Commun. Korean Math. Soc. **23** (2008), No. 1, pp. 1–10

ORDER SYSTEMS, IDEALS AND RIGHT FIXED MAPS OF SUBTRACTION ALGEBRAS

Young Bae Jun, Chul Hwan Park, and Eun Hwan Roh

Abstract. Conditions for an ideal to be irreducible are provided. The notion of an order system in a subtraction algebra is introduced, and related properties are investigated. Relations between ideals and order systems are given. The concept of a fixed map in a subtraction algebra is discussed, and related properties are investigated.

1. Introduction

B. M. Schein [14] considered systems of the form $(\Phi; \circ, \setminus)$, where Φ is a set of functions closed under the composition "$\circ$" of functions (and hence $(\Phi; \circ)$ is a function semigroup) and the set theoretic subtraction "$\setminus$" (and hence $(\Phi; \setminus)$ is a subtraction algebra in the sense of [2]). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. B. Zelinka [15] discussed a problem proposed by B. M. Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. Y. B. Jun et al. [10] introduced the notion of ideals in subtraction algebras and discussed characterization of ideals. In [6], Y. B. Jun and H. S. Kim established the ideal generated by a set, and discussed related results. Y. B. Jun and K. H. Kim [11] introduced the notion of prime and irreducible ideals of a subtraction algebra, and gave a characterization of a prime ideal. They also provided a condition for an ideal to be a prime/irreducible ideal. In this paper, we give conditions for an ideal to be irreducible. We introduce the notion of an order system in a subtraction algebra, and investigate related properties. We provide relations between ideals and order systems. We deal with the concept of a fixed map in a subtraction algebra, and investigate related properties.

2. Preliminaries

By a *subtraction algebra* we mean an algebra $(X; -)$ with a single binary operation "$-$" that satisfies the following identities: for any $x, y, z \in X$,

Received January 27, 2007; Revised November 27, 2007.
2000 *Mathematics Subject Classification.* 03G25, 06B10, 06D99.
Key words and phrases. (weak, complicated) subtraction algebra, (irreducible) ideal, order system, right fixed map, kernel.

(S1) $x - (y - x) = x$;
(S2) $x - (x - y) = y - (y - x)$;
(S3) $(x - y) - z = (x - z) - y$.

The last identity permits us to omit parentheses in expressions of the form $(x - y) - z$. The subtraction determines an order relation on X: $a \leq b \Leftrightarrow a - b = 0$, where $0 = a - a$ is an element that does not depend on the choice of $a \in X$. The ordered set $(X; \leq)$ is a semi-Boolean algebra in the sense of [2], that is, it is a meet semilattice with zero 0 in which every interval $[0, a]$ is a Boolean algebra with respect to the induced order. Here $a \wedge b = a - (a - b)$; the complement of an element $b \in [0, a]$ is $a - b$; and if $b, c \in [0, a]$, then

$$\begin{aligned} b \vee c &= (b' \wedge c')' = a - ((a - b) \wedge (a - c)) \\ &= a - ((a - b) - ((a - b) - (a - c))). \end{aligned}$$

In a subtraction algebra, the following are true (see [10, 11]):

(a1) $(x - y) - y = x - y$.
(a2) $x - 0 = x$ and $0 - x = 0$.
(a3) $(x - y) - x = 0$.
(a4) $x - (x - y) \leq y$.
(a5) $(x - y) - (y - x) = x - y$.
(a6) $x - (x - (x - y)) = x - y$.
(a7) $(x - y) - (z - y) \leq x - z$.
(a8) $x \leq y$ if and only if $x = y - w$ for some $w \in X$.
(a9) $x \leq y$ implies $x - z \leq y - z$ and $z - y \leq z - x$ for all $z \in X$.
(a10) $x, y \leq z$ implies $x - y = x \wedge (z - y)$.
(a11) $(x \wedge y) - (x \wedge z) \leq x \wedge (y - z)$.
(a12) $(x - y) - z = (x - z) - (y - z)$.

As a weak form of a subtraction algebra, Jun et al. discussed the weak subtraction algebras as follows:

Definition 2.1 ([8])**.** By a *weak subtraction algebra* (*WS-algebra*), we mean a triplet $(W, -, 0)$, where W is a nonempty set, $-$ is a binary operation on W and $0 \in W$ is a nullary operation, called *zero element*, such that

(S3) $(\forall x, y, z \in W)$ $((x - y) - z = (x - z) - y)$,
(S4) $(\forall x \in W)$ $(x - 0 = x,\ x - x = 0)$,
(a12) $(\forall x, y, z \in W)$ $((x - y) - z = (x - z) - (y - z))$.

Note that every subtraction algebra is a WS-algebra, but the converse is not true in general (see [8]).

3. Order systems and ideals in WS-algebras

In what follows, let X denote a WS-algebra unless otherwise specified.

Definition 3.1. A nonempty subset A of X is called an *ideal* of X if it satisfies

(b1) $0 \in A$

(b2) $(\forall x \in X)\ (\forall y \in A)\ (x - y \in A \Rightarrow x \in A)$.

The set of all ideals of X will be denoted by $Id(X)$.

Lemma 3.2. *An ideal A of a subtraction algebra X has the following property:*

$$(\forall x \in X)(\forall y \in A)(x \leq y \Rightarrow x \in A).$$

Proof. Straightforward. □

Theorem 3.3. *Let A be a nonempty subset of X. Then the set*

$$K := \left\{ x \in X \;\middle|\; \begin{array}{l} (\cdots((x - a_1) - a_2) - \cdots) - a_n = 0 \\ \text{for some } a_1, a_2, \ldots, a_n \in A \end{array} \right\}$$

is a minimal ideal of X containing A.

Proof. It is similar to the proof of Theorem 3.2 in [6]. □

We say that the ideal K is the *ideal generated* by A, and is denoted by $\langle A \rangle$.

Definition 3.4. An ideal A of X is said to be *irreducible* if for any ideals C and D of X, $A = C \cap D$ implies $A = C$ or $A = D$.

Theorem 3.5. *If $A \in Id(X)$ satisfies the following assertion:*

$$(\forall x, y \in X \setminus A)\,(\exists z \in X \setminus A)\,(z - x \in A,\ z - y \in A), \tag{1}$$

then A is an irreducible ideal of X.

Proof. Assume that $A \in Id(X)$ satisfies (1). Let $C, D \in Id(X)$ be such that $A = C \cap D$, $A \neq C$ and $A \neq D$. Then there exist $x \in C \setminus A \subset X \setminus A$ and $y \in D \setminus A \subset X \setminus A$. It follows from (1) that there exists $z \in X \setminus A$ such that $z - x \in A$ and $z - y \in A$. Since $x \in C$ and $z - x \in A = C \cap D \subset C$, we have $z \in C$ because C is an ideal of X. Also, $y \in D$ and $z - y \in D$, which imply $z \in D$. Hence $z \in C \cap D = A$, which is a contradiction. Hence A is an irreducible ideal of X. □

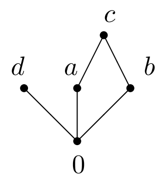
Corollary 3.6 ([11])**.** *Let $A \in Id(X)$. Assume that for any $x, y \in X \setminus A$, there exists $z \in X \setminus A$ such that $z \leq x$ and $z \leq y$. Then A is an irreducible ideal of X.*

Definition 3.7. Let X be a poset. A nonempty subset I of X is called an *order system* of X if it satisfies:

(b3) $(\forall x \in X)\ (\forall y \in I)\ (x \leq y \Rightarrow x \in I)$,
(b4) $(\forall x, y \in I)\ (\exists z \in I)\ (x \leq z,\ y \leq z)$.

The set of all order systems of a poset X will be denoted by $O_s(X)$. Note that if X is a poset with the bottom element $\perp$, then every order system of X contains the bottom element $\perp$.

Example 3.8. Let $X = \{0, a, b, c, d\}$ be a poset with the following Hasse diagram:



Then $I_1 := \{0, a\} \in O_s(X)$, $I_2 := \{0, a, b, c\} \in O_s(X)$, but $J_1 := \{0, b, c\} \notin O_s(X)$ and $J_2 := \{0, a, d\} \notin O_s(X)$.

Theorem 3.9. *For every WS-algebra X, we have $O_s(X) \subset Id(X)$.*

Proof. Let $I \in O_s(X)$. Since I is nonempty, obviously $0 \in I$. Now let $x, y \in X$ satisfy $x - y \in I$ and $y \in I$. Then there exists $z \in I$ such that $x - y \leq z$ and $y \leq z$ by (b4). It follows from (a2) and (a12) that

$$x - z = (x - z) - 0 = (x - z) - (y - z) = (x - y) - z = 0 \in I$$

so from (b2) that $x \in I$. Therefore $I \in Id(X)$, and so $O_s(X) \subset Id(X)$. □

The following example shows that an ideal is not an order system in general.

Example 3.10. (1) Let $X = \{0, a, b, c, d\}$ be a set with the following Cayley table:

$-$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	a
b	b	b	0	0	b
c	c	b	a	0	c
d	d	d	d	d	0

Then $(X, -)$ is a subtraction algebra, and hence a WS-algebra. It is easy to verify that $Q_1 := \{0, a, d\} \in Id(X)$, but $Q_1 := \{0, a, d\} \notin O_s(X)$.

(2) Let $X = \{0, a, b, c, d\}$ be a set with the following Cayley table:

$-$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	a	0
b	b	b	0	b	b
c	c	c	c	0	c
d	d	d	d	d	0

Then $(X, -)$ is a WS-algebra, which is not a subtraction algebra. It is easy to verify that $Q_2 := \{0, a, c\} \in Id(X)$, but $Q_2 := \{0, a, c\} \notin O_s(X)$.

To make an ideal to be an order system, we need more strong condition.

Definition 3.11 ([9])**.** A subtraction algebra X is said to be *complicated* if for any $a, b \in X$ the set

$$\mathscr{G}(a, b) := \{x \in X \mid x - a \leq b\}$$

has the greatest element.

The greatest element of $\mathscr{G}(a, b)$ is denoted by $a + b$.

Lemma 3.12 ([9])**.** *If X is a complicated subtraction algebra, then*

$$(\forall a, b \in X)\,(a \le a + b,\ b \le a + b).$$

Theorem 3.13. *In a complicated subtraction algebra X, every ideal is an order system.*

Proof. Let Q be an ideal of a complicated subtraction algebra X. The condition (b3) follows from Lemma 3.2. Now let $x, y \in Q$. Since $(x+y)-x \le y$, it follows from Lemma 3.2 and (b2) that $x + y \in Q$ so from Lemma 3.12 that (b4) is valid. Hence Q is an order system of X. □

Corollary 3.14 ([9])**.** *Let Q be a nonempty subset of a complicated subtraction algebra X. Then Q is an ideal of X if and only if Q is an order system of X.*

Theorem 3.15. *Let $Q \in O_s(X)$. If Q is irreducible as an ideal of X, then*

$$(\forall a, b \in X \setminus Q)\,(\exists c \in X \setminus Q)\,(c \le a,\ c \le b).$$

Proof. Assume that

$$(\exists a, b \in X \setminus Q)(\forall c \in X)(c \le a,\ c \le b \Rightarrow c \in Q). \tag{2}$$

Let $Q(a)$ and $Q(b)$ be the ideals of X generated by $Q \cup \{a\}$ and $Q \cup \{b\}$ respectively. Then $Q \subset Q(a) \cap Q(b)$. Let $x \in Q(a) \cap Q(b)$. Then $x \in Q(a)$ and $x \in Q(b)$. Thus

$$(\cdots(((x-a)-c_1)-c_2)-\cdots)-c_m = 0$$

and

$$(\cdots(((x-b)-d_1)-d_2)-\cdots)-d_n = 0$$

for some $c_1, c_2, \ldots, c_m, d_1, d_2, \ldots, d_n \in Q$. Since Q is an ideal of X, it follows from (b1) and (b2) that $x - a \in Q$ and $x - b \in Q$ so from (b4) that there exists $z \in Q$ such that $x - a \le z$ and $x - b \le z$. Hence

$$(x-z)-a = (x-a)-z = 0 \text{ and } (x-z)-b = (x-b)-z = 0,$$

and so $x - z \in Q$ by (2). But $Q \in Id(X)$ and $z \in Q$, and thus $x \in Q$ by (b2). Thus $Q(a) \cap Q(b) \subset Q$, and consequently $Q = Q(a) \cap Q(b)$ which is a contradiction. □

4. Right fixed maps

Definition 4.1. A *right fixed map* α of X is defined to be a self map $\alpha : X \to X$ satisfying $\alpha(x - y) = \alpha(x) - y$ for all $x, y \in X$.

Example 4.2. (1) Let $X = \{0, a, b\}$ be a set with the following Cayley table:

$-$	0	a	b
0	0	0	0
a	a	0	a
b	b	b	0

Then $(X, -)$ is a subtraction algebra, and hence a WS-algebra. It can be easily verify that the self map α of X defined by $\alpha(0) = 0$, $\alpha(a) = 0$, and $\alpha(b) = b$ is a right fixed map.

(2) Consider a subtraction algebra, and hence a WS algebra, $X = \{0, a, b, c\}$ with the following Cayley table:

$-$	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

Let $\beta : X \to X$ be defined by $\beta(0) = 0$, $\beta(a) = 0$, $\beta(b) = c$, and $\beta(c) = c$. Then β is not a right fixed map since $\beta(b - c) \neq \beta(b) - c$.

(3) Let $X = \{0, a, b, c, d\}$ be a set with the following Cayley table:

$-$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	b	b
c	c	c	c	0	c
d	d	d	d	d	0

Then $(X, -)$ is a WS-algebra, which is not a subtraction algebra. Let γ be a self map of X defined by $\gamma(0) = \gamma(a) = \gamma(b) = 0$, $\gamma(c) = c$ and $\gamma(d) = d$. Then γ is a right fixed map.

(4) Let $X = \{0, a, b, c, d\}$ be a set with the following Cayley table:

$-$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	0	b
c	c	c	c	0	c
d	d	d	d	d	0

Then $(X, -)$ is a WS-algebra, which is not a subtraction algebra. Let α be a self map of X defined by

$$\alpha(x) = \begin{cases} 0 & \text{if } \ x \in \{0, a\}, \\ x & \text{otherwise.} \end{cases}$$

Then α is a right fixed map of X.

Proposition 4.3. *If α is a right fixed map of X, then*

(i) $\alpha(0) = 0$,
(ii) $(\forall x \in X)\ (\alpha(0 - x) = 0)$,
(iii) $(\forall x \in X)\ (\alpha(x) \leq x)$,
(iv) $(\forall x, y \in X)\ (x \leq y \Rightarrow \alpha(x) \leq y)$.

Proof. (i) For every $x, y \in X$, we have

$$\alpha(0) = \alpha(0 - \alpha(0)) = \alpha(0) - \alpha(0) = 0.$$

(ii) For every $x \in X$, we have $\alpha(0 - x) = \alpha(0) = 0$.

(iii) For any $x \in X$, we get $0 = \alpha(0) = \alpha(x-x) = \alpha(x)-x$, and so $\alpha(x) \leq x$.

(iv) Assume that $x \leq y$ for every $x, y \in X$. Then $0 = \alpha(0) = \alpha(x - y) = \alpha(x) - y$, and so $\alpha(x) \leq y$. □

Definition 4.4. For a right fixed map α of X, the *kernel* of α, denoted by $\ker(\alpha)$, is defined to be the set

$$\ker(\alpha) = \{x \in X \mid \alpha(x) = 0\}.$$

Obviously $\ker(\alpha) \neq \emptyset$ since $0 \in \ker(\alpha)$.

Theorem 4.5. *Let α be a right fixed map of X. Then α is one-to-one if and only if* $\ker(\alpha) = 0$.

Proof. Assume that α is one-to-one and let $x \in \ker(\alpha)$. Then $\alpha(x) = 0 = \alpha(0)$, and thus $x = 0$, i.e., $\ker(\alpha) = \{0\}$. Conversely suppose that $\ker(\alpha) = \{0\}$. Let $x, y \in X$ satisfy $\alpha(x) = \alpha(y)$. Since $\alpha(y) \leq y$, it follows from (a9) that $\alpha(x-y) = \alpha(x)-y \leq \alpha(x)-\alpha(y) = 0$ so that $\alpha(x-y) = 0$. Hence $x-y \in \ker(\alpha)$, and so $x - y = 0$. Similarly, $y - x = 0$. This proves that $x = y$. Therefore α is one-to-one. □

Theorem 4.6. *Let α be a right fixed map of X. Then α is one-to-one if and only if α is the identity map.*

Proof. Sufficiency is obvious. Suppose that α is one-to-one. For every $x \in X$, we have

$$\alpha(x - \alpha(x)) = \alpha(x) - \alpha(x) = 0 = \alpha(0)$$

and so $x - \alpha(x) = 0$, i.e., $x \leq \alpha(x)$. Since $\alpha(x) \leq x$ for all $x \in X$, it follows that $\alpha(x) = x$ so that α is the identity map. □

Theorem 4.7. *Let α be a right fixed map of X. If α is idempotent, i.e., $\alpha(\alpha(x)) = \alpha(x)$ for all $x \in X$, then*

(i) $(\forall x \in X)\,(\alpha(x) = x \Leftrightarrow x \in \mathrm{Im}(\alpha))$.

(ii) $\ker(\alpha) \cap \mathrm{Im}(\alpha) = \{0\}$.

Proof. (i) Necessity is obvious. If $x \in \mathrm{Im}(\alpha)$, then $\alpha(y) = x$ for some $y \in X$. Thus $\alpha(x) = \alpha(\alpha(y)) = \alpha(y) = x$.

(ii) If $x \in \ker(\alpha) \cap \mathrm{Im}(\alpha)$, then $\alpha(x) = 0$ and $\alpha(y) = x$ for some $y \in X$. It follows that

$$0 = \alpha(x) = \alpha(\alpha(y)) = \alpha(y) = x$$

so that $\ker(\alpha) \cap \mathrm{Im}(\alpha) = \{0\}$. □

The following example shows that a WS-algebra X does not satisfy the assertion (a8) in general.

Example 4.8. Let $X = \{0, a, b, c, d\}$ be a WS-algebra, which is not a subtraction algebra, described in Example 4.2(4). We know that $b \leq c$, but there does not exist $w \in X$ such that $b = c - w$.

Theorem 4.9. *Let α be a right fixed map of a subtraction algebra X. Then*

(i) $(\forall x \in X)\ (\exists y \in \ker(\alpha), \exists z \in \mathrm{Im}(\alpha))\ (z = x - y)$.
(ii) *α is idempotent.*

Proof. Since $\alpha(x) \leq x$ for all $x \in X$, it follows from (a8) that $\exists w \in X$ such that $\alpha(x) = x - w$ so from (a6) that

$$x - (x - \alpha(x)) = x - (x - (x - w)) = x - w = \alpha(x).$$

Noticing that $x - \alpha(x) \in \ker(\alpha)$ and $\alpha(x) \in \mathrm{Im}(\alpha)$, we have the result (i). Moreover, using (a1) implies that

$$\alpha(\alpha(x)) = \alpha(x - w) = \alpha(x) - w = (x - w) - w = x - w = \alpha(x)$$

for all $x \in X$, which proves (ii). □

Corollary 4.10. *If α is a right fixed map of a subtraction algebra X, then*

(i) $(\forall x \in X)\,(\alpha(x) = x \Leftrightarrow x \in \mathrm{Im}(\alpha))$.
(ii) $\ker(\alpha) \cap \mathrm{Im}(\alpha) = \{0\}$.

Denote by $RF(X)$ the set of all right fixed maps of X. Let $\ominus$ be a binary operation on $RF(X)$ defined by $(\alpha \ominus \beta)(x) = \alpha(x) - \beta(x)$ for all $\alpha, \beta \in RF(X)$ and $x \in X$. It is easy to verify that if X is a WS-algebra, then $(RF(X), \ominus)$ is a WS-algebra. Let $IRF(X)$ denote the set of all idempotent right fixed maps of X.

Theorem 4.11. *For every $\alpha, \beta \in IRF(X)$, if $\alpha \ominus \beta = 0$ in $RF(X)$, then* $\mathrm{Im}(\alpha) \subset \mathrm{Im}(\beta)$.

Proof. Let $\alpha, \beta \in IRF(X)$ satisfy $\alpha \ominus \beta = 0$. If $y \in \mathrm{Im}(\alpha)$, then $\alpha(y) = y$ by Theorem 4.7, and hence

$$0 = (\alpha \ominus \beta)(y) = \alpha(y) - \beta(y) = y - \beta(y),$$

i.e., $y \leq \beta(y)$. Combining this with Proposition 4.3(iii), we get $y = \beta(y) \in \mathrm{Im}(\beta)$. Hence $\mathrm{Im}(\alpha) \subset \mathrm{Im}(\beta)$. □

Theorem 4.12. *Let $\alpha, \beta \in IRF(X)$. Then*

(i) $\alpha \ominus \beta \in RF(X)$.
(ii) *If $\alpha(\beta(x)) = \beta(\alpha(x))$ for all $x \in X$, then $\alpha \ominus \beta \in IRF(X)$.*
(iii) *If $\mathrm{Im}(\alpha) \subset \mathrm{Im}(\beta)$ and $\alpha(\beta(x)) = \beta(\alpha(x))$ for all $x \in X$, then $\alpha \ominus \beta = 0$ in $RF(X)$.*
(iv) $\mathrm{Im}(\alpha) \cap \ker(\beta) \subset \mathrm{Im}(\alpha \ominus \beta)$.

Proof. (i) For every $x, y \in X$, we have

$$\begin{aligned}(\alpha \ominus \beta)(x - y) &= \alpha(x - y) - \beta(x - y) \\ &= (\alpha(x) - y) - (\beta(x) - y) \\ &= (\alpha(x) - \beta(x)) - y \\ &= (\alpha \ominus \beta)(x) - y,\end{aligned}$$

and so $\alpha \ominus \beta \in RF(X)$.

(ii) Assume that $\alpha(\beta(x)) = \beta(\alpha(x))$ for all $x \in X$. Let $x \in X$. Then

$$\begin{aligned}(\alpha \ominus \beta)((\alpha \ominus \beta)(x)) &= (\alpha \ominus \beta)(\alpha(x) - \beta(x)) \\ &= \alpha(\alpha(x) - \beta(x)) - \beta(\alpha(x) - \beta(x)) \\ &= (\alpha(\alpha(x)) - \beta(x)) - (\beta(\alpha(x)) - \beta(x)) \\ &= (\alpha(x) - \beta(x)) - (\alpha(\beta(x)) - \beta(x)) \\ &= (\alpha(x) - \beta(x)) - \alpha(\beta(x) - \beta(x)) \\ &= (\alpha(x) - \beta(x)) - \alpha(0) \\ &= \alpha(x) - \beta(x) \\ &= (\alpha \ominus \beta)(x),\end{aligned}$$

that is, $\alpha \ominus \beta$ is idempotent. Hence $\alpha \ominus \beta \in IRF(X)$.

(iii) Suppose that $\mathrm{Im}(\alpha) \subset \mathrm{Im}(\beta)$ and $\alpha(\beta(x)) = \beta(\alpha(x))$ for all $x \in X$. Since $\alpha(x) \in \mathrm{Im}(\alpha) \subset \mathrm{Im}(\beta)$ for all $x \in X$, it follows from Theorem 4.7 that

$$\begin{aligned}(\alpha \ominus \beta)(x) &= \alpha(x) - \beta(x) = \beta(\alpha(x)) - \beta(x) \\ &= \alpha(\beta(x)) - \beta(x) = \alpha(\beta(x) - \beta(x)) \\ &= \alpha(0) = 0\end{aligned}$$

for all $x \in X$. Therefore $\alpha \ominus \beta = 0$.

(iv) If $y \in \mathrm{Im}(\alpha) \cap \ker(\beta)$, then $\beta(y) = 0$ and $\alpha(x) = y$ for some $x \in X$. It follows from (a2) that

$$y = \alpha(x) = \alpha(\alpha(x)) - 0 = \alpha(y) - \beta(y) = (\alpha \ominus \beta)(y) \in \mathrm{Im}(\alpha \ominus \beta).$$

Therefore $\mathrm{Im}(\alpha) \cap \ker(\beta) \subset \mathrm{Im}(\alpha \ominus \beta)$. □

We pose a problem: If $\alpha \in RF(X)$, then is $\ker(\alpha)$ an order system (or, an ideal) of X?

References

[1] J. C. Abbott, *Semi-Boolean algebras,* Matemat. Vesnik **4** (1967), 177–198.
[2] ______, *Sets, Lattices and Boolean Algebras,* Allyn and Bacon, Boston 1969.
[3] S. S. Ahn, Y. H. Kim, and K. J. Lee, *A relation on subtraction algebras,* Sci. Math. Jpn. Online **e-2005** (2005), 51–55.
[4] G. Birkhoff, *Lattice Theory,* Amer. Math. Soc. Colloq. Publ., Vol. 25, second edition 1984; third edition, 1967, Providence.
[5] G. Grätzer, *Universal Algebra, 2nd edition,* Springer-Verlag, New York Inc., 1979.
[6] Y. B. Jun and H. S. Kim, *On ideals in subtraction algebras,* Sci. Math. Jpn. Online **e-2006** (2006), 1081–1086.

[7] Y. B. Jun, H. S. Kim, and K. J. Lee, *The essence of subtraction algebras,* Sci. Math. Jpn. Online **e-2006** (2006), 1069–1074.
[8] Y. B. Jun, Y. H. Kim, and K. J. Lee, *Weak forms of subtraction algebras,* Bull. Korean Math. Soc. (submitted).
[9] Y. B. Jun, Y. H. Kim, and K. A. Oh, *Subtraction algebras with additional conditions,* Commun. Korean Math. Soc. (submitted).
[10] Y. B. Jun, H. S. Kim, and E. H. Roh, *Ideal theory of subtraction algebras,* Sci. Math. Jpn. Online **e-2004** (2004), 397–402.
[11] Y. B. Jun and K. H. Kim, *Prime and irreducible ideals in subtraction algebras,* Ital. J. Pure Appl. Math. (submitted)
[12] Y. B. Jun, X. L. Xin, and E. H. Roh, *A class of algebras related to BCI-algebras and semigroups,* Soochow J. Math. **24** (1998), no. 4, 309–321.
[13] Y. H. Kim and H. S. Kim, *Subtraction algebras and BCK-algebras*, Math. Bohemica **128** (2003), no. 1, 21–24.
[14] B. M. Schein, *Difference Semigroups,* Comm. Algebra **20** (1992), 2153–2169.
[15] B. Zelinka, *Subtraction Semigroups,* Math. Bohemica **120** (1995), 445–447.

Young Bae Jun
Department of Mathematics Education (and RINS)
Gyeongsang National University
Chinju 660-701, Korea
E-mail address: skywine@gmail.com

Chul Hwan Park
Department of Mathematics
University of Ulsan
Ulsan 680-749, Korea
E-mail address: chpark@ulsan.ac.kr or skyrosemary@gmail.com

Eun Hwan Roh
Department of Mathematics Education
Chinju National University of Education
Chinju 660-756, Korea
E-mail address: ehroh@cue.ac.kr