Published in International Journal of Heat Exchangers, 2004, Vol 5, Iss 2, P 379-401

OPTIMUM DESIGN OF CROSSFLOW PLATE-FIN HEAT EXCHANGERS THROUGH GENETIC ALGORITHM

Manish Mishra

Mechanical Engineering Department, Indian Institute of Technology, Kharagpur, India –721302

mishra_md@yahoo.com

Phone: 91-3222-81446, Fax: 91-3222-55303

Prasanta Kumar Das*

Mechanical Engineering Department, Indian Institute of Technology, Kharagpur, India –721302

Phone: 91-3222-82916, Fax: 91-3222-55303

pkd@mech.iitkgp.ernet.in

Sunil Sarangi

Cryogenic Engineering Department, Indian Institute of Technology, Kharagpur, India –721302

Phone: 91-3222-83592, Fax: 91-3222-82258

ssarangi@hijli.iitkgp.ernet.in

OPTIMUM DESIGN OF CROSSFLOW PLATE-FIN HEAT EXCHANGERS THROUGH GENETIC ALGORITHM

ABSTRACT

A genetic algorithm based optimisation technique has been developed for crossflow plate-fin heat exchangers. The optimisation program aims at minimising the total annual cost for a specified heat duty under given space and flow restrictions. A multilayer plate-fin heat exchanger has been considered and the optimum values of the design variables consisting of the core and the fin geometrical parameters are obtained for the minimum total cost. For the validation, optimisation of a reduced model of a two-layer heat exchanger has been compared with the solution obtained by the conventional optimisation technique. Comparison of the solutions has been made between the cases when no restriction is there on the upper limit of Reynolds number and with the laminar flow restriction. The effect of fixing the heat exchanger dimensions on the optimum solution has also been studied.

Key Words: crossflow, genetic algorithm, heat exchanger, optimisation, plate-fin, total cost.

1. INTODUCTION

Crossflow plate-fin heat exchangers are widely used in aerospace, automobile and chemical process plants. They offer various advantages like low weight, high efficiency and ability to handle many streams. Often the design of such heat exchangers has to meet the stringent requirements of low initial and operating cost associated with a superior thermal performance. Thus there is a strong motivation for optimising the design of plate-fin heat exchangers to give desired performance with minimum cost, volume or weight or with a combination of these properties.

Optimisation of heat exchangers is a very active field of design research in thermal engineering. A multitude of techniques ranging from classical techniques like Lagrange multiplier, geometric programming, dynamic programming and different non-linear programming methods to various non-classical methods as discrete maximum principle, random search as well as the method of case study have been adopted for such purpose. A comprehensive review of different methods adopted for optimum design of heat exchangers till early 90's has been given by Rao (1991). In this review the merit of all the available techniques have been critically judged and their limitations for the optimisation of heat exchangers have been highlighted.

Rao et al. (1996) obtained the optimal design of shell and tube heat exchanger by a two-stage technique, where optimisation of the geometric design was first done decoupling the geometrical and the heat transfer aspects. Next, the geometric optimisation problem was linked to a thermal rating module to obtain thermal design for any given heat duty. Based on Lagrangian multiplier technique Venkatrathnam (1991) optimised the design of matrix heat exchanger. Abramazon and Ostersetzer (1993) put forward an iterative solution method termed as ‘Omega method’ for optimisation. They have applied this technique for optimum design of plate-fin heat exchanger as well as cold plates for electronic component cooling and compared the results with those obtained from random search algorithms. Dzyubenko et al. (1993) discussed a method based on pressure drop and heat transfer experimental data to select optimum heat transfer surface for space application. Hesselgreaves (1993) suggested an analytical method for calculating optimum size and weight of a plate-fin heat exchanger for a given heat duty using dimensionless design parameters. Muralikrishna and Shenoy (2001) pointed out the difficulties for determining the optimum design of shell and tube heat exchangers. They have suggested a methodology based on graphical technique where a region of feasible design is identified on a pressure drop diagram. On the diagram, curves corresponding to constant heat exchanger area or total cost can be plotted and the optimum solution can be picked up. Gonzales et al. (2001) determined the optimum values of ten operating and geometric variables for determining the minimum cost of an air-cooled heat exchanger using successive quadratic programming. Though the technique produces non-integer values of the integer variables, the authors suggest its use as a good starting point.

Out of the different techniques of optimisation calculus based techniques are well known for their mathematical rigor and elegance. Though they are time tested and suitable for multiple variables, the complexity of any algorithm based on this method increases with the increasing number of variables. In the absence of a proper initial guess the solution may converge onto some local extrema or may even diverge finally. To guard the convergence of the calculus based algorithm onto a local extrema, one may need to try different starting points for the iteration setting initial values of the variables. Moreover, the calculus-based methods are not very convenient for handling discrete variables.

Different versions of branch and bound techniques are suitable for non-linear optimisation problems containing discrete-continuous variables (Gupta and Ravindran, 1983; Salajegheh and Vanderplaats, 1993). These methods in general treat all the variables as continuous and subsequently select feasible discrete solution to identify the optimum. While doing so the original optimisation problem is expanded to a large number of sub-optimisation problem. On the other hand in the methods based on penalty function approach (Gisvold and Moe, 1972), the diversity of local optima may not guarantee convergence to a feasible discrete optimal point. In most of the cases the penalty parameters need further adjustment to continue search iterations.

Different search techniques could be good alternatives for optimisation problem containing discrete or discrete-continuous variables. However, the conventional technique (Stoecker, 1999) becomes very cumbersome and laborious when the extremum is sought for a multivariable problem having a number of constraints. In recent times, some probabilistic search algorithms namely genetic algorithm (GA) and simulated annealing (SA) are being applied to the optimisation of various engineering systems in general and to thermo-processes and fluid applications in particular. These techniques can overcome the above-mentioned difficulties to a large extent.

Genetic algorithm has been applied successfully for the analysis and optimum design of diverse thermal systems and components namely convectively cooled electronic components (Queipo et al., 1994) and cooling channels (Wolfersdorf et al., 1997), flow boiling

(Castrogiovanni and Sforza, 1997), fin profiles (Fabbri, 1997; Younes and Potiron, 2001), finned surface and finned annular ducts (Fabbri, 1998), compact high performance coolers (Schmit et al., 1996) and shell and tube heat exchangers (Tayal et al. 1999). In an effort of predicting heat exchanger performance Pacheco-Vega et al. (2001) recently demonstrated the superiority of GA over the conventional least square technique. The authors commented that as GA works on a global search, it out performs the conventional local gradient-based methods. In case of gradient-based methods there is always a risk to converge at local extrema unless one tries multi-initial values. On the contrary, GA starts with a population of possible solutions, which minimizes the risk of premature convergence. However, it needs to be mentioned that for convergence GA needs a large number of iterations. It posses a great demand on computational time and renders the application of GA unsuitable for simpler problems.

Using the basic framework of GA, a technique for multiconstraint minimisation has been developed in the present work. The technique has been applied to obtain the design of crossflow plate-fin heat exchangers for the lowest total annual cost (TAC). To check the accuracy of the developed method initially a simplified design having only two geometrical variables has been considered. The optimum solutions of this problem as obtained by the present technique agreed closely with an accurate solution obtained by gradient search method. Different cases of optimum design have been studied next. In all these exercises minimisation of TAC has been targeted for a specified heat duty constraint under different combinations of space and flow restrictions. Finally a comparison between the optimum designs attained under different design constraints has been made.

The effect of GA parameters on the optimal solution has been seen. Further, the effect of different constraints on the solution has also been discussed. The methodology used is not new, but the system like plate-fin heat exchanger where it has been applied and the way it has been used is new to the researchers working in this area.

2. OUTLINE OF THE SOLUTION METHODOLOGY

Genetic algorithm is a search procedure based on the principles of genetics and natural selection. An elaborate description of this technique is available in a number of references, for example, Holland (1975), Mitchell (1998) and Goldberg (2000).

2.1 Basic Algorithm

In the simplest form GA can be used to maximise the objective function f(X), which in turn depends on a number of variables. Following is the statement of the problem.

Maximise f(X),

where,

$X = x_i, \quad i=1,2,\ldots,k$

and

$$x_{i,min} \le x_i \le x_{i,max} \tag{1}$$

The vocabulary used in GA belongs originally to genetics. A feasible solution is represented by a binary coded string known as *chromosome*. The variables x_i's are first coded in some string structure. In a simple GA (Goldberg, 2000) binary coded strings consisting of 0's and 1's are mostly used. The length of the string depends on the desired solution accuracy. The variable x_i is coded in a substring s_i of length l_i. The decoded value of a binary substring s_i is calculated as $\sum_{i=0}^{l-1} 2^i s_i$, where $s_i \in (0,1)$ and the string is represented as ($s_{l-1}\, s_{l-2} \ldots s_2\, s_1\, s_0$). For example, a four bit substring (0111) has a decoded value equal to $[(1)2^0 + (1)2^1 + (1)2^2 + (0)2^3]$ or 7. If there are two variables then it needs total 8 bits (0111 0010). A set of feasible solutions is known as *population*. The value of the objective function for a particular member decides its merit (competitiveness) in comparison with its counterparts. In GA language this is termed as *fitness function*. After creating an initial population a simple GA works with three operators: *reproduction*, *crossover* and *mutation*. Reproduction, which constitutes a selection procedure whereby individual strings are selected for mating based on their fitness values relative to the fitness of the other members. Individuals with higher fitness values have a higher probability of being selected for mating and for subsequent genetic production of *offsprings*. This operator, which weakly mimics the Darwinian principal of the survival of the fittest, is an artificial version of natural selection. The reproduction operator used here creates a roulette wheel where each string in the population is assigned a slot in the wheel sized in proportion to its fitness. Since the population size is usually kept fixed, the sum of the probability of each string being selected must be one. Therefore the probability for selecting the i^{th} string is

$$p_i = \frac{f_i}{\sum_{j=1}^{N_p} f_i}, \quad \text{where } N_p \text{ is the population size.} \tag{2}$$

The evolution is achieved by means of crossover and mutation. After reproduction, the crossover operator alters the composition of the offspring by exchanging part of strings from the parents and hence creates new strings. Though different types of crossover techniques are common in practice, in the present analysis single point crossover is used (Figure 1). Crossover operation takes place in two steps. In the first step, selection of two random streams (chromosomes) takes place from the mating pool generated by the reproduction operator. Next a crossover site is selected at random along the string length, and alleles (gene values in a chromosomes) are swapped between the two strings between the crossover site and the end of the strings.

FIGURE 1 HERE

Mutation is a secondary operator, which increases the variability of the population. For a GA using binary alphabet to represent a chromosome, mutation provides variation to the population by changing a bit of the string from 0 to 1 or vice versa with a small mutation probability p_m (Figure 2). The need for mutation is to create point in the vicinity of the current point to prevent the solution from falling into a local optimum, thereby achieving a local search around the current solution, which sometimes is not possible by reproduction and crossover.

FIGURE 2 HERE

A generation or an iteration from the computational point of view is completed when the offspring replaces the parents from the preceding generation. A simple flow chart for a GA based optimisation procedure is given in figure 3.

FIGURE 3 HERE

GA's do not guarantee convergence to global optimum solution and so require suitable stopping criteria. The GA can be terminated when there is no improvement in the objective function (fitness) for a defined number of consecutive generations within a prescribed tolerance range, or when it covers a prespecified maximum number of generations.

2.2 Modification for Constrained Minimisation

If there are number of constraint conditions and the objective function needs to be minimised, the problem is modified as follows:

Minimise $f(X)$, $X=[x_1,\ldots\ldots x_k]$ (3)

Where,

$g_j(X) \leq 0$, $j=1,\ldots\ldots,m$ (4)

and

$x_{i,\,min} \leq x_i \leq x_{i,max}$, $i=1,\ldots\ldots,k.$ (5)

The problem can be recast into unconstrained maximisation problem and the solution may be obtained as outlined earlier. The first step is to convert the constrained optimisation problem into an unconstrained one by adding a penalty function term.

Minimise $f(X) + \sum_{i=1}^{m} \Phi(g_i(X))$, (6)

subject to

$x_{i,\,min} \leq x_i \leq x_{i,max}$, $i=1,\ldots\ldots,k.$ (7)

Where Φ is a penalty function defined as,

$\Phi(g(X)) = R.\langle g(X)\rangle^2.$ (8)

R is the penalty parameter having an arbitrary large value.

The second step is to convert the minimisation problem to a maximisation one. This is done redefining the objective function such that the optimum point remains unchanged. The conversion used in the present work is as follows

Maximise $F(X)$, (9)

where, $F(X) = 1 / \{ f(X) + \sum_{i=1}^{m} \Phi(g_i(X)) \}.$ (10)

The above algorithm can be used for minimising the total annual cost of crossflow plate-fin heat exchangers.

3. GEOMETRICAL, THERMOHYDRAULIC AND COST PARAMETERS OF PLATE-FIN HEAT EXCHANGERS

FIGURE 4 HERE

Figure 4 depicts a schematic view of a crossflow plate-fin heat exchanger with offset-strip fins. The initial and running costs of such equipments depend on the geometrical specifications and thermohydraulic performance parameters. These details are estimated based on the following assumptions.

1. The steady state condition is assumed to be prevailing.
2. Offset-strip fins having the same specifications are used for both the fluids.
3. Heat transfer coefficients and the area distribution are assumed to be uniform and constant.
4. Property variation of the fluids with temperature is neglected.
5. When the design consists more than two layers of finned passages, number of fin layers for fluid ‘b’ (which has a mean temperature closer to atmospheric temperature) is assumed to be one more than that of fluid ‘a’ (N_b=N_a+1).
6. In general, the fin effectiveness for compact plate-fin heat exchangers are quite high (more than 90%). However, in the present exercise, calculations have been done taking

100% fin efficiency. If required one may readily introduce fin efficiency in the present formulation.

3.1 Geometrical Parameters

For the geometrical details shown in figure 4, one may get the free flow areas as

$$Aff_a = (H_a - t_a).(1 - n_a t_a).L_b.N_a, \tag{11}$$

$$Aff_b = (H_b - t_b).(1 - n_b t_b).L_a.N_b. \tag{12}$$

Similarly heat transfer areas for the two sides can be obtained as given below.

$$A_a = L_a.L_b.N_a[1+ 2.n_a.(H_a-t_a)] \tag{13}$$

$$A_b = L_a.L_b.N_b[1+ 2.n_b.(H_b-t_b)] \tag{14}$$

Total heat transfer area, $A_{HT} = A_a + A_b = La.L_b.[\, N_a\{1+ 2.n_a(H_a-t_a)\}+N_b\{1+2.n_b(H_b-t_b)\}]$ (15)

Hydraulic diameter (Joshi and Webb, 1987) of the finned passages is given by

$$Dh = \frac{2(s-t)(H-t)}{\{s+(H-t)\}+\dfrac{(H-t)t}{l_f}}, \tag{16}$$

where $s = (1/n - t)$ (17)

3.2 Thermo-hydraulic Parameters

The rate of heat transfer may be calculated as follows

$$Q = m_a.Cp_a.(T_{a,in} - T_{a,out}) = m_b.Cp_b.(T_{b,in} - T_{b,out}) \tag{18}$$

$$Q = UA(F.LMTD) \tag{19}$$

The LMTD (log mean temperature difference) can be given by

$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\log_e\left(\dfrac{\Delta T_1}{\Delta T_2}\right)}, \tag{20}$$

where

$\Delta T_1 = T_{a,in} - T_{b,out}$ and $\Delta T_2 = T_{a,out} - T_{b,in}$

Neglecting the thermal resistance due to the metal wall, overall heat transfer between the two fluids can be expressed as,

$$\frac{1}{UA} = \frac{1}{(hA)_a} + \frac{1}{(hA)_b} \tag{21}$$

The heat transfer coefficient can be obtained in terms of Colburn 'j' factor as

$$j = St.Pr^{2/3} = \frac{h}{G.Cp} . Pr^{2/3} \tag{22}$$

Substituting h, A and UA in eq. (21) the equality constraint for the heat duty may be expressed as

$$\frac{1}{j_a m_a Cp_a \, Pr\, a_a^{-2/3}} . \frac{(H_a - t_a)(1 - n_a t_a)}{(1 + 2.n_a h_a)L_a} + \frac{1}{j_b m_b Cp_b \, Pr\, a_b^{-2/3}} . \frac{(H_b - t_b)(1 - n_b t_b)}{(1 + 2.n_b h_b)L_b}$$

$$= \frac{F(LMTD)}{Q} = Zq \tag{23}$$

Pressure drop for the two fluid streams can be calculated readily as

$$\Delta P_a = \frac{4.f_a.L_a.G_a{}^2}{2.\rho_a.D_{h,a}} = \frac{2.f_a.m_a^2}{\rho_a{}^2} \frac{L_a}{D_{h,a}.L_b^2.N_a^2.(H_a - t_a)^2(1 - n_a t_a)} , \tag{24}$$

$$\Delta P_b = \frac{4.f_b.L_b.G_b{}^2}{2.\rho_b.D_{h,b}} = \frac{2.f_b.m_b^2}{\rho_b{}^2} \frac{L_b}{D_{h,b}.L_a^2.N_b^2.(H_b - t_b)^2(1 - n_b t_b)} . \tag{25}$$

j and f factors may be evaluated from available correlations (Joshi and Webb, 1987).

For laminar flow ($Re \le 1500$)

$$j = 0.53(Re)^{-0.5}(l_f / D_h)^{-0.15}\{s/(H - t)\}^{-0.14} . \tag{26}$$

$$f = 8.12(Re)^{-0.74}(l_f / D_h)^{-0.41}\{s/(H - t)\}^{-0.02} . \tag{27}$$

For turbulent flow ($Re > 1500$)

$$j = 0.21(Re)^{-0.4}(l_f / D_h)^{-0.24}(t / D_h)^{0.02} . \tag{28}$$

$$f = 1.12(Re)^{-0.36}(l_f / D_h)^{-0.65}(t / D_h)^{0.17}. \quad (29)$$

Where, $$Re = \frac{GD_h}{\mu} = \frac{m.D_h}{Aff.\mu}. \quad (30)$$

3.3 Cost Estimation

The method of defining the total annual cost may vary depending upon the application. However, it should comprise of the initial cost of the equipments namely the heat exchanger and the prime movers for the fluid streams and the running cost. Cost of both the heat exchanger and the prime movers will have a fixed and a variable component as $Z=kA+k_0$ (Zubair et al., 1987). The variable component (kA) for the heat exchanger may be assumed to depend on the total heat transfer area as the type of the heat transfer surface has been specified. In case of prime movers the variable component of the cost will depend on the product of capacity and pressure drop. The running cost on the other hand will depend on the power consumption. Such basis for cost estimation has also been taken by Muralikrishna and Shenoy (2000).

Total annual cost, TAC = Initial cost of (heat exchanger core + pump a + pump b) +

Operating cost of (pump a + pump b)

$$TAC = Af.\left[\{Ca + Cb.A_{HT}{}^{c}\} + \{Ce + Cf.\left(\frac{m_a}{\rho_a}.\Delta P_a\right)^d\} + \{Ce + Cf.\left(\frac{m_a}{\rho_a}.\Delta P_a\right)^d\}\right]$$

$$+ \frac{C_{pow}.(Time/year)}{\eta_{pump}}\left[\frac{m_a}{\rho_a}\Delta P_a + \frac{m_b}{\rho_b}\Delta P_b\right] \quad (31)$$

Where, ΔP_a and ΔP_b are in kPa.

As a specific example following values are selected for the cost factors (Muralikrishna and Shenoy, 2000) and other operating parameters.

c=0.8, d=0.68, Af=0.322, Ca=30000, Cb=750, Ce=2000, Cf=5, C_{pow} = 0.00005 \$/W-hr,

η_{pump} = 0.7, total operation time/year = 8000 hours, specified heat duty, Q = 160 kW

Operating conditions are based on a design problem by Shah (1980).

m_a = 0.8962 kg/s, ρ_a = 0.7468 kg/m^3, Cp_a = 1.022 kJ/kg-C, Pr_a = 0.687, $T_{a,in}$ = 240 °C

m_b = 0.8296 kg/s, ρ_b = 1.3827 kg/m^3, Cp_b = 1.013 kJ/kg-C, Pr_b = 0.694, $T_{b,in}$ = 4 °C

4. OPTIMUM DESIGN THROUGH GA – DIFFERENT CASES

Optimum design of plate-fin heat exchangers have been achieved based on the methodology described in the preceding sections. Three different cases, as elaborated below, have been considered.

4.1 Case I - Heat Exchanger With Two Fin Layers

FIGURE 5 HERE

At the outset an effort has been made to compare the results obtained through GA with those computed using the gradient search technique available with MATLAB. For this the original problem has been simplified substantially. Only two fin layers have been considered with fixed fin geometry and specified coefficients of heat transfer as well as frictional pressure drop. The length (L_a) and breadth (L_b) are the only variables, which are to be optimised. The statement of the problem is as follows.

$N_a = N_b = 1$

Total annual cost can be expressed as

$$TAC = Za + Zb.L_a^{c}.L_b^{c} + Zc.\, L_a^{d}.L_b^{-2d} + Zd.\, L_a^{-2d}.L_b^{d} + Ze.\, L_a.L_b^{-2} + Zf.\, L_a^{-2}.L_b, \qquad (32)$$

where,

$Za = Af.(Ca + 2.Ce)$, $\qquad Zb = Af.Cb$,

$$Zc = Af.Cf.(\frac{m_a}{\rho_a})^d(\frac{Kp_a}{1000})^d, \qquad Zd = Af.Cf.(\frac{m_b}{\rho_b})^d(\frac{Kp_b}{1000})^d,$$

$$Ze = \frac{C_{pow}.Time/year}{\eta_{pump}} \frac{m_a}{\rho_a} \frac{Kp_a}{1000}, \qquad Zf = \frac{C_{pow}.Time/year}{\eta_{pump}} \frac{m_b}{\rho_b} \frac{Kp_b}{1000},$$

$$Kp_a = \frac{2.f_a}{\rho_a.Dh_a} \frac{m_a^2}{K_{Affa}^2}, \qquad Kp_b = \frac{2.f_b}{\rho_b.Dh_b} \frac{m_b^2}{K_{Affb}^2},$$

$K_{Affa} = Aff_a / L_a$, and $K_{Affb} = Aff_b / L_b$.

The optimisation problem then becomes minimisation of the objective function

$$f(X) = Za + Zb.L_a^c.L_b^c + Zc.\ L_a^d.L_b^{-2d} + Zd.\ L_a^{-2d}.L_b^d + Ze.\ L_a.L_b^{-2} + Zf.\ L_a^{-2}.L_b, \tag{33}$$

subjected to constraints:

$$g1(X) \Rightarrow 0.13 \le L_a \le 2; \tag{34}$$

$$g2(X) \Rightarrow 0.12 \le L_b \le 2. \tag{35}$$

When required heat duty is specified, an additional equality constraint comes as,

$$g3(X) \Rightarrow \xi(X) - Zq = 0. \tag{36}$$

Where $\xi(X)$ is the LHS of the equation (23).

Based on the above formulation optimum solution is sought through GA as well as through gradient search technique using the following parametric values.

H=6.35 mm, t=0.152 mm, l_f =3.18 mm, n=615 fins/m (15.62 fins per inch).

$j_a=j_b=0.015$, $f_a=f_b=0.062$

Penalty parameter for GA has been selected as, $R=10^6$.

Optimum dimensions of the heat exchanger and corresponding total cost are obtained using the gradient search technique and GA both without and with the heat duty constraint. In figure 6(a) and 6(b) the optimum solutions obtained from gradient search technique and GA are depicted for cases without and with heat duty constraints. The GA solutions of the above problem have been obtained by changing the GA parameters (like population size 'N_p', crossover probability 'p_c' and mutation probability 'p_m').

FIGURE 6 HERE

As GA is a technique based on stochastic methods the resulting solutions will not be unique one as shown in figure 6. With the variation of GA parameters results are not exactly identical but are very close to one another. To bring out this feature clearly the GA result along with that obtained from MATLAB are once again plotted in figure 7 on a space bound by the physical limits of L_a and L_b used in the problem. Additionally constant cost contours are also plotted on the figures. The close agreement between the solutions obtained from gradient search technique and that from GA is obvious in the figures. All the GA solutions satisfy the constraint conditions while there is a slight variation in the corresponding cost function. Though GA does not produce a unique solution it gives number of near optimal solutions and ultimately offers more flexibility to the designer. Finally an average value of all the GA solutions has been tabulated in table 1. It compares very well with the solution obtained through gradient search technique. However, it needs to be mentioned that time taken for the solution through GA is much more compared to that through gradient search technique.

FIGURE 7 HERE

TABLE 1 HERE

4.2 Case II - Multilayer Heat Exchanger With Inequality Constraints For Heat Duty and Flow Rates

After gaining confidence through a simplified design in the previous example, GA has been applied for the optimum design of the plate-fin heat exchanger having multiple layers. The statement of the optimisation problem is as follows.

$$\text{Minimise } f(X)=TAC, \tag{37}$$

Subjected to the constraints:

$g1(X) \Rightarrow 0.1 \leq L_a \leq 1;$

$g2(X) \Rightarrow 0.1 \leq L_b \leq 1;$

$g3(X) \Rightarrow 0.002 \leq H \leq 0.01;$

$g4(X) \Rightarrow 100 \leq n \leq 1000;$

$g5(X) \Rightarrow 0.0001 \leq t \leq 0.0002;$

$g6(X) \Rightarrow 0.001 \leq l_f \leq 0.010;$

$$g7(X) \Rightarrow 1 \leq N_a \leq 10. \tag{38}$$

The minimum heat duty generated is given by

$$g8(X) \Rightarrow \xi(X) - Zq \leq 0, \tag{39}$$

where $\xi(X)$ is the LHS of the equation (23).

In most of the applications of plate-fin heat exchangers, the flow remains either in laminar or in the lower turbulent range. Therefore, additional constraints have been introduced, to limit the Reynolds number below 1500 for both the fluids.

$g9(X) \Rightarrow Re_a \leq 1500$, and

$$g10(X) \Rightarrow Re_b \leq 1500. \tag{40}$$

Though the designer has some independence in selecting the GA parameters, it has been shown that selection of proper GA parameters (Grefenstette, 1986; Wolfersdorf et al., 1997) renders a quick convergence of the algorithm. The proper GA parameters are problem specific.

Therefore initially an exercise has been made following the methodology of the Wolfersdorf et al. (1997) to select the optimum GA parameters for the present problem. Figure 8 shows the variation of maximum fitness function and the total cost with the population size, crossover and mutation probabilities and penalty parameter. Except for penalty parameter R1, in all the cases the minimum cost corresponds to the maximum value of the fitness function. Taking minimum cost as the selection criteria following parametric values are selected for GA, population size 90, crossover probability 0.8, mutation probability 0.01, and penalty parameter R1=4000, R2=500 and R3=1000.

FIGURE 8 HERE

The optimum solution based on the optimum GA parameters are listed in table 2.

TABLE 2 HERE

4.3 Case III – Effect of Higher Flow Rates and Equality Constraint on Heat Duty

In the present exercise the constraints on Reynolds number have been relaxed while the heat duty constraint is made more restrictive as follows.

Q=160 kW.

The GA parameters for this modified problem has been selected following the methodology described before. The values are as follows: population size = 30, crossover probability = 0.8, mutation probability = 0.01 and penalty parameter = 1000. The optimum solution based on these GA parameters are listed in table 3.

TABLE 3 HERE

5. COMMENTS ON THE RESULTS

On the basis of the optimum solution for different cases given in the earlier section, a few important observations can be summarised as follows.

5.1 Effect of Constraint Conditions on Optimum Design

A comparison of table 2 and 3 reveals a number of interesting points. Restriction on Reynolds number gives a heat exchanger with a large length, width and higher number of fin layers and at the same time provides a larger rate of heat transfer. On the other hand if the restriction on Reynolds number is relaxed, the heat exchanger can be designed for a required lower thermal performance and at the same time its cost can be reduced. This fact can be explained better with help of a constant cost and constant heat duty contours as depicted in figure 9. In this figure iso-cost and iso-heat duty curves are constructed as functions of L_a and L_b while taking all the geometrical parameters of the heat exchanger from table 2. Due to the restriction put in the Reynolds number on the two sides, the solution space is limited to area OABC and the solution is obtained at point O, which gives the limiting values of the Reynolds numbers. Though the solution gives a much higher heat duty it also corresponds to a much higher annual cost for the equipment.

FIGURE 9 HERE

In a similar fashion the cost and the heat duty contours for the second case is shown in figure 10. Corresponding to equality constraint Q=160 kW one gets a much lower cost of the heat exchanger.

FIGURE 10 HERE

5.2 Imposition of Additional Constraints

Due to different practical reasons sometimes there may be space restrictions. As a result one of the dimensions of the heat exchanger may have to be fixed a priori. This acts as an additional constraint. The effect of fixing the heat exchanger lengths on the minimum total cost for case II is shown in figure 11. In general the total cost increases with the increase of L_a as depicted in figure 11(a). Minimum cost is obtained at L_a= 0.509 m, which corresponds to the optimum design when no restriction was put on the heat exchanger length. The same figure gives an additional information of the pressure drops occurring on the two sides of the heat exchanger due to change of L_a. In figure 11(b) the variation of total minimum cost as a function of L_b along with the corresponding pressure drops for the two fluids is depicted. The curves exhibit similar nature to those shown in the previous figure. The total cost increases with L_b, the minimum being at a point where no restriction on length is imposed.

FIGURE 11 HERE

Next, the effect of additional constraints on optimum design for case III has been studied. The total minimum cost is determined varying L_a, L_b and number of layers, N_a individually. The results are shown in figure 12 (a), (b) and (c) respectively. In all these three figures the minimum value of the respective variable corresponds to the optimum value given in table 3. The variations of pressure drop for both the fluids with the variation of L_a, L_b and N_a have also been depicted in the respective curves. It may be noted the pressure variations shown both in figure 11 and 12 do not follow any particular trend. This is because the pressure drop values correspond to the optimum design condition. The optimum design condition gives a combination of parametric values, which may change substantially if a particular parameter is varied. Therefore the observed behaviour of ΔP curves is not unexpected.

FIGURE 12 HERE

Figure 13 gives a comparison of the values of TAC and the corresponding heat duty produced for case II and III for a variation of one of the lengths of the heat exchanger. It shows clearly that the optimum solution is very sensitive to the variation of heat exchanger lengths for case II, where the upper limit of Reynolds number is restricted.

FIGURE 13 HERE

6. CONCLUSION

A methodology based on GA has been developed for the optimisation of multilayer plate-fin heat exchangers with large number of design variables of both discrete and continuous nature. Initially a two-layer heat exchanger with given fin specifications has been considered. The scheme determines optimum values of length and width of the heat exchanger, which minimise the total annual cost. Solution obtained for different combinations of GA parameters gave different set of optimum values for the length and the width. However, the optimum values obtained from all the GA exercises are close enough. The same problem has also been solved graphically as well as through gradient search technique. The solutions generated by GA agree very closely to the graphical solution as well as that obtained from gradient search technique.

Optimisation of multilayer plate-fin heat exchangers has been considered next. Two different cases have been taken up. In the first case the lower limit of the heat duty was specified and the heat exchanger was designed for laminar flow conditions. In the second case the constraint on fluid Reynolds number was relaxed while the design was made to meet the specified heat duty exactly. By imposing the laminar flow constraints, the effective domain in the feasible design space reduces and the size of the heat exchanger increases, which leads to increase in total cost and also the corresponding heat duty produced.

Further, the effect of fixing any of the main geometrical parameters of the heat exchanger on its optimum design has been investigated. In general this additional constraint increases total annual cost of the heat exchanger. However, the effect of this additional constraint is more significant when the design is made for laminar flow conditions.

ACKNOWLEDGEMENT

We would like to acknowledge the reviewers for their valid suggestions which have helped us to modify the communication in the present form.

NOMENCLATURE

A, A_{HT} –heat transfer area, m^2 Af – cost factor

Aff – free flow area, m^2

C – heat capacity rate (mCp), J/K

Ca, Cb, Ce, Cf – cost factors

Cp – specific heat of fluid

C_{pow} - cost of power, \$/W-hr

D_h – hydraulic diameter, m

f – Fanning friction factor

f_{max} – maximum fitness parameter

f(X) - objective function

F – crossflow correction factor

g(X) - constraint

G – mass flux velocity (= m/Aff), kg/ m^2-s

h – heat transfer coefficient

H - height of the fin, m

j - Colburn factor

l_f – lance length of the fin, m

L - heat exchanger length, m

l_i - length of substring

LMTD - log mean temperature difference

m – mass flow rate of fluid, kg/s

n - fin frequency, fins per meter

N_a, N_b – number of layers of finned passages

N_G – number of generations

N_p – population size.

NTU – number of transfer units

p – probability

Pr - Prandtl number

ΔP – pressure drop, N/ m^2

Q – rate of heat transfer, W

R, R1, R2, R3 – penalty parameters

Re – Reynolds number

s – fin spacing (1/n-t), m

s_i - binary sub-string

St – Stanton number [=h/(GCp)]

t – fin thickness, m

T-Temperature, K

TAC – total annual cost, \$

Time/year – annual operational time, hours

U– overall heat transfer coefficient, W/ m^2K

x_i - variable

X – (x_1, x_2,……x_k)

Greek symbols

ρ – density, kg/ m^3

μ - viscosity, N/ m^2-s

$\phi(.)$ – penalty function

η_{pump} – efficiency of pump

Subscripts

a, b – fluid ‘a’ and ‘b’

c - crossover

i - variable number

in – inlet

m – mutation

max -maximum

min – minimum

out – exit

REFERENCES

Abramzon, B. Ostersetzer, S. 1993. Optimal Thermal and Hydraulic Design of Compact Heat Exchangers and Cold Plates for Cooling of Electronic Components, in *Aerospace Heat Exchanger Technology - Proc First International Conference on Aerospace Heat Exchanger Technology*, eds. R.K.Shah and A. Hashemi, pp. 349-368. Palo Alto, CA, USA. Elsevier.

Castrogiovanni, A., Sforza, P.M. 1997. A Genetic Algorithm Model for High Heat Flux Flow Boiling. *Experimental Thermal and Fluid Science*. 15(3):193-201.

Dzyubenko, B.V., Dreitser, G.A., Yakimenko, R.I. 1993. Method of Optimum Heat Transfer Surface Choosing for Space Heat Exchangers, in *Aerospace Heat Exchanger Technology - Proc the First International Conference on Aerospace Heat Exchanger Technology*, eds. R.K.Shah and A. Hashemi, pp. 369-390. Palo Alto, CA, USA. Elsevier.

Fabbri, G. 1997. A Genetic Algorithm for Fin Profile Optimisation. *International Journal of Heat and Mass Transfer.* 40(9):2165-2172.

Fabbri, G. 1998. Heat Transfer Optimisation in finned Annular Ducts Under Laminar Flow Conditions. *Heat Transfer Engineering.* 19(4):42-54.

Gisvold, K.M., Moe, J. 1972. A Method for Nonlinear Mixer Integer Programming and its application to Design Problems. *Journal of Engineering for Industry*. 94(2):353-364.

Goldberg, D.E. 2000. *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison-Wesley Longman, Inc.

Gonzalez, M.T., Petracci N.C., Urbican M. 2001. Air-Cooled Heat Exchanger Design Using Successive Qudratic Programming (SQP). *Heat Transfer Engineering.* 22:11-16.

Grefenstette, J.J. 1986. Optimization of Control Parameters for Genetic Algorithms. *IEEE Trans. Of Systems, Man and Cybernatics.* 16(1):122-128.

Gupta, O.K., Ravindran, A. 1983. Nonlinear Integer Programming and Discrete Optimisation. *Journal of Mechanisms, Transmission and Automation in Design*. 105:160-164.

Hesselgreaves, J.E. 1993. Optimum Size and Weight of Plate-Fin Heat Exchangers, in *Aerospace Heat Exchanger Technology - Proc First International Conference on Aerospace Heat Exchanger Technology*, eds. R.K.Shah and A. Hashemi, pp. 391-399. Palo Alto, CA, USA. Elsevier.

Holland, J. 1975. *Adaptation in Natural and Artificial System*. Ann Arbor: University of Michigan Press.

Joshi, H.M., Webb, R.L. 1987. Heat Transfer and Friction in the Offset Strip-fin Heat Exchanger. *International Journal of Heat & Mass Transfer.* 30(1):69-84.

Mitchell, M. 1998. *An Introduction to Genetic Algorithm*. Prentice Hall of India Pvt. Ltd.

Muralikrishna, K., Shenoy, U.V. 2000. Heat Exchanger Design Targets for Minimum Area and Cost. *Transactions of IchemE.* 78(A):161-167.

Pacheco-Vega, A., Sen, M., Yang, K.T., McClain, R.L. 2001, Correlations of Fin-Tube Heat exchanger Performance data Using Genetic algorithms, Simulated annealing and interval

Methods, *Proc. of ASME International Mechanical Engineering congress and Exposition.* 1-9.

Queipo, N., Devarakonda, R., Humphery, J.A.C. 1994. Genetic Algorithms For Thermosciences Research: Application to the Optimized Cooling of Electronic Components. *International. Journal of Heat and Mass Transfer.* 37(6):893-908.

Rao, K. R. 1991. Optimal Synthesis of Shell and Tube Heat Exchangers. *Ph.D. Thesis,* Indian Institute of Science, Bangalore.

Rao, K. R. Shrinivasa, U. and Srinivasan, J. 1996. Constrained optimisation of heat exchangers. *International Journal of Chemical Engineering.* 103:112-118.

Salajegheh, E., Vanderplaats, G.N. 1993. Efficient Optimum Design of Structures with Discrete Design Variables. *International Journal for Space Structures*. 8(3):199-208.

Schmit, T.S., Dhingra, A.K., Landis, F., Kojasoy, G. 1996. A Genetic Algorithm Optimization Technique for Compact high Intensity Cooler Design. *Journal of Enhanced Heat Transfer*. 3(4): 281-290.

Shah, R.K. 1980. Compact Heat Exchanger Design Procedure, in *Heat exchangers, Thermal Hydraulic Fundamentals and Design*, eds. S. Kakac, A.E. Bergles and F. Mayinger, 495-536. Washington: Hemisphere Publishing Corporation.

Stoecker, W.F. 1999. *Design of Thermal Systems*. McGraw-Hill Book Company.

Tayal, M.C., Fu, Y., Diwekar, U.M. 1999. Optimum Design of Heat Exchangers: A Genetic Algorithm Framework. *Industrial Engineering Chemical Research.* 38:456-467.

Venkatrathnam, G. 1991. Matrix Heat Exchangers. *Ph.D. Thesis,* Indian Institute of Technology, Kharagpur.

von Spakovsky, M.R., Evans, R.B. 1989. The Design and Performance Optimization of Thermal System Components. *Journal of Energy Resources Technology*. 111:231-238.

Wolfersdorf, J.V., Achermann, E., Weigand, B. 1997. Shape Optimization of Cooling Channels Using Genetic Algorithms. *Transactions of ASME Journal of Heat Transfer.* 119:380-388.

Younes, M., Potiron, A. A Genetic Algorithm for the Shape Optimisation of Parts Subjected to Thermal Loading. *Numerical Heat Transfer, Part A*. 39:449-470.

Zubair, S.M., Kadaba, P.V., Evans, R.B. 1987. Second-Law-Based Thermoeconomic Optimisation of Two-Phase Heat Exchangers. *Transactions of ASME Journal of Heat Transfer*. 109:287-294.

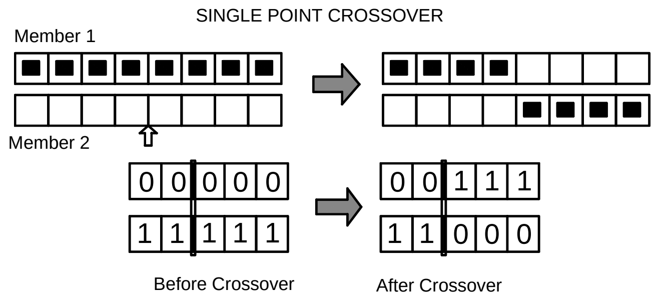


FIGURE 1. Schematic representation of crossover technique

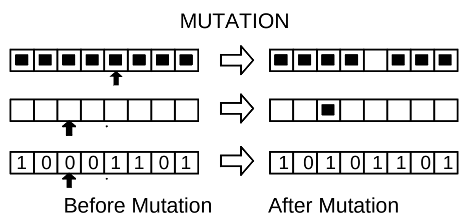


FIGURE 2. Schematic representation of mutation technique

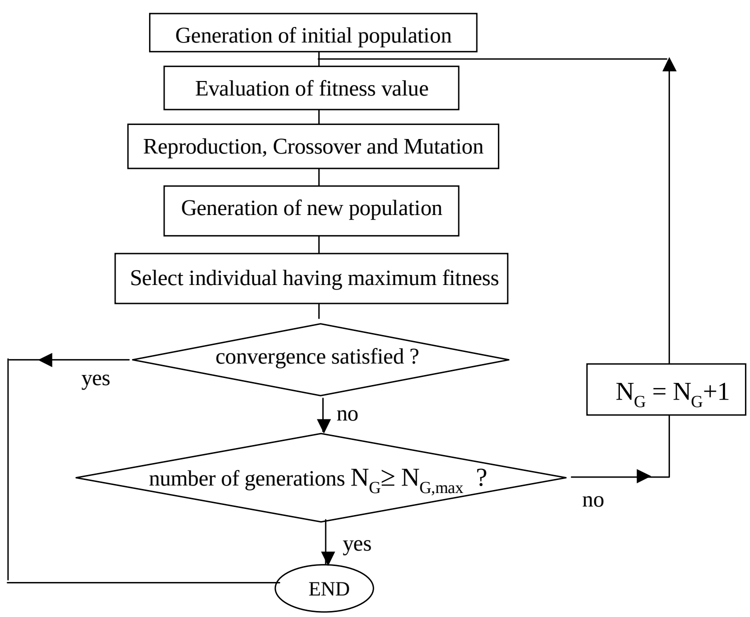


FIGURE 3. Flowchart for a Genetic Algorithm computation

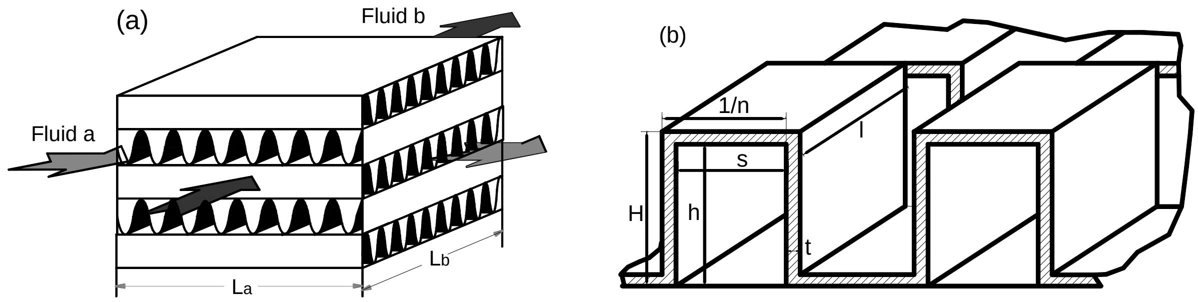


FIGURE 4. (a) crossflow plate-fin heat exchanger (b) offset-strip fin.

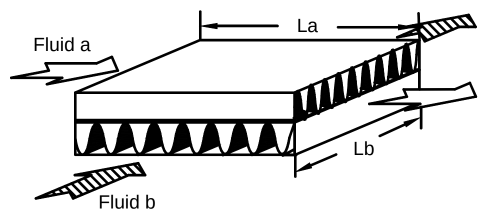


FIGURE 5. Crossflow Plate-Fin Heat Exchanger with two layers.

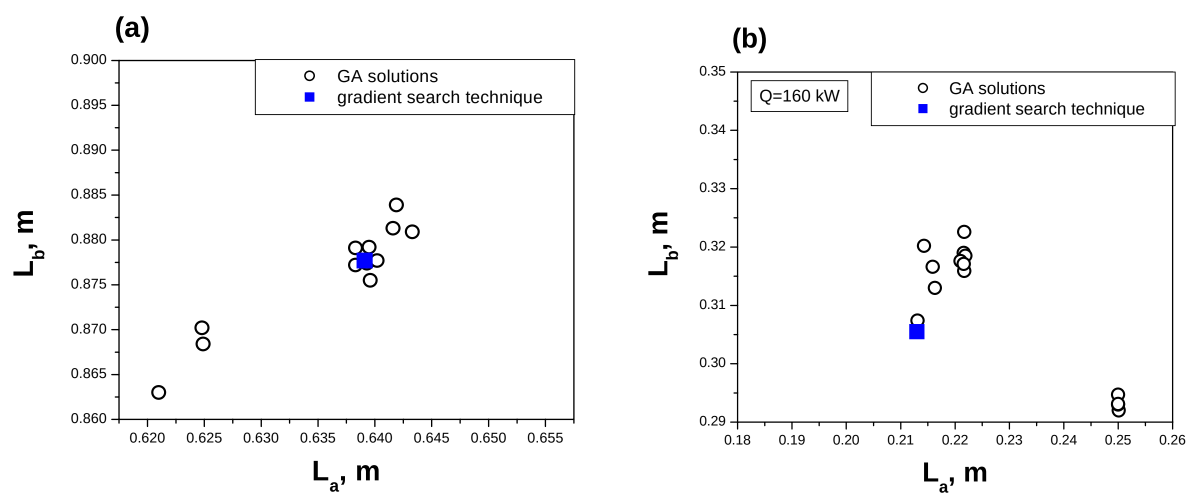


FIGURE 6. Different GA runs (a) without heat duty (b) with heat duty constraint.

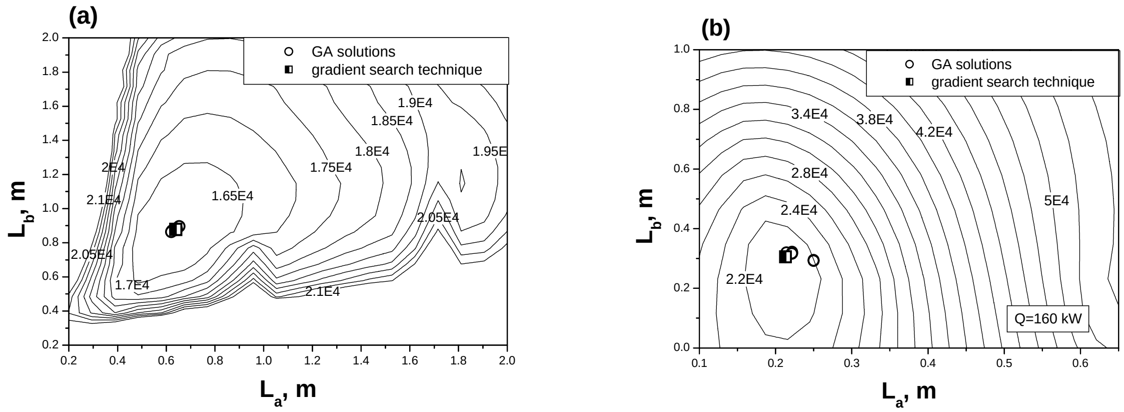
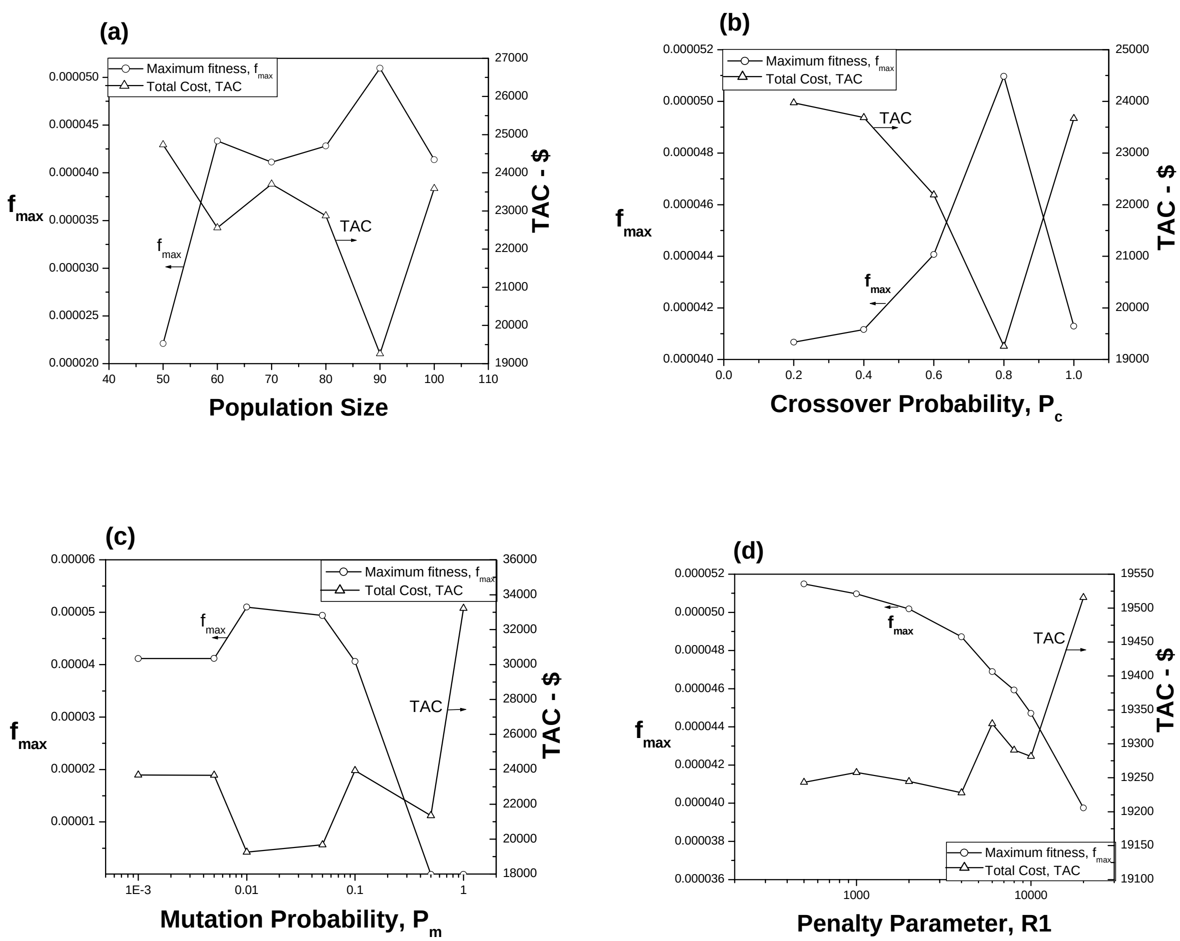


FIGURE 7. Total cost contour (a) without heat duty (b) with heat duty constraint along with conventional and GA solutions.



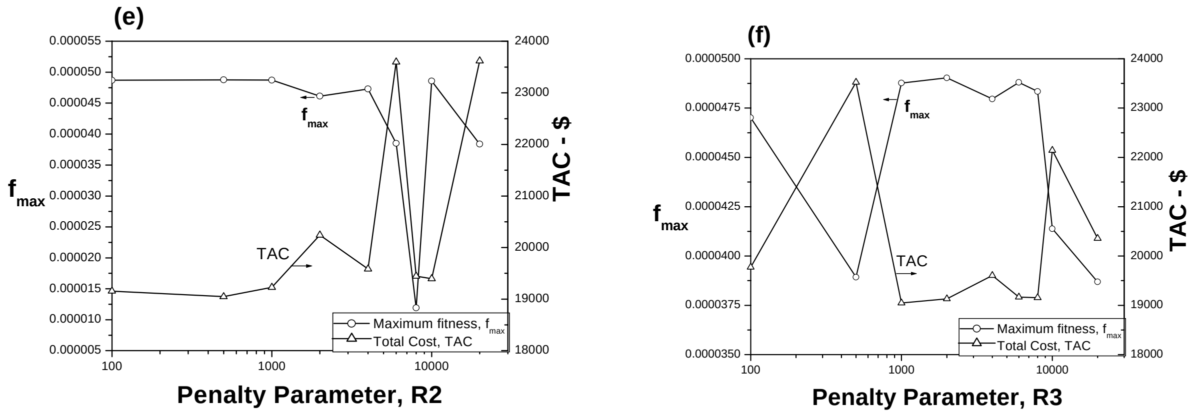


FIGURE 8. Effect of different GA parameters, (a) population size (b) crossover probability (c) mutation probability, and penalty parameters (d) R1, (e) R2, and (f) R3 on maximum fitness and total annual cost.

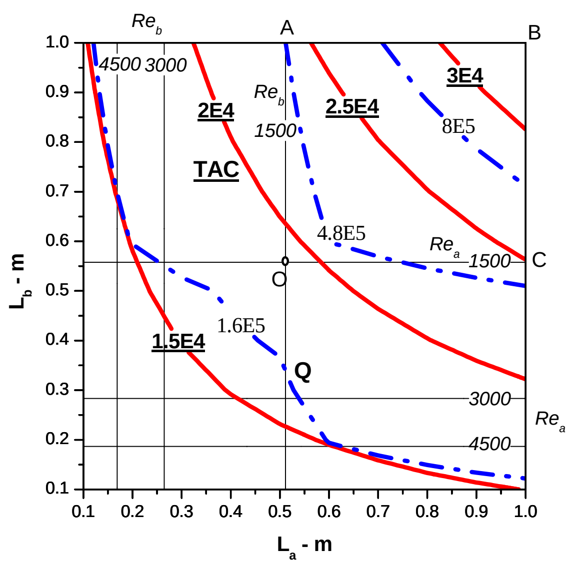


FIGURE 9. Total annual cost and heat duty contours in the design space with flow restrictions.

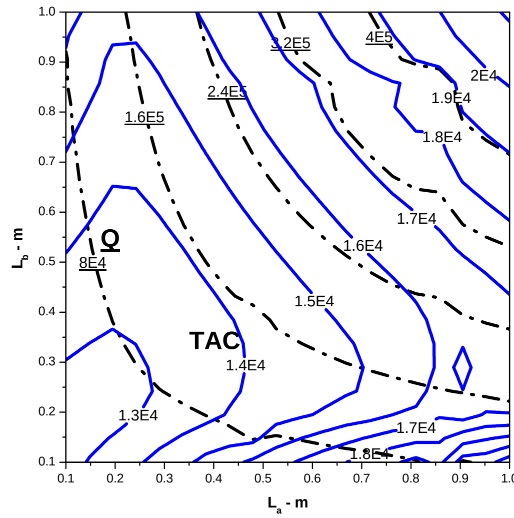


FIGURE 10. Total annual cost and heat duty contours without any flow restriction.

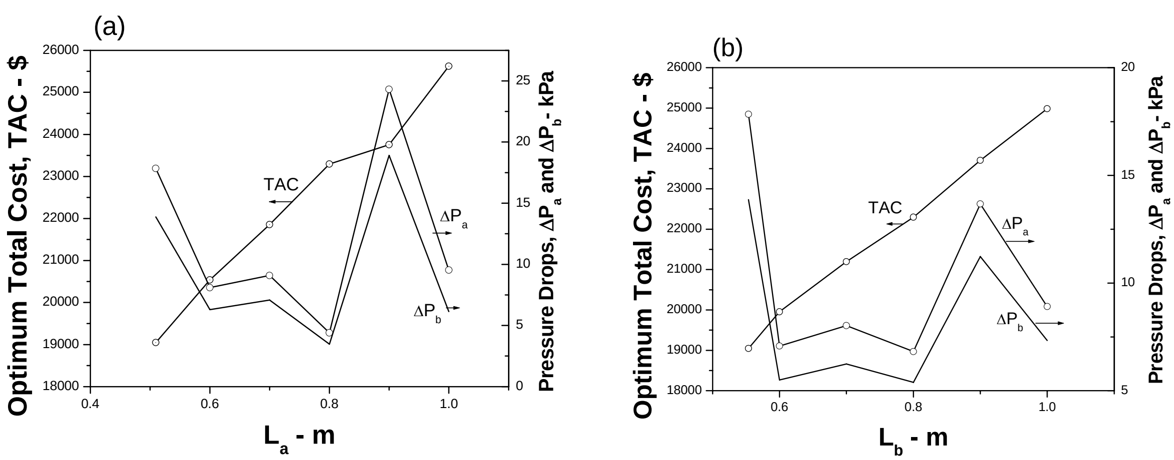


FIGURE 11. Effect of variation of length on total cost and pressure drops. (a) Variation of L_a, and (b) variation of L_b.

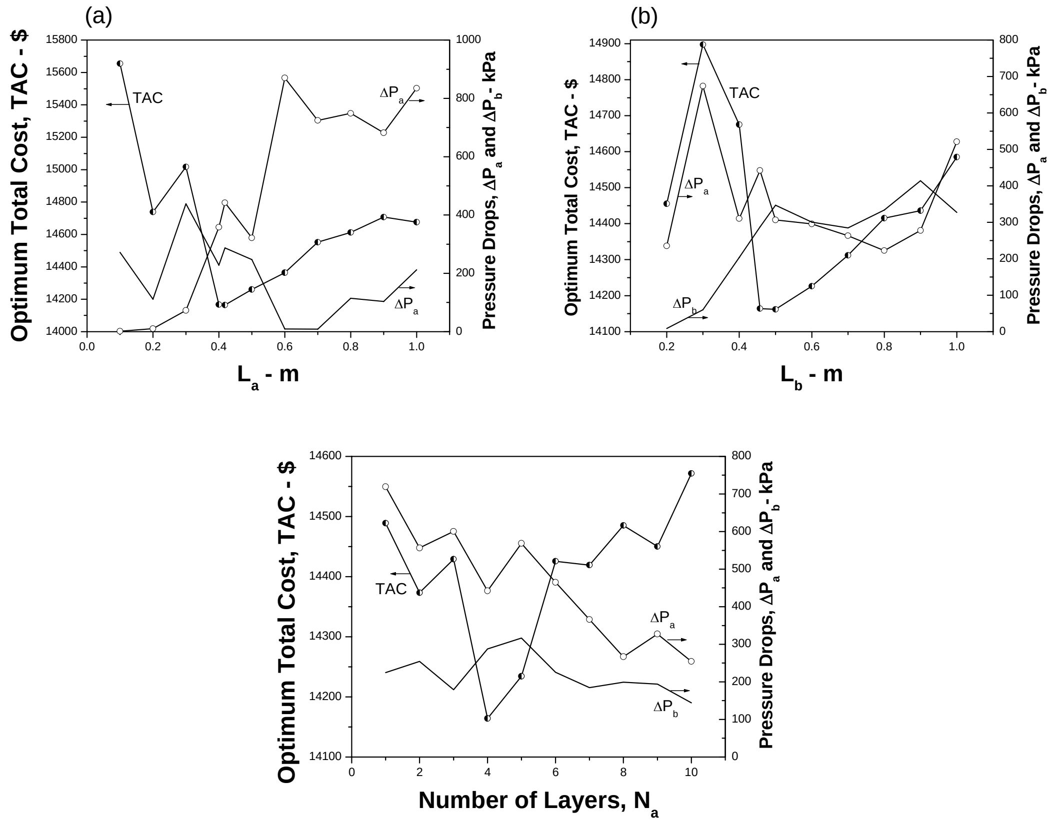


FIGURE 12. Effect of variation of L_a (a), L_b (b), and N_a (c) on optimum total cost and corresponding pressure drops on the two sides.

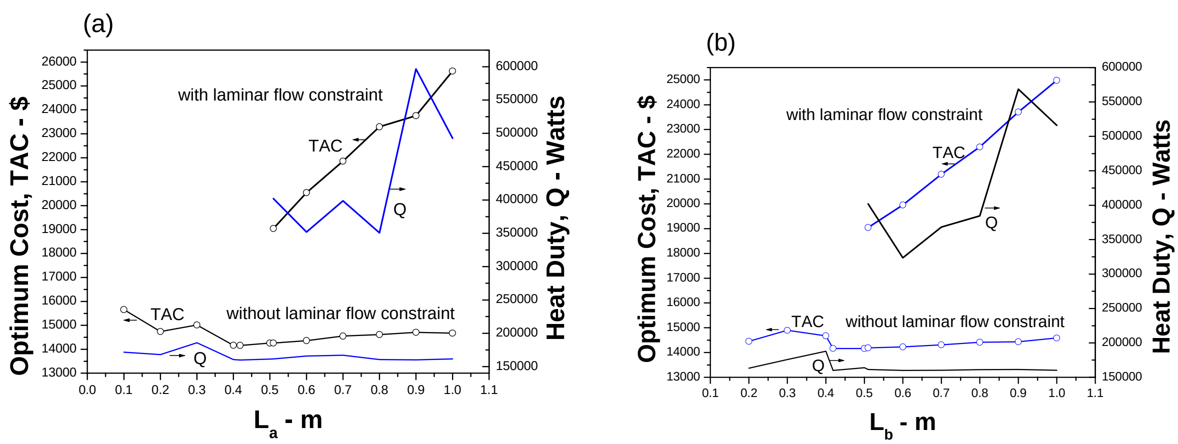


FIGURE 13. Variation of optimum total cost and heat duty with length (a) L_a, and (b) L_b.

TABLE 1. Solution for two layer heat exchanger.

		L_a, m	L_b, m	Total cost (TAC), $
Without heat duty constraint	gradient search technique	0.639	0.877	16180
	GA	0.637	0.877	16180
With heat duty constraint	gradient search technique	0.213	0.306	19963
	GA	0.227	0.310	19967

TABLE 2. Solution for multilayer heat exchanger with flow constraint.

$\mathbf{L_a}$, m	$\mathbf{L_b}$, m	**H**, mm	**n**, fins/m	**t**, mm	$\mathbf{l_f}$, mm	$\mathbf{N_a}$	**TAC**, $	**Q**, kW
0.509	0.554	8.0	891.2	0.168	3.242	9	19046.2	402.05

TABLE 3. Solution for multilayer heat exchanger without flow constraint.

$\mathbf{L_a}$, m	$\mathbf{L_b}$, m	**H**, mm	**n**, fins/m	**t**, mm	$\mathbf{l_f}$, mm	$\mathbf{N_a}$	**TAC**, $	**Q**, kW
0.418	0.457	5.43	992.7	0.182	1.321	4	14164.0	159.97