Optimum Power Control for Successive Interference Cancellation With Imperfect Channel Estimation

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***Abstract*—Successive interference cancellation, in conjunction with orthogonal convolutional codes, has been shown to approach the Shannon capacity for an additive white Gaussian noise channel (Viterbi: 1990). However, this requires highly accurate estimates for the amplitude and phase of each user's signal. In this paper, we derive an optimal power control strategy specifically designed to maximize the overall capacity under the constraint of a high degree of estimation error. This power control strategy presents a general formula of which other power control algorithms are special cases. Even with estimation error as high as 50%, capacity can be approximately doubled relative to not using interference cancellation. In addition, when properly applied to multicell mobile networks, this power control scheme can reduce the handset transmit power, and therefore other-cell interference, by more than an order of magnitude.**

***Index Terms*—Code-division multiple access (CDMA), interference cancellation, multiuser detection (MUD), power control, superorthogonal codes.**

I. INTRODUCTION

FOR A number of reasons, code-division multiple access (CDMA) continues to be a dominant air-interface technology for personal wireless communication systems. While systems based on the widely available commercial standards such as IS-95 and the newer third-generation (3G) standards wideband code-divison multiple access (W-CDMA) and CDMA2000 have proven reasonably robust for low bandwidth applications, it is generally believed that significant increases in capacity and performance are attainable for future CDMA systems. An abundance of theoretical and practical research has been undertaken with this goal in mind. Fundamental work done by Verdu [2] showed the remarkable extent to which the single-user matched filter present in IS-95 systems could be improved upon by using more sophisticated receiver design and signal processing. While the optimal implementations of this work are prohibitively complex for even a modest numbers of users, an assortment of suboptimal methods have been developed which reduce the complexity drastically while still providing large gains over the conventional single-user detector. This field has come to be known as multiuser detection (MUD), and an accessible summary of the field can be found in [3].

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Despite the abundance of academic work on MUD, industry implementations still predominantly use the single-user matched filter. There are several explanations for industry's reluctance to use the results from MUD, and they usually center around continuing questions about the complexity of even the reduced-complexity suboptimal techniques and the robustness of such techniques to the difficulties of the multicell wireless channel. Successive interference cancellation (SIC) as proposed in [1] is a MUD technique that is different from much of the MUD research in that it does not rely on dimensional separation or short-period spreading sequences in order to distinguish users from one another. Further, its entire design as presented in this paper is based on an extremely strong error-correcting code. For these reasons, it is well-suited to an uncoordinated, noisy, asynchronous environment such as the uplink in a cellular system.

There are some serious challenges in making a SIC system feasible in practice. First, the decoding time increases linearly with the number of users. This is because users are decoded successively, as implied in the name of the technique. However, a latency increase that is linear with the number of users is generally considered palatable because processor speed is increasing exponentially. Second, relative to conventional CDMA, a more complicated power control distribution is required to make full use of SIC, because the users must be received with differing powers, dependent on the order of decoding. Third, the amplitude and phase of each user must be accurately estimated. If the estimates are inaccurate, residual interference remains in the composite signal, and the system capacity rapidly erodes. Fourth, as is true of all realistic MUD systems, other-cell interference (OCI) is uncancelable and, thus, proposes a particular problem.

In this paper, we focus on the latter two problems. In the attempt to relax the requirement on accurate amplitude and phase estimation, a novel and general power control algorithm is developed that is shown to be optimal for all CDMA systems. For the sake of receiver simplicity, a channel with only one path from transmitter to receiver is assumed, but the power control results apply to a multipath channel as well, since the power control distribution only depends on the total amount of power received per user. It will be shown in Section IV that if the estimation error is considered when developing a power control distribution, a sizeable amount of estimation error can be tolerated while still maintaining robust bit-error rate (BER) performance at an increased spectral efficiency. In addition, extending the work of [4] and [5], it will be shown in Section V that if the users' relative distances from the cell are considered when assigning powers, OCI can be reduced by approximately an order of magnitude over equal power CDMA systems such as IS-95 and 3G CDMA. In Section VI, the spectral efficiency of a SIC

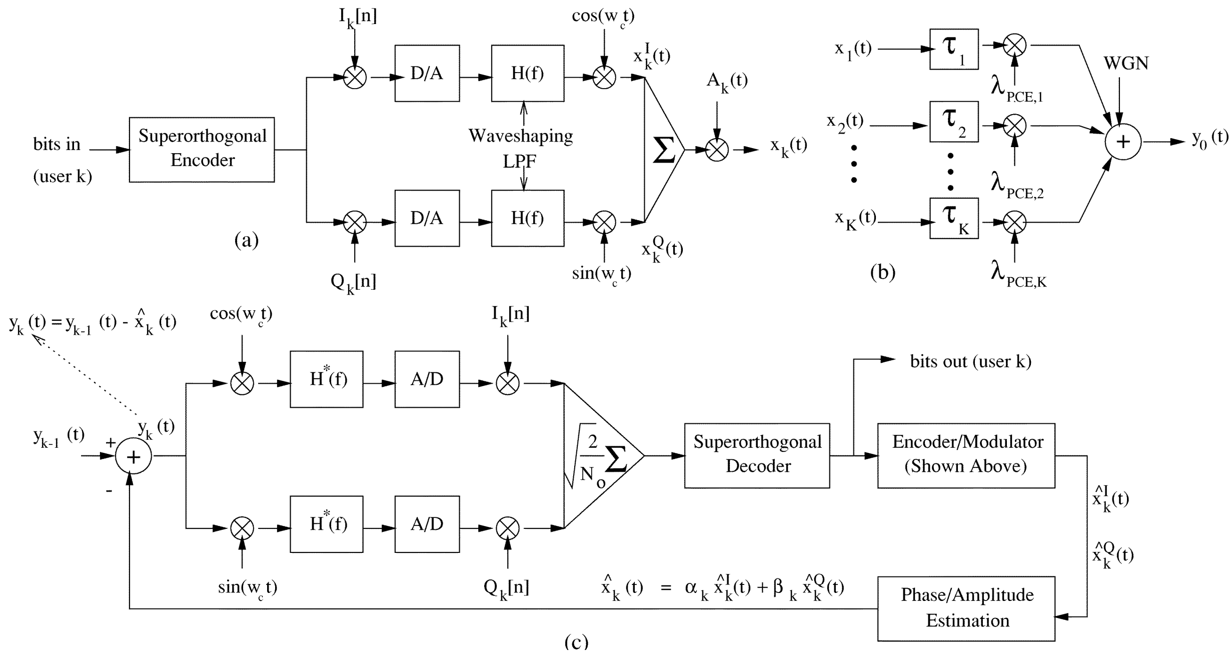

Fig. 1. System block diagram. (a) Transmitter. (b) Channel. (c) Receiver.

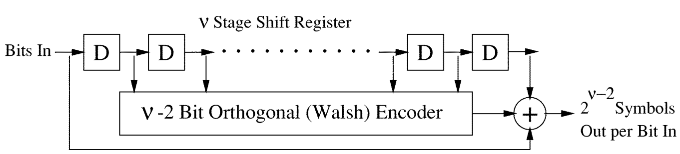

Fig. 2. Superorthogonal encoder

system with optimal power control will be demonstrated and compared with other approaches.

II. System Model

A. Transmitter

The transmitter, channel, and receiver models are shown in Fig. 1. At the transmitter, each user's data bits are encoded by a superorthogonal convolutional encoder. This powerful code is proposed and described in [6] and the encoder is shown in Fig. 2. The spreading gain achieved by a superorthogonal code is $2^{\nu-2}$, where ν is the constraint length of the code. The benefits of the proposed system and power control distribution can also be achieved with other low-rate convolutional codes, such as those recently proposed in [7], which also has slightly superior performance to the superorthogonal code and can accommodate spreading factors that are not powers of two.

After the data has been encoded, it is split into I and Q branches and scrambled by independent binary sequences, in order to ensure that other-user interference produces a random component whose variance is independent of the relative phases between users [1]. The resulting I and Q signals are converted to analog signals and then run through a pulse shaping low-pass filter before being quadrature modulated by the carrier frequency. The resulting transmitted signal for user k is

$$x_k(t) = A_k(t) \sum_{n=-\infty}^{\infty} c_k[n] \cdot [I_k[n] h(t - nT) \cos(\omega_c t) + Q_k[n] h(t - nT) \sin(\omega_c t)] \quad (1)$$

where A_k is the gain factor (due to power control), $c_k[n]$ is the binary serial sequence of encoder output, $I_k[n]$ and $Q_k[n]$ are the binary in-phase and quadrature scrambling sequences, ω_c is the carrier frequency in radians per second, and $h(t)$ is the impulse response of the pulse-shaping filter. $I_k[n]$ and $Q_k[n]$ may have an arbitrarily long period and, thus, are modeled as psuedorandom Bernoulli $\{-1, +1\}$ sequences.

To simplify the analysis and relate it directly to simulated results, we consider the discrete-time baseband transmitted signal as

$$x_k[n] = A_k[n](I_k[n] + jQ_k[n])c_k[n] \quad (2)$$

$$= x_k^I[n] + jx_k^Q[n] \quad (3)$$

Perfect separation between the in-phase and quadrature channels is assumed, so all digital-domain analysis can be considered for uncorrelated $x^I[n]$ and $x^Q[n]$.

B. Channel

The channel is modeled as an asynchronous fading channel with additive white Gaussian noise (AWGN). Each user's signal experiences an independent delay τ_k during transmission, but it is assumed that the receiver can learn the value of this delay through the usual methods employed in commercial systems. Thus, the asynchronicity is relevant in that it demonstrates that no alignment of the users is required for the system. Fast closed-loop power control must be employed in practical CDMA systems in order to mitigate the effects of rapid changes in the received signal strength. In this work, it is assumed that the power control helps neutralize the fading, but some residual power control error (PCE) remains, which can be thought of as unmitigated fading.

The received signal

$$y_0[n] = \sum_{k=1}^{K} \lambda_{\mathrm{PCE},\,k} \cdot x_k[n - \tau_k] + \eta[n] \quad (4)$$

is the sum of the transmitted signals delayed by their respective propagation times (assumed here to be an integer multiple of the sample interval), plus additive noise η, which has noise power N. Path loss is neglected throughout the paper. The power control error λ_{PCE} arises from the imperfect mitigation of fading and will be quantified in detail when the optimum power control distribution is presented.

C. Interference Cancelling Receiver

As implied by the name of the technique, in a SIC system, users' signals are extracted from the composite received signal successively, rather than in parallel. SIC attempts to remove the interference of the kth user (the most recently decoded user) from the current composite received signal $y_{k-1}[n]$, by re-encoding the decoded bit sequence for user k, modulating it with the appropriate amplitude and phase adjustment, and subtracting it out from $y_{k-1}[n]$. This process is illustrated in Fig. 1(c). The forward path is similar to that of a typical CDMA matched filter receiver: The down converted and sampled signal is despread with the synchronized pseudonoise (PN) sequences for user k and then combined and decoded by an appropriately modified Viterbi decoder for superorthogonal codes. It is assumed that synchronization with each user is achieved through the usual methods, namely, an overhead channel with training, and then a phase locked loop. As stated previously however, no cooperation between users is assumed.

Once reliably decoded, user k's decoded bits can then be used to cancel the interference that its signal would cause to later users. The estimated bits for user k are reencoded, and estimates of the amplitude and phase, or equivalently the amplitude of the I and Q branches are formed

$$\alpha_k = \frac{1}{M} \sum_{n=1}^{M} y_k[n] \cdot \hat{x}_k^I[n] \tag{5}$$

$$\beta_k = \frac{1}{M} \sum_{n=1}^{M} y_k[n] \cdot \hat{x}_k^Q[n] \tag{6}$$

where M is the number of symbols in a frame, and α_k and β_k are the amplitude estimates of user k's in-phase and quadrature branches, respectively.

Using these values, an estimate of the received signal from user k can be obtained as

$$\hat{x}_k[n] = \alpha_k \hat{x}_k^I[n] + j\beta_k \hat{x}_k^Q[n]. \tag{7}$$

The stored composite signal may then be updated

$$y_k[n] = y_{k-1} - \hat{x}_{k-1}[n] \tag{8}$$

$$= y_0 - \sum_{i=1}^{k-1} \hat{x}_i[n]. \tag{9}$$

Thus, it is intuitive that the first user is exposed to the most multiple access interference (MAI), while the final user sees a composite signal with a large amount of MAI removed from it. This motivates the discussion of the next session on optimum power control.

III. Optimum Power Control

Power control is required for all realistic CDMA systems because of what is known as the near-far problem: users far from the base station experience far greater path loss than users that are near the base station. Optimum power control is achieved when all users are decoded with the same signal-to-interference ratio (SIR) [8]. Otherwise, a user with a low SIR dominates the BER performance of the system, which is defined as the average BER over all users.

In commercial CDMA systems, the near–far problem is mitigated by controlling the output power of the mobile units with a tight feedback loop, so that the users' signals all arrive at the base station with approximately the same power, which results in a consistent quality of service, as each user experiences an approximate SIR of

$$\mathrm{SIR} = \Gamma_k = \Gamma = \frac{P}{(K-1)P + N} \tag{10}$$

where $P_k = P$ is the received power of each user, K is the number of users, and N is the power of the background AWGN, which can also include OCI, assuming such interference appears as noncoherent additive noise.

When SIC is used, the situation is significantly different. In this case, it is also desirable that each user experiences the same SIR at the time of decoding. However, interference is being subtracted out of the received signal after each user, so the first user to be decoded sees the most interference, the last user the least. Heuristically, the first user to be decoded should be the strongest user, the weakest user should be decoded last.

If the successive cancellation scheme proceeds with no channel estimation error or bit errors, then finding the optimum power control scheme is straightforward as described in [9]. Of course, the amplitude and phase estimation are never perfect and, thus, it is desirable to know the optimum power solution in the presence of imperfect cancellation. If there is cancellation error, the following K equations describe the SIRs for each user:

$$\Gamma_1 = \frac{P_1}{\sum_{k=2}^{K} P_k + N}, \qquad \Gamma_2 = \frac{P_2}{\sum_{k=3}^{K} P_k + \varepsilon_1 P_1 + N}, \ldots$$
$$\Gamma_K = \frac{P_K}{\sum_{k=1}^{K-1} \varepsilon_k P_k + N} \tag{11}$$

where K is again the number of users and ε_k is the fraction of the kth user's power not cancelled. We desire $P_k = \{P_1, P_2, \ldots, P_K\}$ such that $\Gamma_1 = \Gamma_2 = \cdots = \Gamma_K$, since the Γ_k will directly determine the BER.

In (11), there are $K-1$ equations and $K-1$ unknown relative power weightings, since one of the P_k can be set to an arbitrary value depending on the desired receiver sensitivity. These equations can be solved in terms of the SIR Γ as in [5] and [10], but ideally one would like to equalize the users' SIRs in the presence of interference without knowing the target SIR. Hence, a recursive approach was adopted and the derivation is shown in

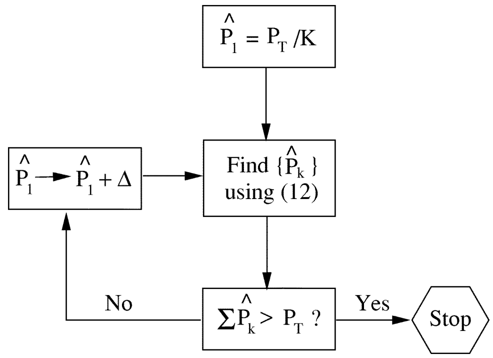

Fig. 3. Algorithm for computing $\{P_k\}$.

the Appendix. The resulting power control distribution for user k can be expressed as

$$P_k = P_{k-1} - \frac{(1-\varepsilon_{k-1})P_{k-1}^2}{V_{k-1}+N} \tag{12}$$

where ε_k is the fractional residual cancellation error for user k, and V_k is the total remaining multiple-access interference (MAI) for user k plus their own power:

$$V_k = \sum_{i=1}^{K} P_i - \sum_{i=1}^{k-1}(1-\varepsilon_i)P_i \tag{13}$$

Note that this is the general optimal solution for CDMA power control, and that although it is possible that $\varepsilon > 1$ if many bit errors are made, all cases of interest are for $\varepsilon \leq 1$. For perfect interference cancellation $\varepsilon_k \rightarrow 0$ and (12) is shown in Appendix II to be identical in this case to the distribution derived in [9]. For no interference cancellation as in a typical equal power commercial CDMA system, $\varepsilon_k \rightarrow 1$ and it can be easily seen that (12) reduces to the familiar equal power solution.

These equations still cannot be solved analytically for all the P_k given P_1, due to the introduced variable $P_T = \sum_{i=1}^{K} P_i$, but they can be solved quickly by iteration to arbitrary accuracy. We provide a simple algorithm in Fig. 3 for computing the optimal power weightings P_k, where $\Delta \ll P_1$ is some chosen step size.

Following the above steps, the $\hat{P}_k$ will converge to the power distribution given in (12) given a total power constraint P_T as $\Delta \rightarrow 0$. Unlike other uplink power control schemes [11], there are no convergence conditions regarding a target SIR in this algorithm. The proposed algorithm simply equalizes the received SIRs and hence always converges for $\varepsilon \leq 1$. Convergence is defined as the ability to make $\delta_P = \sum_{i=1}^{K} |\hat{P}_i - P_i|$ arbitrarily small. The argument that the algorithm presented in Fig. 3 converges to (12) is as follows.

Initially, $\hat{P}_1 = P_T/K$. It is known from simple inspection of (12) that $P_1 \geq P_T/K$ with equality iff $\varepsilon_k = 1\ \forall k$. Thus, by using (12), it can be seen initially that $\hat{P}_k \leq P_k\ \forall k$. Hence, all initial power estimates are conservative, and the initial total power $\hat{P}_T = \sum_{i=1}^{K} \hat{P}_i \leq P_T$. Thus, by increasing $\hat{P}_1$ by Δ, the instant $\hat{P}_T = P_T \Rightarrow \hat{P}_k = P_k$. Thus, by letting $\Delta \rightarrow 0$ and following the algorithm in Fig. 3, $\delta_P \rightarrow 0$. In practice, Δ is a finite value and hence $\delta_P \neq 0$, but δ_P can be made as small as desired by lowering Δ at the cost of increased iterations.

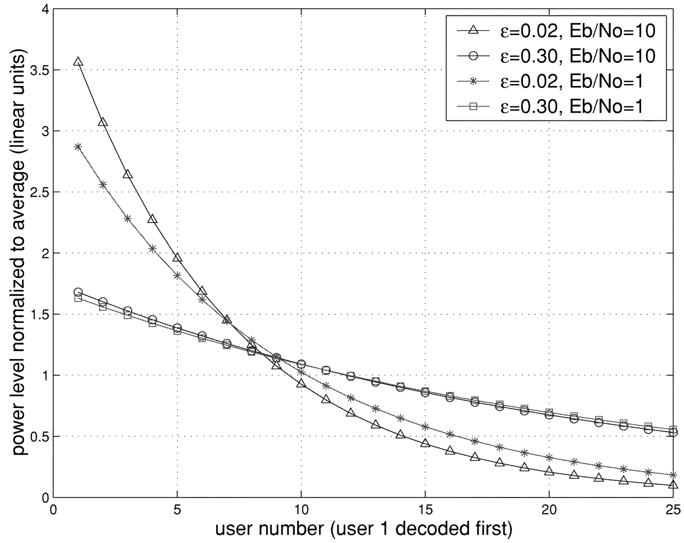

Fig. 4. Sample optimal power distributions for SIC.

It should also be noted that the above reasoning assumes that the fractional cancellation error ε_k is known for each user. Clearly, the amount of cancellation error is rarely known and must instead be estimated or guessed. This will be addressed in Section IV.

A few sample power distributions are shown in Fig. 4, as a function of the amount of uncancelable noise and cancellation error. As can be seen, the relative distribution of powers amongst users is highly dependent on the amount of cancellation error and somewhat dependent on the relative amount of noise power. Note that when the cancellation error and to a lesser extent the noise are kept low, the power differential between earlier and later users is much greater than if those quantities are high, because more successful interference cancellation will take place for earlier users and, thus, the later users will require less power to achieve the same signal to interference ratio. As can be seen in Fig. 4, the dynamic range in received power is less than 10 dB for most cases of practical interest.

IV. Estimation and Power Control Error (PCE) Modeling and Analysis

A. PCE

PCE results when a user is received with a power level that is different than that assigned by the base station. This occurs due to the processing and propagation delay between the transmitter and receiver that makes it difficult to track fast changes in the channel, and also because typically just one bit "up" or "down" commands are sent to the mobile. Nevertheless, fast power control has proven effective for large-scale commercial cellular systems such as IS-95 and WCDMA.

As can be seen in Fig. 1, PCE is applied to the received signal in order to realistically model the received power over a fading channel. PCE has been found to closely follow a lognormal distribution [6], [12] and is defined to have the following normalized variance:

$$\overline{\sigma}_{\mathrm{PCE}}^2 = \frac{1}{P_k^2} E[P_k - \tilde{P}_k]^2 \tag{14}$$

where $\tilde{P}_k$ is the actual received power for user k, and P_k is the assigned (optimal) power for user k. Thus, the received power is modeled as

$$\tilde{P}_k = P_k \cdot \lambda_{\mathrm{PCE}} \tag{15}$$

where

$$\lambda_{\mathrm{PCE}} = e^x,\ x \sim N(0,\ \overline{\sigma}^2_{\mathrm{PCE}}). \tag{16}$$

For simplicity, it is assumed that the fading is uncorrelated from frame to frame, but constant over a frame. Thus, λ_{PCE} takes on a different value over each frame.

B. Estimation Error Modeling

As seen in the power control distribution of Section III, in order to optimally assign powers, the amount of cancellation error per user must be known. Cancellation error has two sources: incorrect bit decisions and imperfect amplitude and phase estimation. Because the BER is assumed to be low, virtually all of the cancellation error ε_k comes from amplitude and phase estimation error. Naturally, estimation error is not typically known either, so in this section, a model for estimation error will be presented. Using this model, it will be shown that despite the lack of knowledge of the exact amount of estimation error, a conservative estimate of the estimation error will produce robust performance.

In the AWGN channel model presented, the amplitude and phase estimation in (5) and (6) will produce an estimate of the received power over each of the I and Q branches of the system. In any realistic system, this estimate will not be perfect for a number of reasons including finite frame length and a dynamic channel, and this estimation error will cause imperfect cancellation of the estimated signal. The amount of residual interference is expressed as a fraction of the user's total power, and this is the fractional residual estimation error ε, as introduced in Section III. In order to create a realistic model for this estimation error for simulation, errors are induced in the estimates of (5) and (6). The estimation error is assumed to follow a log-normal distribution like the PCE and is, thus, modeled similarly

$$\hat{\alpha}_k = \alpha k \cdot \lambda_1, \qquad \hat{\beta}_k = \beta k \cdot \lambda_2 \tag{17}$$

$$\lambda_i = e^{x_i},\ x_i \sim N(0,\ \sigma^2_{\varepsilon}) \text{ i.i.d for } i = \{1,\ 2\}. \tag{18}$$

Thus, σ_ε is the standard deviation of the estimation error for the amplitude estimates of the in-phase and quadrature branches $\{\alpha,\ \beta\}$, and is approximately equal to the total fractional cancellation error ε_k since the amplitude and phase are completely determined by the I and Q amplitudes. We use the notation ε suggestively for both the variance of the estimation error and for the cancellation error because the majority of the cancellation error derives from inaccuracies in the channel estimation, as shall be seen in the next section.

C. Estimation Error Analysis

Using the models of the preceding two subsections, we will now analyze the performance of SIC with the proposed power control algorithm. The BER is plotted as a function of the estimation error in Fig. 5. Each curve represents a different "guess"

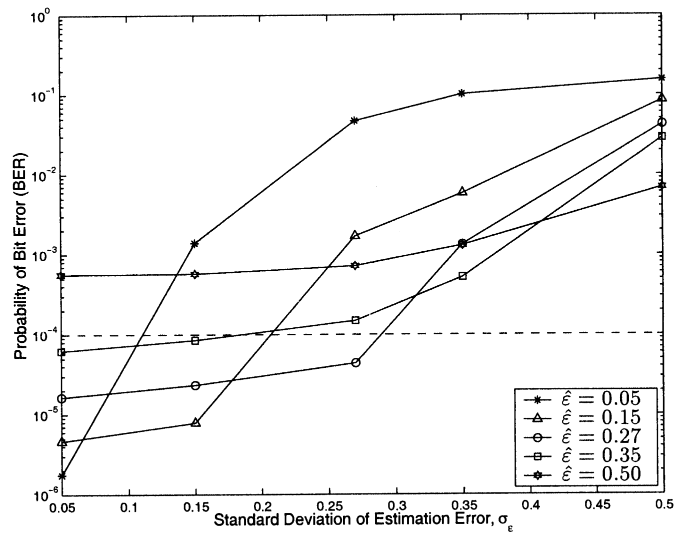

Fig. 5. BER versus estimation error for various values of $\hat{\varepsilon}$.

$\hat{\varepsilon}$ at the cancellation error. Thus, we have three different quantities related to estimation error and for clarity it is important to distinguish between them.

1) the amount of cancellation error ε, which is unknown and expressed as a fraction of the received power;
2) the standard deviation of the amplitude and phase estimation error σ_ε. This is also unknown but modeled in our simulations;
3) a guess at or estimate of the cancellation error ε, which in general will be quite close to σ_ε. This guess is called $\hat{\varepsilon}$ and is the value used for computing the power control distribution in (12).

The system is simulated using parameters in Table I and the results are shown in Fig. 5. While this plot may at first seem confusing, it is easy to understand if two underlying principles are kept in mind.

1) Due to error propagation in SIC, the system is less sensitive to ε being overestimated than underestimated.
2) Because ε relates to a probability distribution, the rare instances when the estimation error is large dominate the BER performance. This is the rationale for choosing $\hat{\varepsilon}$ conservatively.

For example, as can be seen from Fig. 1, a BER of 10^{-4} can be achieved even if the standard deviation of the estimation error is as high as about 0.30, as long as $\hat{\varepsilon}$ is simply chosen to be 0.27. Thus, from this example, no actual knowledge of the cancellation error is required as long as it remains under about 30% of the received signal power.

V. OCI Reduction

In order for any interference cancellation system to operate effectively, uncancelable interference must be kept to a minimum. In most MUD systems, SIC included, OCI cannot be cancelled because the signatures and timing of users in the neighboring cells are unknown to the base station in question. Thus, a method for the reduction of OCI is highly desirable. It will be shown in this section that SIC provides a method

TABLE I
SIMULATION PARAMETERS

Symbol	Description	Value
ν	Superorthogonal code constraint length	7
J	Spreading factor = $2^{\nu-2}$	32
M	Number of symbols/frame (100 bits)	3200
K	Number of full-rate users	
	Fig 5.	40
	Fig 6, Fig 7.	100
	Fig 8.	variable
$\bar{\sigma}_{PCE}$	PCE Normalized std. deviation	0.10
ζ	Path loss exponent	variable
N_c	Number of neighboring cells	6
P_b	Target bit error-rate (BER)	10^{-4}
-	Number of bits used in simulation	200,000
σ_ε	Estimation error, std. deviation	variable
ε_k	Fractional cancellation error for user k	variable
$\hat{\varepsilon}_k$	Receiver estimate of ε_k	variable
N	Noise Power, with $E_b/N_0 \geq 10dB\ \forall\ k$	$\frac{P_K J}{20}$

for drastically reducing OCI relative to that of a commercial CDMA system.

A. Power Assignment Strategy

A well-known model for path loss in cellular systems is given by

$$\frac{P_R}{P_T} = P_0 \left(\frac{d_0}{d}\right)^\zeta \tag{19}$$

where P_R is the received power, P_T is the transmit power, d_0 is some reference distance (typically one meter), P_0 is the path loss at a distance d_0, d is the separation distance of the transmitter and receiver, and ζ is the "path loss exponent," usually between 2.5 and 6, often taken for macrocells to be four in the absence of empirical data.

Because of the typically large path loss exponent, users that are far away from the base station must transmit at a much higher power level than those close to the base station, if their power levels are to be comparable at the receiver. This is known as the "near–far problem" and necessitates power control in any practical CDMA system. However, in a CDMA system using SIC as described in this paper, disparate powers amongst users are actually preferable. Some work has argued that this relaxes the need for accurate power control: Simply decode first the users with the strongest receive powers [14]. In this paper, we propose a different approach. The users far away from the base station can be assigned the lower power levels (and, thus, be decoded later), while the users close to the base station can be assigned the higher power levels. While this strategy does not relax the need for accurate power control, it has several beneficial effects on OCI and capacity that more than compensate for the increase in complexity.

First and most importantly, the users closest to the neighboring cells are the users who are typically farthest from the desired base station. They must raise their powers to reach the base station but in doing so cause increased interference to the neighboring cell. By assigning these users the lower received power levels, they will transmit with significantly less power in a SIC system than a conventional system. Thus, they will cause less interference to neighboring cells, and this reduction will be quantified in the next section. Second, because far-away users can now lower their power levels, the dynamic range required for accurate power control will be reduced. Third, maintaining an accurate power control distribution is important for system capacity, as will be shown in Section VI. Thus, it is preferred to tightly control the users' power levels, relative to simply estimating the power levels and then ordering the decoding.

B. OCI Reduction

In order to quantify the OCI reduction, the OCI in a SIC system with power levels as in (12) shall be compared with the OCI in a conventional equal power system. The total average OCI reduction $\overline{\Omega}$ is the average OCI reduction in each cell times the number of neighboring cells

$$\overline{\Omega} = N_c \cdot \frac{\frac{1}{K}\sum_{k=1}^{K} (\mathrm{OCI}_{\mathrm{Conv}})_k}{\frac{1}{K}\sum_{k=1}^{K} (\mathrm{OCI}_{\mathrm{SIC}})_k} \tag{20}$$

$$= N_c \cdot \frac{\sum_{k=1}^{K} (P_{T,\mathrm{Conv}})_k \cdot P_0 \left(\frac{d_0}{d_k}\right)^\zeta}{\sum_{k=1}^{K} (P_{T,\mathrm{SIC}})_k \cdot P_0 \left(\frac{d_0}{d_k}\right)^\zeta} \tag{21}$$

$$= N_c \cdot \frac{\sum_{k=1}^{K} \frac{(P_{T,\mathrm{Conv}})_k}{d_k^\zeta}}{\sum_{k=1}^{K} \frac{(P_{T,\mathrm{SIC}})_k}{d_k^\zeta}} \tag{22}$$

$$= N_c \cdot \frac{\sum_{k=1}^{K} (P_{R,\mathrm{Conv}})_k \left(\frac{r_k}{d_k}\right)^\zeta}{\sum_{k=1}^{K} P_k \left(\frac{r_k}{d_k}\right)^\zeta} \tag{23}$$

where N_c is the number of neighboring cells, d_k is the distance to a neighboring base station, r_k is the distance to the desired base station, $(P_{R,\mathrm{SIC}})_k = P_k$ is the received power for user k from (12), $(P_{R,\mathrm{Conv}})_k$ is the received power for user k in a conventional system, and OCI_k is the interference power received at the neighboring base station from user k. It is assumed that the users are uniformly positioned throughout cells with a circular coverage area of radius R and that the neighboring base station is a distance $2R$ from the desired base station.

For fairness, $(P_{R,\mathrm{Conv}})_k$ and P_k are constrained such that the conventional system and a SIC system have an equivalent SIR for each user at the time of decoding, that is, by setting

$$\mathrm{SIR}_{\mathrm{Conv}} = \mathrm{SIR}_{\mathrm{SIC}} \tag{24}$$

$$\mathrm{SIR}_{\mathrm{Conv}} = \frac{(P_{R,\mathrm{Conv}})_k}{(K-1)(P_{R,\mathrm{Conv}})_k + N} \tag{25}$$

$$\mathrm{SIR}_{\mathrm{Conv}} = \frac{P_k}{\sum_{i=k+1}^{K} P_i + \sum_{i=1}^{k-1} \varepsilon_i P_i + N} \tag{26}$$

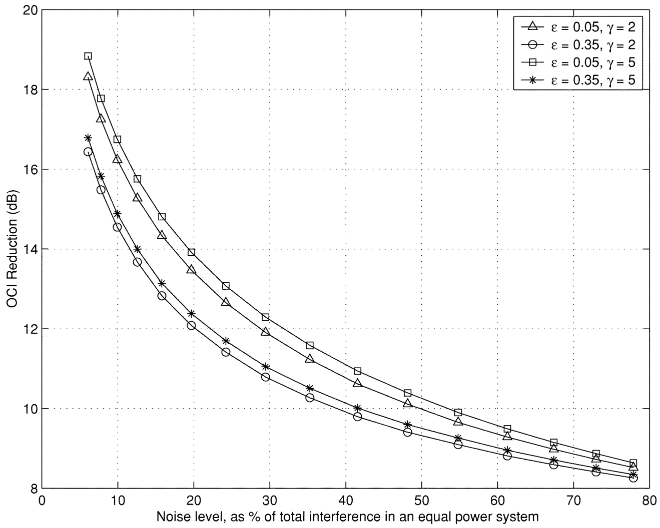

Fig. 6. OCI reduction.

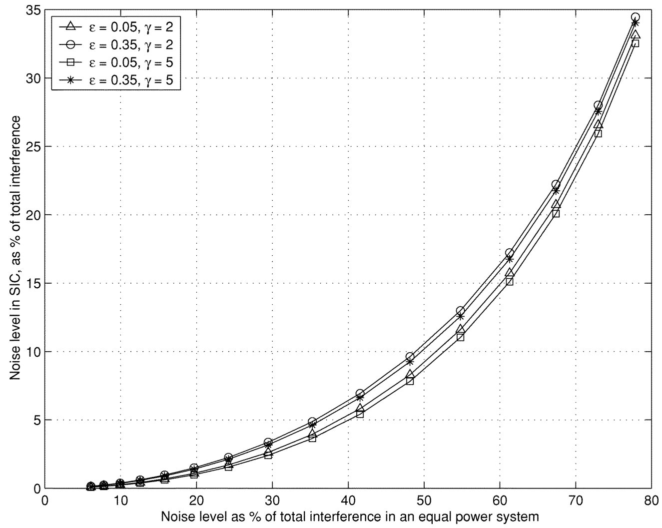

Fig. 7. SIC OCI versus conventional OCI.

Constraining $\mathrm{SIR}_{\mathrm{Conv}} = \mathrm{SIR}_{\mathrm{SIC}}$ is actually conservative since at a given SIR and spreading factor, a CDMA system based on superorthogonal codes will outperform a system with concatenated convolutional codes, Walsh modulation, and repetition codes, as used in IS-95 [6].

The conventional power distribution $(P_{R,\mathrm{Conv}})_k$ is easily found empirically since

$$(P_{R,\mathrm{Conv}})_k = \frac{1}{\sum_{i=1}^{K} (P_{R,\mathrm{Conv}})_i} = \text{Constant}\ \forall k. \tag{27}$$

However, the optimal SIC power distribution $(P_{R,\mathrm{SIC}})_k$ is found recursively, so closed-form results for the OCI reduction are not presented.

Note that since the OCI reduction considers a ratio of average OCI reduction over all K users and that the users are uniformly distributed throughout the cell, neither the radius of the cell, nor the number of users K affect the results. As can be seen in Fig. 6, the OCI reduction is an inverse function of two quantities: the uncancelable interference (OCI and thermal noise) and the amount of estimation error ε. It is an inverse function of these quantities because as noise and estimation error increase, the power distribution becomes tighter because less interference cancellation is possible. Since this makes the SIC system approach an equal power system, clearly the gain from SIC decreases in this case. Contrary to what intuition might predict, the OCI reduction is only very weakly dependent on the path loss exponent ζ. It can be seen that an improvement of about 10 dB can be made even at high noise levels, when half of all interference comes from AWGN and users in neighboring cells, as is typical in equal-power commercial systems [14].

It is also of interest to know what a certain percentage of OCI in an equal power system translates into in a SIC system. This is shown in Fig. 7. As can be seen, a normal CDMA system with as much as 50% of its total interference coming from users in other cells is reduced to under 10% in a SIC system using the power assignment strategy proposed in this paper.

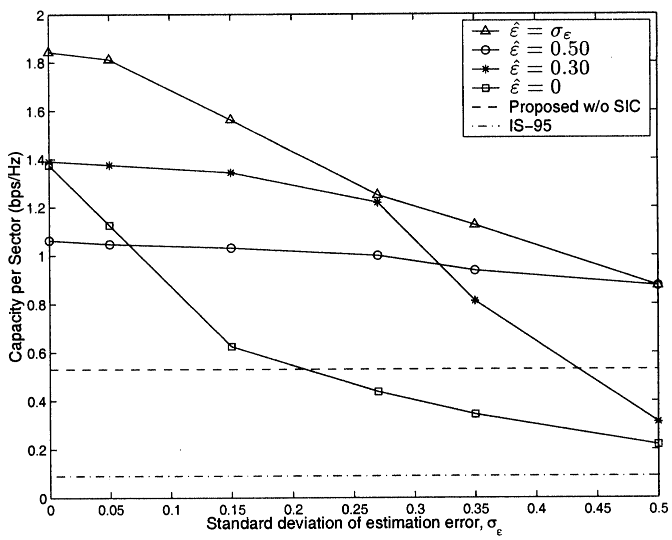

Fig. 8. Spectral efficiencies for SIC at $\mathrm{BER} = 10^{-4}$.

VI. Capacity

In the previous two sections, it has been demonstrated that the proposed SIC system can be designed to be robust to estimation error while simultaneously reducing the amount of OCI occurring throughout the system. Capacity, or equivalently spectral efficiency, is defined in this paper as the amount of traffic that can be accommodated in a fixed bandwidth at a specified BER. The BER specification is taken to be 10^{-4}. Commercial CDMA systems typically deploy sectorized antennas, which further increase capacity by approximately the number of sectors. For generality, in this work, just the capacity per sector is considered, so the achievable capacity per cell would be an integer (often three) multiple of the capacity presented here [15].

Simulation of the proposed SIC system in a low OCI environment ($Eb/No = 10$ dB for all users) resulted in spectral efficiencies summarized in Fig. 8 as a function of the amount of estimation error. The top curve is for the proposed system, using the power control scheme shown in Section III and assuming that the variance of the estimation error is known, i.e., $\hat{\varepsilon} = \sigma_\varepsilon$. Agreeing with intuition and Fig. 5, the $\hat{\varepsilon} = 0.5$ and $\hat{\varepsilon} = 0.3$ curves show that there is a capacity penalty for choosing $\hat{\varepsilon}$ conservatively when there is a small amount of estimation error but that the system capacity remains far more robust as estimation error increases.

If it is assumed that there is no estimation error when computing the power control distribution, as is generally done in the literature, it is seen in Fig. 8 that the capacity is greatly reduced if estimation error does occur. At $\sigma_\varepsilon = 0.5$ (estimation error of approximately 50%), a system with optimum power control still has about twice the capacity of the same system without SIC. On the other hand, a system which falsely assumes that there is no estimation error when designing the power control distribution is far worse off with SIC than with no interference cancellation at all.

The two "curves" which do not change as a function of estimation error are provided for reference. The higher value ($C \approx 0.5$ b/s/Hz) consists of the transmitter in Fig. 1, but the receiver does not perform any interference cancellation. This line serves as a comparison basis to show how much SIC improves the system performance. As can be seen, with optimum power control, SIC adds significantly even as the estimation error grows large. On the other hand, if the estimation error is incorrectly assumed to be zero, for $\sigma_\varepsilon > 0.22$, the system is better off without SIC.

The spectral efficiency for a commercial IS-95 system is shown as a comparison. A typical IS-95 system with three-sector antennas is reported by Qualcomm [16] to allow around 85 users per cell with an average data rate of 4 kb/s in a bandwidth of 1.25 MHz, at an approximate BER of 10^{-3} to 10^{-4}. This corresponds to a spectral efficiency of 0.09 b/s/Hz/sector. It is important to note that this number was quoted for a real-world channel (not flat fading), but it shows the extent to which sophisticated signal processing may be able to improve the capacity of CDMA systems.

VII. Conclusion

In order for SIC to work properly, a power control algorithm which takes inevitable channel estimation error into account is required. It is our contention that fast and appropriately designed power control is a key element in allowing a SIC system to achieve high performance in practice. While this introduces complexity into the system, the potential rewards for doing so are considerable. A general formula for the optimum power control distribution for SIC and conventional CDMA was derived in this paper. Using this distribution, it was shown that channel estimation error up to 50% can be tolerated, while still at least doubling the capacity of a system without SIC. On the other hand, using suboptimal power control results in greatly reduced capacity. In addition to this large gain in capacity, OCI can be simultaneously reduced by around an order of magnitude if users are assigned power levels based on their distance from the base station.

Appendix I
Derivation of Optimal Power Allocation

Defining the total power $P_T = \sum_{i=1}^{K} P_i$ and cancellation efficiency $\eta_k = 1 - \varepsilon_k$, the first equation for SIR $\Gamma_1 = \Gamma_2$ becomes

$$\frac{P_1}{P_T - P_1 + N} = \frac{P_2}{P_T - \eta_1 P_1 - P_2 + N}. \quad (28)$$

Solving for P_2 gives the power for the second user as a function of the power of the first user and the efficiency of cancellation

$$P_2 = P_1 - \frac{\eta_1 P_1^2}{P_T + N}. \quad (29)$$

The subsequent equations are less straightforward. We will find P_3 as an example, and then, the general result for P_k will be presented by induction. The second equation for SIR is

$$\frac{P_2}{P_T - \eta_1 P_1 - P_2 + N} = \frac{P_3}{P_T - \eta_1 P_1 - \eta_2 P_2 - P_3 + N}. \quad (30)$$

Introducing a convenient notation for the total remaining MAI to user k plus their own power

$$V_k = P_T - \sum_{i=1}^{k-1} \eta_i P_i \quad (31)$$

Substituting V_2 into (30) gives

$$\frac{P_2}{V_2 - P_2 + N} = \frac{P_3}{V_2 - \eta_2 P_2 - P_3 + N} \quad (32)$$

which results in a solution for the third user's power which is dependent on the second user's power and cancellation efficiency η_2:

$$P_3 = P_2 - \frac{\eta_2 P_2^2}{V_2 + N}. \quad (33)$$

The remaining equations can be solved in an identical fashion, resulting in a recursive relation

$$P_k = P_{k-1} - \frac{\eta_{k-1} P_{k-1}^2}{V_{k-1} + N} \quad (34)$$

$$= P_{k-1} - \frac{(1 - \varepsilon_{k-1}) P_{k-1}^2}{V_{k-1} + N}. \quad (35)$$

Appendix II
Proof That (12) Is Equivalent to Previous Result for Perfect Cancellation

Here, it is proven that (12) converges to the solution in [9] when $\varepsilon_k \to 0$: Letting $\varepsilon_k = 0\, \forall\, k$, which represents the perfect cancellation case, (12) becomes

$$P_k = P_{k-1} \left(\frac{1}{1 + \gamma} \right) \quad (36)$$

where γ is a target SIR, and $\gamma = \gamma_k\ \forall\, k$, with

$$\gamma_k = \frac{P_k}{I_k} \quad (37)$$

$$I_k = \sum_{i=1}^{k-1} \varepsilon_k P_i + \sum_{i=k+1}^{K} P_i + N. \quad (38)$$

Using this result, (36) becomes

$$P_k = P_1 \left(\frac{1}{1+\gamma} \right)^{k-1}. \tag{39}$$

By definition of the target SIR γ

$$P_1 = \gamma \left(\sum_{i=2}^{K} P_i + N \right). \tag{40}$$

By using the expression in (39) and summing the series, with some additional algebra it can be shown that

$$P_1 = \gamma N (1+\gamma)^{K-1}. \tag{41}$$

Inserting (41) into (39) results in the perfect interference cancellation case derived in [9]

$$P_k = \gamma N (1+\gamma)^{K-k}. \tag{42}$$

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