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Ordering Generalized Hexagonal Fuzzy Numbers Using Rank, Mode, Divergence and Spread

P. Rajarajeswari[1], A.Sahaya Sudha [2]
[1]*(Department of Mathematics, Chikkanna Government Arts College, Tirupur, India.)*
[2]*(Department of Mathematics, Nirmala College for women, Coimbatore, India.)*

Abstract: *The Fuzzy set theory has been applied in many fields such as management, engineering and almost in every business enterprise as well as day to day activities. Ordering fuzzy numbers plays an important role in approximate reasoning, optimization, forecasting, decision making, risk analysis, controlling, scheduling and various other usages in our day to day activities. This paper describes a ranking method for ordering fuzzy numbers based on Area, Mode, divergence, Spreads and Weights of generalized (non-normal) hexagonal fuzzy numbers.*

Keywords: *Ranking function, Hexagonal fuzzy numbers, Centroid points, Area*

I. Introduction

Ranking fuzzy number is used mainly in data analysis, artificial intelligence and various other fields of operations research. In fuzzy environment ranking fuzzy numbers is a very important in decision making procedure. Ranking fuzzy numbers were first proposed by Jain [1] for decision making in fuzzy situations by representing the ill- defined quantity as a fuzzy set. Some of these ranking methods have been compared and reviewed by Bortolan and Degani [2], and more recently by Chen and Hwang [3] .Lee and Li [4] proposed the comparison of fuzzy numbers. Liou and Wang [5] presented ranking fuzzy numbers with interval values. The centroids of fuzzy numbers have been examined recently. One of the most commonly used methods under the class of fuzzy scoring is the centroid point method .Cheng [6] used a centroid based distance method to rank fuzzy numbers in 1998.Then Chu and Tsao[7] utilized the area between the centroid point and the origin to rank fuzzy numbers in 2002.Abbasbandi and Asady[8] suggested a sign distance method for ranking fuzzy numbers in 2006. Wang Y.J and Lee.H.S[9] proposed the revised method of ranking fuzzy numbers with an area between the centroid and original points in2008.Since then several methods have been proposed by various researchers which includes ranking fuzzy numbers using maximizing and minimizing set[10] decomposition principle and signed distance [11] ,different heights and spreads[12] , rank, mode ,divergence and ,spread [13],area compensation distance method[14],Ordering of trapezoidal fuzzy numbers[15]

II. Preliminaries

2.1 DEFINITION: [16]

The Characteristic function $\mu_{\bar{A}}(x)$ of a crisp setA $\subseteq$ X assigns a value of either 1 or 0 to each individual in the universal setX.This function can be generalized to a function $\mu_{\bar{A}}$ such that the value assigned to the element of the universal set X fall within a specified range (i.e) $\mu_{\bar{A}} : X \rightarrow [0,1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\bar{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \in X\}$ defind by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

2.2 DEFINITION: [16]

A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R is said to be a fuzzy number of its membership function has the following characteristics

(i) $\mu_{\tilde{A}} : R \rightarrow [0,1]$ is continuous

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in [-\infty, a_1]U[a_4, \infty]$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on [a_1, a_2] and strictly decreasing on [a_3, a_4]

(iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [a_2, a_3]$, where $a_1 \le a_2 \le a_3 \le a_4$

2.3 DEFINITION: [16]

A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R is said to be generalized fuzzy number of its membership function has the following characteristics

(i) $\mu_{\tilde{A}} : R \to [0,1]$ is continuous

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in [-\infty, a_1] U [a_4, \infty]$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on [a_1, a_2] and strictly decreasing on [a_3, a_4]

(iv) $\mu_{\tilde{A}}(x) = w$ for all $x \in [a_2, a_3]$, where $0 < w \le 1$

2.4 DEFINITION: [16]

A fuzzy number A is a trapezoidal fuzzy number denoted by (a_1 , a_2, a_3 ,a_4) and its membership function is given below , where $a_1 \le a_2 \le a_3 \le a_4$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \frac{(a_4-x)}{(a_4-a_3)} & for\ a_3 \le x \le a_4 \\ 0 & for\ x > a_4 \end{cases}$$

2.5 DEFINITION: [16]

A generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; w)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} w(\frac{x-a_1}{a_2-a_1}), a_1 \le x \le a_2 \\ w \quad , a_2 \le x \le a_3 \\ w(\frac{a_4-x}{a_4-a_3}), a_3 \le x \le a_4 \end{cases}$$

III. Hexagonal Fuzzy Numbers

3.1 DEFINITION: [17]

A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by $\tilde{A}_H$ (a_1, a_2, a_3, a_4, a_5, a_6) where a_1, a_2, a_3, a_4, a_5, a_6 are real numbers and its membership function $\mu_{\tilde{A}_H}$(x) is given below.

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & for\ a_2 \le x \le a_3 \\ 1 & for\ a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & for\ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & for\ a_5 \le x \le a_6 \\ 0 & for\ x > a_6 \end{cases}$$

3.2 DEFINITION: [17]

A generalized fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by $\tilde{A}_H$ =(a_1, a_2, a_3, a_4, a_5, $a_{6,w}$) where a_1, a_2, a_3, a_4, a_5, a_6 are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given below.

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0 & for \ x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & for \ a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\,w\left(\frac{x-a_2}{a_3-a_2}\right) & for \ a_2 \le x \le a_3 \\ W & for \ a_3 \le x \le a_4 \\ 1-\frac{w}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & for \ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & for \ a_5 \le x \le a_6 \\ 0 & for \ x > a_6 \end{cases}$$

3.3 ORDERING OF HEXAGONAL FUZZY NUMBER: [18]

Let $\tilde{A}_H$= (a_1, a_2, a_3, a_4, a_5, a_6) and $\tilde{B}_H$ = (b_1, b_2, b_3, b_4, b_5, b_6) be in F(R) be the set of all real hexagonal fuzzy numbers

i) $\tilde{A}_H \approx \tilde{B}_H$ if and only if $a_i = b_i$, i=1,2,3,4,5,6

ii) $\tilde{A}_H \le \tilde{A}_H$ if and only if $a_i \le b_i$, i=1,2,3,4,5,6

iii) $\tilde{A}_H \ge \tilde{A}_H$ if and only if $a_i \ge b_i$, i=1,2,3,4,5,6

3.4 RANKING OF HEXAGONAL FUZZY NUMBERS: [18]

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function R: F (R) → R, where F (R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into a real number, where a natural order exists. For any two hexagonal fuzzy numbers $\tilde{A}_H$=(a_1,a_2,a_3,a_4,a_5,a_6)and $\tilde{B}_H$=(b_1,b_2,b_3,b_4,b_5,b_6) we have the following comparison

i) $\tilde{A}_H \approx \tilde{B}_H \Leftrightarrow R(\tilde{A}_H) = R(\tilde{B}_H)$

ii) $\tilde{A}_H \ge \tilde{B}_H \Leftrightarrow R(\tilde{A}_H) \ge R(\tilde{B}_H)$

iii) $\tilde{A}_H \le \tilde{B}_H \Leftrightarrow R(\tilde{A}_H) \le R(\tilde{B}_H)$

III. Proposed ranking Method

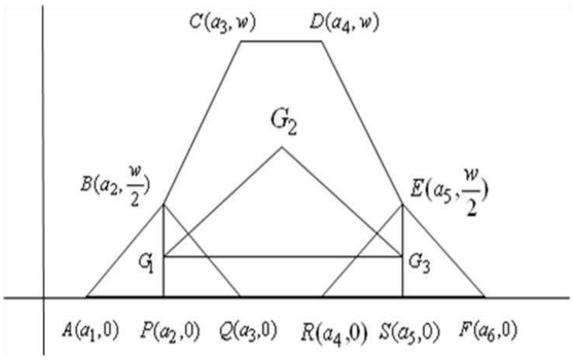

Fig.1 Generalized hexagonal fuzzy number

The centroid of a hexagonal fuzzy number is considered to be the balancing point of the hexagon (Fig.1).Divide the hexagonal into three plane figures .These three plane figures are a Triangle ABQ, Hexagon CDERQB and again a triangle REF respectively. The circumcenter of the centroids of these three plane figures is taken as the point of reference to define the ranking of generalized Hexagonal fuzzy numbers. Let the centroid of the three plane figures be G_1, G_2, G_3, respectively.

The centroid of the three plane figures is

$$G_1 = \left(\frac{a_1+a_2+a_3}{3}, \frac{w}{6}\right) ; G_2=\left(\frac{a_2+2a_3+2a_4+a_5}{6}, \frac{w}{2}\right) \quad G_3=\left(\frac{a_4+a_5+a_6}{3}, \frac{w}{6}\right) \text{ respectively.}$$

Equation of the line $\overline{G_1 G_3}$ is $y=\frac{w}{6}$ and G_2 does not lie on the line $G_1 G_3$.

Therefore G_1,G_2 and G_3 are non- collinear and they form a triangle. We define the centroid $G_{\tilde{A}_H}(\overline{x_0},\overline{y_0})$ of the triangle with vertices G_1 ,G_2 and G_3 of the generalized hexagonal fuzzy number $\tilde{A}_H$ =(a_1, a_2 ,a_3, a_4,a_5, a_6;w) as

$$G_{\tilde{A}_H}(\overline{x_0},\overline{y_0}) = \left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}, \frac{5w}{18}\right) \quad \text{----------------} \quad \textbf{(1)}$$

The ranking function of the generalized hexagonal fuzzy number $\tilde{A}_H$ =(a_1, a_2 ,a_3, a_4,a_5, a_6;w),which maps the set of all fuzzy numbers to a set of real numbers is defined as:

$$R(\tilde{A}_H) = (\overline{x_0})(\overline{y_0}) = \left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}\right)\left(\frac{5W}{18}\right) \quad \text{-------------} \quad \textbf{(2)}$$

This is the area between the centroid of the centroids $G_{\tilde{A}_H}(\overline{x_0},\overline{y_0})$ as defined in (1) and (2) the original point.

The mode of the generalized hexagonal fuzzy number $\tilde{A}_H$=(a_1, a_2 ,a_3, a_4,a_5, a_6;w) is defined as:

$$Mode = \frac{1}{2}\int_0^w (a_3 + a_4)dx ------(3)$$

The divergence of the generalized hexagonal fuzzy number $\tilde{A}_H$=(a_1, a_2 ,a_3, a_4,a_5, a_6;w) is defined as

$$Diverence = \int_0^w (a_6 - a_1)dx..........(4)$$

The left spread of the generalized hexagonal fuzzy number $\tilde{A}_H$=(a_1, a_2 ,a_3, a_4,a_5, a_6;w) is defined as

$$Leftspread \ \ ls = \int_0^{\frac{w}{2}} (a_2 - a_1)dx + \int_{\frac{w}{2}}^{w} (a_3 - a_2)dx$$

$$ls = \int_0^w (a_3 - a_1)dx \text{-----------}(5)$$

The right spread of the generalized hexagonal fuzzy number $\tilde{A}_H$=(a_1, a_2 ,a_3, a_4,a_5, a_6;w) is defined as

$$Rightspread \ \ rs = \int_0^{\frac{w}{2}} (a_6 - a_5)dx + \int_{\frac{w}{2}}^{w} (a_5 - a_4)dx$$

$$rs = \int_0^w (a_6 - a_4)dx ------(6)$$

4.1. PROPOSITION:

If $\tilde{A}_H$ =(a_1, a_2 ,a_3, a_4,a_5, a_6;w_1) and $\tilde{B}_H$=(b_1,b_2,b_3,b_4,b_5,b_6;w_2) are two generalized Hexagonal fuzzy numbers then

(i) $R(\tilde{A}_H) = R(\tilde{B}_H)$

(ii) $Mode(\tilde{A}_H) = Mode(\tilde{B}_H)$

(iii) $Divergence(\tilde{A}_H) = Divergence(\tilde{B}_H)$ then,

(a) Left spread($\tilde{A}_H$) > Left spread($\tilde{B}_H$) if $w_1 a_3 > w_2 b_3$
(b) Left spread($\tilde{A}_H$) < Left spread($\tilde{B}_H$) if $w_1 a_3 < w_2 b_3$
(c) Left spread($\tilde{A}_H$) = Left spread($\tilde{B}_H$) if $w_1 a_3 = w_2 b_3$

Proof:
From the assumptions
(i) R($\tilde{A}_H$) =R($\tilde{B}_H$)

$$(i.e) w_1 \left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}\right)\left(\frac{5}{18}\right) = w_2 \left(\frac{2b_1+3b_2+4b_3+4b_3+3b_5+2a_6}{18}\right)\left(\frac{5}{18}\right)$$

(i.e.) $w_1(2a_1+3a_2+4a_3+4a_4+3a_5+2a_6) = w_2(2b_1+3b_2+4b_3+4b_4+3b_5+2b_6) --(7)$

(ii) Mode($\tilde{A}_H$) = Mode($\tilde{B}_H$)

(i.e.) $w_1(a_3+a_4) = w_2(b_3+b_4) ------(8)$

(iii) Divergence($\tilde{A}_H$) = Divergence($\tilde{B}_H$)

(i.e.) $w_1(a_6-a_1) = w_2(b_6-b_1)$ --------------- (9)

Solving (7),(8) and (9)

$w_1 a_1 = w_2 b_1$
$w_1 a_6 = w_2 b_6$
$w_1(a_3+a_4) = w_2(b_3+b_4)$
$w_1(a_2+a_5) = w_2(b_2+b_5)$

(a) Left spread($\tilde{A}_H$) > Left spread($\tilde{B}_H$)
$\Leftrightarrow w_1(a_3-a_1) > w_2(b_3-b_1)$
$\Leftrightarrow w_1 a_3 > w_2 b_3$ ($\because w_1 a_1 = w_2 b_1$)
Hence Left spread($\tilde{A}_H$) > Left spread($\tilde{B}_H$) iff $w_1 a_3 > w_2 b_3$

(b) Left spread($\tilde{A}_H$) < Left spread($\tilde{B}_H$)
$\Leftrightarrow w_1(a_3-a_1) \prec w_2(b_3-b_1)$
$\Leftrightarrow w_1 a_3 < w_2 b_3$ ($\because w_1 a_1 = w_2 b_1$)
Hence Left spread($\tilde{A}_H$) < Left spread($\tilde{B}_H$) iff $w_1 a_3 < w_2 b_3$

(c) Left spread($\tilde{A}_H$) = Left spread($\tilde{B}_H$)
$\Leftrightarrow w_1(a_3-a_1) = w_2(b_3-b_1)$
$\Leftrightarrow w_1 a_3 = w_2 b_3$(($\because w_1 a_1 = w_2 b_2$)
Hence Left spread($\tilde{A}_H$) = Left spread($\tilde{B}_H$) iff $w_1 a_3 = w_2 b_3$

4.1.1 COROLLARY:
All the results of proposition 4.1 also hold for right spread.

4.2 PROPOSED APPROACH FOR RANKING GENERALIZED HEXAGONAL FUZZY NUMBER:
If $\tilde{A}_H=(a_1, a_2, a_3, a_4, a_5, a_6; w_1)$ and $\tilde{B}_H=(b_1,b_2,b_3,b_4,b_5,b_6;w_2)$ are two generalized hexagonal fuzzy numbers then

Step 1

Find R($\tilde{A}_H$) and R ($\tilde{B}_H$)

Case (i) If R($\tilde{A}_H$) > R ($\tilde{B}_H$)then $\tilde{A}_H > \tilde{B}_H$
Case (ii) If R($\tilde{A}_H$) < R ($\tilde{B}_H$)then $\tilde{A}_H < \tilde{B}_H$

Case (iii) If R($\tilde{A}_H$) = R ($\tilde{B}_H$)then $\tilde{A}_H = \tilde{B}_H$then go to Step 2

Step 2

Find mode ($\tilde{A}_H$) and mode ($\tilde{B}_H$)

Case (i) If mode ($\tilde{A}_H$) > mode ($\tilde{B}_H$)then $\tilde{A}_H > \tilde{B}_H$
Case (ii) If mode ($\tilde{A}_H$) < mode ($\tilde{B}_H$)then $\tilde{A}_H < \tilde{B}_H$
Case (iii) If mode ($\tilde{A}_H$) = mode ($\tilde{B}_H$)then $\tilde{A}_H = \tilde{B}_H$then go to Step 3

Step 3

Find divergence ($\tilde{A}_H$) and mode ($\tilde{B}_H$)

Case (i) If divergence ($\tilde{A}_H$) > divergence ($\tilde{B}_H$)then $\tilde{A}_H > \tilde{B}_H$
Case (ii) If divergence ($\tilde{A}_H$) < divergence ($\tilde{B}_H$)then $\tilde{A}_H < \tilde{B}_H$
Case (iii) If divergence ($\tilde{A}_H$) = divergence ($\tilde{B}_H$)then $\tilde{A}_H = \tilde{B}_H$then go to Step 4

Step 4

Find Left spread ($\tilde{A}_H$) and Left spread ($\tilde{B}_H$)

Case (i) Left spread($\tilde{A}_H$) > Left spread($\tilde{B}_H$)
i.e. $w_1 a_3 > w_2 b_3$ then $\tilde{A}_H > \tilde{B}_H$ (from proposition 4.1.)
Case (ii) Left spread($\tilde{A}_H$) < Left spread($\tilde{B}_H$)
ie $w_1 a_3 < w_2 b_3$ then $\tilde{A}_H < \tilde{B}_H$ (from proposition 4.1)
Case (iii) Left spread($\tilde{A}_H$) = Left spread($\tilde{B}_H$)
i.e $w_1 a_3 = w_2 b_3$ then $\tilde{A}_H = \tilde{B}_H$ (from proposition 4.1)

Step 5

Find w_1 and w_2

Case (i) If $w_1 > w_2$ then $\tilde{A}_H > \tilde{B}_H$
Case (ii) If $w_1 < w_2$ then $\tilde{A}_H < \tilde{B}_H$
Case (iii) If $w_1 = w_2$ then $\tilde{A}_H = \tilde{B}_H$

IV. Result and Discussions

Example: 1

Let $\tilde{A}_H$ =(0.02, 0.03 ,0.05, 0.06,0.08, 0.09;0.7) and
$\tilde{B}_H$=(0.04,0.06,0.1,0.12,0.16,0.18;0.35)

Step 1

R($\tilde{A}_H$)=0.693, R ($\tilde{B}_H$)=0.693, Since R($\tilde{A}_H$) = R ($\tilde{B}_H$) go to Step 2

Step 2

Mode($\tilde{A}_H$)=0.077, Mode ($\tilde{B}_H$)=0.077, Since Mode($\tilde{A}_H$) = Mode ($\tilde{B}_H$) go to Step 3

Step3

Divergence($\tilde{A}_H$)=0.049, Divergence ($\tilde{B}_H$)=0.049, Since Divergence($\tilde{A}_H$) = Divergence ($\tilde{B}_H$) go to Step 4

Step4

Left spread($\tilde{A}_H$)=0.021, Left spread ($\tilde{B}_H$)=0.021, Since Left spread($\tilde{A}_H$) = Left spread ($\tilde{B}_H$) go to Step 5

Step5

w_1=0.7, w_2=0.35 Since $w_1 > w_2$, $\tilde{A}_H > \tilde{B}_H$

Example:2
Let $\tilde{A}_H$ =(0.17,0.2,0.25,0.38,0.4,0.45;0.4) and $\tilde{B}_H$=(0.84,0.4,0.5,0.76,0.8,0.9;0.8)

Step 1
R($\tilde{A}_H$)=2.2 R ($\tilde{B}_H$)=8.896, Since R($\tilde{A}_H$) < R ($\tilde{B}_H$) , $\tilde{A}_H < \tilde{B}_H$

Example:3
Let $\tilde{A}_H$ =(0.3,0.5,0.5,0.7,0.8,0.9;1) and $\tilde{B}_H$=(0.1,0.5,0.6,0.7,0.8,0.9;1)

Step 1
R($\tilde{A}_H$)=11.1, R ($\tilde{B}_H$)= 11.1, Since R($\tilde{A}_H$) = R ($\tilde{B}_H$) , go to Step 2

Step2
Mode($\tilde{A}_H$)=1.2, Mode ($\tilde{B}_H$)=1.3, Since Mode($\tilde{A}_H$) < Mode ($\tilde{B}_H$) , $\tilde{A}_H < \tilde{B}_H$

Example:4
Let $\tilde{A}_H$ =(0.1,0.2,0.3,0.6,0.7,0.8;1) and $\tilde{B}_H$=(0.2,0.2,0.4,0.5,0.7,0.8;1)

Step 1
R($\tilde{A}_H$)=8.3 R ($\tilde{B}_H$)=8.3, Since R($\tilde{A}_H$) = R ($\tilde{B}_H$) , go to Step 2

Step2
Mode($\tilde{A}_H$)=0.9, Mode ($\tilde{B}_H$)=0.9, Since Mode($\tilde{A}_H$) = Mode ($\tilde{B}_H$) , go to Step3

Step3
Divergence($\tilde{A}_H$)=0.7 Divergence ($\tilde{B}_H$)=0.6,

Since Divergence($\tilde{A}_H$) > Divergence ($\tilde{B}_H$) , $\tilde{A}_H > \tilde{B}_H$

V. Conclusion

In this paper, a hexagonal fuzzy ranking is used for centroid of a triangle from an earlier version. [17, 18] In this ranking method we have used rank, mode, divergence and spreads are used and we infer the results found were satisfactory and the conditions are satisfied and this has been proved with illustrative examples and if more number of parameters also taken we can make the decision making process simple in working on risk and uncertainty with optimized ranking order. This ranking procedure can be applied in various decision making problems such as, fuzzy optimization, Fuzzy AHP.

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