness Criteria

The minimum cover thicknesses are decided based on the (a)qualities of concrete, (b)the diameter of reinforcing bars,(c) the construction errors, and (d)the importance of structures.
According to the concrete structural design code in our country[14)], the minimum cover thicknesses are specified to be:

$$C_{\min} = \alpha C_0 \qquad (1)$$

in which $C_{\min}$=minimum cover thickness of concrete, and α is defined as:

$$\alpha = \begin{cases} 1.2 & (f'_{ck} \le 17.6MPa) \\ 1.0 & (17.6Mpa \le f'_{ck} \le 34.3Mpa) \\ 0.8 & (34.3MPa \le f'_{ck}) \end{cases} \qquad (2)$$

in which f'_{ck} = design compressive strength of concrete, and C_0=standard cover thicknesses which are shown in **Table 1** depending upon the state of natures of circumstances where the structures are placed. This requirement is necessary

Table 1 Minimum Cover Thicknesses for State of Natures

State of Natures	Structural Members: Slab (cm)	Beam (cm)	Column (cm)
Normal	2.5	3.0	3.5
Corrosive	4.0	5.0	6.0
Severely Corrosive	5.0	6.0	7.0

to prevent the reinforcing bars from corrosion. If C_{min} is smaller than the diameter of reinforcing bars, then it is replaced by certain values greater than the diameteres.

(2) Crack Width with Bending

The lateral crack widths occurring in concrete slab are given to be[15]:

$$w = l(\varepsilon_{sm} + \varepsilon'_{cs}) \tag{3}$$

in which, w=the crack width, l =the distances between cracks, ε'_{cs}=the strain estimating the increments of crack widths caused by creep or shrinkage of concrete, and ε_{sm}=the average strain in reinforcing bar expressed as:

$$\varepsilon_{sm} = \varepsilon_s - \Delta\varepsilon_s = \frac{\sigma_s}{E_s} \tag{4}$$

in which ε_s=the strain when reinforcing bar can move freely without any constraint, $\Delta\varepsilon_s$= the average value of reduced strain in reinforcing bar after having the concrete shared the tension stress, σ_s=the stress induced in reinforcing bar, E_s=the Elastic modulus of bars.

Substitutes Eq.4 to Eq.3, the crack widths w become:

$$w = l\left(\frac{\sigma_s}{E_s} - \Delta\varepsilon_s + \varepsilon'_{cs}\right) \tag{5}$$

In our country, $\Delta\varepsilon_s$ is taken to be 0 for simplicity and also for safe side estimation. And ε'_{cs} is estimated as 150×10^{-6}, approximately, in the design code[14].

(3)Allowable Crack Width

According to the experimental data ever obtained shows that the damages of corrosion of reinforcements are affected by the cover thicknesses, significantly[16]. The deeper cover thicknesses of tension side concrete lead to the wider crack widths. In order to make the crack widths to be narrow, the cover thicknesses should be taken to be small. However, it affects the corrosion, severely.

From the view point of the serviceability limit design, the allowable crack widths are specified as **Table 2** in Japan, related to the real cover thicknesses C.

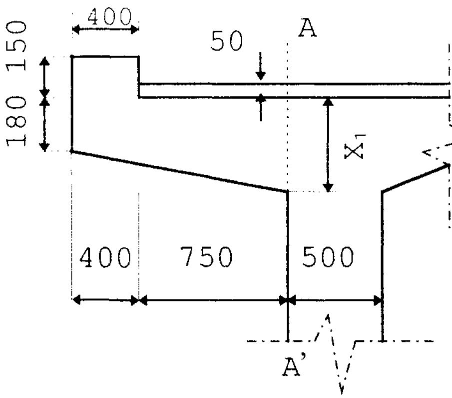
In Table 2, the strings of * mean that the allowable widths are not specified.

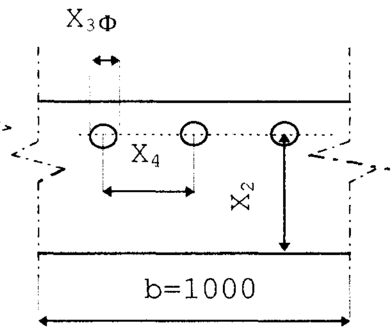
(4)Design Conditions

As the basic design conditions of slab shown in **Fig.1**, the span length and the lane width of bridge are assumed to be 11.6m, and 7.5m with three main girders, respectively.

Table 2 Allowable Crack Widths for Cover Thicknesses

	State of Natures		
Materials	Normal	Corrosive	Severely Corrosive
Deformation or Round Bar	0.005C	0.004C	0.0035C
PC Bar	0.004C	***	*****





(Dimension in mm)

Fig.1 Cross Section of Model

Fig.2 A-A' Section

The design compressive strength and the working stress of concrete are taken to be f'_{ck}=23.5MPa and σ_{ca}=7.8MPa. and the working stress of reinforcing bar is taken as σ_{sa}=137.2MPa, based on the Highway Bridge Design Code[17]. The densities of concrete and asphalt for pavement and reinforcing bar are estimated to be γ_c=24.5kN/m^3, γ_{as}=22.5kN/m^3, γ_s=76.4kN/m^3, respectively.

(5)The Constraints

The optimum design is formulated to the section A-A' in **Fig.1**. The constraints are usually assigned to the stresses induced in concrete and reinforcing bars, in addition to side constraints like the sizes of slabs. The design variables to be optimized are shown in **Fig.2**. In **Fig.1,2**, x_1 is the total depth of the slab, x_2 is the effective depth , $x_{3\Phi}$ is nominal diameter and x_{3D} is the nominal sectional area of reinforcing bars, and x_4 is the spacing between bars, respectively.

The side constraints are given to be:

$$x_1 \geq 0.18 \tag{6}$$

$$d_{\min} \leq x_2 \leq 100x_1 - x_{3\Phi}/2 - C_{\min} \tag{7}$$

in which $d_{\min}$ is expressed as:

$$d_{\min} = C_c\sqrt{\frac{M}{\sigma_{ca}b}} \approx 2.26\sqrt{8.45x_1 + 64.4} \tag{8}$$

in which $k_1=E_s/E_c$=15, E_c=the modulus of elasticity of concrete,

$k_2=\sigma_{sa}/\sigma_{ca}=17.5$, and $k_3=k_1/(k_1+k_2)$, $C_c=\sqrt{2/(k_3(1-k_3/3))}$, b=unit length (=100cm), respectively. The total amount of bending moment M including live, dead and impact load is estimated approximately as $(5.15+0.676x_1)\times 9.8$kN-m per unit length.

According to this moment M, required reinforcement area is shown to be:

$$A_s=\frac{M}{(7/8)x_2\sigma_{ca}} \tag{9}$$

Assuming that the reinforcing bars with x_{3D} sectional areas are placed in every x_4 spacings in unit length b, then the required sectional area A_s' is shown as:

$$A_s'=bx_{3D}/x_4 \tag{10}$$

From Eq.9 and 10, the requirement for sectional area of reinforcement per unit length is, therefore, shown to be:

$$A_s'\geq A_s \tag{11}$$

In succession, the stresses induced in concrete and reinforcing bars are examined. The constraints for these requirements are given as:

$$\frac{(10.3+1.35x_1)\times 10^3}{\kappa_A x_2^2\eta_A}\leq\sigma_{ca} \tag{12}$$

$$\frac{x_4(5.15+0.676x_1)\times 10^3}{x_2^2x_3\eta_A}\leq\sigma_{sa} \tag{13}$$

in which,

$$\kappa_A=\sqrt{2k_1p+(k_1p)^2}-k_1p \tag{14}$$

$$\eta_A=1-\frac{\kappa_A}{3} \tag{15}$$

$$p=A_s'/bx_2 \tag{16}$$

In common designs, without checking the crack widths, the optimum solution minimizing the cost or the volume is solved subjected to the constraints Eq.6 , 7, 11, 12, 13. However, it is recognized that the obtained solution sometimes violates the allowable value of crack widths. Therefore, Eq.5 is added to the constraints. In Eq.5, the maximum value of l is obtained as the function of the real cover thickness C and x_4 and $x_{3\Phi}$. It takes the form[18]:

$$l=4C+0.7(x_4-x_{3\Phi}) \tag{17}$$

Concludingly, the crack widths in **Table 2** are restricted to the allowable width w_a. This is shown as:

$$w=\mu\{4C+0.7(x_4-x_{3\Phi})\}\left(\frac{\sigma_s}{E_s}+\varepsilon_{cs}'\right)\leq w_a \tag{18}$$

in which μ=the constant that expresses the cohesion ability of reinforcement to the concrete, and it is taken as 1.0 for deformed bar and is 1.3 for round or PC bars. The stress σ_s in Eq. 18 is given to be equal to σ_{sa}(=137.2MPa)[15].

(6)Objective Function

As the objective function Z, the cost of A-A'sectional area per unit length is minimized. It takes the form:

$$Z=\mathrm{cost}_c\cdot V_c+\mathrm{cost}_s\cdot V_s+\mathrm{cost}_f\cdot F_{ab} \tag{19}$$

Table 3 The Nominal Diameters and Sectional Areas of reinforcing bars available in Japan.

$x_{3\Phi}$	x_{3D}	$x_{3\Phi}$	x_{3D}	$x_{3\Phi}$	x_{3D}
D6	0.3167	D22	3.871	D38	11.40
D10	0.7133	D25	5.067	D41	13.40
D13	1.267	D29	6.424	D51	20.27
D16	1.986	D32	7.942		
D19	2.865	D35	9.566		

in which, $cost_c$ and $cost_s$ are the material cost of concrete and bars, and $cost_f$ expresses fabrication cost, V_c and V_s are the volume of concrete and reinforcing bar, F_{ab} is fabrication cost function, respectively. The third term in Eq.19 expresses the incremental cost by fabrications. Since the narrow reinforcement spacings x_4 will increase the expenses for fabrication labor cost . The function F_{ab} is, therefore, assumed as the function of x_4 to be:

$$F_{ab} = \sqrt{100 / x_4} \tag{20}$$

It is very hard to estimate these cost, exactly . By examing the real common designs performed in our country, the construction and material costs including the labor cost overhead are estimated as the ratio $cost_s / cost_c$, and it varies from 100 to 200, and also assumed as $cost_f / cost_c$=500, approximately.

Adopting these values, for $cost_s / cost_c$=150, the objective function per unit length is expressed to be:

$$Z = 100x_1 + 1.5 \times 10^4 x_{3\Phi} + 500\sqrt{100 / x_4} \tag{21}$$

NUMERIC EXAMPLE OF CONCRETE SLAB

As the design variables, following are used in the numeric example. The total depth x_1 is varied from 25cm to 40cm, and x_2 is from 20cm to 35cm, in every 0.5cm increment, respectively. The reinforcement materials x_3 listed in **Table 3** are only available in Japan[18]. In **Table 3,** $x_{3\Phi}$ and x_{3D} are defined as the nominal diameters and the nominal sectional areas, respectively. The reinforcement spacing x_4 is varied from 6.0cm to 37cm, in every 1.0cm increment.

The lengths of binary strings corresponding to these variables are 5 bits for x_1 and x_2, 4 bits for x_3, and 5 bits for x_4. The individual chromosomes, therefore, are expressed by totally 19 bits string length. Each bit positions are given 0 or 1 randomly. For example, a string {0 1 0 1 1 0 0 1 1 0 0 1 0 1 0 1 1 1 0} expresses that the first 5 binary digits for x_1 is 11, therefore it corresponding to the value of depth x_1=30.5cm.

In the same manner, the next 5, 4, 5 bits expresses x_2=23.0cm, x_3=D22, and x_4=20.0cm, respectively.

In this paper, SGA is performed when state of nature is assumed to be normal.

The initial populations(number of individuals) are taken to be 500. In the selection processes, the population size is fixed to 70 in each generations.

Table 4 Optimum Solutions By SGA

Solutions (Cost Ratio)	x_1 (cm)	x_2 (cm)	x_3	x_4 (cm)	σ_s (Mpa)	σ_c (Mpa)	w (mm)	w_a (mm)
100	30.0	23.0	D22	20.0	136	5.9	2.94	2.95
150	32.5	26.5	D19	17.0	135	4.9	2.51	2.52
200	35.0	28.5	D19	18.0	133	4.5	2.73	2.77

Table 5 Exact Optimum Solutions

Optimum Solutions	x_1 (cm)	x_2 (cm)	x_3	x_4 (cm)
100	26.5	21.5	D19	14.0
150	32.5	26.5	D19	17.0
200	34.5	28.5	D19	18.0

The probability of mutation is assumed as 1/2000, and the fitness is fixed to be 3.0. This means that the chromosomes, who make the objective values greater than 3.0 times to the smallest objective value in current generations, are selected in evolution processes. Severe fitness may sometimes retard the progress.

The convergences of object functions are shown in **Fig.3(a,b,c)** with generations. In **Fig.3**, the cost ratios ($\cos t_s / \cos t_c$) are estimated to be 100, 150 and 200, respectively. The vertical axis expresses the non dimensional cost $\tilde{Z}$ = Z /$\cos t_c$.

In **Table 4**, the solutions by the SGA are shown with the cost ratios at generation 1000. From **Table 4,** it appears that the total depths and effective depths become large when cost ratio increases, while the diameters of reinforcing bars become relatively thin and the spacings become to be narrow .

It shows that thin bars would be preferable as the reinforcement when the cost of bars is estimated expensive compare with the concrete. While narrow spacings make the crack widths be narrow to be equal to specified allowable values.

However, too thin bars are not necessarily so advantageous. For example, the total cost of reinforcing bar per ton is estimated as about ¥166,000(¥52,000 material cost plus ¥40,000 manufacturing cost plus ¥74,000 fabrication cost, $1 ≒¥188) for bar diameters less than or equal to 13mm.

On the other hand, for bar diameters from 16mm to 25mm, it is estimated to be ¥147,000(¥51,000 plus ¥34,000 plus ¥62,000). For more thick diameters greater than 29mm, it is estimated to be more cheaper like ¥109,000, but usually they are rarely used in our country in real slab designs.

In this meaning, diameter 19mm reinforcing bar seems to be common and familiar one in our country.

In order to ensure the solution obtained here, all combinations of design variables are investigated.

In this case, the number of total combinations 2^{19} are searched to find exact

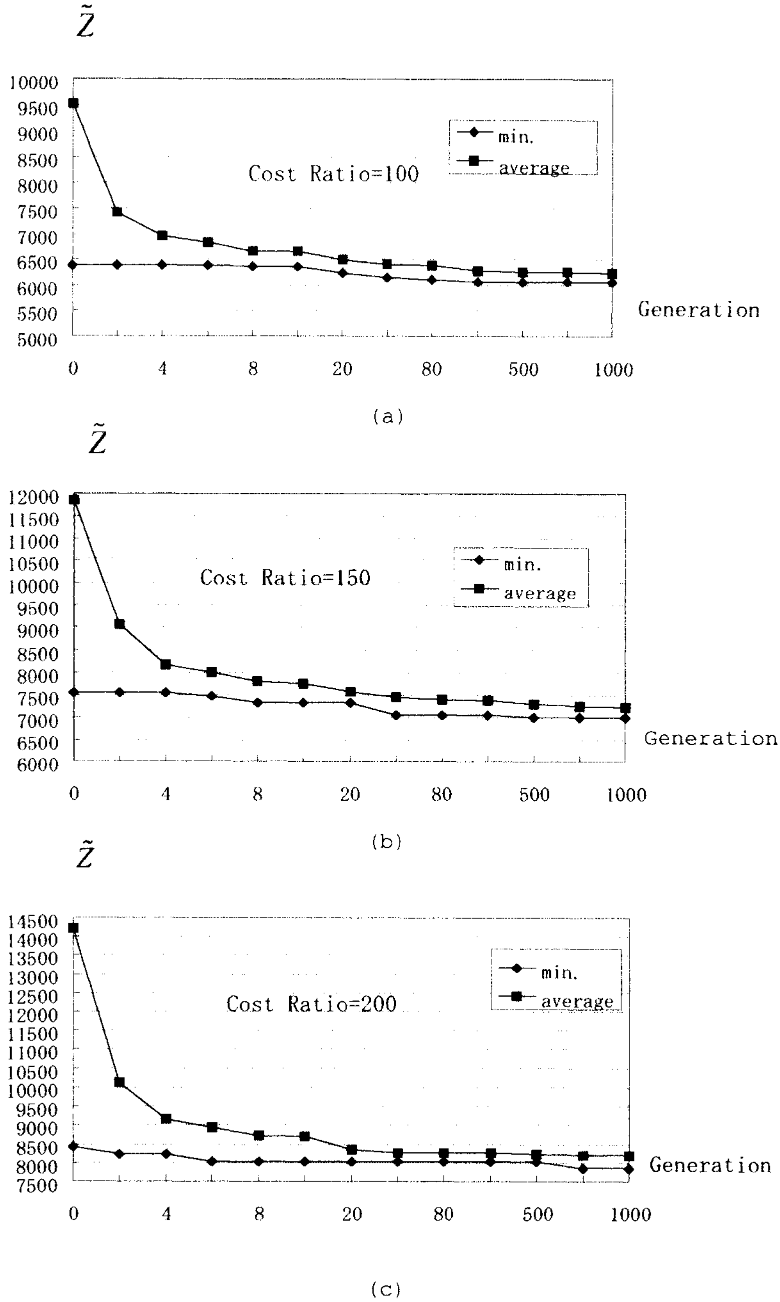


Fig. 3 Covergencies of Objective Functions with Generations

solution. The exact solutions are shown in **Table 5** as the cost ratio being equal.

Excluding one case when the cost ratio is 100, exact and SGA solutions seem to be equal. The CPU time, searching for exact solution, was about 110sec , while it was about 50sec in the SGA schemes. The differences of time can be expected to be significant when the string lengths become much longer.

SUMMARY AND CONCLUSIONS

In the structural design processes, one of the most important thing is to obtain good or optimum solution. In order to obtain it, mathematical models of real structures are solved using linear or nonlinear programming techniques. It was very difficult or sometimes impossible to obtain the exact optimum solution when design variables are expressed as discrete type variables.

Recently, the GA scheme seems to improve these difficulties. In the GA, exact optimum solution can not be obtained, however, good or preferable solution can be obtained.

In this paper the optimum design of concrete slab is performed using the SGA, and the solutions are investigated. The necessity for restricting the crack width is emphasized for optimum cncrete slab designs. It is also very important to obtain the durable concrete structures from the view point of the serviceability limit states designs. One of the most advantage of the GA scheme is that the computer algorithm is very simple, and it may more speed up the CPU time than ever done, if it is applied to the optimum designs which have many design variables .

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