Optimum Design of Laminated Composite Plates for Maximum Thermal Buckling Loads

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ABSTRACT: Buckling temperatures of graphite/epoxy laminated composite plates are maximized for a given total thickness considering fiber-directions and relative thicknesses of layers as design variables. Thermal buckling analysis is carried out using the finite element method with 4 node shear deformable plate element, while genetic algorithm (GA) is employed to optimize as many as ten variables for the five layered plates. In addition to traditional three-operator approach, i.e., reproduction, crossover and mutation, other variants of GAs such as the elitist model, two-point crossover models are also discussed. The study includes composite plates of three different aspect ratios, two support conditions and three different numbers of layers. The presented results reveal that the buckling loads can be increased significantly with appropriately orienting the fiber directions and varying the thickness of different layers. The authors also recommend the judicious selection of specific thicknesses and fiber orientations needed for the practical implementation highlighting the importance of their theoretical values so that they may also be made available in near future for fabrication.

INTRODUCTION

THE PROPERTIES OF composites such as high strength, high stiffness-to-weight ratio, damping, corrosion resistance and ease with which it can be tailored to any shape have made it especially suitable for aircraft and spacecraft structures. These composite structural components are subjected to severe thermal loads and are found to buckle at high temperature without the application of mechanical loads. Hence, in aircraft industry it has become very important to optimize the composite structures so that they can withstand higher thermal loads.

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Many investigators [1–4] in the past studied the thermal buckling behavior of laminated plates. Chai et al. [5] optimized the ply-angles of anti-symmetric laminated composite plates subjected to in-plane compression using complex optimization technique. Shin et al. [6] optimized the relative ply-thicknesses of symmetric laminated plates for maximum buckling load. Optimality equations were solved by homotopy method, which permits tracing the optima as a function of total thickness. Riche and Haftka [7] optimized the stacking sequence of composite plate in buckling employing genetic algorithm (GA) and reported the advantages of using GA in such optimization problems. However, little attention has been given to the optimum design of layered plates for thermal buckling loads. Tauchert [8] showed that the buckling temperature can be maximized by properly orienting the fiber directions in different layers. They used Powel's method of conjugate directions for optimizing thermal buckling temperature. Thermal buckling temperature was optimized for simply supported antisymmetric laminates by Adali and Duffy [9]. In another related development, Walker et al. [10] studied the optimal design of symmetrically laminated composite plates for maximum thermal buckling temperature considering nonuniform temperature distributions with simply supported and clamped boundary conditions. They used golden section search method to find the optimum ply-angles.

Genetic algorithm (GA) is being used for structural optimization recently. The GAs work with a set of initial population instead of a single point. Each member of the population is the potential solution to the problem. Three essential steps in GA are: (a) reproduction, (b) crossover and (c) mutation. The possible solutions to the problem are represented by bit-string manner. An initial population is generated by random selection of the individual bits in the binary strings of given length. The string represents the design variables i.e., fiber orientations and thickness of various layers. The procedure is well documented [11,12] and hence its repetition is avoided here.

It may be noted that most of the researchers have considered only one or two independent design parameters (ply-angles θ_1 and θ_2), layup sequence being fixed beforehand. But the thermal buckling load can further be optimized by providing different ply-angles and ply-thicknesses for different layers. To the authors' knowledge, no work is reported in the literature on thermal buckling load maximization by considering both ply-angles and ply-thicknesses as design variables. The present work is an endeavor to consider as many as $2n-1$ independent design variables for an n-layered composite plate. The thermal buckling analysis is carried out using the finite element method with a four node rectangular shear deformable plate bending element. Buckling temperature is obtained for the eigenvalue analysis. Uniform temperature distribution is considered in the analysis. The genetic algorithm is employed for the reason mentioned next to maximize the buckling temperature. The frequency with which the genetic material is exchanged and the initial population size will have significant effect on the outcome. There is an optimum value for each problem. In the present case, by varying the crossover and the mutation probabilities and starting from different population sizes the optimum values are obtained. Also different variants of the genetic algorithm such as elitist and two-point crossover methods are used. The whole process is repeated with three different random seeds. The results are reported in the form of tables. The effects of aspect ratio, support condition and number of layers on the maximum buckling temperature are investigated.

WHY GA IS EMPLOYED

Of all the available optimization techniques, genetic algorithm (GA) has been chosen in the present case to maximize the buckling temperature for its obvious merits. In the present case, the variation of buckling load with design variables (ply-angles and ply-thicknesses) is not known beforehand. Hence, GA has been chosen which does not need the first derivative of the cost function like the gradient-based methods.

For multimodal and non-convex design problems, the superiority of the GA is already established ([Deb [12], Dhingra and Lee [13]). It is known that the gradient-based method may be employed for smooth, unimodal and convex functions. They yield locally optimum solutions for non-convex design problems or multimodal functions. On the other hand, gradient-based methods are mostly adopted for optimization problems with continuous design variables and are found to be inadequate for discrete/discrete continuous design problems. Riche and Haftka [7] reported the advantages of using GA in stacking sequence optimization of composite plate in buckling. In the present case, the design variables (ply-angles and thicknesses) are discretised between the lower and upper limits adopting binary coding. Though in the present case the design variables are continuous, the gradient-based methods could be used with a limitation that it may not converge to the true optimum. Thus, though the objective function is continuous the gradient-based methods will be inadequate for the reasons mentioned above and hence, the GA is employed for this investigation. Moreover several near-optimum solutions of GA may be considered for the practical implementation as alternatives, which is an added advantage of this method.

FINITE ELEMENT FORMULATION

The first order shear deformation theory, used in the analysis, assumes a linear variation of in-plane displacement fields, u and v through the depth of the plate. The transverse displacement $w(x,y)$ is assumed to be constant throughout the thickness of the plate. The displacement field of a rectangular shear deformable plate can be expressed as

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) + z\{w_{,x} + \gamma_x(x,y)\} \\
v(x,y,z) &= v_0(x,y) + z\{w_{,y} + \gamma_y(x,y)\} \qquad (1) \\
w(x,y,z) &= w_0(x,y)
\end{aligned}
$$

From the theory of large displacement, the strain-displacement relations can be written as

$$
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{,x} \\ v_{,y} \\ v_{,x} + u_{,y} \end{Bmatrix} + \begin{Bmatrix} w_{,x}^2/2 \\ w_{,y}^2/2 \\ w_{,x}\, w_{,y} \end{Bmatrix} - z \begin{Bmatrix} w_{,xx} + \gamma_{x,x} \\ w_{,yy} + \gamma_{y,y} \\ 2.w_{,xy} + \gamma_{y,x} + \gamma_{x,y} \end{Bmatrix} \qquad (2a)
$$

$$
= \{\varepsilon^0\} + z\{\kappa\}
$$

and

$$
\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} -\gamma_x \\ -\gamma_y \end{Bmatrix} \qquad (2b)
$$

where $()_{,x}$ and $()_{,y}$ represent partial differentiation with respect to x and y. The relationship between stress resultants and the strain terms for the laminated plate may be written as

$$
\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^T \end{Bmatrix} \qquad (3)
$$

and the shear resultants may be written as

$$
\begin{Bmatrix} R_x \\ R_y \end{Bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{54} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} dz \qquad (4)
$$

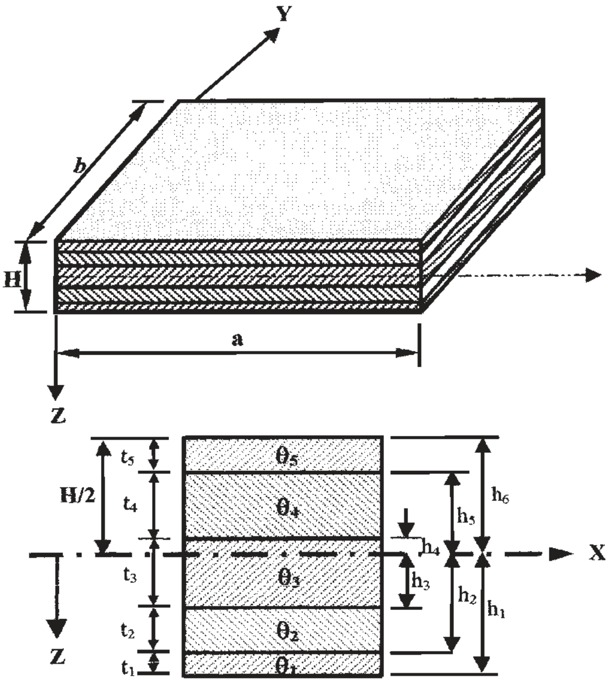
where stretching, stretching-bending and bending stiffnesses are defined as

$$
(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} [Q_{ij}]^k (1, z, z^2) dz \qquad (5)
$$

where $[Q_{ij}]^k$ represents transformed plane stress reduced stiffness matrix of the kth lamina, which is a function of ply-angles (θ_1, θ_2, θ_3 . . . θ_n) and $(h_k - h_{k-1})$ is the thickness of the lamina (Figure 1).

The thermal load vector is given by the expression

$$
\{N^T, M^T\} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} [Q_{ij}] \cdot \{\alpha\} \cdot T(z) \cdot (1, z) \cdot dz \qquad (6)
$$



Authors' problems: a/b = *1.0, 1.5 and 2.0*
Number of layers 3, 4 and 5
Boundary conditions: simply supported and clamped

Figure 1. *Geometry of laminated composite plate.*

A four noded rectangular plate bending element is developed for this purpose. It has 10 degrees of freedom per node, namely u_0, v_0, w, w_x, w_y, w_{xx}, w_{xy}, w_{yy}, γ_x and γ_y. The displacement fields are given by the following expressions.

$$u_0 = [1, x, y, xy]\{c_i\}, \qquad i = 1,4$$

$$v_0 = [1, x, y, xy]\{c_i\}, \qquad i = 5,8$$

$$w = [1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^4, x^3y, x^2y^2, xy^3, y^4, x^5, x^4y, x^3y^2, x^2y^3,$$

$$xy^4, y^5, x^5y, x^3y^3, xy^5]\{c_i\},$$

$$i = 9,32$$

$$\gamma_x = [1, x, y, xy]\{c_i\}, \qquad i = 33,36$$

$$\gamma_y = [1, x, y, xy]\{c_i\}, \qquad i = 37,40$$

where c_i are constants and can be expressed in terms of nodal displacements. The 4×4 Gaussian quadrature is adopted for computing the bending stiffness matrices, thermal load vector and geometric stiffness matrix. A 2×2 Gaussian quadrature is used for computing the shear stiffness matrix.

Following the procedure given in Reference [16] (equating the first variation of total potential energy to zero) the governing equation of the problem may be written as

$$\{[K_s] + \lambda[K_g]\}\{\delta\} = \{0\} \tag{7}$$

where $[K_s]$ and $[K_g]$ are the linear and geometric stiffness matrices respectively. The lowest eigenvalue (λ) gives the buckling temperature T_c.

FORMULATION OF THE OPTIMIZATION PROBLEM

The present problem is to maximize the buckling temperature (T_c) for the optimum combination of ply-angles $\theta_1, \theta_2, \theta_3 \ldots \theta_n$ and ply-thicknesses $t_1, t_2, t_3 \ldots t_n$ of the laminated plate having n layers. The total thickness of the plate H is constant. This results in $2n - 1$ design variables. Hence, the problem can be written mathematically as

$$T_c^{opt} = \max\{T_{ij}(\theta_i, t_j)\} \qquad \theta, t \subset \Omega \;\; (i, j = 1, 2, \ldots, n)$$

Subject to following constraints:

- Design variables (domain Ω):

$$-90° \le \theta_i \le 90° \qquad i = 1, \ldots, n$$

$$0.05 \le t_i / H \le 0.8 \qquad i = 1, \ldots, n$$

$$\sum_{i=1}^{n} t_i / H = 1.0$$

The equality constraint has been implemented by adding a penalty function (Deb [12]) to the objective function, which is proportional to square of constraint violation. Sadagopan and Pitchumani [14,15] also employed penalty function approach in solving optimization problems using GA. The other inequality constraints are taken care of during the decoding of variables of the binary strings.

NUMERICAL WORK AND DISCUSSION

A computer program is developed as per the above formulation and has been validated with the existing results available in the literature. The boundary conditions for which the results of both comparative problems and authors' own problems have been obtained are as follows:

1. Simply supported case (s1 boundary condition):
   $u = v = w = 0$ at $x = 0,a$
   $u = v = w = 0$ at $y = 0,b$
2. Clamped edge (c1 boundary condition):
   $u = v = w = w_{,y} = 0$ at $x = 0,a$
   $u = v = w = w_{,x} = 0$ at $y = 0,b$

After the convergence study the 6 × 6 mesh is adopted for the whole plate. Critical temperatures of simply supported and clamped isotropic square plates subjected to a uniform temperature rise are observed to compare well with the existing results of Gossard et al. [1] as given in Table 1. The optimum ply-angles (θ) of a 4 layered [θ/–θ/–θ/θ] symmetric laminated composite plate for the maximum buckling temperature with simply supported and clamped boundary conditions obtained by the present genetic algorithm are compared with those of Walker et al. [10] in Table 2. The material properties as used by them are $E_1 = 181$ GPa, $E_2 = 10.34$ GPa, $G_{12} = 7.17$ GPa, $\mu_{12} = 0.28$, $\alpha_1 = 0.02 \times 10^{-6}$ and $\alpha_2 = 22.3 \times 10^{-6}$. It may be noted from the table that the results compare well.

Graphite/epoxy laminated composite plate considered in the present analysis has the following properties:

$$E_1 = 141.0\text{ GPa}, \quad E_2 = 3.1\text{ GPa}, \quad G_{12} = 9.31\text{ GPa},$$

$$\mu_{12} = 0.28, \quad \alpha_1 = 0.018 \times 10^{-6} \text{ and } \alpha_2 = 21.6 \times 10^{-6}$$

Genetic algorithm is then used to optimize the design variables, ply-angles (θ_i) and ply-thicknesses (t_i), which are coded in binary string structures. Ten bits are used to code θ_i and five bits are used to code t_i. In the present case, by varying the crossover and the mutation probabilities, and starting from different population sizes the optimum values are obtained. Also different variants of the genetic algorithm such as elitist and two-point crossover methods are used. The whole process is repeated with three different random seeds. Improvements of optimum solution (buckling temperature) of a 3 layered square simply supported angle-ply plate with generation number are given in Tables 3, 4 and 5 for different crossover probabilities, mutation probabilities, and population sizes. It is observed from Table 5 that the near optimum solution may be reached in about 150, 70 and 24 generations for population sizes of 75, 125 and 225 respectively. Further improvement of the solution is very slow with the increasing generation number as observed

Table 1. Critical buckling temperature (in °C) for isotropic square plates (a/H = 100.0, $\alpha = 2.0 \times 10^{-6}$, μ = 0.3).

	Simply Supported (s1)		Clamped (c1)	
Temperature Distribution	**Analytical Reference [1]**	**Present Study**	**Analytical Reference [1]**	**Present Study**
Uniform temperature rise	63.27	63.236	168.71	167.569

Table 2. Comparison of optimum ply-angle (θ) of a 4-layered [$\theta/-\theta/-\theta/\theta$] composite plate for maximum thermal buckling load.

Simply Supported		Clamped	
Reference [10]	**Present Study**	**Reference [10]**	**Present Study**
45.1°	45.0°	54.3°	52.9°

Table 3. Convergence of optimum buckling temperature (in °C) in the case of a 3-layered square simply supported plate for different crossover probabilities (population size 125, mutation probability = 0.01).

Crossover Probability 0.7		Crossover Probability 0.75		Crossover Probability 0.8	
Generation Number	**Optimum Solution (T_c^{opt})**	**Generation Number**	**Optimum Solution (T_c^{opt})**	**Generation Number**	**Optimum Solution (T_c^{opt})**
1	55.81	1	55.81	1	55.81
3	56.36	7	56.19	5	57.94
5	59.71	10	59.09	7	61.23
11	60.48	16	62.32	8	68.01
14	64.66	21	63.58	21	68.39
18	66.97	29	67.21	67	68.74
32	67.67	30	67.93	70	69.01
36	68.40	50	68.26	91	69.14
53	68.98	53	68.77	105	69.25
194	69.21	128	69.17	113	69.26
338	69.29	319	69.28	363	69.36

Table 4. Convergence of optimum buckling temperature (in °C) in the case of a 3-layered square simply supported plate for different mutation probabilities (population size 125, crossover probability = 0.8).

Mutation Probability 0.01		Mutation Probability 0.015		Mutation Probability 0.02	
Generation Number	Optimum Solution (T_c^{opt})	Generation Number	Optimum Solution (T_c^{opt})	Generation Number	Optimum Solution (T_c^{opt})
1	55.81	1	55.81	1	55.81
5	57.94	3	60.69	3	60.48
7	61.23	6	64.25	8	66.03
8	68.01	16	66.98	15	67.29
21	68.39	39	67.87	28	68.77
67	68.74	40	68.37	129	68.82
70	69.01	90	69.03	210	69.00
91	69.14	161	69.13	217	69.12
105	69.25	530	69.20	456	69.15
113	69.26				
363	69.36				

Table 5. Convergence of optimum buckling temperature (in °C) in the case of a 3-layered square simply supported plate (crossover probability = 0.8, mutation probability = 0.01).

Population Size 75		Population Size 125		Population Size 225	
Generation Number	Optimum Solution (T_c^{opt})	Generation Number	Optimum Solution (T_c^{opt})	Generation Number	Optimum Solution (T_c^{opt})
1	58.8	1	55.81	1	60.63
4	60.74	5	57.94	10	66.38
7	66.81	7	61.23	12	66.54
11	67.21	8	68.01	23	67.44
33	67.24	21	68.39	24	69.03
35	68.57	67	68.74	86	69.08
56	68.84	70	69.01	96	69.17
150	69.16	91	69.14	143	69.21
179	69.17	105	69.25	172	69.25
405	69.26	113	69.26	195	69.28
		363	69.36	610	69.35

from the table. Two-point crossover is found to be detrimental in this case. Elitist model of GA is found to be beneficial but does not have significant effect on the optimum solution. After few generations, comparing the fitness values and noting the optimum solution the search domain may be decreased. In this way the optimum value can be obtained with a greater accuracy as shown in Table 6. The limiting of search domain is indicated by"local search."

Optimum buckling temperatures along with the optimum ply-orientations and ply-thicknesses for simply supported and clamped boundary conditions are presented in Tables 7 and 8 respectively for three different aspect ratios (1.0, 1.5, 2.0) of 3-layered composite plates. In the first row, the buckling temperatures of angle-ply (45/–45/45) plates are given. The results obtained by optimizing the ply-orientations alone are given in the second row, which show that the buckling temperature increases by about 12% in the case of simply supported square plate and by about 21.8% in the case of clamped square plate. However, it may be noted from the results of third row of Tables 7 and 8 that when both fiber orientations and thicknesses are optimized, buckling temperature increases by 54% and 62.7% respectively in the cases of simply supported and clamped square plates. The limiting of search domain after observing answers from the global search is indicated by "local" in the fourth row. Better solution is expected to be obtained in a less number of generations if the local search is carried out around possible optimum solution. But this needs a prior idea about the optimum point for which global search should be carried out. When a/b ratio is 2, the optimum values increase by more than 125% and 95% in the cases simply supported clamped plates, respectively.

It is worth mentioning here that the buckling temperatures obtained from the "local search," though higher, are very close to their corresponding optimum point values obtained from the "global search." This observation is true for both the simply supported and clamped 3-layered plates with a/b ratios of 1.0, 1.5 and

Table 6. Local search method applied to the problem considered in Table 2 (population size = 125).

Angle	Relative Thickness (t_i/H)	Buckling Temperature (in °C)	Generation Number
Global Search			
$-90° \le \theta_i \le 90°$	$0.05 \le t_i/H \le 0.8$	$i = 1, 2, 3$	
45.04/–45.13/44.42	0.1258/0.7548/0.1194	69.01	70
Local Search			
$43° \le \theta_1 \le 47°$ $0.1 \le t_1/H \le 0.2$	$-47° \le \theta_2 \le -43°$ $0.7 \le t_2/H \le 0.8$	$43° \le \theta_3 \le 47°$ $0.1 \le t_3/H \le 0.2$	
45.04/–45.13/44.78	0.1161/0.7678/0.1161	69.45	75

Table 7. Simply supported 3-layered plate (b/H = 100.0).

a/b	Optimization	Angle	Relative Thickness (t_i/H)	Buckling Temperature (in °C)
1.0		45/–45/45	Equal	45.21
	Angle	45.7/–44.3/–44.3	Equal	50.68
	Angle & thickness	45.39/–45.48/45.48	0.1097/0.7774/0.1129	69.03
	Local	45.04/–45.13/44.78	0.1161/0.7678/0.1161	69.45
1.5		45/–45/45	Equal	32.19
	Angle	57.41/–82.6/–70.6	Equal	48.36
	Angle & thickness	65.45/–67.47/60.61	0.1258/0.7774/0.0968	61.21
	Local	61.61/–69.35/60.61	0.1065/0.7903/0.1032	61.51
2.0		45/–45/45	Equal	27.35
	Angle	63.0/–88.3/–53.36	Equal	45.96
	Angle & thickness	59.29/–65.89/62.02	0.1387/0.7323/0.1290	61.20
	Local	60.32/–68.06/53.22	0.1097/0.7871/0.1032	61.55

Table 8. Clamped 3-layered plate (b/H = 100.0).

a/b	Optimization	Angle	Relative Thickness (t_i/H)	Buckling Temperature (in °C)
1.0		45/–45/45	Equal	93.13
	Angle	0.1/–69.6/30.7	Equal	113.49
	Angle & thickness	13.46/–73.98/11.35	0.1460/0.6994/0.1549	151.51
	Local	0/90/0	0.15/0.7/0.15	151.83
1.5		45/–45/45	Equal	73.32
	Angle	73.44/–4.75/–72.03	Equal	101.35
	Angle & thickness	61.32/–47.94/61.67	0.1085/0.7311/0.1604	139.89
	Local	62.98/–46.45/62.25	0.1452/0.7096/0.1452	142.98
2.0		45/–45/45	Equal	68.81
	Angle	72.56/3.34/–74.5	Equal	96.61
	Angle & thickness	71.08/–38.09/81.9	0.1387/0.6871/0.1742	128.77
	Local	70.0/–37.42/70.0	0.1468/0.7064/0.1468	134.52

Table 9. Simply supported 4-layered plate (b/H = 100.0).

a/*b*	Optimization	Angle	Relative Thickness (t_i/H)	Buckling Temperature (in °C)
1.0		45/–45/45/–45	Equal	65.17
	Angle	45/–45/45/–45	Equal	65.17
	Angle & thickness	47.40/–46.72/46.53/–46.71	.1359/.2877/.4284/.1480	69.30
1.5		45/–45/45/–45	Equal	44.76
	Angle	67.0/–54.0/66.9/–69.1	Equal	57.74
	Angle & thickness	56.74/–68.88/66.42/66.07	.1516/0.3710/.3484/.129	60.43
2.0		45/–45/45/–45	Equal	35.69
	Angle	67.98/–56.88/57.59/–63.23	Equal	57.66
	Angle & thickness	55.86/–59.91/64.13/–73.1	.1516/.3032/.3484/.1968	60.16

2.0 as seen from Tables 7 and 8. Considering further that the local search is time consuming, it is being carried out only for 3-layered plates based on the results of the global search. Accordingly, the respective optimum buckling temperatures of 4 and 5-layered plates are computed up to the optimum point of the global search. Further studies on the effects of number of layers, angles of orientation and thicknesses of different layers along with the role of simply supported and clamped boundary conditions are made with the results of the global search only.

The corresponding global search results of 4-layered angle-ply plates with simply supported and clamped boundary conditions are reported in Tables 9 and 10

Table 10. Clamped 4-layered plate (b/H = 100.0).

a/*b*	Optimization	Angle	Relative Thickness (t_i/H)	Buckling Temperature (in °C)
1.0		45/–45/45/–45	Equal	133.3
	Angle	0/90/90/0	Equal	147.12
	Angle & thickness	23.48/–85.42/–14.34/83.66	.0557/.2129/.5578/.1736	150.97
1.5		45/–45/45/–45	Equal	99.12
	Angle	65.16/–32.23/26.6/–67.45	Equal	131.2
	Angle & thickness	57.7/–51.1/0.26/–89.12	.1266/.2982/.3616/.2136	139.21
2.0		45/–45/45/–45	Equal	90.25
	Angle	75.91/–22.01/29.06/–64.64	Equal	123.75
	Angle & thickness	76.01/–25.42/–13.81/82.78	.1871/.3194/.3226/.1710	132.02

Table 11. Simply supported 5-layered plate (b/H = 100.0).

a/b	Optimization	Angle	Relative Thickness (t_i/H)	Buckling Temperature (in °C)
1.0		45/–45/45/–45/45	Equal	60.8
	Angle	45.7/44.3/50.0/47.1/–47.1	Equal	67.77
	Angle & thickness	45.83/–45.13/–6.07/43.9/–45.48	0.1387/0.3419/0.1097/0.1645/0.1452	69.4
1.5		45/–45/45/–45/45	Equal	42.02
	Angle	65/–59.35/90/63.93/–67.1	Equal	60.43
	Angle & thickness	55.86/–67.47/72.228/66.95/–68.0	0.102/0.250/0.356/0.095/0.197	60.869
2.0		45/–45/45/–45/45	Equal	34.06
	Angle	59.3/–61.5/–85.1/56.1/–69.2	Equal	60.41
	Angle & thickness	56.13/–67.47/74.51/72.22/–63.43	0.1387/0.1516/0.1871/0.3806/0.1419	63.76

respectively and the same for 5-layered simply supported and clamped angle-ply plates are given in Tables 11 and 12 respectively. From the tables it is clear that the effect of optimization is significant on the buckling temperature. It is observed from Table 13, where optimum buckling temperatures are compared for all the cases studied here, that increasing the number of layers does not increase the optimum buckling temperature considerably. Also it may be noted that optimum ply-angles and relative ply-thickness are dependent on boundary conditions.

Table 12. Clamped 5-layered plate (b/H = 100.0).

a/b	Optimization	Angle	Relative Thickness (t_i/H)	Buckling Temperature (in °C)
1.0		45/–45/45/–45/45	Equal	128.9
	Angle	83.0/–15.0/9.8/–8.7/–88.2	Equal	149.98
	Angle & thickness	18.74/–79.27/–25.6/–78.56/25.07	0.1/0.2258/0.3903/0.1774/0.1065	151.22
1.5		45/–45/45/–45/45	Equal	98.12
	Angle	61.3/–55.4/5.5/36.1/–69.6	Equal	136.92
	Angle & thickness	69.32/–30.17/51.46/62.72/–54.28	0.1960/0.4372/0.1381/0.1135/0.1152	140.75
2.0		45/–45/45/45/–45	Equal	90.42
	Angle	63.4/–38.2/–15.3/45.9/–66.4	Equal	132.71
	Angle & thickness	50.76/–63.25/–2.55/60.26/–65.71	0.1290/0.2129/0.3032/0.1968/0.1581	135.42

Table 13. Comparison of optimum buckling temperatures (in °C) for different cases.

		Simply Supported			Clamped		
a/b	**Number of Layers:**	**3**	**4**	**5**	**3**	**4**	**5**
1.0		69.03	69.30	69.4	151.51	150.97	151.22
1.5		61.21	60.43	60.87	139.89	139.21	140.75
2.0		61.20	60.16	63.76	128.77	132.02	135.42

CONCLUSIONS

In the present paper, the optimum fiber orientations in different layers and optimum thickness of each layer are obtained for 3, 4 and 5-layered rectangular graphite/epoxy composite plates. The optimization of the design variables is achieved by genetic algorithm in which cost function i.e., the critical buckling temperature is obtained using the finite element method. It is observed from the numerical work that in some cases buckling temperature can be increased by 125% by suitably taking the design variables. It is also noted that the gain in optimization of the design variables is comparatively less as the layer number increases. The optimum ply-angles and relative thickness are dependent on boundary conditions and a/b ratio. For the practical implementation, however, the optimum values of the design variables (ply-angles and ply-thicknesses) are recommended to be rounded up judiciously to their respective nearest values, notwithstanding the fact that the academic knowledge of true optimum design is very much essential with a view to making them practically available in future.

NOTATION

a,b = dimensions of the plate in x and y directions
$[A]$, $[B]$, $[D]$ = extensional, coupling and bending stiffness matrices
E_L, E_T = Young's modulus in the fiber and transverse to the fiber directions
G_{LT} = Shear modulus
H = thickness of the plate
$h_{i,}$ = distance of bottom surface of ith layer from the middle surface of the plate
$[K_s^e]$ = element stiffness matrix
$[K_g^e]$ = geometric stiffness matrix
$\{N, M\}^T$ = force and moment resultants
$\{N^T, M^T\}$ = thermal load vector
$[Q_{ij}]$ = transformed reduced stiffness matrix

$\{R_x, R_y\}$ = shear resultants
T_c = buckling temperature (in °C)
t_i = thickness of ith layer of the plate
$\{\alpha\}$ = thermal expansion coefficient
u_0, v_0, w_0 = mid-surface displacement of the plate in x, y and z directions, respectively
$w_{,x}$, $w_{,y}$ = slope of the plate in x and y directions respectively
$w_{,xx}$, $w_{,xy}$, $w_{,yy}$ = second derivatives of w with respect to x and y
γ_x, γ_y = rotation due to shear in x and y directions respectively
$\{\varepsilon_x^0, \varepsilon_y^0, \varepsilon_{xy}^0\}^T$ = mid-surface membrane strain components
$\{\kappa_x, \kappa_y, \kappa_{xy}\}^T$ = curvature components
μ_{LT} = Poisson's ratio
$\theta_1, \theta_2, \theta_3 \ldots \theta_n$ = ply-angles of the layers
λ = eigenvalue
$\{\delta\}$ = global displacement vector

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