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# Order Relations of Sets and its Application in Socio-Economics

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**Abstract**

The order relations of comparative sets and an application to emission influences on sets of urban building conflict situations are investigated with non-ordered as-built properties and their effects on the property market. Sets of points, independent of convexity properties and varying according to the consideration of socio-economic properties, are comparatively examined, concerning minima and maxima.

**Keywords**: set optimization, order relations, socio-economics

## 1 Introduction

The technical records of engineering specialists justify the practice of public administrations working according to the optimality concept of the minimizer and the deviations from this for urban building issues in Germany many times over, similar to scientific publications in the mathematical field of set optimization. Here, the definition of a set-valued mapping $F: S \subset X \rightrightarrows Y$ with real linear spaces $X$ and $Y$ and a nonempty subset $S$ considers only a minimal element $\bar{y}$ in the set $\bigcup_{x \in S} F(x)$ and then defines the set $F(\bar{x})$ with $\bar{y} \in F(\bar{x})$ for arbitrary $\bar{x} \in S$ as the "best". However, in general a minimal element $\bar{y}$ does not mean that the entire set $F(\bar{x})$ is minimal for all sets $F(x)$ with $x \in S$ is (e.g. the most favourable value in a rent index says nothing about whether the entire range in which this value occurs is also the least expensive).

This concept, or this understanding of the minimizer, is found again in socio-economics. The Federal Association of German Realtors (IVD) [9] analyzes the real estate market over a defined period. Individual data from the residential building structure, purchasing power and real estate price index, without set-valued structure, are given separately and the "best" result always indicated. Hagedorn [6] investigates the extent to which the building of another runway at the Munich Airport (Germany) has influenced real estate prices in the surrounding area. Here again, the predicted set inventory is individually viewed as "minimized" and positive and negative quality values accordingly assigned.

The Institute for Organization Communication (IFOK) [7] investigates the relationship between noise disturbance and quality of life in the region surrounding the Frankfurt Airport (Germany). With these data, the present paper studies new optimal sets with order relationships for sets with a view to an individually chosen thematic focus. The as-built properties of this urban building conflict situation are explored on the basis of set-valued considerations.

This paper is structured as follows. Chapter 2 studies specific order relationships of Jahn and Duc-Ha [11] on the basis of a "setless" order relationship and investigates the strength of the different order relationships according to implications. Their intersection and as-built properties find application in a new practical, socio-economic example, the building conflict situation in the surroundings of the Frankfurt Airport. The order relationships allow a new, scientifically-founded, optimal, comparative set consideration.

## 2 Order Relations in Set Optimization

Order relations of sets and their use in the power set $\mathcal{P}(Y) = \{A \subset Y \mid A \text{ is non-empty}\}$ with $Y$ as an arbitrary real vector space and in a certain subset $\mathcal{M}$ relating to one of the minima and maxima will now be introduced.

**Definition 2.1** Let $A, B \in \mathcal{P}(Y)$ be arbitrarily chosen sets.

(a) The *setless* or *Kuroiwa-Nishnianidze-Young (KNY)* – order relation $\preccurlyeq_s$ is defined as

$$A \preccurlyeq_s B \;:\Leftrightarrow\; (\forall a \in A \; \exists b \in B : a \leq b) \text{ and } (\forall b \in B \; \exists a \in A : a \leq b).$$

(b) The *l-typeless* order relation $\preccurlyeq_l$ is defined as

$$A \preccurlyeq_l B \;:\Leftrightarrow\; \left(\forall b \in B \;\exists a \in A:\; a \le b\right).$$

(c) The *u-typeless* order relation $\preccurlyeq_u$ is defined as

$$A \preccurlyeq_u B \;:\Leftrightarrow\; \left(\forall a \in A \;\exists b \in B:\; a \le b\right).$$

The order relation $\preccurlyeq_s$ was introduced independently by Young [20] in algebra and Nishnianidze [14] by using this in fixed point theory. Chiriaev and Walster [3] use this order relation in the interval arithmetic and SUN Microsystems [15] implements this concept in the f95 FORTRAN compiler. Kuroiwa [12] describes this order relation as a natural criterion.

In general $\preccurlyeq_s$ is not anti-symmetric.

**Example 2.2** The sets $\{1,2,4\}$ and $\{1,3,4\}$ are given as subsets of $\mathbb{N}$. If $\le$ denotes the natural order, then $\{1,2,4\} \preccurlyeq_s \{1,3,4\}$ and $\{1,3,4\} \preccurlyeq_s \{1,2,4\}$, and the two sets are not identical.

The disadvantage of an arbitrary spread of the setless order relation becomes evident in $\mathbb{R}^2$. We shall examine the "peripheral regions" which cannot be further considered with a chosen optimization problem and accordingly can vary arbitrarily.

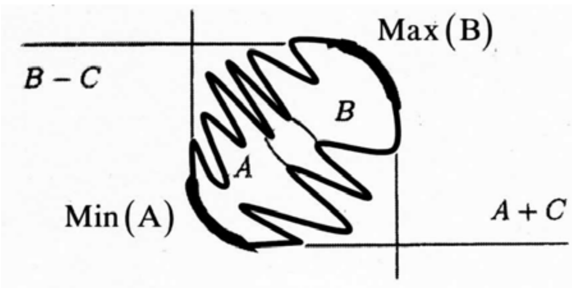



Fig. 1: "Variable peripheral regions" in $\preccurlyeq_s$

The definition of the setless order relation considers only one minimal point and one maximal point and then characterizes the two sets. This is a disadvantage. The other value regions of these sets can be arbitrarily distributed, leading to an infinite number of arrangement models. The question arises, how and whether these outlier values must be considered in a dominant comparison.

A stronger order relation than that last mentioned is referred to as a certainly less order relation.

**Definition 2.3** Let $A,\ B \in \mathcal{P}(Y)$ be arbitrarily chosen sets. The *certainly less* order relation $\preccurlyeq_c$ is defined as

$$A \preccurlyeq_c B \ :\Leftrightarrow\ (A = B) \text{ or } (A \neq B,\ \forall a \in A\ \forall b \in B : a \leq b).$$

The presentation of a weaker concept follows next.

**Definition 2.4** Let $A,\ B \in \mathcal{P}(Y)$ be arbitrarily chosen sets. The *possibly less* order relation $\preccurlyeq_p$ is defined as

$$A \preccurlyeq_p B \ :\Leftrightarrow\ \exists a \in A\ \exists b \in B :\ a \leq b.$$

Possibly less thus means that one set is "possibly" smaller than the other. The following order relations of Jahn and Duc-Ha [11] utilize the mininimal and maximal elements of a set. It is known that in a real linear topological space $Y$ minimal and maximal elements exist for every compact set in $\mathcal{P}(Y)$. This leads us to the definition of the minmax less order relation.

We shall consider sets $A$ of $\mathcal{M}$, where

$$\mathcal{M} := \left\{ A \in \mathcal{P}(Y) \middle| \min(A) \text{ and } \max(A) \text{ are non-empty} \right\}$$

is assumed to be nonempty. There $\min(A)$ and $\max(A)$ mean the sets of minimal and maximal elements of $A$. The following definitions refer specifically to this.

**Definition 2.5** Let $A,\ B \in \mathcal{M}$ be arbitrarily chosen sets. The *minmax less* order relation $\preccurlyeq_m$ is defined as

$$A \preccurlyeq_m B \ :\Leftrightarrow\ \min(A) \preccurlyeq_s \min(B) \text{ and } \max(A) \preccurlyeq_s \max(B).$$

The lower index *m* stands for the term *minmax*.

**Definition 2.6** Let $A,\ B \in \mathcal{M}$ be arbitrarily chosen sets. The *minmax certainly less* order relation $\preccurlyeq_{mc}$ is defined as

$$A \preccurlyeq_{mc} B \ :\Leftrightarrow$$

$$(A = B) \text{ or } (A \neq B,\ \min(A) \preccurlyeq_c \min(B) \text{ and } \max(A) \preccurlyeq_c \max(B)).$$

The lower index *mc* stands for the term *minmax certainly.*

In practice the *minmax certainly less* order relation is extremely strong: $\min(A)$ must in any case be less than $\min(B)$; the same procedure applies for the maximum of both sets.

**Proposition 2.7** *Let* $A,\ B \in \mathcal{P}(Y)$ *with* $A \neq B$ *be arbitrarily given. Then:*

$$A \preccurlyeq_s B \Rightarrow A \preccurlyeq_l B \Rightarrow A \preccurlyeq_p B \text{ and } A \preccurlyeq_s B \Rightarrow A \preccurlyeq_u B \Rightarrow A \preccurlyeq_p B.$$

**Proof** (by [11]): Let $C$ be a convex cone. The implications $A \preccurlyeq_s B \Rightarrow A \preccurlyeq_l B$ and $A \preccurlyeq_s B \Rightarrow A \preccurlyeq_u B$ follow directly from Definition 2.1. The implications $A \preccurlyeq_l B \Rightarrow A \preccurlyeq_p B$ and $A \preccurlyeq_u B \Rightarrow A \preccurlyeq_p B$ result from the fact that

$$A \preccurlyeq_l B \Leftrightarrow B \subset A + C \Rightarrow B \cap (A + C) \neq \varnothing \Leftrightarrow A \cap (B - C) \neq \varnothing \Leftrightarrow A \preccurlyeq_p B$$

and

$$A \preccurlyeq_u B \Leftrightarrow A \subset B - C \Rightarrow A \cap (B - C) \neq \varnothing \Leftrightarrow B \cap (A + C) \neq \varnothing \Leftrightarrow A \preccurlyeq_p B$$

are true. □

**Definition 2.8** Let $C$ be a convex cone in the real vector space $Y$. A set $A \in \mathcal{M}$ is called *quasi-dominated* iff

$$\min(A) + C = A + C \text{ and } \max(A) - C = A - C.$$

**Proposition 2.9** Let $A,\ B \in \mathcal{M}$ with $A \neq B$ be arbitrarily given, let $C$ be a convex cone in the real vector space $Y$, $A \subset \min(A) + C$ and $A \subset \max(A) - C$. Assuming that $A$ and $B$ have the quasi-dominated property, then

$$A \preccurlyeq_c B \Rightarrow A \preccurlyeq_{mc} B \Rightarrow A \preccurlyeq_m B \Rightarrow A \preccurlyeq_s B.$$

**Proof** (by [11]): "$A \preccurlyeq_c B \Rightarrow A \preccurlyeq_{mc} B$": By definition, $A \preccurlyeq_c B$ with $A \neq B$ means $B - A \subset C$. As $\min(A),\ \max(A)$ are subsets of $A$ and $\min(B),\ \max(B)$ subsets of $B$.

$\min(B) - \min(A) \subset C$ and $\max(B) - \max(A) \subset C$ are equivalent to $A \preccurlyeq_{mc} B$.

"$A \preccurlyeq_{mc} B \Rightarrow A \preccurlyeq_m B$": $A \preccurlyeq_{mc} B$ with $A \neq B$ implies that $\min(B) - \min(A) \subset C$ and $\max(B) - \max(A) \subset C$.

Thus, $\min(B) \subset \min(A) + C$, $\min(A) \subset \min(B) - C$ and $\max(B) \subset \max(A) + C$, $\max(A) \subset \max(B) - C$ are equivalent to $A \preccurlyeq_m B$.

"$A \preccurlyeq_m B \Rightarrow A \preccurlyeq_s B$": For arbitrary sets $A, B \in \mathcal{M}$ the order relation $A \preccurlyeq_m B$ means with the proberty of the set less order relation which follows from its definition

$$\min(A) \preccurlyeq_s \min(B) \Leftrightarrow$$

$$\min(A) \subset \min(B) - C \wedge \min(B) \subset \min(A) + C = A + C$$

and

$$\max(A) \preccurlyeq_s \max(B) \Leftrightarrow$$

$$\max(A) \subset \max(B) - C = B - C \ \wedge \ \max(B) \subset \max(A) + C.$$

This implies with Definition 2.9 that $\min(B) \subset A + C$ and $\max(A) \subset B - C$, so that $B + C = \min(B) + C$ and $A - C = \max(A) - C \subset B - C$. We therefore have $B \subset A + C$ and $A \subset B - C \Leftrightarrow A \preccurlyeq_s B$. □

The above implications of the order relations can be applied as to a socio-economic problem given in the following section.

## 3 Socio-economic Application of Order Relations: the conflict situation

For historical reasons the term conflict situation has a number of meanings. In the following the meaning of "conflict situation" in relation to the present model will be explained and accordingly applied. A conflict situation in the meaning of the present paper refers to a region in which different, opposing, occasionally conflicting uses come together. In greater detail, this is concerned with regions in which widely different interests are in close proximity. This characterizes a special form of settlement and represents the normal case with urban building. An example could be residential use next to an industrial area or various mixtures ("melange") of commercial-industrial and residential use. The different interests of the parties concerned lead to conflicts and problems. The symptomatic case of problems and conflicts in conflict situations, particularly in older sites, and the limited facts of the matter for addressing the issues can be found everywhere. The known and new order relations already introduced in this chapter offer a possible characterization of such a conflict.

**Origin of the problem**

Two conflicting uses (e.g. residential use vs. commercial/industrial use) must come together, converge or mix in order that a conflict situation and the resulting tensions arise. The causes of convergent conflicting interests are, for example, new residential buildings close to commercial and industrial areas, expansion of existing residential areas by new building or extensions of previously unused spaces, and expansion of existing commercial-industrial usage. Besides the convergence of different uses, a change in the use of space can also lead to conflict situations: commercial objects are converted to residential space due to environmental incompatibilities, previous residential use is partly or entirely converted to commercial use, or mutual blockage of commercial and residential use occurs. Such convergence represents the most important causes for a conflict situation. A problem for an intended works expansion which nears the residential area is that, due to the presence of the neighbouring purely residential area, the noise emissions of the company must be kept extremely low.

**Consequences**

Due to the related problem of fundamentally different guideline values and limit values for immissions the incompatible uses constitute a conflict potential. This leads to possible consequences for a company with commercial-industrial orientation. The lack of site perspectives makes the further development of the company impossible. As a result, limiting the ability to respond to customer wishes and market requirements, together with unreasonably imposed obligations and operating restrictions disadvantage the company in relation to competitors not subjected to such a conflict situation. In the worst case, an increasingly aggravated conflict situation and increasingly restrictive requirements dictated by environmental protection can lead to the destruction of a company. The persons concerned, who belong to a residential community constituting part of a conflict situation then see the following negative impacts: immission disturbance (e.g. noise evolution and increased waste gases) and the compelling need to tolerate a certain level of disturbances.

**Legal situation**

Nearly all conflict situations are the result of sites not foreseen in urban building planning, that is evolved structures. In the case of problematical planned sites, on the other hand, imbalanced planning and planning errors are responsible for the resulting conflict situation or an already existing emergent conflict situation which cannot be completely resolved by planning is consciously considered. In this case the status quo of the conflict situation is "frozen".

**Resolution of the conflict**

When a conflict situation cannot be resolved at the planning level alone (obligatory tolerance and preservation of the status quo), as a rule this is considered in the planning (master plan, development plan). Here the actual uses must be considered. A long-term restructuring of the incompatible uses according to plan (structural changes) to compatible uses resolves the conflict situation. In the past location problems with different interests, such as in the case of Tammer [16], were interpreted as distance minimization, that is the weighted sum of the distances between the different given, but freely selectable, locations. The choice of weighting factors is important here. With the intersection emphasized in Jahn and Duc-Ha [11], however, particularly for $\preccurlyeq_m$, $\preccurlyeq_{mc}$ and $\preccurlyeq_{mn}$ areas optimized in respect to different properties can be examined for fixed, non-variable criteria. Figure 2 shows the urban planning instruments and thus the emergence of a resulting conflict situation. Strategies with cooperative approaches are preferable to policy instruments.


Fig. 2: Measures and their intervention quality

The extent to which noise pollution as a result of the extension and reconstruction of the Frankfurt Airport affects nearby built-up areas will be investigated below. The disturbance due to noise is one of the central variables in the area of research into the effects of noise. The disturbance due to noise is documented in empirical investigations of the effect of environmental noise on human beings. This is based upon the pronounced correlation between acoustical burden factors and disturbance of the population. Disturbance is the effective dependent variable, for which very comprehensive dose-effect curves exist (see EU Directive 2002/49/EC, [17]). Although the disturbance of human beings has often been confirmed to be one of the principal effects of noise and has been systematically investigated, there is little consensus in the question of defining the term noise disturbance. Most researchers do not distinguish between the different aspects relating to noise disturbance: disturbance of planned activities, emotional stress or the level of acceptance of environmental conflict situations. Noise disturbance is a feeling of being disturbed without having any control over the situation. The term implies the disturbances resulting from this situation and the emotional evaluation thereof. This includes the assessment of noise prevention and the evaluation of possibly required adaptation and coping processes, and comparisons with earlier and future disturbance situations (see Hagedorn [6]).

From the psychological point of view, stress is the result of an individually perceived deliberation process between situative requirements and the individual capabilities and possibilities to deal with these demands. The less possibilities the person affected sees for adequately coping with the noise and its negative effects, the more disturbing is the perception of unusual and undesirable noises. Global noise disturbance represents the integration of individual disturbance processes. This stress concept can be extended to include social concepts. Here, noises resulting from other persons and institutions are perceived as noise and evaluated differently. In respect of the scale employed, the measurement of noise disturbance is based upon international agreements of the International Commission on Biological Effects of Noise (ICBEN) and Fields [5]. Subjectively perceived environmental disturbances in relation to health-oriented quality of life, such as air traffic noise, are difficult to measure, because other environmental stress factors – such as road traffic or air pollution – also influence their perception by an individual. In the following, the previously introduced order relations for the description of disturbing effects will be employed on an ICBEN scale from 1 to 5. Everyday experience shows that the need for quiet, and therefore the reaction to the effect of environmental noise, varies periodically with daily activities.

The following data from the IFOK study [7] are viewed as quasi-empirical. The different forms of quasi-experiments are based upon the relevant opinions or behavioural patterns (in respect of the parameter noise disturbance) of different groups of persons differing in the combinations of the conditions of interest (specific disturbance levels with air traffic noise) but otherwise as far as possible comparable and measurable. The term "quasi-experiment" is a selection procedure. The large-scale survey model for the investigation includes 2304 persons (9 noise level classes à 2.5 dB, 4 sites, 2 areas per noise level class, and 32 persons per area) from several residential areas over a total of nine air traffic noise disturbance steps at different locations in the vicinity of the Frankfurt Airport. For the determination of the acoustical parameters of the air traffic noise the mean air traffic noise disturbance level over one year – averaged over the six most congested months of the year as reported by the Umweltbundesamt (Federal Environment Agency) [17] – and the disturbance determined on specific days of investigation at the homes of selected test persons. The areas investigated were chosen, for example, according to the criterion that the calculated difference at the next site of measurement was not more than 2.5 dB. For the examination of the effects of air traffic noise, within the scope of the evaluations the opinions of the test persons were classified according to noise level classes with a minimum width of 2.5 dB and a maximum width of 5 dB.

In the following the order relations are used according to a specific procedure. With this "100/100" rule an envelope of the noise values for east and west operation during the six most congested months (in Frankfurt, May to October 2006) was constructed. This represents the respective worst case scenario. For the investigation of a possible relationship between air traffic noise and noise disturbance, as well as various parameters of the living quality and quality of life of persons living near the airport, the areas investigated are defined principally according to the gradient of the air traffic noise disturbance. The choice of investigated areas and area clusters is graduated according to the intensity of the air traffic noise disturbance in consideration of different factors, such as the social and spatial structure, environmental conditions within a level of 45 dB to 62.5 dB (16 hours day level) relevant for the air traffic – related disturbance. The following criteria were considered for the choice of areas:

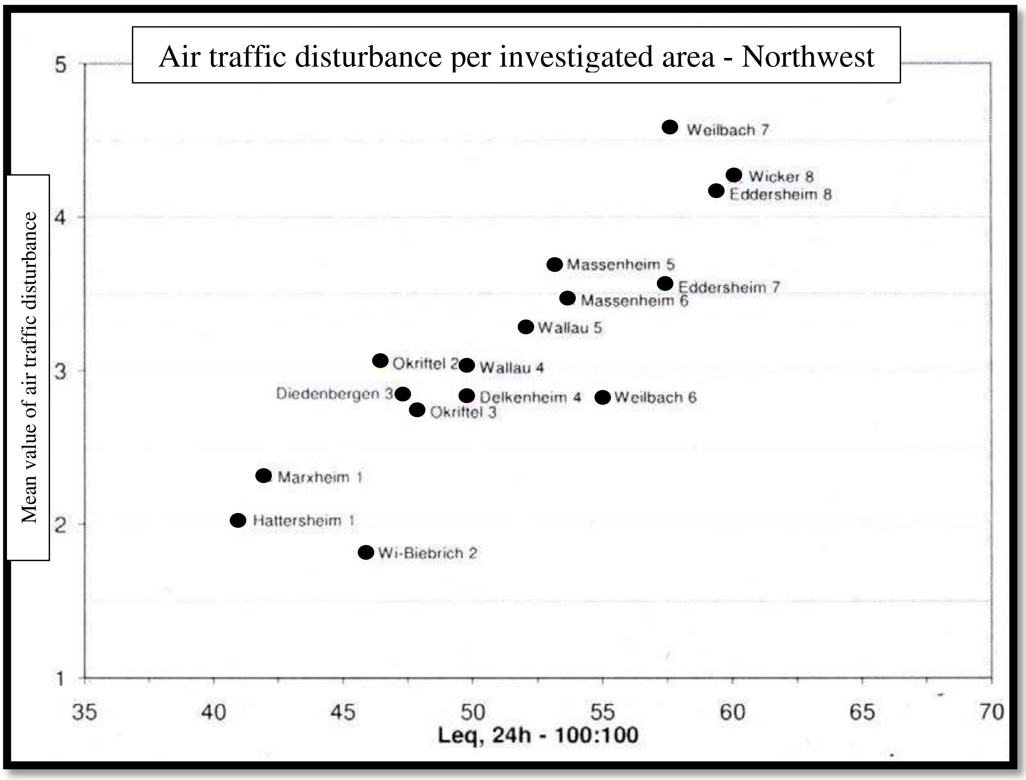
General criteria - sites: northwest, west, south and east

Acoustical criteria: In addition to the different actual noise level steps areas were chosen for which a higher disturbance level, a constant disturbance (fluctuation range $\pm$ 2 dB) or a reduction of the air traffic noise disturbance is predicted following the planned increase in air traffic. Other sources of traffic noise (rail and road traffic) and selected areas without additional emission sources in the immediate vicinity of the airport were taken into consideration.

Social structure criteria: The residential units within a given area had to be spatially coherent in order to ensure analogous structural conditions (consideration of development and social structure). An area including at least 160 residential units was taken as a reference size. Within the chosen residential areas a random selection was made from all persons registered in the respective investigated area with a minimum age of 16 years. The test persons were selected according to a step-wise, representative random sampling process with the following three steps:


Fig. 3: Selection of test persons

Figures 4 and 5 show the areas investigated at the four sites in the vicinity of the Frankfurt Airport in relation to the air traffic noise disturbance (24 hours). The noise disturbance levels are determined according to the 100/100 rule. Leq, 24h is the continuous sound level over 24 hours. This serves for the identification of the extent of change in the air traffic noise per person queried in each area. This objective change is compared with the subjectively expected change (current and future) for the given disturbance level. Leq, 3 is the increase in sound level during the time from 6 a.m. to 10 p.m.



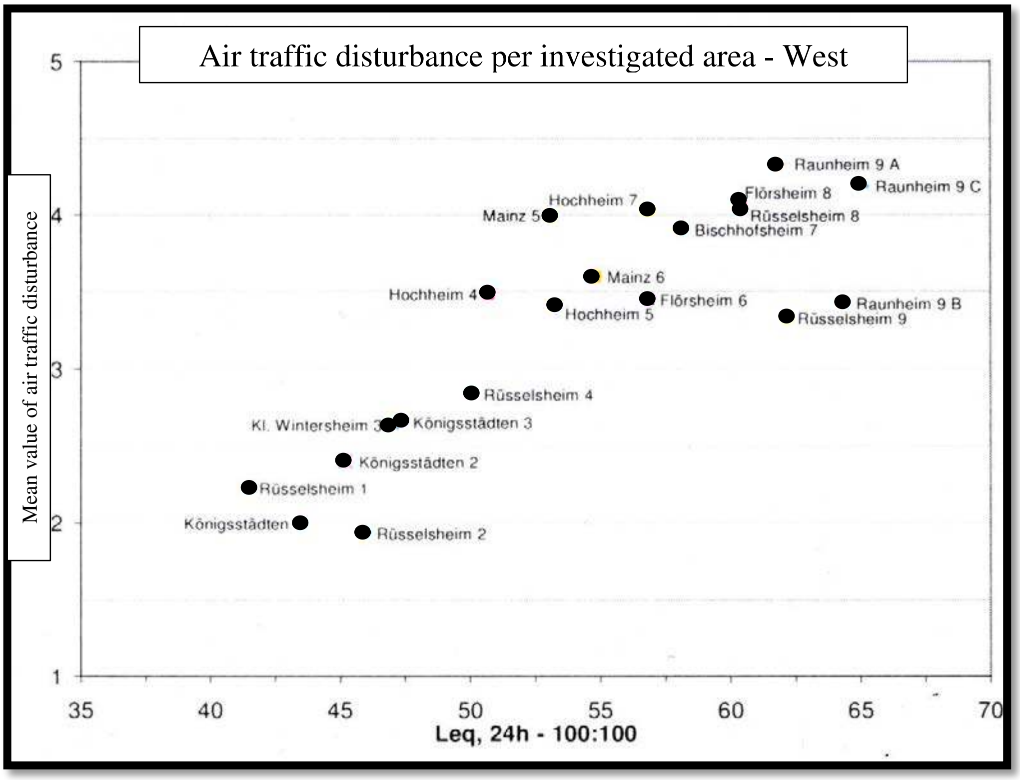
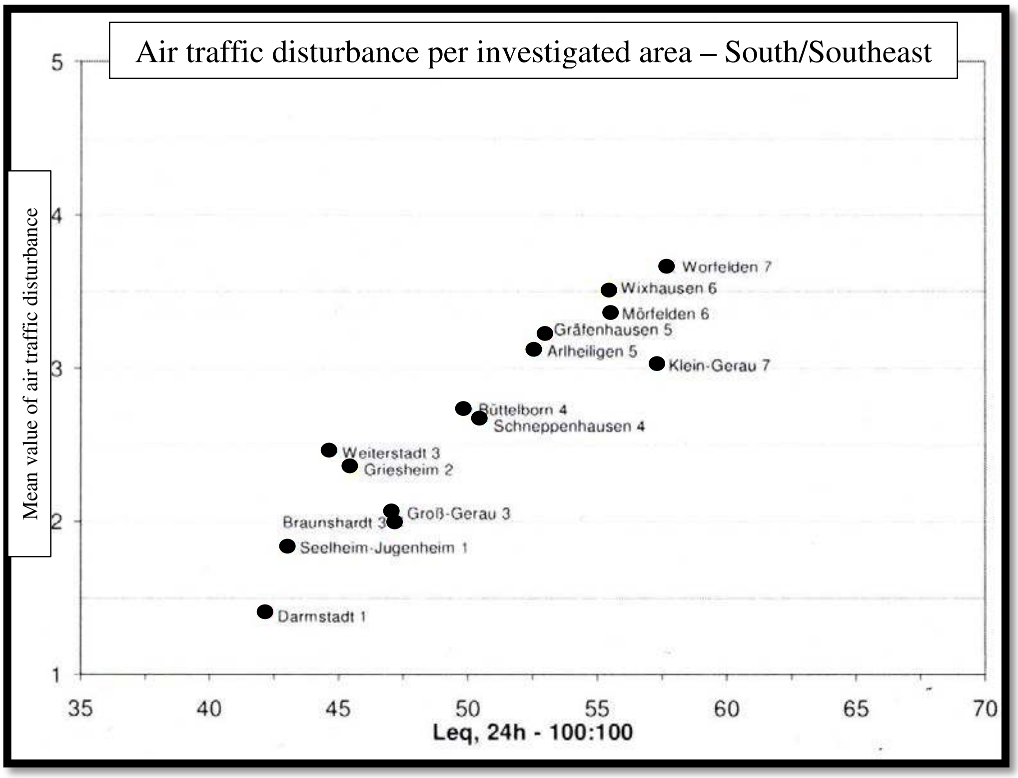




Fig. 4: Air traffic noise disturbance and air traffic noise level in the northwest/west area investigated with the data from [7]



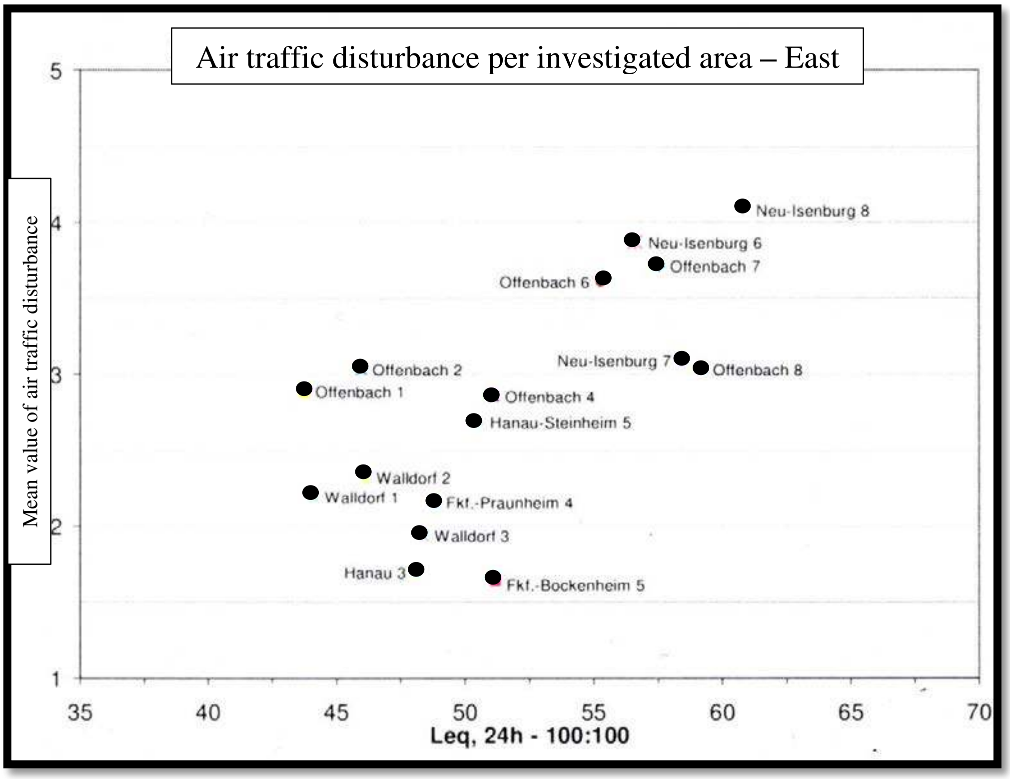




Fig. 5: Air traffic noise disturbance and air traffic noise level in the southeast/east area investigated with the data from [7]

The figures make clear in which areas - separately shown for the different sites – the highest overall disturbance values are found. The figures above show the mean disturbance values for each area investigated. The areas most seriously disturbed by air traffic noise are therefore: in the northwest region the residents of Weilbach, Wicker and Eddersheim; in the west region overall the residents of Raunheim, Flörsheim, Hochheim and Rüsselsheim are subjected to the greatest air traffic noise disturbance; in the south/southeast region the residents of Worfelden, Wixhausen and Mörfelden are most seriously affected; in the east investigated region the residents of Offenbach and have the highest noise disturbance values.

With the order relations of Jahn and Duc-Ha [11] the following specific relationships then result:

$$\min(\text{northwest}) \preccurlyeq_p \min(\text{west}) \text{ and } \max(\text{northwest}) \preccurlyeq_p \max(\text{west}),$$
$$\min(\text{south/southeast}) \preccurlyeq_u \min(\text{northwest}) \text{ and}$$
$$\max(\text{south/southeast}) \preccurlyeq_c \max(\text{northwest}),$$
$$\min(\text{northwest}) \preccurlyeq_p \min(\text{east}) \text{ and } \min(\text{east}) \preccurlyeq_p \min(\text{northwest}),$$
$$\min(\text{south/southeast}) \preccurlyeq_p \min(\text{west}) \text{ and } \max(\text{south/southeast}) \preccurlyeq_c \max(\text{west}),$$
$$\min(\text{west}) \preccurlyeq_p \min(\text{east}) \text{ and } \max(\text{east}) \preccurlyeq_c \max(\text{west}),$$
$$\text{south/southeast} \preccurlyeq_{mc} \text{east}.$$

Conclusion: The south/southeast region is the most favourable site in respect of the individual perception of noise and the actual noise level, as precisely in the order-relevant comparison south/southeast vs. east "minmax certainly less" prevails as a strong condition. By comparison, the east and west regions are characterized by extreme dissatisfaction due to the high noise level.

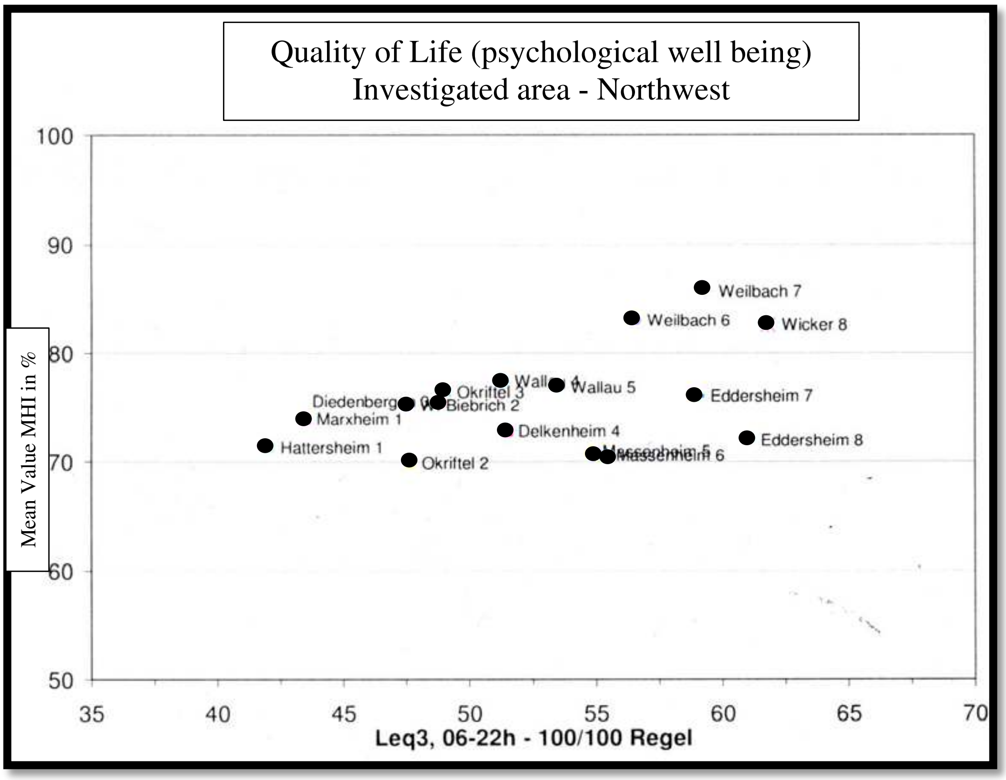
The most frequently implemented measures and efforts to cope with this situation with which the persons queried react to noise disturbance when an airplane flies over them aim to minimize the noise disturbance (soundproof windows) or maximize the speech to noise ratio (speaking louder, turning up the TV or radio volume, moving closer to the partner in conversation) inside. Avoidance behaviour (ending a conversation, postponing activities, withdrawing) are seldom found in the investigation statistics, as is plugging the ears or taking sedatives and tranquilisers. Actively used possibilities for active response to the disturbance arising from air traffic noise in the investigated areas are: generating protest lists and formal complaints. These activities increase with increasing noise level. While, in particular, efforts to achieve the reduction of air traffic noise via the air traffic noise protection delegate, airport operator and the Federal State of

Hesse are demanded, one expects the least cooperation from the industrial operations at the airport. The set volume of the investigated areas in Figures 4 to 5 accordingly allows a characterization of the resident structure.

Methods for health-related quality of life: *HRQoL* are originally disease-specific and have proven useful for the determination of the quality of results in science (see Bullinger [2]). Health-related quality of life is equivalent to subjective health indicators and describes a multi-dimensional psychological construct. This is comprised of the physical condition (disease- and therapy-related complaints), psychological state of being (mood), social integration (number and quality of relationships with other persons), and functional competence (professional, household and leisure time activities) (see Bullinger [1]).

In Figures 6 and 7 below the SF-12 was used as the essential instrument for the measurement of the health-related quality of life. The SF-12 is a questionnaire authorised in 1992 by Ware and Sherbourbe [18] as a short, economical, disease-spanning instrument for the measurement of world-wide quality of life. The following physical and psychological criticisms (Mental Health Inventory MHI) are considered in the evaluation: moderately difficult activity / energy, less work (physical and mental), dealing only with selected tasks / minimum care, general health / hindrance due to pain, adverse contacts (frequency), and calm and serenity.

An overall examination of the results of the health-related quality of life initially reveals that in all psychological areas of quality of life the standard values were achieved. Accordingly, the SF-12 questionnaire is not able to indicate differences in respect of the sound level. Subjective adverse health effects due to the effects of air traffic noise were not demonstrated. Another principal effect is the expectation in relation to the extension of the airport: Persons not expecting a worsening of their situation report a higher quality of life – independently of the sensitivity to noise.



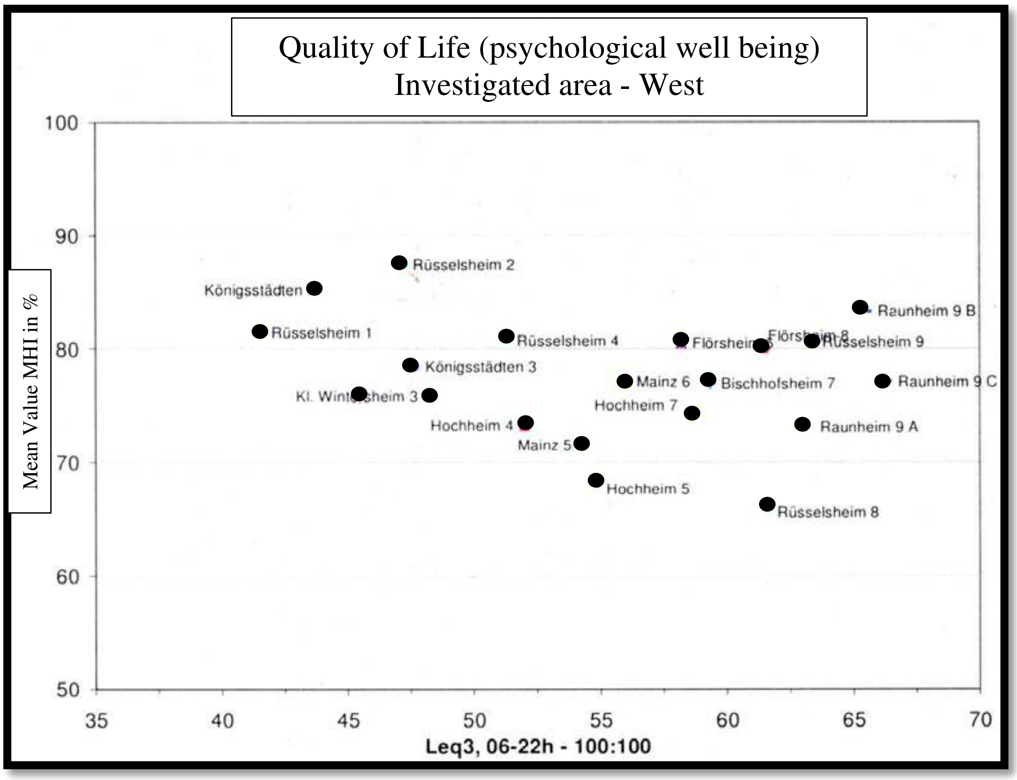
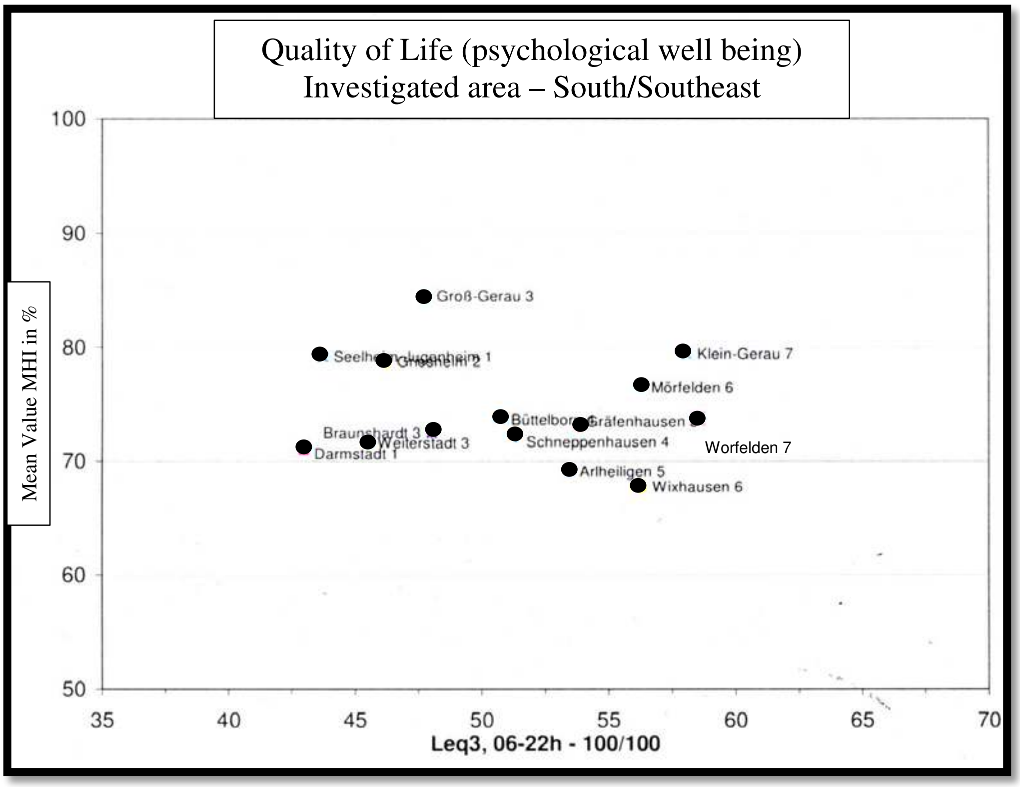




Fig. 6: Quality of life and psychological well-being of northwest/west area with the data from [7]



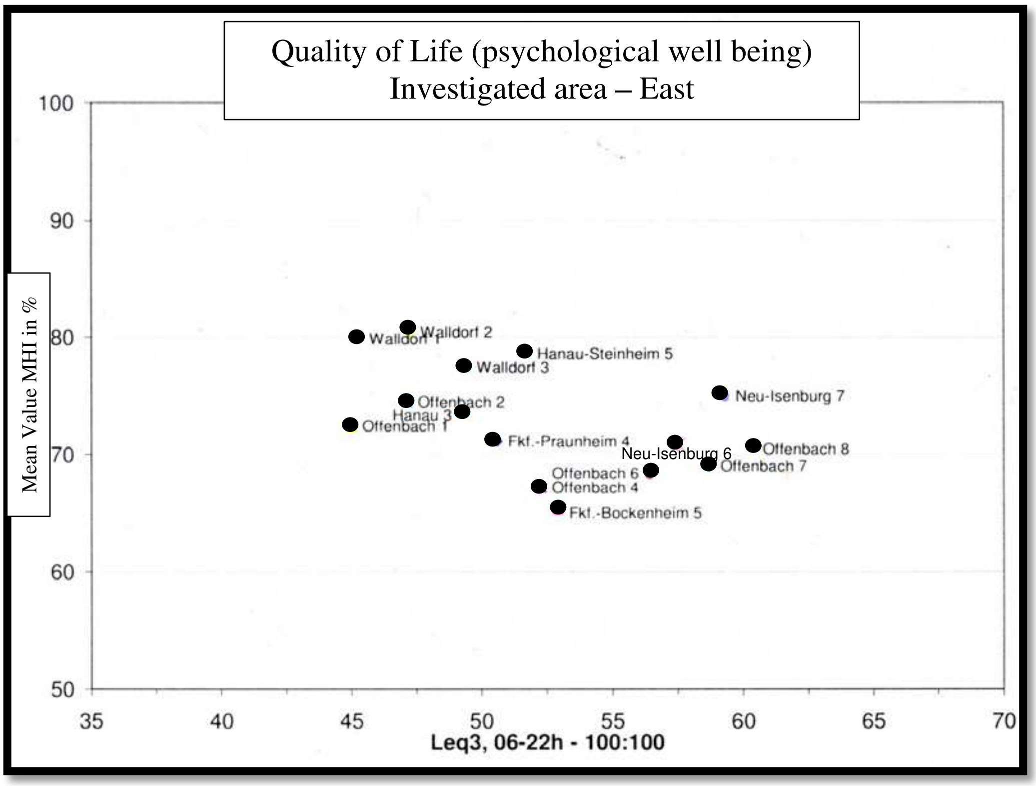




Fig. 7: Quality of life and psychological well-being of south/southeast area with the data from [7]

The comparative, however weak, order relations here are:

$$\max(\text{south/southeast}) \preccurlyeq_u \max(\text{northwest}),$$
$$\min(\text{northwest}) \preccurlyeq_p \min(\text{east}) \text{ and } \max(\text{east}) \preccurlyeq_p \max(\text{northwest}),$$
$$\max(\text{east}) \preccurlyeq_p \max(\text{west}),$$
$$\min(\text{east}) \preccurlyeq_p \min(\text{south/southeast}), \min(\text{south/southeast}) \preccurlyeq_p \min(\text{east}).$$

Conclusion: This example there offers far less possibilities for the use of order relations. Conspicuous: The south/southeast region has a low level of psychological well-being because of the Frankfurt Airport – with subjectively lower air traffic noise disturbance.

In the following the optimal set is investigated in terms of the mean value and standard deviation for the west region, as peripheral areas with overly positive and negative data have only a negligible influence on the conflict situations. For example, if the air traffic noise disturbance is low and the quality of life high, these values are uninteresting for the experiments. On the other hand, if the air traffic noise disturbance is high with a simultaneously low quality of life, the communities are under pressure to improve this situation. For this purpose, one can make use of the following optimal values as an interval in relation to a conflict situation.

| Investigated area WEST | Leq, 24 hrs. | Air traffic noise disturbance | Quality of life |
|---|---|---|---|
| Hochheim 4 | 50.72 | 3.49 | 74 |
| Hochheim 5 | 53.29 | 3.41 | 68 |
| Hochheim 7 | 56.87 | 4.04 | 75 |
| Kl. Winternheim 3 | 47.05 | 2.64 | 76 |
| Försheim 6 | 56.87 | 3.45 | 80.5 |
| Flörsheim 8 | 60.43 | 4.06 | 80 |
| Bischofsheim 7 | 58.2 | 3.9 | 77 |
| Rüsselsheim 1 | 41.45 | 2.23 | 82 |
| Rüsselsheim 2 | 45.98 | 1.93 | 87 |
| Rüsselsheim 4 | 50.1 | 2.84 | 81 |
| Rüsselsheim 8 | 60.41 | 4.02 | 66 |
| Rüsselsheim 9 | 62.16 | 3.33 | 80 |
| Königsstädten | 43.45 | 2 | 85 |
| Königsstädten 2 | 45.21 | 2.4 | 76 |
| Königsstädten 3 | 47.39 | 2.66 | 78 |
| Raunheim 9A | 61.94 | 4.33 | 74 |
| Raunheim 9B | 64.4 | 3.43 | 84 |
| Raunheim 9C | 65.15 | 4.19 | 77 |
| Mainz 5 | 53.13 | 4 | 72 |
| Mainz 6 | 54.85 | 3.6 | 77 |
| | | | |
| Mean values: | 53.9525 | 3.2975 | 77.475 |
| Standard deviations: | 7.28715299 | 0.764459734 | 5.290196992 |

| Investigated area WEST | Leq, 24 hrs. | Air traffic noise disturbance | Quality of life |
|---|---|---|---|
| Hochheim 4 | 50.72 | 3.49 | 74 |
| Hochheim 5 | 53.29 | 3.41 | 68 |
| Hochheim 7 | 56.87 | 4.04 | 75 |
| Kl. Winternheim 3 | 47.05 | 2.64 | 76 |
| Försheim 6 | 56.87 | 3.45 | 80.5 |
| Bischofsheim 7 | 58.2 | 3.9 | 77 |
| Rüsselsheim 4 | 50.1 | 2.84 | 81 |
| Königsstädten 3 | 47.39 | 2.66 | 78 |
| Mainz 6 | 54.85 | 3.6 | 77 |

The point sets examined up to now are discrete, as no convexity is assumed here. In order to obtain these one would have to cut away areas, which would be contrary to the principle of the conflict situation.

Below each of the four areas investigates, northwest, east, south/southeast and west, is divided according to geographic considerations into three new sets. The stronger order relations now obtained reveal a new characterization of the respective areas.

| Investigated area NORTHWEST | Leq, 24 hrs. | Mean value - air traffic noise disturbance |
|---|---|---|
| NORTH | | |
| Marxheim 1 | 42.04 | 2.31 |
| Diedenbergen 3 | 47.34 | 2.86 |
| Hattersheim 1 | 41.08 | 2.03 |
| MIDDLE | | |
| Wiesbaden-Biebrich 2 | 45.93 | 1.82 |
| Wallau 4 | 49.85 | 3.03 |
| Wallau 5 | 52.07 | 3.29 |
| Okriftel 2 | 46.51 | 3.05 |
| Okriftel 3 | 47.94 | 2.74 |
| Weilbach 6 | 55.02 | 2.83 |
| Weilbach 7 | 57.74 | 4.57 |
| SOUTH | | |
| Delkenheim 4 | 49.84 | 2.83 |
| Massenheim 5 | 53.21 | 3.69 |
| Massenheim 6 | 53.75 | 3.46 |
| Eddersheim 7 | 57.49 | 3.55 |
| Eddersheim 8 | 59.48 | 4.15 |
| Wicker 8 | 60.13 | 4.27 |

The following northwest order relations then result in respect of Leq, 24 hours and the mean value of the air traffic noise disturbance:

$$\max\left(\text{north}_{\text{northwest}}\right) \preccurlyeq_c \max\left(\text{middle}_{\text{northwest}}\right),\ \text{north}_{\text{northwest}} \preccurlyeq_u \text{middle}_{\text{northwest}},$$
$$\min\left(\text{middle}_{\text{northwest}}\right) \preccurlyeq_c \min\left(\text{south}_{\text{northwest}}\right),\ \text{middle}_{\text{northwest}} \preccurlyeq_l \text{south}_{\text{northwest}},$$
$$\text{north}_{\text{northwest}} \preccurlyeq_{mc} \text{south}_{\text{northwest}}.$$

| Investigated area | Leq, 06-22 hrs. | Mean value - MHI |
|---|---|---|
| NORTHWEST | | |
| NORD | | |
| Marxheim 1 | 43.49 | 74 |
| Diedenbergen 3 | 48.86 | 76 |
| Hattersheim 1 | 42.01 | 72 |
| MIDDLE | | |
| Wiesbaden-Biebrich 2 | 47.6 | 76 |
| Wallau 4 | 51.34 | 78 |
| Wallau 5 | 53.55 | 77.5 |
| Okriftel 2 | 47.68 | 70 |
| Okriftel 3 | 49.01 | 77 |
| Weilbach 6 | 56.6 | 83,5 |
| Weilbach 7 | 59.32 | 86 |
| SOUTH | | |
| Delkenheim 4 | 51.5 | 73 |
| Massenheim 5 | 54.81 | 71.5 |
| Massenheim 6 | 55.36 | 71 |
| Eddersheim 7 | 59.01 | 77 |
| Eddersheim 8 | 61.01 | 72.5 |
| Wicker 8 | 61.81 | 83 |

This now gives us the following northwest order relations in respect of Leq, 06-22 hours and the mean value of the MHI:

$$\text{north}_{\text{northwest}} \preccurlyeq_{mc} \text{middle}_{\text{northwest}},$$
$$\min\left(\text{middle}_{\text{northwest}}\right) \preccurlyeq_{l} \min\left(\text{south}_{\text{northwest}}\right),$$
$$\max\left(\text{north}_{\text{northwest}}\right) \preccurlyeq_{c} \max\left(\text{south}_{\text{northwest}}\right),\ \text{north}_{\text{northwest}} \preccurlyeq_{u} \text{south}_{\text{northwest}}.$$

| Investigated area | Leq, 24 hrs. | Mean value - air traffic noise disturbance |
|---|---|---|
| EAST | | |
| EAST | | |
| Hanau 3 | 48.13 | 1.7 |
| Hanau-Steinheim 5 | 50.56 | 2.69 |
| MIDDLE | | |
| F-Bockenheim 5 | 51.17 | 1.65 |
| F-Praunheim 4 | 48.84 | 2.16 |
| Offenbach 1 | 43.75 | 2.89 |
| Offenbach 2 | 45.98 | 3.03 |
| Offenbach 4 | 51.12 | 2.86 |
| Offenbach 6 | 55.28 | 3.61 |
| Offenbach 7 | 57.56 | 3.71 |
| Offenbach 8 | 59.21 | 3.03 |
| WEST | | |
| Walldorf 1 | 44.11 | 2.19 |
| Walldorf 2 | 46.12 | 2.32 |
| Walldorf 3 | 48.39 | 1.94 |
| Neu-Isenburg 6 | 56.61 | 3.87 |
| Neu-Isenburg 7 | 58.42 | 3.09 |
| Neu-Isenburg 8 | 60.8 | 4.08 |

The following east order relations in respect of Leq, 24 hours and the mean value of the air traffic noise disturbance are then found to be:

$$\max\left(\text{east}_{\text{east}}\right) \preccurlyeq_c \max\left(\text{middle}_{\text{east}}\right),\ \text{east}_{\text{east}} \preccurlyeq_u \text{middle}_{\text{east}},$$
$$\max\left(\text{middle}_{\text{east}}\right) \preccurlyeq_c \max\left(\text{west}_{\text{east}}\right),\ \text{middle}_{\text{east}} \preccurlyeq_u \text{west}_{\text{east}},$$
$$\max\left(\text{east}_{\text{east}}\right) \preccurlyeq_c \max\left(\text{west}_{\text{east}}\right),\ \min\left(\text{east}_{\text{east}}\right) \preccurlyeq_l \min\left(\text{west}_{\text{east}}\right),\ \text{east}_{\text{east}} \preccurlyeq_u \text{west}_{\text{east}}.$$

| Investigated area EAST EAST | Leq, 06-22 hrs. | Mean value - MHI |
|---|---|---|
| Hanau 3 | 49.27 | 73 |
| Hanau-Steinheim 5 | 51.7 | 78 |
| MIDDLE | | |
| F-Bockenheim 5 | 52.92 | 65 |
| F-Praunheim 4 | 50.59 | 71 |
| Offenbach 1 | 45 | 72 |
| Offenbach 2 | 47.14 | 74 |
| Offenbach 4 | 52.27 | 66.5 |
| Offenbach 6 | 56.42 | 67 |
| Offenbach 7 | 58.7 | 68 |
| Offenbach 8 | 60.33 | 69 |
| WEST | | |
| Walldorf 1 | 45.27 | 79.5 |
| Walldorf 2 | 47.22 | 80.5 |
| Walldorf 3 | 49.38 | 77 |
| Neu-Isenburg 6 | 57.49 | 71 |
| Neu-Isenburg 7 | 59.29 | 75 |
| Neu-Isenburg 8 | 61.69 | 75 |

The resulting east order relations in respect of Leq, 06-22 hours and the mean MHI value are then:

$$\begin{gathered}\min\left(\text{middle}_{\text{east}}\right) \preccurlyeq_l \min\left(\text{east}_{\text{east}}\right),\\ \max\left(\text{middle}_{\text{east}}\right) \preccurlyeq_l \max\left(\text{west}_{\text{west}}\right),\ \text{middle}_{\text{east}} \preccurlyeq_u \text{west}_{\text{west}},\\ \text{east}_{\text{east}} \preccurlyeq_p \text{west}_{\text{east}},\ \text{west}_{\text{east}} \preccurlyeq_p \text{east}_{\text{east}}.\end{gathered}$$

| Investigated area SOUTH/SOUTHEAST NORTH | Leq, 24 hrs. | Mean value - air traffic noise disturbance |
|---|---|---|
| Wixhausen 6 | 55.47 | 3.53 |
| Seelheim-Jugenheim 1 | 43.09 | 1.83 |
| Gräfenhausen 5 | 53.07 | 3.22 |
| Schneppenhausen 4 | 50.57 | 2.67 |
| Mörfelden 6 | 55.53 | 3.36 |
| Worfelden 7 | 57.74 | 3.67 |
| MIDDLE | | |
| Braunshardt 3 | 47.29 | 2 |
| Groß-Gerau 3 | 47.22 | 2.04 |
| Klein-Gerau 7 | 57.32 | 3.03 |
| SOUTH | | |
| Darmstadt 1 | 42.21 | 1.41 |
| Arlheiligen 4 | 52.62 | 3.12 |
| Weiterstadt 3 | 44.72 | 2.45 |
| Büttelborn 4 | 49.98 | 2.74 |
| Griesheim 2 | 45.56 | 2.34 |

The following south/southeast order relations in respect of Leq, 24 hours and the mean value of the air traffic noise disturbance are then found to be:

$$\min\left(\text{north}_{\text{SSE}}\right) \preccurlyeq_c \min\left(\text{middle}_{\text{SSE}}\right),\ \max\left(\text{middle}_{\text{SSE}}\right) \preccurlyeq_c \max\left(\text{north}_{\text{SSE}}\right),$$
$$\text{north}_{\text{SSE}} \preccurlyeq_l \text{middle}_{\text{SSE}},$$
$$\min\left(\text{south}_{\text{SSE}}\right) \preccurlyeq_c \min\left(\text{middle}_{\text{SSE}}\right),\ \text{south}_{\text{SSE}} \preccurlyeq_l \text{middle}_{\text{SSE}},$$
$$\text{south}_{\text{SSE}} \preccurlyeq_{mc} \text{north}_{\text{SSE}}.$$

| Investigated area SOUTH/SOUTHEAST NORTH | Leq, 06-22 hrs. | Mean value - MHI |
|---|---|---|
| Wixhausen 6 | 56.28 | 68 |
| Seelheim-Jugenheim 1 | 43.72 | 78.5 |
| Gräfenhausen 5 | 53.86 | 73.5 |
| Schneppenhausen 4 | 51.34 | 72.5 |
| Mörfelden 6 | 56.25 | 76 |
| Worfelden 7 | 58.45 | 74 |
| MIDDLE | | |
| Braunshardt 3 | 48.01 | 73 |
| Groß-Gerau 3 | 47.80 | 85 |
| Klein-Gerau 7 | 58.07 | 79 |
| SOUTH | | |
| Darmstadt 1 | 42.95 | 71 |
| Arlheiligen 4 | 53.42 | 69 |
| Weiterstadt 3 | 45.46 | 72 |
| Büttelborn 4 | 50.74 | 74 |
| Griesheim 2 | 46.24 | 78 |

The south/southeast order relations in respect of Leq, 06-22 hours and the mean MHI value are then:

$$\begin{gathered}
\text{north}_{\text{SSE}} \preccurlyeq_p \text{middle}_{\text{SSE}},\ \text{middle}_{\text{SSE}} \preccurlyeq_p \text{north}_{\text{SSE}}, \\
\min\left(\text{north}_{\text{SSE}}\right) \preccurlyeq_p \min\left(\text{middle}_{\text{SSE}}\right),\ \max\left(\text{north}_{\text{SSE}}\right) \preccurlyeq_p \max\left(\text{middle}_{\text{SSE}}\right), \\
\min\left(\text{south}_{\text{SSE}}\right) \preccurlyeq_l \min\left(\text{middle}_{\text{SSE}}\right),\ \max\left(\text{south}_{\text{SSE}}\right) \preccurlyeq_u \max\left(\text{middle}_{\text{SSE}}\right), \\
\text{south}_{\text{SSE}} \preccurlyeq_s \text{middle}_{\text{SSE}}, \\
\min\left(\text{south}_{\text{SSE}}\right) \preccurlyeq_l \min\left(\text{north}_{\text{SSE}}\right),\ \max\left(\text{south}_{\text{SSE}}\right) \preccurlyeq_p \max\left(\text{north}_{\text{SSE}}\right).
\end{gathered}$$

| Investigated area | Leq, 24 hrs. | Mean value - air traffic noise disturbance |
|---|---|---|
| WEST | | |
| WEST | | |
| Kl. Winternheim 3 | 47.05 | 2.64 |
| Mainz 5 | 53.13 | 4 |
| Mainz 6 | 54.85 | 3.6 |
| MIDDLE | | |
| Hochheim 4 | 50.72 | 3.49 |
| Hochheim 5 | 53.29 | 3.41 |
| Hochheim 6 | 56.87 | 4.04 |
| Bischofsheim 7 | 58.20 | 3.9 |
| EAST | | |
| Flörsheim 6 | 56.87 | 3.45 |
| Flörsheim 8 | 60.43 | 4.06 |
| Rüsselsheim 1 | 41.45 | 2.23 |
| Rüsselsheim 2 | 45.98 | 1.93 |
| Rüsselsheim 4 | 50.1 | 2.84 |
| Rüsselsheim 8 | 60.41 | 4.02 |
| Rüsselsheim 9 | 62.16 | 3.33 |
| Königsstädten 1 | 43.45 | 2 |
| Königsstädten 2 | 45.21 | 2.4 |
| Königsstädten 3 | 47.39 | 2.66 |
| Raunheim 9A | 61.94 | 4.33 |
| Raunheim 9B | 64.4 | 3.44 |
| Raunheim 9C | 65.15 | 4.19 |

The resulting west order relations in respect of Leq, 24 hours and the mean value of the air traffic noise disturbance are then:

$$\min\left(\text{west}_{\text{west}}\right) \preccurlyeq_c \min\left(\text{middle}_{\text{west}}\right),\ \max\left(\text{west}_{\text{west}}\right) \preccurlyeq_s \max\left(\text{middle}_{\text{west}}\right),$$
$$\text{east}_{\text{west}} \preccurlyeq_{mc} \text{middle}_{\text{west}},$$
$$\min\left(\text{east}_{\text{west}}\right) \preccurlyeq_c \min\left(\text{west}_{\text{west}}\right),\ \max\left(\text{west}_{\text{west}}\right) \preccurlyeq_c \max\left(\text{east}_{\text{west}}\right),$$
$$\text{east}_{\text{west}} \preccurlyeq_l \text{west}_{\text{West}},\ \text{west}_{\text{west}} \preccurlyeq_u \text{east}_{\text{west}}.$$

| Investigated area WEST | Leq, 06-22 hrs. | Mean value - MHI |
|---|---|---|
| WEST | | |
| Kl. Winternheim 3 | 48.21 | 76 |
| Mainz 5 | 54.28 | 72 |
| Mainz 6 | 55.99 | 77 |
| MIDDLE | | |
| Hochheim 4 | 52 | 74 |
| Hochheim 5 | 54.87 | 68 |
| Hochheim 6 | 59 | 75 |
| Bischofsheim 7 | 59.33 | 77 |
| EAST | | |
| Flörsheim 6 | 58.19 | 80.5 |
| Flörsheim 8 | 61.5 | 80 |
| Rüsselsheim 1 | 41.53 | 82 |
| Rüsselsheim 2 | 47.16 | 87 |
| Rüsselsheim 4 | 51.27 | 81 |
| Rüsselsheim 8 | 61.6 | 66 |
| Rüsselsheim 9 | 63.34 | 80 |
| Königsstädten 1 | 43.65 | 85 |
| Königsstädten 2 | 45.46 | 76 |
| Königsstädten 3 | 47.65 | 78 |
| Raunheim 9A | 63.14 | 74 |
| Raunheim 9B | 65.54 | 84 |
| Raunheim 9C | 66.34 | 77 |

The west order relations in respect of Leq, 06-22 hours and the mean MHI value are then:

$$\max\left(\text{west}_{\text{west}}\right) \preccurlyeq_c \max\left(\text{middle}_{\text{west}}\right),\ \text{west}_{\text{west}} \preccurlyeq_u \text{middle}_{\text{west}},$$
$$\max\left(\text{middle}_{\text{west}}\right) \preccurlyeq_u \max\left(\text{east}_{\text{west}}\right),\ \text{middle}_{\text{west}} \preccurlyeq_u \text{east}_{\text{west}},$$
$$\max\left(\text{west}_{\text{west}}\right) \preccurlyeq_u \max\left(\text{east}_{\text{west}}\right),\ \text{west}_{\text{west}} \preccurlyeq_u \text{east}_{\text{west}}.$$

## 4 Summary and Outlook

Although up to now no large-scale systematic noise research studies exist in relation to the fourth runway of the Frankfurt Airport – the noise levels have been known only since the end of 2011 / beginning of 2012 – subjectively the highly irritated citizens affected are beset with anxiety: deafening noise, resulting in diseases and worthless real estate which they can no longer sell – and all of this in spite of an overall rise in the demand for real estate, with a turnover of 5.1 billion Euros. Together with Munich, Dresden and Leipzig, Frankfurt a.M. has the largest population increase of all German cities with a population of more than 500,000, of course with rising prices. In no other large German city more new condominiums have been sold in relation to the overall residential property market than in Frankfurt: 40%. In relation to the population, with around 1600 new condominiums Frankfurt occupies the second place after Munich (see [19]). Nevertheless, a spill-over effect is still noticeably felt (precarization from one district to another with decreasing purchasing power). A migration trend

Försheim $\rightarrow$ Hattersheim $\rightarrow$ Liederbach $\rightarrow$ Kelkheim $\rightarrow$ Königstein/Taunus

is becoming evident, which on the basis of a corresponding data situation would result in still stronger comparative order relations, as in Chapter 3. From the scientific point of view, the question of order-relevant comparative relations remains. Here also, it is not enough to selectively examine only a specific district value and arrive at a "global" result on the basis of this. Here also, provided that relevant serious databases are available, it is possible to work descriptively with order relations. It is precisely this comparison which allows exact conclusions in respect of "new" air traffic noise.

Assuming site-independent value adjustments poses problems, just as the assumption of a linear function for the relationship between loss in value and air traffic intensity. Nevertheless, there is an order-relative relationship between the appropriate properties for real estate in the vicinity of the airport. At the same time, however, a nation-wide price and value loss factor does not exist.

There is no shortage of investigations and analyses of price and value developments for real estate. However, the usefulness of the results is generally limited, as order-relevant viewpoints are not used for these. The difficulty lies above all in extracting the influence of air traffic noise on values. The interpretation of the results is mostly ambivalent: besides the disadvantages, a nearby airport also has advantages, such as good infrastructure, which also affect real estate values. Furthermore, a certain "getting used to effect" to the current situation can compensate somewhat for prices. The Federal Environment Agency has nevertheless compiled publications and has attempted to summarize these results in the form of a general. A value decrease factor of 0.87% per decibel increase in air traffic noise above a continuous sound level of 55dB (see Immobilienzeitung (Real Estate Journal) [8]). If one considers the interval treatments of the optimized sets in Chapter 3, this factor influences three of the nine sites in the west region. A study of the Chemnitz University of Technology [4] leads to a different result. The researchers report a value decrease of 0.83% per decibel above 40dB for residential property in the Rhine-Main region in the vicinity of the airport. For the optimized sets this results in drastic changes: all sites of the west region are then affected.

The characterization of the conflict situation elaborated in the foregoing chapters therefore has an influence on the social and monetary factor in respect of real estate: a site-dependent value adjustment. Order relations with different focal themes were determined in order to obtain the essential structure of non-ordered as-built properties as a set-theoretical and order-relevant exploration.

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## References

[1] M. Bullinger, Lebensqualität – Entscheidungshilfen durch ein neues evaluiertes Konzept? Soziale Gerechtigkeit im Gesundheitswesen, Springer 1992.

[2] M. Bullinger, Zum Einfluss wahrgenommener Umweltbedingungen auf die subjektive Gesundheit, Umwelt und Gesundheit, 1998, 83-98.

[3] A. Chiriaev, G.W. Walster, Interval Arithmetic Specification, Technical Report, 1998.

[4] K.H. Eger, K. Köhler, D.T.G. Rübbelke, S. Schnorr, F. Thießen, Immobilien und Fluglärm, WWDP 69 (2005).

[5] J.M. Fields, Standardized General-Purpose Noise Reaction Questions for Community Noise Surveys: Research and a Recommendation, Journal of Sound and Vibration 242/4 (2001), 641-679.

[6] W. Hagedorn, Untersuchungsbericht zur Analyse von Immobilienpreisveränderungen im Umfeld des Flughafens München, 2010.

[7] Ifok GmbH, Belästigung durch Fluglärm im Umfeld des Frankfurter Flughafens, 2006.

[8] Immobilienzeitung – Fachzeitung für die Immobilienwirtschaft, 5 (2012).

[9] Federal Association of German Realtors (IVD), IVD Real Estate Price Index for 2008/2009, IMMO Ideen GmbH for the Federal Association of German Realtors (IVD) Association of Real Estate Consultants.

[10] J. Jahn, Vector Optimization – Theory, Applications, and Extensions, Springer, Second Edition, 2011.

[11] J.Jahn, T.X.D Ha, New Order Relations in Set Optimization, J. Optim. Theory Appl. 148 (2011).

[12] D. Kuroiwa, The Natural Criteria in Set-Valued Optimization, Research on Nonlinear Analysis and Convex Analysis, Surikaisekikenkyusho Kokyuroku 1031 (1998), 85-90.

[13] D. Kuroiwa, Some Duality Theorems of Set-Valued Optimization with Natural Criteria, Nonlinear Analysis and Convex Analysis, 1999, 221-228.

[14] Z.G. Nishnianidze, Fixed Points of Monotonic Multiple-Valued Operators. Bull. Georgian Acad. Sci. 114 (1984), 489-491.

[15] SUN Microsystems, Inc., Interval Arithmetic Programming Reference, 2000.

[16] C. Tammer, Site Problems in Urban Planning (research project), 1999-2003.

[17] Federal Environment Agency, Guidelines for the Calculation of Air Traffic Noise (AzB-99): Draft of the Federal Environment Agency for the description of noise emissions of current aircraft groups covered by the draft revisions of the Air Traffic Noise Act, 1999.

[18] J. Ware, C. Sherbourbe, The MOS 36-item Short-Form Health Survey (SF-36), I. Conceptual Framework and Item Selection, Medical Care 30 (1992), 473-483.

[19] www. frankfurt.de, Evaluation Team – Debus, Purchase Price Meeting 2011.

[20] R.C. Young, The Algebra of Many-Valued Quantities, Math. Ann. 104 (1931), 260-290.

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