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## OR PRACTICE

### DETERMINING LOT SIZES AND RESOURCE REQUIREMENTS: A REVIEW

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Inventory planning can become extremely complicated in situations with nontrivial setup times and/or costs. The pervasiveness of this type of problem in actual practice, combined with its computational complexity, has long attracted researchers' attention. Although the most general model formulations still defy optimization when applied to realistically sized problems, many recent contributions are both innovative and promising. This article concentrates on the lot-sizing and capacity dimensions of production planning. We review selected contributions in four major categories, drawing from both practitioner- and research-based literature. We assess the strengths and weaknesses of these contributions, and suggest promising research avenues. We conclude that capacity limitations, as well as other realities of plant environments, such as scrap, demand uncertainties or inaccurate data, are the Achilles heel of lot-sizing research. Directing attention to these complex environments is a challenge that offers great rewards for actual implementation.

Lot sizing when setup times and/or costs are significant has been one of the most renowned issues in production planning, and dates to 1915 with F. W. Harris, who developed the concept of an Economic Order Quantity. When extended to multiple items routed through several capacitated work centers, the problem becomes particularly formidable. Recent studies of the Japanese "just-in-time" system (Krajewski et al. 1983; Ritzman, King and Krajewski 1984) underscore the importance of setups. When setup times can be reduced to modest levels, or when resources can be dedicated to a particular product family, lot sizing is of secondary interest. Even a simple reorder point system, combined with extremely small lot sizes, does quite well. If setup times and costs can be ignored, capacitated lot sizing problems can be solved optimally with linear programming, as evidenced by a wide array of aggregate planning models (Bowman 1963, Hanssman and Hess 1960, Holt et al. 1960, Jones 1967, Taubert 1968). Unfortunately, there are limits on reducing setup times in many

manufacturing environments. Dedicated production lines, group technology and flexible automation often are not feasible alternatives. Large setups suggest a mixed-integer programming formulation (Ritzman et al. 1979), with binary variables representing setups for each item-period combination. However, one survey shows that a typical manufacturer must deal with thousands of items, 30-period time horizons and many capacitated work centers (Anderson and Schroeder 1979). The number of binary variables required to model such situations far exceeds the capability of current optimization codes.

The theory of computational complexity provides a more formal measure of the problem's complexity. Florian, Lenstra and Rinnooy Kan (1980) show that several versions of even a single-item problem are NP-hard when setups are significant and there is just one capacitated work center. This result implies that it is unlikely that any computationally efficient optimal algorithm can be developed. Bitran and Yanasse (1982) go on to show that several single-item cases

*Subject classification* 331 lot sizing and capacity policies, 366 research on more complex environments.

solvable by a polynomial time algorithm become NP-hard as soon as a second item with an independent setup is introduced.

Despite these difficulties, researchers have made progress, which seems to be accelerating, in addressing these problems. Exact algorithms to subproblems have been developed that can be applied routinely to large problems. Disaggregation and aggregation procedures have been proposed to reduce the complexity addressed at any one decision level. More general formulations are being solved heuristically, sometimes with exact algorithms as components. Finally, research has identified the elements in the manufacturing environment that are most crucial to successful production planning.

Interest in capacitated lot sizing with significant setups goes beyond academic curiosity. A casual look at recent issues of a practitioner journal such as *Production and Inventory Management* suggests that practicing professionals are concerned about inventory investment, capacity lost to setups, and on-time delivery. This interest has intensified since 1970, when individuals such as Orlicky (1975, 1976) and Plossl and Wight (1973) launched the "MRP Crusade." Savings reported from implementing better production planning systems have been impressive (Anderson and Schroeder).

Managerial decisions can be represented as a hierarchy of choices, as shown in Figure 1. Some decision parameters are fixed by decisions made at higher levels.

This paper focuses on the second level, the medium-range decisions with a planning horizon of 6–18 months (Buffa and Miller 1979, Peterson and Silver 1979). We do not explicitly consider the interaction between levels, although the interested reader can refer to Hax and Meal 1975, and Ritzman et al. We review the literature only when setups are significant, and classify the work into four major categories. Figure 2 shows our four categories, recognizing two particularly important environmental characteristics—type of demand and resource constraints. Within each category, we review the research according to five criteria: computational effort, generalization, optimality, simplicity, and testing of the proposed methods. We conclude the discussion for each category with unresolved questions and suggestions for further work.

The simplest problem in Figure 2 is the Single-Level, Unconstrained-Resources (SLUR) problem. Products are simple (such as castings and forgings), and are made directly from purchased materials with no intermediate stocking points or subassemblies. Product demands are assessed from customer orders

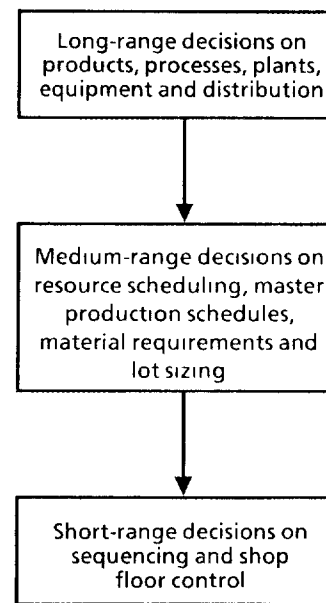


Figure 1. A hierarchy of decisions in a manufacturing organization.

or market forecasts. This situation is often called "independent demand," since an item's requirements do not depend upon the lot-sizing decisions for other items. We assume that there is repeat business for each item, in contrast to one-for-one production of customized products. At the same time, demand volumes are not sufficiently high to justify dedicated resources (which obviates setups). Each product is manufactured by being routed through one or more work centers. Resources are available in abundance, although the time or cost of setups is nontrivial. The ability to solve the SLUR problem has transfer value to more complex problems. The problem also may be sufficiently close to some real manufacturing environments, which have flexible resources (workforce and equipment) and considerable capacity slack.

The Single-Level, Constrained-Resources (SLCR) problem recognizes that the resources are limited, while still assuming independent demand. Capacity requirements introduce an interdependency between items. This problem is of interest for two reasons. First, it is found in several manufacturing settings. Plants with shallow bills of material—such as assembly processes, bottling operations or breweries—are good examples. Second, a common approach to the Multiple-Level, Constrained-Resources (MLCR) problem is to first develop a lot-sizing plan, called a "master schedule," for "end items" (final products). This master schedule is essentially a SLCR problem.

The third category, labeled Multiple-Level, Unconstrained-Resources (MLUR), recognizes more than one level of produced items. End items are made from intermediate items. Intermediate items, in turn, have other intermediate items or purchased items as components. Although end items still have independent demands, all lower-level items experience dependent demand. Dependent demands derive from the lot-size decisions for parents, with parent-component relationships captured by "bills of material." Multiple-level lot sizing is the most common manufacturing environment. Anderson and Schroeder found firms reporting an average of 6 levels in their bills of material. As with SLUR, it is usually unrealistic to assume that resources are unrestricted and available in abundance. However, many approaches to the MLUR problem begin by finding lot sizes without consideration of capacity constraints. The solution is then modified somehow to ease capacity infeasibilities. Structural solutions, such as a cross-trained workforce or considerable capacity slack, make the unlimited resource assumption less critical. In addition, some MLUR algorithms show promise for MLCR problems.

The fourth problem, MLCR, is the most realistic. Given its complexity, the practitioner response has been to subdivide the problem into (1) master production scheduling, (2) material requirements planning, (3) capacity requirements planning and (4) sequencing decisions. As is recognized, decisions to these subproblems must somehow be linked together. The subsequent section on MLCR further reviews current practice, along with applied mathematical programming and simulation methodologies. The section concludes with a discussion of the applicability to real-life problems.

Our review of the literature is limited, for the most part, to work done since 1970. Owing to the sheer volume of contributions, some relevant work is inevitably omitted. This paper updates a 1977 survey (Krajewski and Ritzman) with more recent developments. It also supplements Collier's (1982) survey, which had Material Requirements Planning (MRP) as its primary focus. The production planning review reported in Gelders and Van Wassenhove (1981) is broad, whereas our review is more focused and also provides a baseline for future research.

### 1. Single-Level Lot Sizing without Resource Constraints (SLUR)

The main concern for this class of problems is determining production lot sizes of a single item for several future periods that minimize the sum of setup costs and inventory holding costs over the planning horizon, while satisfying known demand in discrete time periods. There is no interdependency between items, either because of capacity constraints or parent-component relationships. Lot sizing decisions therefore can be made for one item at a time. After F. W. Harris proposed the Economic Order Quantity (EOQ), researchers developed several models of increasing complexity and realism (Tinarelli 1983). For manufactured items, typical models include those of Collier, Gorham 1968, Orlicky 1975 and Wagner and Whitin 1958: Lot-For-Lot, Modified Economic Order Quantity, Periodic Order Quantity (POQ), Least Unit Cost, Part Period Balancing (PPB), and the Wagner-Whitin model (WW). More recent contributions include the Silver-Meal (1973) heuristic, the reformulation of the WW model to account for schedule-change costs (Carlson, Jucker and Kropp 1979), and

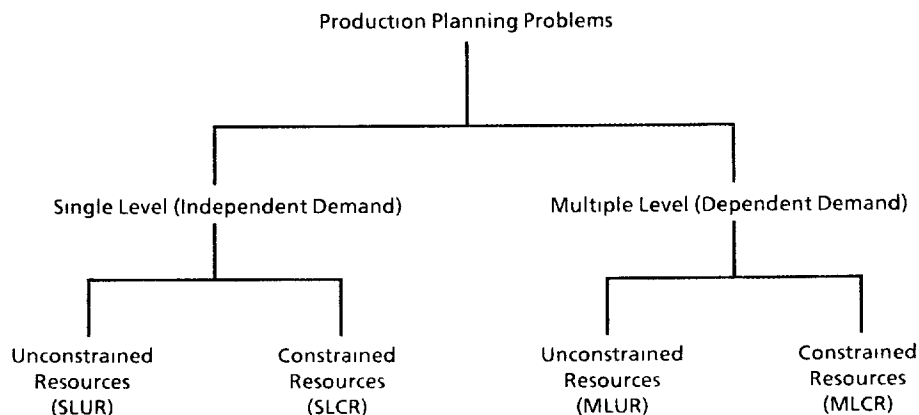


Figure 2. A classification of production planning.

worst-case error bounds for selected heuristics and aggregation procedures (Bitran, Magnanti and Yanasse 1984.)

**Research Issues**

Table I classifies a selection of research work according to five criteria:

- (a) *Computational effort.* This is the amount of computation time required for one solution, compared to other SLUR techniques. Of course, even the most time-consuming SLUR technique poses little difficulty, when compared to the most efficient procedures for MLCR problems.
- (b) *Generalization.* Can the technique be applied equally well to all SLUR problems? For example, EOQ assumes constant demand, whereas WW allows discrete demand patterns.
- (c) *Optimality.* Does the technique guarantee an optimal solution, or is it a heuristic?
- (d) *Simplicity.* Is the technique easily understood and transparent? The WW algorithm has been criticized on this basis, which might partially explain

why a 1974 survey by the American Production and Inventory Control Society found no respondents using it.

- (e) *Testing.* If the technique is heuristic, has it been compared with competing techniques over a wide range of manufacturing settings?

The SLUR problem is not NP-hard, and optimal solutions can be found using the WW algorithm. Limited comparisons of heuristics by Berry 1972, Blackburn and Millen 1980 and Kaimann 1969 suggest that little is lost by using simpler techniques, such as PPB (or its modification with the look-ahead and look-back feature), the Silver-Meal heuristic and POQ. This result implies that little can be gained from future research on the SLUR problem. However, two unresolved questions still remain:

- (a) How should the SLUR problem be solved if there are uncertainties in the environment? These uncertainties can be caused by demand forecast errors, unreliable vendors, scrap losses or inventory record errors. The remnants generated by EOQ lot sizing can be advantageous in an uncer-

**Table I**  
 Selected Research on the SLUR Problem

Description of Work	Classification Criteria <sup>a</sup>					Strength	Weakness
	C	G	O	S	T		
<i>Harris (1915):</i> Economic Order Quantity (EOQ)	Good	Poor	Yes	Fair	N/A	First quantitative formulation for lot sizing	Optimal only for constant demand rate and static cost parameters
<i>Wagner-Whitin (WW) (1958):</i> Optimal lot sizing with varying demand pattern	Poor	Good	Yes	Poor	N/A	Optimal for discrete demand pattern	Compared to other SLUR techniques, more difficult to understand and requires more computation effort
<i>Kaiman (1969):</i> Compares EOQ with WW	N/A	N/A	N/A	N/A	Fair	Tests control for demand variations and setup times	Tests limited to 25 sample problems
<i>Berry (1972):</i> Compares EOQ, POQ, PPB and WW	N/A	N/A	N/A	N/A	Fair	Tests control for demand variations and EOQ/average demand ratio	As above
<i>Silver and Meal (1973):</i> Proposes heuristic based on average cost per period	Good	Good	No	Good	Fair	Simplicity and computational effort make it attractive for capacity-constrained problems	As above
<i>Carlson, Jucker and Kropp (1979):</i> Propose heuristic to dampen system nervousness	Poor	Good	Yes	Poor	N/A	Revise WW to include schedule-change cost	Same as WW
<i>Blackburn and Millen (1980):</i> Compare PPB, Silver-Meal and WW	N/A	N/A	N/A	N/A	Good	Test in rolling schedule environment	Tests assume perfect demand forecasts

<sup>a</sup>C = computational effort, compared to other SLUR procedures, G = generality to all SLUR problems, O = optimality, S = simplicity in understanding, T = thorough testing, if heuristic.

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tain environment, at least if on-time delivery is considered. Whybark and Williams (1976) address some of these issues, although more research would help.

- (b) The most frequent applications of SLUR techniques appear to be for the MLUR and MLCR problems. How effective are they in situations for which they were not developed or tested? Even the best SLUR technique might compare quite unfavorably if evaluated as a module of a MLCR solution procedure.

## 2. Single-Level Lot Sizing with Resource Constraints (SLCR)

Lot sizing with resource constraints and independent demand items has also been of research interest for several years. Considerable work has been completed on the economic lot scheduling problem (ELSP), which allows for multiple items but is limited to a single resource and constant demand rates. Elmaghraby (1978) offers an excellent literature survey of this research, while Schweitzer and Silver (1983) point out pitfalls in some of the formulations already offered. Haessler (1979) has provided a simple and efficient heuristic for the ELSP problem.

One of the earlier formulations for *varying* demand rates was by Zangwill (1966), who formulated dynamic programming algorithms for facilities in parallel or in a serial order. However, only very small problems could be solved by these methods. Relaxing the assumption of constant demand rates has also been the primary motivation of several recent heuristic methods. The basis of most of these methods has been the Silver-Meal heuristic, which essentially combines production for several periods into one lot as long as average cost per period keeps decreasing. These methods have been described as the Eisenhut (1975) heuristic, the "extended Eisenhut" (Lambrecht and Vanderveken 1978), the "further extended Eisenhut" (Dogramci, Panayiotopoulos and Adam 1981) and the Dixon and Silver (1981) heuristic. Karni and Roll (1982) also extended the heuristic for single-item lot sizing (Karni 1981) to constrained problems, and report good results.

The generality of these approaches presents several difficulties. First, only one resource can be capacitated. This limitation is a severe restriction in many plants that contain several capacitated work centers. Second, setups are introduced only as a cost, rather than also creating downtime at the capacitated work centers. Ignoring setup times can distort the capacity con-

straints. In the short run, capacity levels are often fixed. The question becomes how to best use existing capacity when setups are a significant element of capacity. Third, these approaches do not permit capacity to be adjusted, such as by overtime, subcontracting or changing the number of shifts. Fourth, some problems are correctly modeled only when the decision variables are changeovers rather than setups (Glassey 1968, Lasdon and Terjung 1967), or lot-sizing and sequencing choices are made simultaneously. Producing an item in two consecutive periods does not necessarily require a second setup. This requirement depends on whether the item being produced at the end of one period is the same one starting off the next period. These four difficulties are better handled by the work traced to Manne in 1958. He proposed a formulation in which binary setup variables are replaced by binary variables for each possible production sequence. Although this approach significantly increases the number of decision variables, it imposes only a modest number of constraints. The great advantage, however, is that the model can be solved with linear programming rather than mixed integer linear programming. The number of fractional binary variables so obtained is small. Having to round the fractional values to 0 or 1 means that Manne's approach does not guarantee optimal solutions. However, simulation tests (Dzielinski, Baker and Manne 1963) showed it compares favorably to simply applying an EOQ model. Subsequent work by Bahl 1983, Bahl and Ritzman 1984a, Dzielinski and Gomory 1965, and Lasdon and Terjung has improved the computational efficiency of Manne's formulation through decomposition, column generation techniques, and restrictions on the number of production sequences.

Two final heuristics for SLCR problems qualify as the most general to date. Aras and Swanson (1982) provide a heuristic algorithm that incorporates both lot-sizing and sequencing decisions for the single-resource case. The capacity constraint accounts for setup time. Daniels (1983) goes one step further by allowing several capacitated work centers, as long as they are arranged serially. Setup times can be sequence dependent. Another feature is that the method correctly captures idle time caused by waiting on work from upstream work stations.

### Research Issues

Table II classifies some of the research on the SLCR problem. Although no available optimization technique will solve realistically sized problems, it is clear

**Table II**  
 Selected Research on SLCR Problem

Description of Work	Classification Criteria <sup>a</sup>					Strength	Weakness
	C	G	O	S	T		
<i>Manne (1958)</i> : Lot sizing with resource constraints	Poor	Good	No	Fair	Poor	Novel problem formulation amenable to linear programming	Large number of variables; nonoptimal solution
<i>Dzielinski, Baker and Manne (1963)</i> : Simulation tests of using EOQ vs. Manne's model	N/A	N/A	N/A	N/A	N/A	Tests Manne's approach	Tested with only one real-life problem
<i>Dzielinski and Gomory (1965)</i> : Applies Dantzig-Wolfe Decomposition to Manne's model	Fair	Good	No	Poor	Fair	Larger problems are amenable to solution	Lacks simplicity in understanding
<i>Lasdon and Terjung (1967)</i> : Applies Column Generation method to Manne's model	Good	Good	No	Poor	Fair	Computational efficiency over earlier approaches; better bounds on solution	Lacks simplicity in understanding
<i>Newson (1975)</i> : Heuristic method for lot-sizing problem	Good	Fair	No	Fair	Fair	Computational savings	Not tested for multiple work centers
<i>Haessler (1979)</i> : Simple heuristic for ELSP	Good	Poor	No	Good	Poor	Simple and efficient	Assumes constant demand and single resource
<i>Dixon and Silver (1981)</i> : Single-resource heuristic	Good	Poor	No	Good	Good	Very good solutions with little computational effort	Single resource; setup times assumed negligible
<i>Aras and Swanson (1982)</i> : Single-resource heuristic to expanded problem	Good	Good	No	Good	Fair	Incorporates sequencing decisions; setup times apply to capacity constraints	Single resource and limited testing
<i>Bahl and Ritzman (1983)</i> : Heuristic method for Manne's formulation	Good	Good	No	Good	Fair	Additional computational savings	Limited testing
<i>Bahl (1983)</i> : Heuristic method for Manne's formulation	Good	Good	No	Poor	Fair	Efficiency gains over column generation	Limited testing
<i>Daniels (1983)</i> : Multiple-resource heuristic to expanded problem	Good	Good	No	Fair	Fair	Incorporates sequencing decisions; sequence-dependent setups are part of capacity constraints; multiple resources	Heuristic method, and lacks transparency in understanding

<sup>a</sup>C = computational effort, compared to other SLCR procedures, G = generality to all SLCR problems, O = optimality, S = simplicity in understanding, T = thorough testing, if heuristic.

that progress is being made on computationally feasible heuristics for a widening spectrum of SLCR problems.

In contrast to the situation with SLUR problems, the following future research opportunities on SLCR problems are more wide ranging and offer larger payoffs:

(a) Develop ways to enrich the single-resource heuristics that assume setups without any loss of capacity, perhaps through some type of iterative solution procedure.

(b) Intensify research efforts on more general SLCR problems that recognize multiple capacity constraints, the interaction between lot sizing and sequencing, and uncertain events (such as forecast errors or process yield losses).

(c) Identify the manufacturing environments in which the departure from optimality is greatest when applying the less general heuristics.

(d) Ascertain the usefulness of SLCR heuristics for developing master schedules for MLCR problems.

(e) Assess how SLCR heuristics can be applied to MRP environments as a replacement of the cur-

rent Capacity Requirements Planning (CRP) module.

- (f) Reduce problem dimensionality by aggregation/disaggregation, particularly in terms of the number of items, work centers, and time periods.
- (g) Intensify efforts to test existing and new heuristics as to the quality of their solutions and their computational efficiency. Ascertain whether some have a comparative advantage in certain manufacturing environments.

### **3. Multiple-Level Lot Sizing without Resource Constraints (MLUR)**

Multiple-level lot sizing introduces dependent demands: the lot-sizing and timing decisions for items at one level in the product structure depend on the decisions made for their parents (successors). Most work on MLUR problems assumes deterministic demands for the "end items," which are the final products having no parents. Research to date also focuses mainly on the case in which each item has at most one parent; commonality is disallowed. One of the first approaches to this problem is Schussel's Economic Lot Release Size model (Schussel 1968), which allows for only one end item. His heuristic begins at the lowest level and proceeds up the product structure to the end item. Once we find the lot size and total cost for the end item, we go down the product structure imposing integer requirements. The algorithm iterates between these two steps until two successive end-item lot sizes and associated costs are within a prespecified percentage of one another. His conclusion that a component's lot size should be an integer multiple of its parent was reinforced by the findings from Taha and Skeith's (1970) enumeration procedure and subsequently proved by Crowston, Wagner and Williams (1973) when there is instantaneous production, constant end-item demand rates, and an infinite planning horizon. Dynamic programming formulations have also been developed, such as those by Zangwill (1969) for the single end-item case and by Crowston, Wagner and Williams, and Crowston and Wagner (1973), for multiple end items.

The computational time required by dynamic programming algorithms increases exponentially with the number of items or number of time periods. This feature has redirected interest toward heuristic algorithms. Crowston, Wagner and Henshaw (1972) evaluate three heuristics: a single-pass, multi-pass and modified multi-pass procedure. They found that the modified pass heuristic generates solutions within

0.6% of optimality. In other words, the difference in total costs generated by applying this technique to sample problems, when compared to the cost of optimal solutions, averaged only 0.6% above the cost for optimal solutions. McLaren (1976) tested several SLUR lot-sizing rules for a single-pass procedure, including the WW, POQ, Modified EOQ, and Adjusted Order Moment rules. The "adjusted" rules revise setup costs between successfully higher adjacent levels, based on the ratio of expected time between orders at adjacent levels of the product structure. The effect is to increase parent item lot sizes. McLaren's heuristic, though very simple, obtained results close to optimality. The best performers were the WW adjusted rule, which resulted in solutions within 0.8% of optimal, and the adjusted Order Moment rule, which averaged 2.2% of optimality. This notion of simple procedures to modify cost parameters, which can then be incorporated in SLUR heuristics using a single-pass procedure, has been extended by Blackburn and Millen (1982). By making cost parameters a function of product structure characteristics, their procedure resulted in costs within 1.0% of optimality, although deviations from optimality increased with the number of product levels.

Another promising development is by Afentakis, Gavish and Karmarkar (1984), who reformulate the MLUR problem to facilitate Lagrangian relaxation methods. The bounds so obtained are then used for an efficient branch-and-bound algorithm. This exact procedure was tested for problems ranging from 5 to 50 items, with a 12-period planning horizon. Maximum CPU time on the IBM 3032 was less than one minute and seemed to increase only slightly faster than problem size. This approach should be able to handle at least moderately-sized problems and also can serve as a benchmark for testing heuristic procedures.

All of the MLUR approaches discussed so far are limited to problems in which there is no commonality. This assumption is unrealistic for many plant environments. Steinberg and Napier (1980) were the first to consider commonality by proposing a formulation that is a constrained generalized network with fixed charge arcs and side constraints. This work brings out the importance of commonality and serves as a benchmark for evaluating heuristic algorithms. However, the model is solved with a mixed integer linear programming code, which limits its application to small problems. Graves (1981) also presents a clever multi-pass heuristic for MLUR problems with commonality. His improvement algorithm applies WW to each item in such a way that the current schedule is revised in



iterative fashion until no further improvements are found. He tested the algorithm for 250 test problems, with optimal solutions found in approximately 90% of the cases and the average departure from optimality being less than 0.5%. However, the tests were limited to 5-item problems without commonality. His conjecture is that computational effort will be proportional to a single-pass heuristic, although this result has yet to be confirmed.

### Research Issues

Table III helps summarize some of the past research on MLUR problems. Significant progress has been made on MLUR problems when commonality is disallowed. Exact and near-optimal heuristics are now available. However, some serious gaps in our knowledge base need attention:

- (a) Work is just beginning on problems allowing for commonality. We are not yet sure whether single- or multi-pass heuristics such as Graves' will be near-optimal and computationally feasible for large problems. Perhaps optimization procedures can even be developed for the more general MLUR problem. Afentakis and Gavish (1986) extended their approach to general product structures, but restricted testing to three end items with 15 stages since the new procedure requires more computer time as the number of end items increases.
- (b) Determine the robustness of MLUR procedures when applied to uncertain plant environments, where there are forecast errors, lead time delays, inaccurate inventory records, rolling planning horizons and scrap losses.
- (c) Adapt MLUR procedures to the situation found at most plants (Anderson and Schroeder), where there is a mix of customized items made-to-order and standard items made-to-stock.
- (d) There is a need now to do more comparative testing of currently available algorithms. Tests so far have been limited to a few problems, with little attention paid to experimental factors such as problem size, number of product levels, number of components per parent, commonality and cost structures.
- (e) Reduce problem dimensionality by aggregation and disaggregation techniques, making optimization techniques more attractive.

Undoubtedly the research questions of greatest importance deal with the assumption of unlimited capacity. Having unlimited capacity normally is a major

departure from reality. MLUR procedures implicitly recognize one portion of resource requirements through setup cost parameters. However, there is no assurance that the sum of processing and setup times for MLUR solutions will be within each period's capacity limitation. This is particularly true when setup costs (and therefore lot sizes) are large and there are multiple work centers. Large lot sizes create erratic demands for resources. Collier (1980) shows that even if capacity levels can be varied at will to accommodate requirements, the capacity change costs can be prohibitive. The fundamental question, therefore, is how MLUR procedures can be made sensitive to capacity limitations. It may well be possible that the underlying structure of some existing MLUR heuristics can be transplanted to capacitated environments.

### 4. Multiple-Level Lot Sizing with Resource Constraints (MLCR)

Models and solution procedures indeed become formidable when multi-level problems are complicated by capacitated facilities. Capacity constraints introduce a new question—when to adjust capacity with overtime, subcontracting and variations in workforce size. They also make lot-sizing choices strongly interdependent with sequencing (or “priority planning”) decisions. A casual look at practitioner-oriented literature such as the *Production and Inventory Management* journal and *APICS Conference Proceedings* strongly suggests that most real-life environments are MLCR problems. This section begins by reviewing these reports on current practice, which in many ways has been more pioneering than research on MLCR problems. We then discuss recent research in the areas of simulation and mathematical programming.

#### Current Practice

Reports on current practice center on two computer-based systems: Material Requirements Planning (MRP) and Optimized Production Technology (OPT). The OPT software (Creative Output, Inc.; Goldratt 1982) is quite recent and seems to be the first package using quantitative techniques to make simultaneous decisions involving lot sizing, capacity and sequencing. The proprietary nature of this heuristic approach has so far inhibited research on its performance. OPT is currently being tried at several companies, which is an indirect test of its merits. It is interesting that practicing professionals at such plants are willing to make such a sizeable investment in this “black box,” even though it has less transparency than the various

**Table III**  
 Selected Research on MLUR Problem

Description of Work	Classification Criteria <sup>a</sup>					Strength	Weakness
	C	G	O	S	T		
<i>Schussel (1968)</i> : Proposes Economic Lot Release Size model	Fair	Fair	No	Good	Poor	Introduces notion of multi-pass heuristic	One end item, no commonality, and untested
<i>Zangwill (1969)</i> : Dynamic programming model with back-logging	Poor	Poor	Yes	Poor	N/A	Provides optimal solution to specialized MLUR subproblem	Limited to one-parent, one-component system and excessive computational requirements
<i>Crowston, Wagner and Henshaw (1972)</i> : Compares single, multi-pass and modified multi-pass procedure	Fair	Fair	No	Fair	Fair	Demonstrates near optimality of heuristic procedures	No commonality; limited set of test problems
<i>Crowston and Wagner (1973)</i> ; <i>Crowston, Wagner and Williams</i> . Dynamic programming models when demands are at a constant rate, or vary	Poor	Fair	Yes	Poor	N/A	Provides benchmark for testing subsequent heuristics	No commonality; excessive computational requirements
<i>McLaren (1976)</i> : Develops cost parameter adjustment rules for single-pass application of SLUR heuristics	Good	Fair	No	Good	Good	Simplicity in use for single-pass procedures	Commonality not allowed
<i>Steinberg and Napier (1980)</i> : Constrained generalized network formulation	Poor	Good	Yes	Poor	N/A	Allows for commonality	Excessive computational requirements
<i>Afentakis, Gavish and Karmarkar (1984)</i> : Branch-and-bound algorithm using Lagrangian relaxation	Fair	Good	Yes	Poor	Good	Optimal solutions for at least moderately sized problems	Excessive computational requirements
<i>Graves (1981)</i> : Multi-pass heuristic allowing for commonality	Fair	Good	No	Fair	Fair	Allows for commonality and computational time may be attractive	Untested for larger problems with commonality
<i>Blackburn and Millen (1982)</i> : Extends McLaren's ideas on modifying cost parameters	Good	Fair	No	Good	Good	Allows a single-pass procedure with simple SLUR rules	Commonality not allowed. Departures from optimality increase with product levels

<sup>a</sup> C = computational effort, compared to other MLUR procedures, G = generality to all MLUR problems, O = optimality, S = simplicity in understanding, T = thorough testing, if heuristic.

algorithms reported here. Perhaps transparency is less of an issue than having techniques that are truly isomorphic with MLCR problems.

Certainly most attention during the last decade has been given to MRP systems, which have been widely implemented. MRP is a hierarchical system that provides key information to planners for making (a) master production schedule (MPS) decisions (end item lot sizes), (b) lot-sizing choices for lower-

level-produced components and purchased items, (c) capacity decisions, and (d) "priority" or sequencing decisions for open orders. It also can shed light on future revenues, production costs, inventory levels and capacity needs. A variety of software packages are available for these purposes (Bourke 1980, IBM 1972).

*Master production scheduling (MPS)*. The major input to MRP, along with bills of material and inventory status files, is the Master Schedule. The most

common suggestions are to freeze a portion of the MPS and also to test out MPS alternatives with "resource requirements planning." This rough-cut-planning device estimates time-phased capacity requirements at key resource centers, using load profiles for each end item (Orlicky 1975, Smolens 1977). The accuracy of these projections is an open question, partially because they ignore component lot sizing and inventory positions. Most publications on MPS deal with its importance or general relationship to other decision areas (Darnton and Garton 1976; Kohankie, Waterbury and Morency 1976; Malko 1976; and Maranka 1976). Berry, Vollmann and Whybark 1979 do provide an excellent summary of current MPS practice at selected plants, while pointing out the trade-offs involved. Miller (1977) has also argued that safety stocks might best be positioned at levels below end items when there is considerable demand uncertainty and item commonality. With these exceptions, there is little agreement on how to decide on end-item lot sizes. Furthermore, none of the SLCR procedures of Table II have been considered, even though they seem quite applicable.

*Lower-level lot sizing.* In our terminology for the MLUR problem, MRP uses a top-down, single-pass heuristic to develop the "material requirements plan" for all lower level items. The resulting "planned order releases" for an item projects its lot-sizing schedule for several periods into the future. The average firm uses around 30 periods in its time horizon (Anderson and Schroeder). For each item, the planner must pre-specify a SLUR rule, a lead-time offset and any safety stock provision. The literature offers little guidance on how to make these selections, although survey results do show a preference for larger lot sizes at the top and bottom levels of the product structure. Thiesen (1974) found that most firms use fixed lot sizes at the end-item level and SLUR heuristics such as PPB or POQ at the lowest levels of produced components. Items at intermediate levels tend to have a lot-for-lot rule. Wemmerlov (1979) reports similar findings. One interpretation is that setup costs for intermediate items are insignificant. The other is that excess inventory at intermediate levels provides little additional buffering and efforts are made not to "pyramid" inventory at all product levels.

*Capacity decisions.* MRP generates the material requirements plan oblivious to capacity constraints. However, planned order releases and information on open orders can be translated into workload profiles, typically for 8–12 weeks ahead. This procedure is called "capacity requirements planning" (CRP). It

leaves to the planner's judgment the question of how to cope with requirements that exceed available capacity. This judgmental process has been termed "closed-loop" planning, since one option is to somehow revise the planned lot sizes to better meet capacity limits. Other options are to change the MPS, subcontract or authorize overtime. CRP packages are classified as either infinite or finite loading (Burlingame 1974; IBM; LaRobardier 1974), with infinite loading being most prevalent. Finite loading systems assign production orders to time periods so that capacity is not exceeded. This assignment is done by attempting to reschedule lower priority orders into earlier or later periods. Most articles support an infinite loading system (Paul 1975; Plossl and Wight), although one company has experienced success with a capacity smoothing approach (Graziano 1974).

*Priority decisions.* Lot sizing with capacity limitations necessarily introduces the question of how to sequence orders already released (Tatsiopoulus and Kingsman 1983). If an open order for an item is delayed because of capacity problems, a parent order might not be released as planned. Most plants sequence jobs using rules based on due dates (Orlicky 1975). Simple mechanisms that are available for updating due dates alert planners when an open order must be expedited or can be delayed.

*Simulation.* Several simulation models for studying MLCR problems have been developed during the last decade. One of the most recent versions allows the researcher to experiment with any of the environmental or system variables shown in Table IV (Krajewski et al. 1982a, b). This version has been validated at real plants, and its predictions have correlated with current plant performances. A wide range of lot-sizing, capacity and sequencing strategies can be analyzed, as can their interaction with various factors in the plant environment. The simulator is currently sized to handle 200 items, arranged in any product structure configuration, and up to 200 work stations. Each item has a preassigned routing through one or more work stations, which is maintained throughout the simulation. Each work station is capacitated.

Early studies focused on the relative performance of various SLUR lot-sizing heuristics in a MRP system, as well as lead time and priority choices (Berry 1978; Biggs 1979; Biggs, Goodman and Hardy 1977). The last two papers used the same lot-sizing rule for all items, whereas Collier (1980) tested 25 combinations of rules in two-level product structures. These researchers found that (1) there is a significant interaction between lot-size and priority rules, and (2) an

**Table IV**  
 Environmental and System Variables for MLCR Problem

<i>System Variables</i>	<i>Environmental Uncertainties</i>
Type of system (MRP, ROP, lot requirements planning, KANBAN)	Scrap and rework
Type of lot sizing rule and magnitude of lot sizes	Forecast errors
—End items (MPS)	Frequency of rush jobs
—Components and purchased items	Vendor reliability
Capacity changes	—Shipment quantities
Worker assignment rules	—Lead time variability
Priority and sequencing rules	Inventory record inaccuracies
Order splitting	Service parts
Stability of MPS	Delays for material handling and inspection
<i>Product Characteristics</i>	<i>Shop Characteristics</i>
Number of items and BOM shape	Repeatability and degree of equipment specialization
—end items	Shop emphasis
—intermediate items	—fabrication
—purchased	—assembly
Cumulative lead time	Routing complexity
Number of BOM levels	—job shop
Commonality	—flow shop
Number of components per parent	Shop size (number of work centers)
Number of specials	Length of routings
Length of vendor lead times	Gateway operations
Average and variability in independent demands	Number of “breaks” or storage points in process
<i>Buffers</i>	Equipment failures
Safety stock and safety lead times	Alternate routings and preemption
Capacity slack	Size of setup vs. processing times
—total	Length of work week
—variability between work centers	
Worker flexibility	

EOQ rule performs well, compared to other SLUR rules, in multilevel systems. When there are capacity limits, the remnants avoided by almost all SLUR and MLUR heuristics become an asset in terms of on-time delivery of end items. Bott and Ritzman (1983) tested 7 potential factors causing irregular capacity requirements, which in turn hurts inventory and on-time delivery performance. They found the major culprits to be unstable master schedules, specialized resources and a large quantity of product levels. Harl and Ritzman (1985) tested a capacity-sensitive lot-sizing heuristic that makes selected changes to the material requirements plan. They found that it improved customer service by an average of 32% and to be particularly useful when large lot sizes must be used. They also showed that resource requirements planning at the MPS level is an insufficient tool for coping with capacity problems. More recent experiments (Krajewski et al. 1983) on 16 representative plants tested the importance of 7 clusters of factors that encompass most of the individual factors in Table IV. The most important factors were the *magnitude* of lot sizes (not

the type of SLUR rule), setup times, scrap losses, workforce flexibility, product structure, degree of customized production and capacity slack. Less important factors, at least over the range of parameter settings selected by a panel of managers, were equipment failures, forecast errors, order splitting, length of routings, routing patterns (flow versus job shop), vendor lead times and vendor quality. One implication of these studies is that lot-sizing strategies depend strongly on management’s ability to “shape” the environment—e.g., reducing setup times or increasing worker flexibility.

Comparisons have also been made between the systems now used in practice. Ritzman and Krajewski (1983) have shown that MRP has a distinct advantage over a reorder point (ROP) system when there are more product levels and larger lot sizes. The Japanese “just-in-time” systems such as KANBAN has been shown to perform no better than ROP or MRP (Ritzman, King and Krajewski 1984). KANBAN’s virtue is that it is a simple catalyst for spotting areas for shaping the shop environment. It pays off

only when setup times can be made insignificant. When this happens, lot sizes can be reduced to very small quantities, which also simplifies capacity and sequencing decisions.

### Mathematical Programming

Current MRP literature offers no definitive answers on (a) how to develop the master schedule, (b) how to preselect each item's lead time, SLUR lot-sizing rule, and safety stock, and (c) how to revise the MPS and material requirements plan, recognizing capacity and sequencing constraints. Two studies (Bahl and Ritzman 1984; Bitran, Haas and Hax 1983) show that mathematical programming approaches can lead to significant improvements in certain manufacturing environments.

It is only recently that mathematical programming models have been proposed (Haehling von Lanzenhauer 1970; Krajewski and Ritzman). Due to problem dimensionality and the difficulty in handling setups, these models make simplifying assumptions. Some models are limited to very specialized product structures. Lambrecht and Vandervecken (1979) consider a single end-item structure in which each component in turn has one parent and one component. Each item is produced at only one work center, and only the one devoted to the end item is capacitated. Gabbay (1979) retains the same assumptions except that there can be several end items, and each work center is capacitated. To do so, he makes a restrictive assumption on processing times.

More general MLCR formulations are also available. Bitran, Haas and Hax extend single-stage hierarchical production planning (Hax and Meal 1975) and propose a two-stage model by aggregating products into product types and product families and by aggregating parts into part types. Billington, McClain and Thomas (1983) propose another intriguing way to reduce dimensionality. Along with allowing capacitated work centers and a generalized product structure, they treat some lead times as decision variables. Their approach is similar to the OPT philosophy (Creative Output, Inc.) in that they assume that only a few work centers are capacitated and that these are easily identified. Their simplifying scheme eliminates from a mixed integer programming formulation all lot-sizing variables except those associated with items produced at capacitated centers or those sharing a common component that is produced at a capacitated center. After their approach has solved the simplified formulation it assigns all eliminated items lot-for-lot production.

Although many formulations such as Bitran, Haas and Hax assume that each item is produced at just one work center, Bahl and Ritzman (1984b) allow multiple operations per item. Their heuristic procedure simultaneously makes lot-sizing and capacity decisions, with no limitations on product structures. Their limiting assumptions are (1) that lead times can be predetermined, and (2) that lot-for-lot production (or any production rates prescribed by prespecified SLUR heuristics) occurs at all levels except the end-item and lowest levels. This second assumption is in line with reports on current practice (Thiesen, Wemmerlov).

### Research Issues

Table V demonstrates the embryonic nature of MLCR research. The generalized MLCR problem is much more complex than the SLCR, which has been shown to be NP-hard even in simple cases. No optimizing techniques are available for realistically-sized problems. Simple principles—such as making component lot sizes an integer multiple of parent lot sizes or avoiding, when there is any starting inventory, solutions where production occurs—have not been established. There has been progress across a variety of manufacturing environments in developing new heuristic procedures and evaluating those currently used in practice. MLCR models are the most promising areas for further research. We outline but a few research directions:

- Develop optimization techniques, such as decomposition and Lagrangian relaxation, for special cases of the MLCR problem. This research could provide benchmarks for evaluating heuristics and even components of heuristic problems coping with more general MLCR problems.
- Introduce new research questions through empirical studies that articulate current practice. These studies could also help establish which environments are most attractive for alternate production planning techniques.
- As Ritzman et al. (1979) point out, work is needed on hierarchical procedures. For many MLCR problems, it is unlikely that lot-sizing, sequencing and capacity decisions can be made simultaneously. This situation suggests sequential decision making, with feedback loops.
- Assess how the factors in Table IV impact the performance of heuristic procedures.
- Determine the robustness of MLUR problems in capacitated environments.

**Table V**  
 Selected Research on MLCR Problem

Description of Work	Classification Criteria <sup>a</sup>					Strength	Weakness
	C	G	O	S	T		
<i>Haehling von Lanzenhauer (1970)</i> : Formulates MLCR problem	Poor	Fair	Yes	Fair	N/A	One of first statements of problem	Computationally feasible solution procedure not available. Setups not considered
<i>Berry (1972); Biggs (1979); Biggs, Goodman and Hardy (1977); and Collier (1980)</i> : First simulation studies of SLUR heuristics, lead time and sequencing rules in MLCR environment	Good	Poor	No	Good	Poor	Identifies interactions between rules. Considers SLUR heuristics easily implemented in practice.	Limited range of environmental settings from Table IV
<i>Lambrecht-Vandervecken and Gabbay (1978-1979)</i> : Formulation for serial production environment	Poor	Poor	Yes	Poor	N/A	Provides an alternative to current MRP practice	Limited to special product structures
<i>Berry, Vollman and Whybark (1979)</i> : Summarizes current MPS practice	N/A	N/A	No	N/A	N/A	Provides richer understanding of real problems	Limited to MPS component of MRP
<i>Billington McClain and Thomas (1983)</i> : Proposes way to compress product structure	Fair	Good	Yes	Fair	N/A	Introduces lead time offsets as decision variables	Solving by mixed integer linear programming limits compressed problem to very small size
<i>Harl and Ritzman (1985)</i> : Proposes capacity-sensitive SLUR heuristics to MLCR problem	Good	Good	No	Good	Fair	Easily implemented in current MRP systems	Priority rules remain capacity-insensitive
<i>Bahl and Ritzman, (1983, 1984b)</i> : Heuristic procedure for lot-sizing and capacity choices with any product structure	Fair	Good	No	Fair	Fair	Identifies environments when coordination between MPS, lower-level lot-sizing, and capacity planning is most crucial	Assumes predetermined lead time offsets and lot-for-lot production at middle levels of BOM
<i>Bitran, Haas and Hax (1983)</i> : Two-stage heuristic procedure for making MLCR decisions	Fair	Fair	No	Fair	Fair	Aggregation procedure for simplifying the problem	Limited testing and not generalized for all types of MLCR problems
<i>Bott and Ritzman (1983)</i> : Identifies key factors combining with lot sizing to create capacity problems	N/A	Good	No	N/A	N/A	Brings out importance of capacity constraints when lot sizing	Does not propose new methods for making lot-sizing and capacity decisions simultaneously
<i>Krajewski et al. (1982a, b, 1983); Ritzman, King and Krajewski (1984)</i> : Compares impact of environmental factors and system choice on performances	N/A	Good	No	N/A	N/A	Widest range of factors studied to date, with settings established by panel of managers	Cannot be generalized for all possible plant settings. Limited to current systems used in practice
<i>Ritzman and Krajewski (1983)</i> : Compares MRP and ROP systems	N/A	Good	No	N/A	N/A	Finds MRP's advantage is strongest with more BOM levels and larger lot sizes	Limited testing and restricted to systems currently used in practice

<sup>a</sup> C = computational effort, compared to other MLUR procedures, G = generality to all MLUR problems, O = optimality, S = simplicity in understanding, T = thorough testing, if heuristic.

- Apply SLCR procedures to master production scheduling and assess the impact on component plans.
- Determine how much product structure compression and aggregation are possible in real-world problems without losing too much in solution quality.
- Develop more automated MLCR systems. Today's production systems are dominated by robotics, CAD, CAM and computers: with the backup of the data bases and computer hardware and further research in models and methodologies in MLCR systems, it may be possible to automate decisions concerning master production scheduling, lot sizing, capacity requirement levels, order releases, scheduling and sequencing.

### 5. Conclusion

We have grouped lot-sizing research in situations with significant setups into four categories, depending on the number of product levels and presence of resource constraints. After reviewing contributions to date, we suggested issues for future research. It is clear that the soft spot in the field is how to solve SLCR and MLCR problems—which, paradoxically, are the ones most prevalent in practice.

We expect future research to take on more realities of actual production environments. The first step is to recognize capacity constraints and capacity alternatives, followed by giving more attention to the environmental factors in Table IV. This choice implies an increasing focus on such topics as hierarchical planning, methods of coupling sequential decision modules, aggregation techniques, the embedding of optimization procedures into more encompassing heuristics, and empirically assessing the merits of procedures currently used in practice.

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