Research Report

Optimum Vehicle Trajectory Control for Obstacle Avoidance Problem

Yoshikazu Hattori, Eiichi Ono, Shigeyuki Hosoe

Abstract

In this paper, a new vehicle control algorithm for avoiding an obstacle within the shortest possible travel distance is proposed. The algorithm consists of two steps. In the first step, the optimal vehicle trajectory and the corresponding force and moment of the vehicle are determined using second-order cone programming. In the second step, the computed force and moment are distributed into each tire force, while using sequential quadratic programming with a pseudo-inverse matrix for the derivation.

Keywords Vehicle dynamics, Predictive control, Optimum control, Collision avoidance, Preview information, Trajectory control

1. Introduction

Over the last several decades, there has been an ongoing and rapid development in the field of vehicle dynamics controls. This began with anti-lock brake systems (ABS), traction control systems (TCS), and then continued with the development of electronic stability control (ESC) that helps prevent side slip of a vehicle. These systems have dramatically enhanced the stability of vehicles.[2)] Recently, attention has focused on vehicle dynamics integrated management (VDIM). VDIM incorporates a range of functions for vehicle dynamics control and actually performs them by using actuators. The authors have proposed a hierarchical control algorithm (H-VDIM) that is effective for realizing VDIM by using the steering and braking systems[3)] to control each tire force. VDIM enables excellent controllability and stability based on seamless control in a range of situations from ordinary to critical.

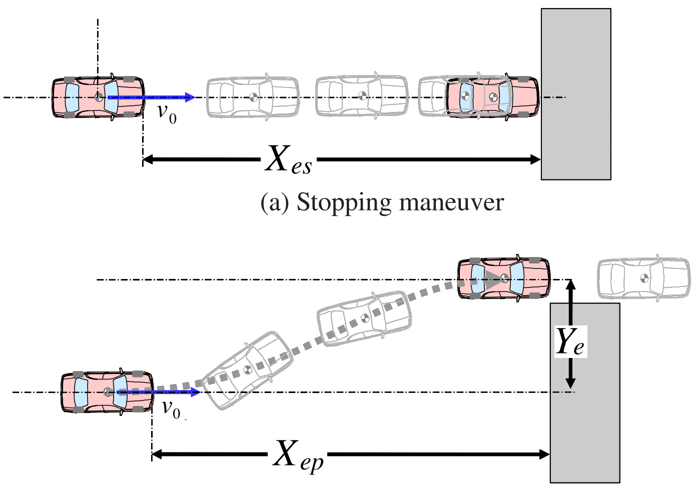
In a previously developed version of VDIM, the target vehicle dynamics presented as the motion that the driver desired were calculated from the driver's operation of the steering, and brake/accelerator pedals. With the evolution of technology, however, it has become possible to obtain preview information such as the driving environment by using a camera, radar and/or other sensors, or by using an information service provided by the infrastructure. As a result of these changes, the next generation of VDIM is expected to bring enhancements to safety, ride comfort and energy efficiency, through a predictive control that utilizes this preview information.

This report discusses an obstacle avoidance problem as a first step toward realizing the next-generation VDIM. A new control algorithm for avoiding an obstacle within the shortest possible distance is proposed. In this approach, the control algorithm is constructed within the framework of H-VDIM, in which the problem is classified into the following two steps. In the first step, the optimal trajectory and corresponding force and moment of the vehicle are determined based on second-order cone programming. The details of this method are described in **Sec. 4**. In the second step, the computed force and moment are optimally distributed to each tire. An effective algorithm was described in our previous papers.[4, 5)] The proposed method is confirmed by simulation.

2. Collision avoidance problem

When an active safety system finds an obstacle in the path of the vehicle, the system either alerts the driver or takes action to avoid the obstacle. In these systems, one of the most important issues involves determining the timing at which the system should become active. Of course, there are other factors that affect the timing, such as the driver's sensibilities, safety margins, and so on. Although, we can apply an absolute index to indicate whether it is physically possible to determine whether it is possible to avoid the obstacle. In this paper, therefore, we consider an avoidance problem that requires us to determine the distance to an obstacle such that, if that distance were any shorter, the vehicle would not be able to avoid the collision. This distance is called the minimum avoidable distance.

The obstacle avoidance problem is depicted in **Fig. 1**. The vehicle has an initial speed v_0, while Y_e denotes the lateral distance that the vehicle has to move to avoid the detected obstacle. This lateral distance is, of course, equal to the lateral size of the obstacle. Note that there are two types of maneuver for avoiding a collision. One is a stopping maneuver,



(a) Stopping maneuver

(b) Passing maneuver

Fig. 1 Obstacle avoidance problem.

where the brakes are applied to make the vehicle just stop before it reaches the obstacle (Fig. 1(a)). The second is a passing maneuver, where the driver steers the vehicle so that it passes by the obstacle such that the lateral speed v_{ye}, yaw angle θ_e, and yaw angle velocity $\dot{\theta}_e$ are all zero, as shown in Fig. 1(b). Also, X_{es} and X_{ep} indicate the distances covered by the vehicle during the stopping and passing maneuvers. Then, the minimal avoidable distance X_e equals $\min\{X_{es}, X_{ep}\}$.

3. Hierarchical vehicle dynamics integrated management algorithm: H-VDIM

The functions that people demand from a VDIM are becoming more and more. As the result, the actuators built into the vehicle are also increasing. Accordingly, the issue of compatibility between algorithms and system configurations is becoming substantial.

A hierarchical vehicle dynamics integrated management (H-VDIM) algorithm has been proposed to satisfy the above requirements (**Fig. 2**). The H-VDIM algorithm consists of the following layers, each connected hierarchically.

[Vehicle Dynamics Control]

This layer calculates the desired longitudinal and lateral forces and yaw moment of the vehicle. The forces and moment are determined so as to achieve the desired vehicle motion while maintaining stability. The desired motion is estimated by the driver's pedal inputs and the steering wheel angle.

[Force & Moment Distribution]

This layer determines the distribution of each tire force, so that the total of the tire forces produces the desired force and moment for the vehicle.

[Wheel Control]

This layer calculates the target values for each actuator, such as those for the engine, braking, steering and others. The target values are determined so as to generate the desired tire forces.

[Actuator Control]

Each actuator system has a corresponding control unit. The braking system, for example, has actuated pressure valves to control the braking torque.

The upper layer outputs the target values to the lower layer, while the lower layer feeds back the results of applying the calculated values. The upper layer then recalculates the target values depending on the feedback. This two-way communication enables each layer to cooperate with the other and maintains higher robustness against the characteristic change of the controlled system and the variable environment. Adding preview information, H-VDIM is enhanced as in **Fig. 3**. We can extend the role of the vehicle dynamics control layer so that it includes trajectory control based on predictive control. The actuators used for vehicle control are strongly coupled through body dynamics that include nonlinear tire characteristics. Thus, the predictive control of the trajectories is generally very complex and difficult. The proposed H-VDIM approach helps to separate the control into simpler tasks and makes on-line control easier.

4. Trajectory control

4. 1 Formulation of problem

The collision avoidance problem described in

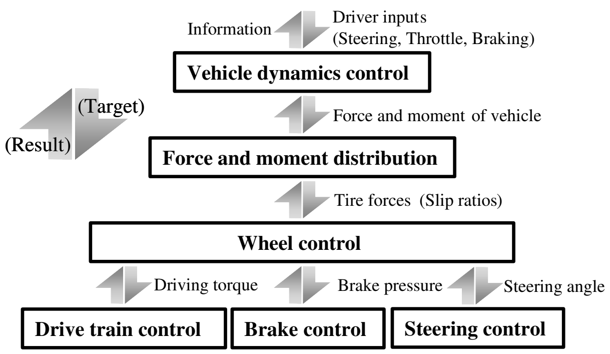


Fig. 2 Hierarchical vehicle dynamics integrated management algorithm: H-VDIM.

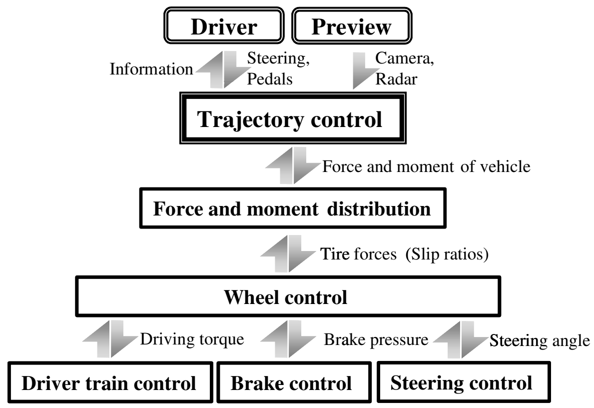


Fig. 3 H-VDIM with preview information.

Chapter 2 is first solved as the trajectory control for a rigid body within the framework of H-VDIM (**Fig. 4**). Where, $F_x(t)$, $F_y(t)$, $F(t)$ are the longitudinal and lateral forces and the resultant force of the rigid body, and $M_z(t)$ is the moment around the Z-axis. These are constrained by a relationship that consists of the friction circles of each tire. Further, m and I_z are the mass and inertia around the Z-axis of the rigid body.

In the stopping maneuver, the minimum avoidable distance is achieved when F_x is minimized, where that minimized F_x produces the maximum deceleration without lateral and rotational motion. In such a situation, the distance X_{es} that the vehicle must travel until it stops can be solved relatively easily. The calculation is described later. For the passing maneuver, however, the optimum control and the avoidable distance X_{ep} are described as the problem for minimizing the performance function J under the following dynamic equations. Further, the problem incorporates the equality constraints expressed by Eqs. (5)-(7), as well as the inequality constraints for the control inputs (Eq. (8)). Note that, the terminal time of the avoidance is unknown.

[Dynamics equations]

$$\dot{\boldsymbol{x}}(t) = \mathrm{diag}\left(\frac{1}{m}, \frac{1}{m}, \frac{1}{I_z}\right)\boldsymbol{u}(t) \quad \cdots\cdots (1)$$

$$\boldsymbol{x}(t) = [v_x(t), v_y(t), r(t)]^T \quad \cdots\cdots (2)$$

$$\boldsymbol{u}(t) = [F_x(t), F_y(t), M_z(t)]^T \quad \cdots\cdots (3)$$

[Performance function]

$$J = \int_0^{T_e} x_1(t)dt = \int_0^{T_e} v_x(t)dt \quad \cdots\cdots (4)$$

[Constraints]

$$\boldsymbol{x}(0) = [v_0, 0, 0]^T \quad \cdots\cdots (5)$$

$$[x_2(T_e), x_3(T_e)] = [0, 0] \quad \cdots\cdots (6)$$

$$\left[\int_0^{T_e} x_2(t)dt, \int_0^{T_e} x_3(t)dt\right] = [Y_e, 0] \quad \cdots\cdots (7)$$

$$C(\boldsymbol{u}(t)) \leq 0 \quad \cdots\cdots (8)$$

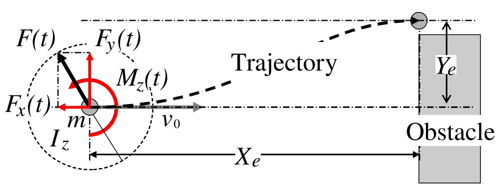


Fig. 4 Collision avoidance problem for a rigid body.

Several sequential approximate solutions have been proposed to solve the above problem as a two-point boundary value problem, but these involve a large amount of calculation. Accordingly, we solve this problem by using a two-step optimization approach. First, the terminal time T_e is assumed to already be known as T'_e, such that the problem can be converted to a discrete time, and that the optimum solutions X'_e, $\boldsymbol{u}'_{opt}$ can be solved. The second step is to find the terminal time T_e minimizing X'_e. The former problem can be formulated as a multi-dimensional optimization problem using convex programming and the latter becomes a simple nonlinear optimization with only one unknown variable T_e. With this separation of solving process, we can obtain the solution with comparative ease.

4. 2 Constraint of force and moment

It is well-known that the longitudinal and lateral forces generated between the tires and the road surface are constrained within a circle called the "friction circle." Here, we consider the range of $F_x(t)$, $F_y(t)$, $M_z(t)$ as the resultant force of each tire. Using the force and moment distribution algorithm shown in Chapter 5, the range of $F_x(t)$, $F_y(t)$, $M_z(t)$ can be determined. The calculated result is shown in **Fig. 5**.

From the figure, we can see that the range of force

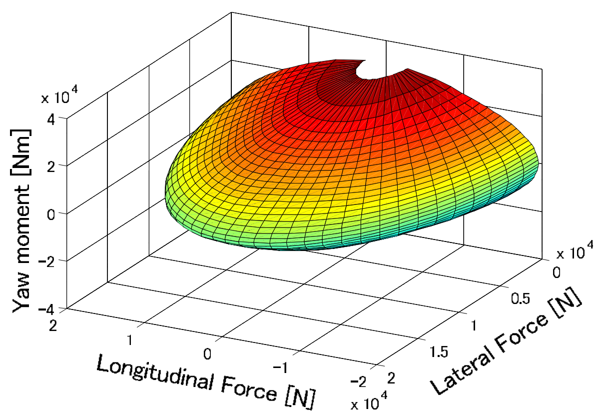


Fig. 5 Constraint on resultant force and moment of the vehicle.

and moment is a convex set and it is approximately represented as

$$C(\boldsymbol{u}(t)) = F_y^2(t) + F_y^2(t) + W_r^2(t)M_z^2(t) - F^2(t) \leq 0 \quad \cdots\cdots (9)$$

where W_r is a constant, and $F(t)$ the maximum force of the vehicle at time t. In this paper, we assume that $F(t)$ is a constant during the control interval, i.e.

$$F(t) = \bar{F} \quad \text{for all } t \quad \cdots\cdots (10)$$

4. 3 Second-order cone programming problem

If a set $\mathcal{K} \subseteq \mathbb{R}^n$ satisfies $\lambda \boldsymbol{x} \in \mathcal{K}$ for $\forall \boldsymbol{x} \in \mathcal{K}$ and $\forall \lambda > 0$, then it is called a cone. A second-order cone in the N-dimensional space R^N is defined as follows.

$$\mathcal{B}(N) \overset{\text{def}}{=} \left\{ \boldsymbol{x} \in \mathbb{R}^N \;\middle|\; x_1 \geq \sqrt{\sum_{j=2}^{N} x_j^2} \right\} \quad \cdots\cdots (11)$$

For a tuple of p vectors $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_p)$, $\boldsymbol{x}_i \in R^{N^i}$ $(i = 1, \cdots, p)$, the set $\mathcal{B}$ defined by

$$\mathcal{B} \overset{\text{def}}{=} \{ \boldsymbol{x}_i \in \mathcal{B}(N_i) \, (i = 1, \cdots, p) \} \quad \cdots\cdots (12)$$

is also called a second-order cone.

A second-order cone program is an optimizing problem involving the minimizing of the linear performance function under the affine and second-order cone constraints over optimizing variables. The second-order cone programming problem is a special case of a semi-definite programming problem. Consequently, a primal-dual interior-point method, which is a powerful numerical algorithm for a semi-definite programming problem, can be applied.[6, 7)]

Now, we will reformulate the problem given by Eqs. (1)-(8) as a second-order cone programming problem. For this, let us assume that terminal time T'_e is already known and let us discretize the problem by sampling period Δ_t. The sampling period should be small enough so that we can assume that the control inputs $\boldsymbol{u}(t)$ are constant during each sampling period. By taking an appropriate integer L, the control interval T'_e can be expressed as

$$T'_e = L\Delta_t. \quad \cdots\cdots (13)$$

Next, let us define discrete values $\{F_x(k), F_y(k), M_z(k), F(k), v_x(k), v_y(k), x(k), y(k), r(k), \theta(k), (k = 0, \cdots, L)\}$, where $F_x(k)$, $F_y(k)$ are the longitudinal and lateral forces, $M_z(k)$ the rotational moment, $F(k)$ the maximum force, $v_x(k)$, $v_y(k)$, the X and Y directional speeds, $x(k)$, $y(k)$ the positions in the X and Y directions, and $r(k)$, $\theta(k)$ the yaw angle velocity and the yaw angle, each at the k-th sampling interval.

The optimizing variables $\boldsymbol{X}$ are defined by the following equations.

$$\boldsymbol{X}(k) = [F(k), F_x(k), F_y(k), W_r M_z(k)] \quad \cdots\cdots (14)$$

$$\boldsymbol{X} = [\boldsymbol{X}(0), \cdots, \boldsymbol{X}(k), \cdots, \boldsymbol{X}(L-1)]^T \quad \cdots\cdots (15)$$

As shown in Eq. (9), $\boldsymbol{X}$ is an element of the second-order cone ($\boldsymbol{X} \in \mathcal{B}$). Further, the performance function J' is described by equation

$$J' = x(L). \quad \cdots\cdots (16)$$

The relationships between $x(k)$, $v_x(k)$ and $F_x(k)$ are expressed by the equations

$$x(k) = x(k-1) + \Delta_t v_x(k-1) + \frac{1}{2m}\Delta_t^2 F_x(k-1) \quad (k = 1, 2, \cdots, L) \quad \cdots\cdots (17)$$

$$v_x(k) = v_x(0) + \sum_{i=0}^{k-1} \frac{\Delta_t}{m} F_x(i) \quad (k = 1, 2, \cdots, L). \quad \cdots\cdots (18)$$

Similarly, v_y, y, r, θ satisfy some linear relations of $\boldsymbol{X}$. In addition, at the terminal sampling time L, $\boldsymbol{X}$ should fulfill the following affine constraints.

$$v_x(0) = v_0 \quad \cdots\cdots (19)$$

$$v_y(0) = 0 \quad \cdots\cdots (20)$$

$$x(0) = 0 \quad \cdots\cdots (21)$$

$$y(0) = 0 \quad \cdots\cdots (22)$$

$$r(0) = 0 \quad \cdots\cdots (23)$$

$$\theta(0) = 0 \quad \cdots\cdots (24)$$

$$v_y(L) = 0 \quad \cdots\cdots (25)$$

$$y(L) = Y_e \quad \cdots\cdots (26)$$

$$r(L) = 0 \quad \cdots\cdots (27)$$

$$\theta(L) = 0 \quad \cdots\cdots (28)$$

$$u_F(k) = F \quad (k = 0, 1, \cdots, L-1) \quad \cdots\cdots (29)$$

Hence, the collision avoidance problem has been formulated as an optimizing problem that has the linear performance function and constraints for variables $\boldsymbol{X} \in \mathcal{B}$ in the second-order cone space. It can be solved efficiently by applying the second-order cone programming method.

4. 4 Search for terminal time

First, we consider the stopping maneuver. If the maximum decelerating force for ideal straight braking is $\bar{F}$, the distance X_{es} needed for the vehicle to stop assuming an initial speed v_0 and the corresponding maneuvering time T_{es} are described by the equations.

$$T_{es}=\frac{v_0}{\left(\frac{\overline{F}}{m}\right)}=\frac{mv_0}{\overline{F}} \quad \cdots\cdots (30)$$

$$X_{es}=v_0T_{es}-\frac{1}{2}\frac{\overline{F}}{m}T_{es}^2=\frac{1}{2}\frac{m}{\overline{F}}v_0^2 \quad \cdots\cdots (31)$$

Next, we consider the passing maneuver. T_{ep} is the time needed for the passing maneuver to avoid the obstacle. In searching for T_{ep} that gives the minimum avoidable distance, we should note that we can restrict the range of the search to

$$T_{ep}\le T_{es}. \quad \cdots\cdots (32)$$

This is because, in the stopping maneuver, full braking in the longitudinal direction always gives the slowest velocity in the X-direction and therefore, if $T_{ep} > T_{es}$, then the stopping maneuver obviously gives a shorter avoidable distance.

On the other hand, T_{ep} should be greater than or equal to the minimum time T_{min} needed for the vehicle to move only in the lateral direction to the position $y = Y_e$ and stop there. T_{min} is attained if the vehicle is maximally accelerated up to $y = Y_e/2$ and maximally decelerated until $y = Y_e$. With the elementary calculation, T_{min} is given by

$$T_{min}=2\sqrt{\frac{m}{\overline{F}}Y_e}. \quad \cdots\cdots (33)$$

Thus, if the shortest avoidance is achieved by the passing maneuver, T_{ep} should lie within the range

$$2\sqrt{\frac{m}{\overline{F}}Y_e}\le T_{ep}\le\frac{mv_0}{\overline{F}}. \quad \cdots\cdots (34)$$

Accordingly, in the search for the terminal time, the existing area of the solution is limited, and the optimal solution can be found with comparative ease by using a general line-search method.

5. Simulation

The results of collision avoidance using the above algorithm were confirmed by simulation. For the simulation, the vehicle was assumed to be a rigid body. In this problem, however, the initial and terminal conditions of the yaw angle and yaw angle velocity are all 0. Therefore, the shortest avoidance is achieved, obviously, when all of the tire forces are used for translational motion without yaw motion as $M_z\equiv 0$. Consequently, it is sufficient to assume the vehicle to be a mass point. Then, the following results of the trajectory control are solved for the mass point. The minimum avoidable distance for the initial speed is shown in **Fig. 6**, where Y_e = 3 [m]. At an initial speed in excess of 18.6 [m/s], the passing maneuver produces the minimum avoidable distance. Then, the minimum avoidable distance approaches the result for pure side movement asymptotically.

The target trajectory calculated by the trajectory control and the result achieved by the force and moment distribution are shown in **Fig. 7**, for an initial speed of 20 [m/s]. The arrows indicate the resultant force for each tire force produced by the distribution control. The distribution control considers the movement of the weight and the nonlinear characteristics of each tire for the weight. Therefore, the target force and moment are not achieved completely. Accordingly, there is some tracking error between the target and the result trajectory. When the data for a general passenger car is used, the standard deviation of the force error is 4.3 [%], the results of the terminal conditions are

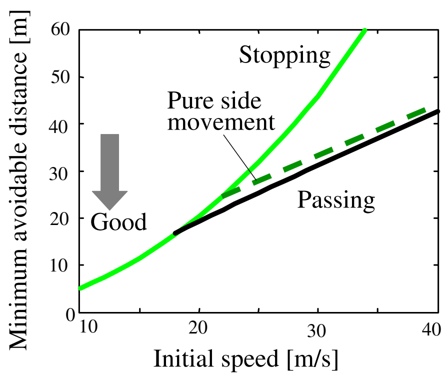


Fig. 6 Minimum avoidable distance.

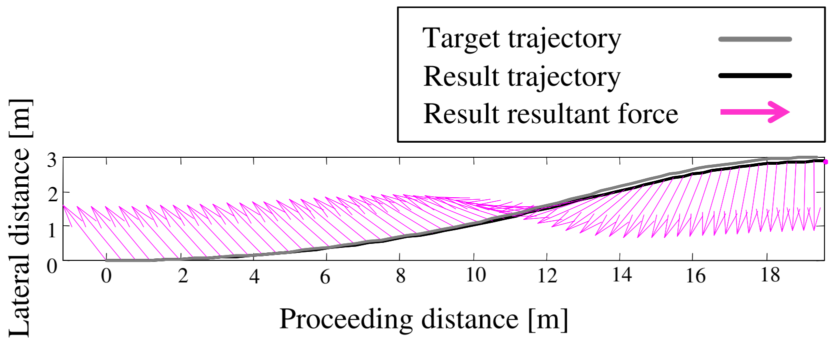


Fig. 7 Control result.

Y_e = 2.88 [m] and $v_y(L)$ = 0.049 [m/s]. All are less than 5 [%]. The difference in the avoidable distance is 0.2 [m] which is almost 1 [%].

6. Conclusion

As a first step toward developing a next-generation VDIM with preview information, a new control algorithm for an obstacle avoidance problem was proposed. Based on the framework of H-VDIM, the problem is separated into two subsystems that can be solved comparatively easily. The vehicle trajectory errors between theoretical optimum solutions and control results by using the proposed algorithm are acceptably small.

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