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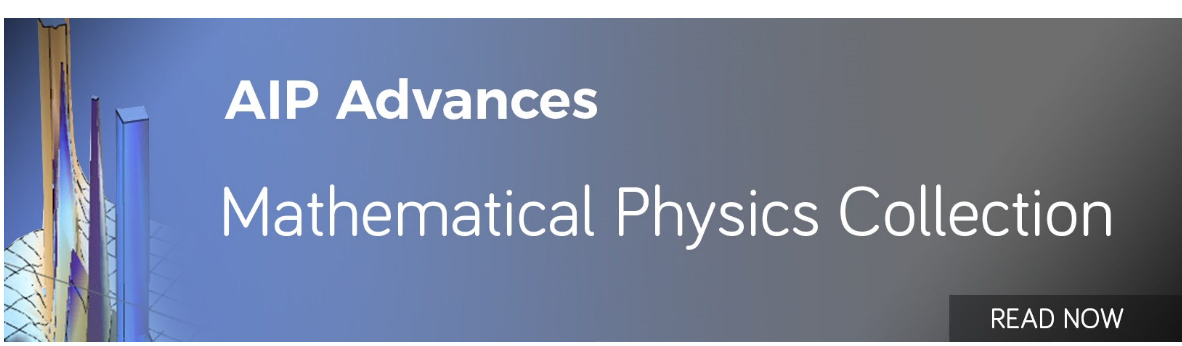

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Order of chemical reaction and convective boundary condition effects on micropolar fluid flow over a stretching sheet

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The study of chemical reaction and convective boundary conditions on thermosolutal behavior of magneto hydrodynamic (MHD) fluid flow above a stretching sheet has been investigated in the current investigation. A mathematical model is established and the governing partial differential system is modified into a set of nonlinear coupled ordinary differential system and it is evaluated by using the numerical scheme of RK method. The physical importance of different flow variables such as the magnetic parameter, vortex viscosity parameter, order of chemical reaction, Prandtl, skin friction coefficient and Eckert numbers are discussed in detailed through graphical representation. © *2018 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).* https://doi.org/10.1063/1.5053445

I. INTRODUCTION

Micropolar fluids are some special type of fluids, those responded to influence of spin inertia. This type of structures may appear in polymer technology and which are existing in the stretching of plastic sheets. The practical importance of these fluids in industrial application have initiated many researchers. Eringen[1,2] was the first to introduce the Micropolar fluid theory. Later several experimental investigation for various parameters on micropolar fluid can be seen in Migun et al.[3] and Kolpashchikov et al.[4] Ariman et al.[5] analyzed basic structure of viscous flow in micropolar fluids. In this article, they have been studied micropolar fluids with free surface flows in between two parallel plates, and they are investigated an exact solution for velocity and microrotation components. The motivation of this fluid theory appears in particle suspension,[6] animal blood (Ariman et al.[7]) and liquid crystals (Lockwood et al.[8]) etc.

There are remarkable applications in the current investigation on theory of boundary layer over a stretching sheet. First time Crane[9] introduced the investigation on the theory of boundary layer over a two dimensional viscoelastic stretching sheet. There after, various authors have extended this results on stretching/shrinking sheet. The viscous in-compressible fluid flow of thermal distribution with corresponding to homogenous heat flux on a stretching sheet is notified by Dutta et al.[10] They identified that, thermal profiles are decreasing with enhancing the Prandtl number. Kelson and Desseaux[11] find the influence of surface condition on the fluid flow of a micropolar fluid, due to strong injection or suction. They observed that suction parameter reduces the width of boundary layer. The influence of mixed convective on a porous stretching sheet in a micropolar fluid flow, it is examined by Bhargava et al.[12] They given that, the thermal transfer rate reduces with enhance in injection of the flow, hence, it improves with enhance the Grashof number and it's suction variable. Ali et al.[13] examined steady, 3-dimensional micropolar fluid passing through a stretching surface. Vajravelu et al.[14] investigated the thermal convection in the MHD flow on a nonuniform stretching sheet and thermal radiation. It is noted

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that, by increasing the magnitudes of radiation parameter reduces the thermal convection in the fluid flow region. Khedr et al.[15] examined the influences of injection/suction and heat absorption/generation effects for the MHD flow through a vertical semi-infinite permeable plate. They identified that the Nusselt number enhances with the enhance in suction or injection. Govardhan et al.[16] investigated unsteady MHD incompressible micropolar fluid on a stretching sheet. They reported that skin friction coefficient reduces due to enhance the vortex viscosity parameter.

Thermosolutal convection with chemical reaction has many applications in real life, for example, thermal distribution of cooling tower, drying, freezing the crops damaged and also in volcanic systems and geophysics.[17–19] Ali et al.[20] have reported the thermal generation/absorption, chemical reaction and heat dispersion effects. The effect of chemical reaction includes variable viscosity on Hiemenz-flow through saturated porous layer with the impact of magnetic field investigated by Seddeek et al.[21] Damseh et al.[22] studied primary order chemical reaction, viscous and temperature generating or absorbing effects for the natural convective micropolar fluid flow through a vertical permeable infinitely long surface. They noted that magnetic field, variable viscosity and mass transfer coefficient increases due to increase the Prandtl number. Modather et al.[23] examined the thermal and solutal transfer of micropolar fluid flow passing along the porous layer in an infinite varying permeable porous plate, in presence of chemical reaction of first order. They concluded that as the chemical reaction parameter enhances, the skin-friction coefficient decrease at the wall. Rahman and Lawatia[24] notified it, the rate of enhance in mass transfer function for lower order reaction is greater than as compared to higher order reaction. Ali et al.[25] examined an unsteady, MHD natural convection micropolar fluid flow through vertical permeable plate with effect of Joule-heating. Rawat et al.[26] find the boundary layer theory and solutal convection of a micropolar fluid passes through a linearly stretching sheet in a porous bed. Patil and Chamkha[27] examined the influence of heat source and first order chemical reaction on the thermosolutal convection of a polar fluid upon a porous layer. They investigated that the flow velocity profiles reduces in the porous layer as compared to with out porous layer. Ferdows and Qasem[28] examined the effect of chemical reaction on linearly stretching sheet in a viscous fluid. They reported that, the flow profiles reduces with the enhance of Schmidt number. The work conducted by Rashad et al.[29] in a moving isothermal vertical surface for a mixed convective micropolar fluid immersed thermal and solutal stratified porous bed with a first order chemical reaction. The earlier studies in this area have been motivated by above literature and physical applications, which creates interest in this study explores the higher-order chemical reactions and convective boundary condition on a convective micropolar fluid.

The intention of this article is to examine the influence of higher-order chemical reactions and convective boundary condition on steady micropolar fluid for different types of physical variables. The flow velocity, thermal and solutal profiles on the stretching sheet are studied by using the boundary layer theory.

II. DESCRIPTION AND MODEL FORMULATION

Let us choose a two dimensional fluid flow up on a linearly stretching surface. The physical system is shown in below in Fig. 1. The working fluid is an electrically conducting micropolar

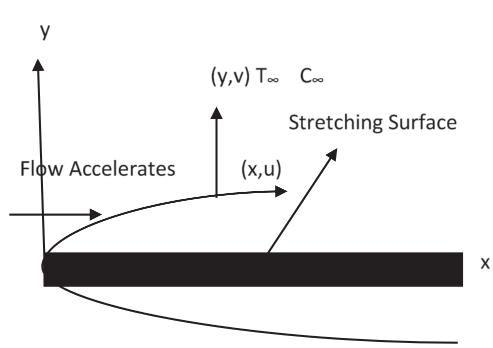

FIG. 1. The physical system.

fluid over a plane at $y = 0$. The linear velocity of the stretched sheet is $q_x = u_w(x) = cx$ and the heat flux at the wall is $T_w(x) = T_\infty + bx$. B_0 is a homogenous external magnetic field, which acts in the perpendicular way to the surface of the sheet. Here, x and y are coordinate axes, $x-$ axis is one end to other end of the surface of the sheet and, $y-$ axis is chosen as perpendicular to it. Under these implications the governing equations of the flow, thermal and solutal fields are given below

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0, \tag{1}$$

$$q_x \frac{\partial q_x}{\partial x} + q_y \frac{\partial q_y}{\partial y} = \left(\nu + \frac{\kappa}{\rho}\right)\frac{\partial^2 q_x}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial N}{\partial y} - \frac{\sigma_e B_0^2}{\rho}u, \tag{2}$$

$$q_x \frac{\partial N}{\partial x} + q_y \frac{\partial N}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho j}\left(2N + \frac{\partial q_x}{\partial y}\right), \tag{3}$$

$$q_x \frac{\partial T}{\partial x} + q_y \frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial q_x}{\partial y}\right)^2, \tag{4}$$

$$q_x \frac{\partial C}{\partial x} + q_y \frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_1(C - C_\infty)^n. \tag{5}$$

The boundary conditions for Eqs. (1)–(5) are assumed as

$$\left.\begin{array}{l} q_x = u_w(x) = cx, \quad q_y = 0, \quad N = -S\frac{\partial u}{\partial y}, \\ -k_f\frac{\partial T}{\partial y} = h_f\left(T_f - T\right) \quad C = C_w \end{array}\right\} \text{ at } \quad y = 0, \tag{6}$$

$$q_x \to 0, \quad N \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{at} \quad y \to \infty. \tag{7}$$

Where, q_x and q_y are the seepage velocities with respect to Cartesian coordinate axises x and y. N is micro-rotation in xy-direction, the temperature of the fluid inside the boundary is T and T_∞ at the far field of stream temperature, and the corresponding fluid concentrations are C and C_∞. The density is ρ, kinematic viscosity is ν, κ is vortex viscosity, and also, γ and j are spin gradient and micro-inertia they are used as $\gamma = \mu\left(1 + \frac{\kappa}{2\mu}\right)j$ and here $j = \left(\frac{\nu}{c}\right)$, c_p is specific heat, n is the order of chemical reaction parameter, α is thermal diffusivity, D is solutal diffusivity, k_1 is reaction rate constant and S is boundary parameter (which is consider as $S = 0.5$ (Govardhan et al.[16])). The thermal convection coefficient is h_f and it is proportional to $x^{-\frac{1}{2}}$, that is $h_f = ax^{-\frac{1}{2}}$, where a is a constant. We introducing the a stream function $\psi(x, y)$, it is satisfied the continuity equation (1) such that $q_x = \frac{\partial \psi}{\partial y}$ and $q_y = -\frac{\partial \psi}{\partial x}$, to evaluate the governing Eqs. (1)–(5), applied the transformation as given the below

$$\left.\begin{array}{l} \eta = \sqrt{\frac{c}{\nu}}y, \quad q_x = cxg'(\eta), \quad q_y = -\sqrt{c\nu}g(\eta), \\ N = cx\sqrt{\frac{c}{\nu}}h(\eta)\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \end{array}\right\} \tag{8}$$

On using Eq. (8) and Eqs. (2)–(7) converted into boundary value problem:

$$(1 + R)g''' + gg'' - (g')^2 - Mg' + Rh' = 0, \tag{9}$$

$$\left(1 + \frac{R}{2}\right)h'' + gh' - g'h - R(2h + g'') = 0, \tag{10}$$

$$\theta'' + Pr(g\theta' - g'\theta) + EcPr(g'')^2 = 0, \tag{11}$$

$$\phi'' + Sc\left(g\phi' - g'\phi - Cr\phi^n\right) = 0, \tag{12}$$

$$\left.\begin{array}{c} g(0)=0, \quad \phi(0)=1, \quad h(0)=-Sg''(0), \\ \theta'(0)=-Bi(1-\theta(0)), \quad g'(0)=1, \quad g'(\infty)=0, \\ h(\infty)=0, \quad \theta(\infty)=0, \quad \phi(\infty)=0. \end{array}\right\} \tag{13}$$

The non-dimensional constants in Eqs. (9)–(13) are the Vortex viscosity R, the Magnetic parameter M, Sc is the Schmidt number, Pr and Ec are the Prandtl and Eckert number, Cr is the parameter of chemical reaction and Bi is the Biot number. They are introduced as, respectively

$$\left.\begin{array}{c} R=\frac{\kappa}{\mu}, \quad M=\frac{\sigma_e}{c\rho}B_0^2, \quad Pr=\frac{\nu}{\alpha}, \quad Sc=\frac{\nu}{D}, \\ Cr=\frac{k_1(C_w-C_\infty)^{n-1}}{c}, \quad Ec=\frac{c^2x^2}{c_p(T_f-T_\infty)}, \quad Bi=\frac{h_f}{k_f}\sqrt{\frac{\nu}{c}}. \end{array}\right\} \tag{14}$$

A. Wall couple stress, skin friction, thermal and solutal convection coefficients

The major interest on the physical parameters are the wall couple stress parameter M_w, C_f is skin friction coefficient, Nu_x and Sh_x are local Nusselt number and Sherwood number. These are representing the surface drag, wall thermal and solutal convection rates, respectively. At $S = 0.5$, the corresponding wall shear stress on the sheet is defined as

$$\tau_w = \left[\frac{\partial u}{\partial y}(\mu+\kappa)+N\kappa\right]_{y=0}. \tag{15}$$

The couple stress parameter is

$$M_w = \left(\mu+\frac{\kappa}{2}\right)j\left(\frac{\partial N}{\partial y}\right)_{y=0}. \tag{16}$$

The non-dimensional couple stress parameter is

$$M_x = \left(1+\frac{R}{2}\right)h'(0). \tag{17}$$

The dimensionless local skin friction coefficient is:

$$C_f = \frac{\tau_w}{\rho u_w^2} = Re_x^{-\frac{1}{2}}\left(1+\frac{R}{2}\right)g''(0). \tag{18}$$

Here, local Reynolds number $Re_x = \frac{cx^2}{\nu}$. The dimensionless thermal and solutal transfer coefficients are defined as

$$\left.\begin{array}{c} Nu_x = \frac{xq_w}{\alpha(T_f-T_\infty)} = -Re_x^{\frac{1}{2}}\theta'(0), \\ Sh_x = \frac{xq_m}{D(C_w-C_\infty)} = -Re_x^{\frac{1}{2}}\phi'(0), \end{array}\right\} \tag{19}$$

where $q_w = -\alpha\frac{\partial T}{\partial y}|_{y=0}$ and $q_m = -D\frac{\partial C}{\partial y}|_{y=0}$.

III. RESULTS AND ANALYSIS

In this study, examined the influence of higher-order chemical reaction and convective boundary condition with respect to possible physical parameters on thermosolutal convection in a micropolar fluid up on a stretching sheet. The velocity, thermal and solutal profiles on the stretching sheet are studied by using the boundary layer theory. The non-linear differential system of Eqs. (9)–(13) are evaluated numerically by applying the shooting and RK method. The numerical computations are carried out by taking some fixed specified values at $M = 5$, $R = 0.5$, $Pr = 0.71$, $Sc = 1$, $Cr = 1$, $Bi = 0.5$, $Ec = 0.05$ and $n = 1$. Our results are very good agreement with the article of Ref. 28. The effect of the vortex viscosity, Schmidt number, magnetic parameter, order of chemical reaction, Eckert number, Prandtl number, boundary parameter and the Biot number are shown in Figs. 2–13 for different fluid flow properties through graphical representation.

Figs. 2–3 represent the response of the vortex viscosity on the microrotation and velocity, respectively. It is cleared that, the vortex viscosity (R) increased, then the micropolar fluid velocity field also enhances, whereas the microrotation decreases near the wall. It is evident from these figures that

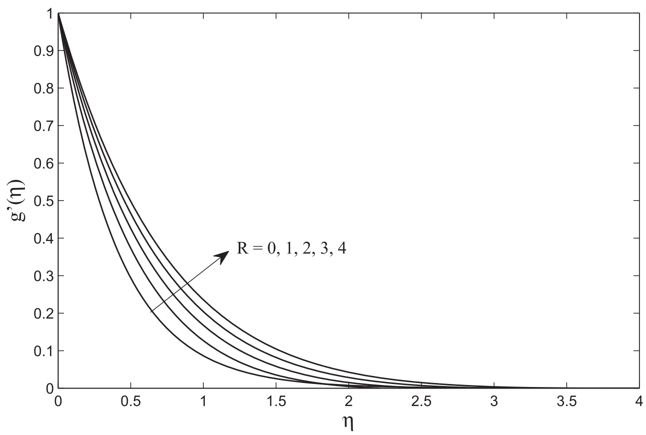

FIG. 2. Response of $g'(\eta)$ with η.

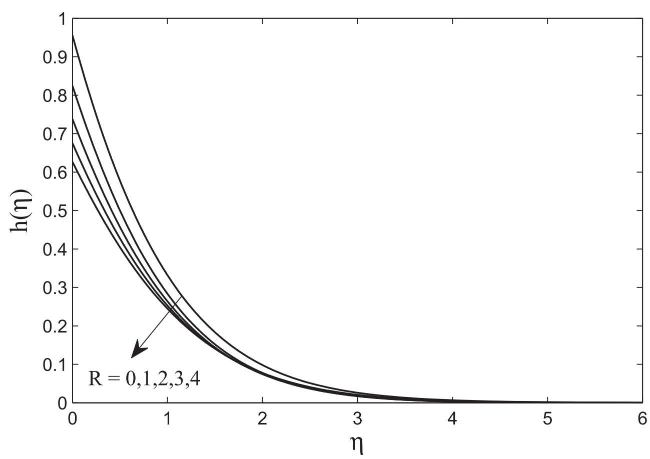

FIG. 3. Response of $h(\eta)$ with η.

as R is increasing then the boundary layer thickness also enhances. Fig. 2 shows that the stretching sheet applies a drag force on the micropolar fluid. It is not surprising to see this behavior due to the improvement of the boundary layer.

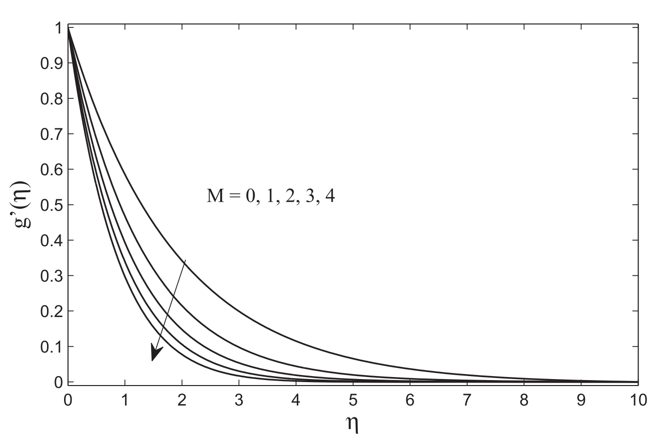

FIG. 4. Response of $g'(\eta)$ with η.

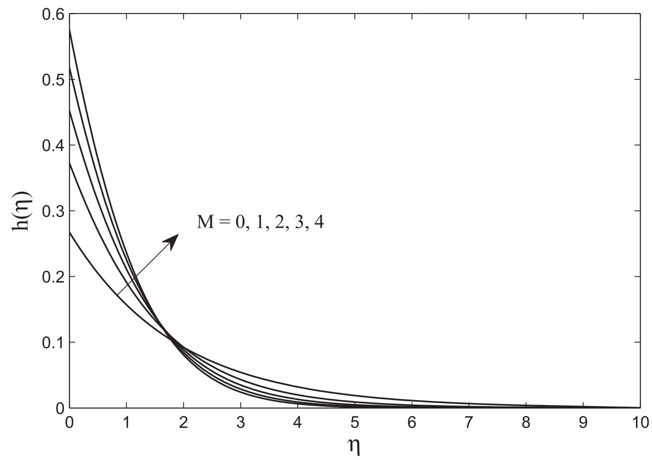

FIG. 5. Response of $h(\eta)$ with η.

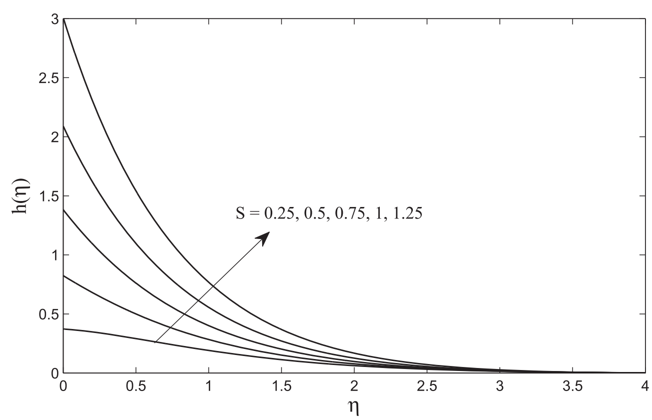

FIG. 6. Response of $h(\eta)$ with η.

Figs. 4–5 represent the influence of M on the flow velocity field and microrotation profiles as function of η, respectively. From Fig. 4, it is clear that as M increased, then the seepage velocity profiles are decreased. From Fig. 5, it can be notified that as the magnetic parameter M enhances, up

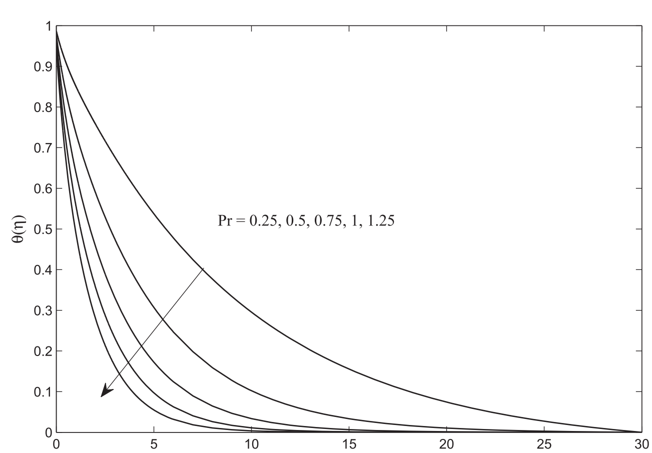

FIG. 7. Response of $\theta(\eta)$ with η.

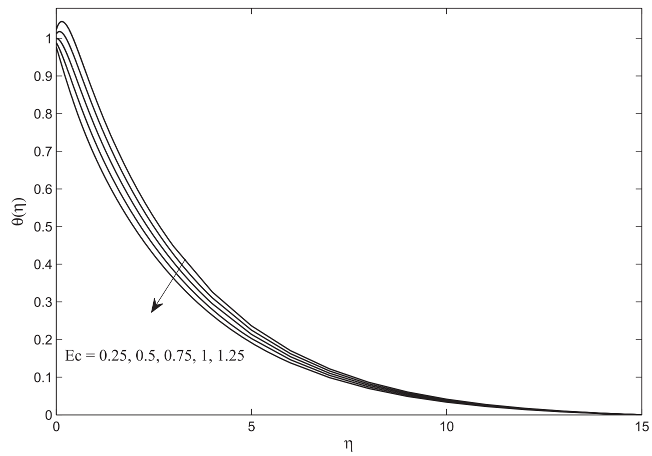

FIG. 8. Response of $\theta(\eta)$ with η.

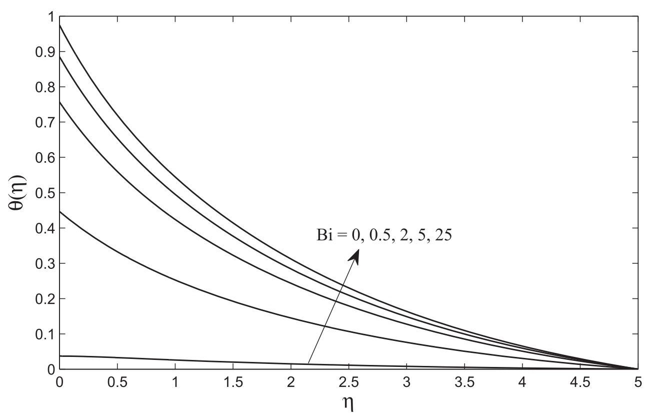

FIG. 9. Response of $\theta(\eta)$ with η.

to nearly $\eta = 1.5$, micropolarity increases and there after it is decreased. In presence of magnetic field, the body force is produced, it opposes the fluid motion and therefore the thickness of boundary layer is enhanced. The importance of the surface parameter S on the flow filed profile is represented in Fig. 6. As S increases, the microrotation enhances substantially near the wall and the micro-elements rotate with an increased intensity which leads to the higher possible angular velocity g. All the microrotation

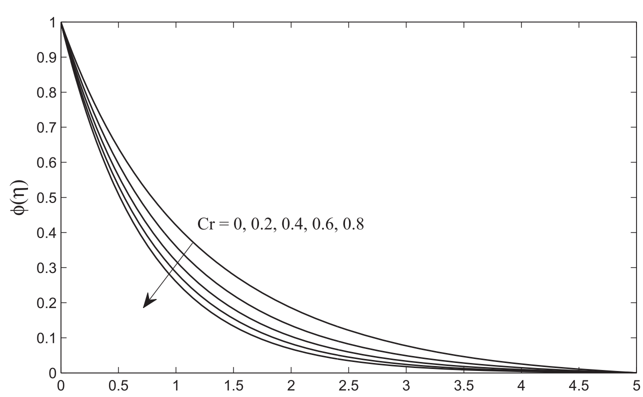

FIG. 10. Response of $\phi(\eta)$ with η.

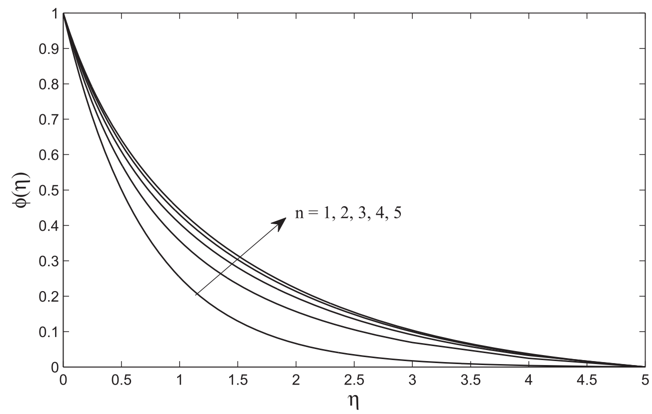

FIG. 11. Response of $\phi(\eta)$ with η.

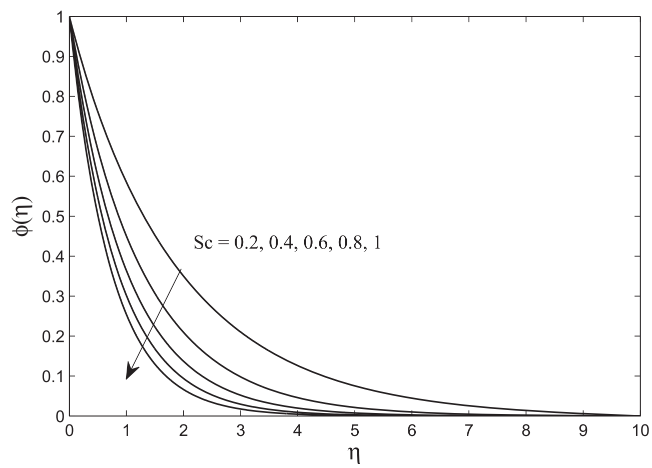

FIG. 12. Response of $\phi(\eta)$ with η.

profiles reach to a particular worth of η and that locality is very far from the boundary wall, and the values of S cease to have some effect on the microrotation field near and elsewhere.

The response of Pr on the thermal convection is shown in Fig. 7. Higher values of Pr indicate a more uniform thermal convection across the boundary layer and a thinner temperature boundary layer thickness also appears. Lower values of Pr have the highest thermal conductivity, for that

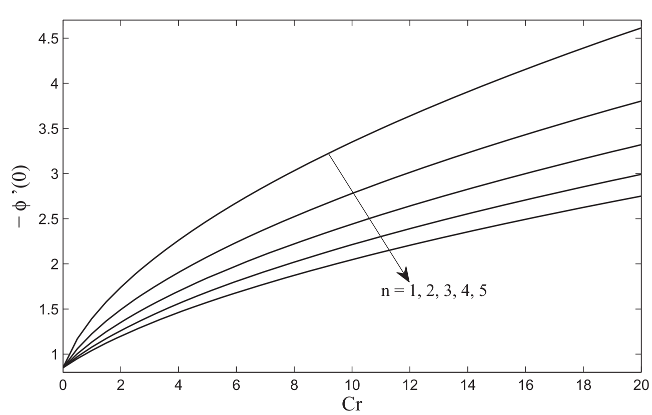

FIG. 13. Response of $-\phi'(0)$ with Cr.

TABLE I. Response of C_f, Nu_x and Sh_x for various parameters at $Ec = 0.05$, $n = 1$ and $R = 0.5$.

Bi	M	Pr	Sc	Cr	$C_f Re_x^{-\frac{1}{2}}$	Nu_x	Sh_x
0	5	0.72	1	1	-2.6449	0	1.1264
5	5	0.72	1	1	-2.6449	0.2994	1.1264
10	5	0.72	1	1	-2.6449	0.3098	1.1264
1	1	0.72	1	1	-1.5622	0.2925	1.1623
1	2	0.72	1	1	-1.8972	0.2725	1.1493
1	3	0.72	1	1	-2.1776	0.2576	1.1399
1	5	0.4	1	1	-2.6449	0.2044	1.1264
1	5	0.8	1	1	-2.6449	0.2443	1.1264
1	5	1.2	1	1	-2.6449	0.2827	1.1264
1	5	0.72	0.3	1	-2.6449	0.2364	0.5994
1	5	0.72	0.6	1	-2.6449	0.2364	0.8595
1	5	0.72	0.9	1	-2.6449	0.2364	1.0652
1	5	0.72	1	0	-2.6449	0.2364	0.4271
1	5	0.72	1	0.5	-2.6449	0.2364	0.8586
1	5	0.72	1	1	-2.6449	0.2364	1.1264

temperature can diffuse far from the vertical surface much quicker than for a larger magnitude Pr fluids. This causes a denser boundary layer along the surface of the sheet. The smaller magnitudes of Pr (Pr 0.25, 0.6) indicate physically mixtures of noble gases, air or hydrogen are represent by $Pr = 0.7$ and for water is $Pr = 1$. It is notified that as Pr enhances substantially, the temperature of the micropolar fluid decreases. In all these cases, θ inclines to zero as $\eta \to \infty$, even though for higher value of ($Pr = 1$) is parabolic. The influence of the Eckert number Ec on the thermal distribution is appeared in Fig. 8. As the parameter Ec increases, θ increases rapidly near the wall and $\theta \to 0$ as $\eta \to \infty$. The temperature profiles for a clear fluid at $Pr = 0.72$ in Table I match with the literature results given by Aziz.[30] The thermal contours for various values of Bi are presented in Fig. 9. The h_f is inversely proportional to the temperature resistance on the heated fluid side. The heated fluid side convection resistance reduces due to enhancement of the Biot number Bi values and consequently, the surface temperature enhances.

Fig. 10 illustrates the changes of the solutal profiles with respect to η for different values of Cr. It shows that as Cr enhances, the reaction in between the species that the result of the solutal convection reduces in a micropolar fluid. The absence of the chemical reaction is represented by $Cr = 0$. The variations of mass transfer effects in a micropolar fluid is almost identical for larger values of Cr. Fig. 11 reports the effect of the solutal profiles for different values of the n. It remarks that the mass transfer increases as n enhances. The lower-order chemical reaction enhances the thermal convection rate as compared to a higher-order reaction. The effect of Sc on the solutal effects for the reactive fluid flow ($n = 1$) case is shown in Fig. 12. So, it may be concluded that, the reaction decreases with mass transfer rate. The variations of the surface solutal convection $-\phi'(0)$ for different values of Cr and n are observed in Fig. 13. It shows that, the surface solutal convection for those fluids reduces with the enhancement of n and it is increased quite rapidly with the increases of Cr. It clearly remarks that, the surface mass transfer decreased for higher-order reactions (See in Fig. 13).

IV. CONCLUSION

A detailed study of two-dimensional laminar thermosolutal convection over a stretching sheet has been examined. The impact of a higher-order chemical reaction, thermal convection on the improvement of the boundary layer flow considered into the account. The influence of the various physical parameters S, M, Ec, Pr, R, Sc and Cr on the fluid flow characteristics are discussed. Tables have also been made to evaluate the values of the skin friction, thermal and solutal convection rates. From the acquired solution, the following conclusions are obtained:

- It has been concluded from the graphs that the enhancement in the viscosity R increases the flow velocity but reduces the microrotation of the fluid.

- In the presence of M, the velocity is reduced.
- The micropolar fluid temperature increased, due to increase in Bi and Ec but decrease with Pr.
- The concentration of micropolar fluid reduces, because of the enhancements in varying the values of Sc and Cr.
- The mass transfer rate for a lower-order chemical reaction is maximum than that for a higher-order chemical reaction.

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