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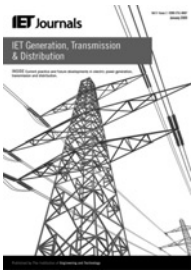

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Ordinal optimisation approach for locating and sizing of distributed generation

R.A. Jabr[1] *B.C. Pal*[2]

[1]*Electrical, Computer and Communication Engineering Department, Notre Dame University, P.O. Box 72, Zouk Mikhael, Zouk Mosbeh, Lebanon*
[2]*Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2BT, UK*
E-mail: rjabr@ndu.edu.lb

Abstract: This study presents an ordinal optimisation (OO) method for specifying the locations and capacities of distributed generation (DG) such that a trade-off between loss minimisation and DG capacity maximisation is achieved. The OO approach consists of three main phases. First, the large search space of potential combinations of DG locations is represented by sampling a relatively small number of alternatives. Second, the objective function value for each of the sampled alternatives is evaluated using a crude but computationally efficient linear programming model. Third, the top-*s* alternatives from the crude model evaluation are simulated via an exact non-linear programming optimal power flow (OPF) programme to find the best DG locations and capacities. OO theory allows computing the size *s* of the selected subset such that it contains at least *k* designs from among the true top-*g* samples with a pre-specified alignment probability AP. This study discusses problem-specific approaches for sampling, crude model implementation and subset size selection. The approach is validated by comparing with recently published results of a hybrid genetic algorithm OPF applied to a 69-node distribution network operating under Ofgem (UK) financial incentives for distribution network operators.

1 Introduction

Distributed generation (DG) technologies are among the main vehicles for the reduction of carbon emissions in accordance with the Kyoto agreement on climate change. Other benefits of DG include the reduction of losses and the deferment of investment for network upgrades. The different technologies that are classified as DG in the UK are categorised into renewable and fossil fuel-based sources. The renewable sources comprise wind turbines, biomass, photovoltaics, geothermal, small hydro, and tidal and wave. Fossil fuel-based sources consist of fuel cells, internal combustion engines and combustion turbines. The installation of DG in distributed networks has to be coordinated such that early connections do not effectively sterilise parts of the network by constraining the development of other, potentially larger, plant connections [1]. This necessitates providing the distribution network operator with tools for determining the DG locations and capacities that make best use of the existing network infrastructure. The information provided by such tools could be transferred to potential developers in the form of spare connection sites and sizes.

The problem of DG planning has recently received much attention by power system researchers. Different formulations have been solved using calculus-based methods, search-based methods and combinations of the previous techniques. The calculus-based methods include linear programming (LP) [2], second-order algorithms [3] and OPF-based approaches [1, 4]. These optimisation methods treat the DG capacities as continuous variables while their locations remain fixed. However, the problem of placing DG in practical networks belongs to the complexity category of non-deterministic polynomial (NP) complete, that is, it is almost certain that solving for its global optimum cannot be done efficiently on a computer. Solutions can be obtained by analytical approaches only under simplifying assumptions, such as placing a single DG [5]. Search-based methods have been proposed to seek the

optimal (or near-optimal) DG locations and capacities from candidate sites and sizes. The search-based methods comprise various artificial intelligence techniques such as genetic algorithms (GA) [6, 7], combined fuzzy-GA [8], multi-objective evolutionary approaches [9] and tabu search [10]. Combinations of search-based and calculus-based techniques that handle discrete and continuous variables have been also proposed to improve the modelling of DG sites and sizes. An example of such a hybrid technique is the combination of GA and optimal power flow (OPF) [11–13]. Another approach is the heuristic method of [14] for DG investment planning. It is also possible to handle both discrete and continuous variables via Benders decomposition. Such an approach has been proposed in [15] within the context of placing static var compensators for maximising the loading margin.

This paper presents an ordinal optimisation (OO) approach for solving the DG planning problem with discrete and continuous variables. A mathematical description of the OO technique is given in Appendix 1. OO is based on the idea that the relative order (instead of the cardinal value) of the performance of different alternatives in a decision problem is robust with respect to estimation noise [16]. This implies that if a set of alternatives is very approximately evaluated and ordered according to this approximate evaluation, then there is high probability that the actual good alternatives can be found in the top-s estimated good choices. As an example, consider the limiting case where the estimation noise associated with the approximate evaluation (crude model) has infinite variance, that is, the top-s alternatives are randomly picked. Moreover, assume that the search space has $N = 1000$ alternatives and the actual good enough alternatives are considered to be the top-50 ($g = 50$). By blindly picking $s = 86$ samples from the search space, the alignment probability that at least one good enough alternative ($k = 1$) is in the 86 samples is given by (1) from [17]

$$\mathrm{AP}(k=1) = \sum_{i=k}^{\min(g,s)} \frac{\binom{g}{i}\binom{N-g}{s-i}}{\binom{N}{s}} = \sum_{i=1}^{50} \frac{\binom{50}{i}\binom{914}{86-i}}{\binom{1000}{86}} \cong 0.99 \quad (1)$$

This implies that if any alternative in the top-50 is considered satisfactory, there is no need to do an exhaustive search to locate good alternatives. The previous example demonstrates more than 10-fold reduction in the search space. Moreover, the crude model is usually constructed such that the ordering according to the approximate evaluation is biased in favour of the actual good alternatives. Therefore the number of samples ($s = 86$) of the previous example is an upper bound on the size of the selected subset that contains at least one good enough alternative with 99% chance.

The utility of OO is in dealing with NP-complete problems such as DG planning with discrete and continuous variables. Previous power systems research has reported successful applications of OO for OPF [18] and constrained state estimation [19] with discrete controls, multiyear transmission expansion planning [20] and capacitor placement in transmission systems [21]. The contributions of this research lie in the specific algorithmic choices for the application of OO in DG planning, namely

1. the number of samples N from the entire search space and the corresponding heuristic rule for sampling;

2. the crude model for the fast and approximate evaluation of the N samples; and

3. the size s of the selected subset of choices that will be subjected to more elaborate and accurate analysis, for instance, an exact OPF solution. The analysis can also include stability simulations and short-circuit current computations.

The proposed OO approach is validated by comparing with recently published results from a combined GA–OPF approach [13].

2 Distributed generation planning

The evaluation of a given set of capacity expansion locations is dependent on the choice of the performance measure, typically the extreme value of an objective function in a given feasible region. The objective can take several forms among which are DG capacity maximisation [1, 2, 12], loss minimisation [3, 8, 10] or a combination of both [13].

Without loss of generality, this paper considers the specific UK distribution network operator requirements that specify incentives for DG connections and losses [13]

$$\text{maximise } C_{\mathrm{DG}} \sum_{j=1}^{N_{\mathrm{DG}}} P_{\mathrm{DG}j} + C_{\mathrm{L}}(P_{\mathrm{L(target)}} - P_{\mathrm{L(actual)}}) \quad (2)$$

where C_{DG} is the incentive for every MW of new DG in £/MW-year converted into £/MWh, C_{L} is the incentive/penalty for every MW decrease/increase of loss relative to the target in £/MWh, N_{DG} is the number of DG installed in the network, $P_{\mathrm{DG}j}$ is the real power output of the jth distributed generator in MW, $P_{\mathrm{L(target)}}$ is the target power loss level in MW and $P_{\mathrm{L(actual)}}$ is the actual power loss after DG installation in MW. It is computed as the summation of the real power injections at all the nodes.

Renewable DG sources including combined heat and power (CHP) stations that use renewable fuels (e.g. sewage gas) may also benefit from the climate change levy

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exemption for renewables [22]. Good quality CHP stations are also eligible to benefit from the climate change levy exemption specifically for CHP. Other incentives related to quality of service such as when DG participates into frequency and voltage control services are not considered in this paper.

The feasible region is formed of the typical OPF constraints

$$\left.\begin{array}{l}\sum P_i = 0\\ \sum Q_i = 0\end{array}\right\} \quad \text{at each node } i \text{ (net power injections)} \tag{3}$$

$$\left.\begin{array}{l}P_{Gj\min} \le P_{Gj} \le P_{Gj\max}\\ Q_{Gj\min} \le Q_{Gj} \le Q_{Gj\max}\end{array}\right\} \quad \begin{array}{l}\text{for each generator } j\\ \text{(power generation limits)}\end{array} \tag{4}$$

$$V_{i\min} \le V_i \le V_{i\max} \quad \text{for each node } i \text{ (voltage limits)} \tag{5}$$

$$S_{mn} \le S_{mn\max} \quad \begin{array}{l}\text{for each circuit from node } m \text{ to node } n\\ \text{(branch flow limits)}\end{array} \tag{6}$$

Following [12, 13], distributed generators are also assumed to operate in power factor control mode thus requiring an additional constraint

$$\frac{Q_{\mathrm{DG}j}}{P_{\mathrm{DG}j}} = \tan\phi_j \tag{7}$$

where $Q_{\mathrm{DG}j}$ is the real power output of the jth distributed generator in MVAr and ϕ_j is the constant power factor angle of the jth distributed generator. It is possible to further include constraints on DG penetration limit and permitted fault levels [4]. The above OPF problem can be solved using the MATPOWER software package [23]. It constitutes the exact model for evaluating the different alternatives of DG locations.

3 Ordinal optimisation approach

The search for the exact global solution to the problem of locating and sizing of DG requires for non-trivial cases an immense computational time. For instance, choosing 9 DG locations out of 68 potential candidates necessitates the evaluation of

$$\binom{68}{9} = \frac{68!}{9!(68-9)!} \cong 4.93 \times 10^{10} \text{ combinations}$$

With around 0.15 s per OPF solution on a Core-Duo 1.73 GHz processor, the evaluation of the above combinations sequentially takes more than 200 years to complete. The OO theory provides a probabilistic framework for reducing the search space and the computational effort involved in ranking the different alternatives. The OO algorithm for locating and sizing a pre-specified number of distributed generators can be summarised as follows:

1. Select N designs (i.e. combinations of DG locations) from the search space such that at least one of them is in the top α percent of all solutions with a probability level P. Typical values of α and P are 0.1 and 99%, respectively.

2. Evaluate and order the N designs using a crude and computationally cheap model.

3. Compute the number of top-s samples in the N designs that contain at least one actual good alternative, for instance one in the true top-50, with a given alignment probability level $\mathrm{AP}(k=1) \ge 0.95$. Evaluate each of the s samples using an exact OPF model and determine the good enough solution.

The implementation details relating to each of the above steps are discussed below.

3.1 Search space

Let the set Θ denote the population of all designs and assume that the solutions have been ordered with respect to their objective function (2) values from best to worst. Let Λ consist of the top α percent designs in Θ, that is,

$$\alpha = \frac{|\Lambda|}{|\Theta|} \tag{8}$$

where $|\ |$ denotes the cardinality of a set. If N^* designs are blindly picked from Θ, the probability that none of these designs is in Λ is $(1-\alpha)^{N^*}$. Thus, the probability that at least one of the N^* designs is in Λ is $1-(1-\alpha)^{N^*}$. If this probability is desired to be no less than P, then [24]

$$N^* \ge \frac{\ln(1-P)}{\ln(1-\alpha)} \tag{9}$$

Table 1 shows the minimum required values of N^* for different values of P and α. For instance, if at least one of the N^* solutions is desired to be in the top 0.1% of the search space with no less than 99% probability, then N^* should be at least 4603.

In practice, a heuristic rule is used to rank the nodes in terms of their suitability for DG placement, and the minimum number of top candidate locations N_C among the nodes is chosen such that the number of sample

Table 1 Required number of designs for applying OO

P, %	Top 0.1%	Top 0.01%
99	4603	46 050
99.9	6905	69 075
99.99	9206	92 099
99.999	11 508	115 124

designs is

$$N = \binom{N_C}{N_{DG}} = \frac{N_C!}{N_{DG}!(N_C - N_{DG})!} \geq \frac{\ln(1-P)}{\ln(1-\alpha)} \quad (10)$$

The heuristic rule operates on the complete set of nodes that are eligible for DG placement. At each stage, the rule uses the OPF described in Section 2 to identify the most suitable node for locating a distributed generator, that is, the node that results in the maximal value of the objective function (2). The ranking procedure is described in detail below.

Let T denote the list of nodes that are eligible for DG placement. The list TS will contain the sorted list of the top N_C DG locations when the procedure terminates. Initially, TS is empty. Unless otherwise specified, step $(j+1)$ follows step j.

Step 1: Given that the nodes in TS are DG sites, use the OPF solver to identify the node in T that when used as an additional DG site would yield the maximal value of the objective function (2). This requires executing an OPF solution for each DG location in T. In each OPF execution, the nodes in TS are assumed to be DG sites. After executing the OPF solutions, the DG location in T that yields the best objective function value is the desired node.

Step 2: Remove the node identified in Step 1 from T. Add it at the bottom of TS.

Step 3: If the length of list TS is N_C, print TS and terminate the search. Otherwise, go to Step 1.

The above procedure terminates after executing

$$\frac{N_C(2N_T - N_C + 1)}{2} \quad (11)$$

OPF solutions, where N_T is the initial length of list T. The search space formed of the N designs is denoted by Θ^N. Θ^N is representative of Θ [17].

3.2 Crude model

To speed up the search process, OO theory advocates the use of a crude model to evaluate each of the N designs in Θ^N. In the context of DG sizing, the crude model is formed of a linear version of the OPF in Section 2. The model is constructed by making use of an approximation formula that relates an arbitrary load flow variable y to the vector of independent injections $\boldsymbol{z}$ of the load flow problem. The vector $\boldsymbol{z}$ is formed of the squared voltage magnitudes at the slack and PV nodes, the real power injections at the PV and PQ nodes and the reactive power injections at the PQ nodes. The load flow variable y can be the squared voltage magnitude at a PQ node, the real power injection at the slack node, the reactive power injection at the slack or a PV node or the real/reactive line power flow. The expression of the variable y in terms of $\boldsymbol{z}$ is

$$y = \boldsymbol{\beta}^T \boldsymbol{z} \quad (12)$$

where $\boldsymbol{\beta}$ is a vector of coefficients given by

$$\boldsymbol{\beta} = [\boldsymbol{L}(\boldsymbol{x}_0)^T]^{-1} \boldsymbol{R}\boldsymbol{x}_0 \quad (13)$$

Appendix 2 includes a clarification of the notation and the derivation of (12) and (13).

By using (12), the operational constraints (4)–(7) in the OPF definition can be therefore approximated by a set of linear inequalities in terms of $\boldsymbol{z}$. Moreover, the variables in $\boldsymbol{z}$ are limited to the squared voltage magnitude at the slack and PV nodes, the real power generation at the PV nodes and the real power produced by DG at PQ nodes. The reactive power generation associated with DG at PQ nodes does not appear as a variable in $\boldsymbol{z}$ because the constant power factor condition allows it to be substituted out of the problem. As compared to the OPF model, the crude model can be therefore optimised using an LP solver [25] with considerably less computational effort.

3.3 Selected subset

Assume that the N alternatives in Θ^N are ordered according to the crude model from best to worst. Let S denote the subset of the top-s alternatives among the above crudely ordered N designs and let the actual top-g designs denote the good enough subset G, that is, the truly good enough designs are the top-g alternatives among the accurately ordered N designs in Θ^N. By assuming an infinite variance of the estimation noise of the crude model relative to the exact model, it is possible to compute the value s such that at least k truly good enough alternatives are in S with a given value of the alignment probability [17]

$$\mathrm{AP}(k) = \Pr[|G \cap S| \geq k] = \sum_{i=k}^{\min(g,s)} \frac{\binom{g}{i}\binom{N-g}{s-i}}{\binom{N}{s}} \quad (14)$$

Typically, the good enough subset G is formed of the true top-50 in Θ^N, the minimum alignment level k between the selected subset S and the good enough set G is one, and the alignment probability AP $(k=1)$ is set to at least 0.95. The size of S for a given N is then the minimum value of s which satisfies

$$\mathrm{AP}(k=1) = \sum_{i=1}^{\min(50,s)} \frac{\binom{50}{i}\binom{N-50}{s-i}}{\binom{N}{s}} \geq 0.95 \quad (15)$$

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To simplify the computation of s, it is possible to employ in (14) the binomial approximation to the hyper-geometric distribution [26]

$$\mathrm{AP}(k) = \sum_{i=k}^{\min(g,s)} \binom{s}{i} \rho^i (1-\rho)^{s-i} \tag{16}$$

where

$$\rho = \frac{g}{N} \tag{17}$$

For example, the size of the selected subset S corresponding to $N^* = 4603$ is 275. This size was obtained by incrementing s until finding the first value that satisfies (15) while employing the binomial approximation. In practice, the variance of the estimation noise is not infinite, that is, the crude model is biased in favour of the actual good alternatives. This implies that the value of s computed by (15) serves as an upper bound on the size of S that contains at least one good enough alternative $(k = 1)$ with a given $\mathrm{AP}(k = 1) \geq 0.95$. For $N = 1000$, Edward Lau and Ho [27] have tabulated the required size of the selected subset as a function of k, g, the estimation noise level and the class of the ordered performance curve.

Once the set S is formed, the designs in it are evaluated by using the exact OPF model. The combination of DG locations that produces the highest objective function value is declared as a good enough solution with a high probability to the problem of locating and sizing of DG.

It is important to note that when both Θ^N and Θ are large enough (which is typical in the DG problem) the selected subset S that is selected from Θ^N also has a high probability to contain good enough designs in Θ, and the difference between the two alignment probabilities can be ignored for engineering purposes [17].

4 Numerical results

The OO algorithm for locating and sizing of DG has been programmed in MATLAB running on an Intel® Core™ 2 Duo Processor T5300 (1.73 GHz) PC with 1 GB RAM. The approach was tested on a 69-node distribution network and the results are compared with those obtained from a GA–OPF method and recently published in [13]. The line and load data together with the one line diagram of the network appear in [13, 28]. The network was assumed to be operated under UK voltage regulation limits of $\pm 6\%$ of the nominal value and with thermal limits of 3 MVA for all lines. Moreover, all DG units were assumed to operate with 0.9 PF lagging and the loss target was set to 0.228 MW, the network loss prior to DG installation [13]. The coefficients in the objective function were based on the incentive figures employed in the UK [13]; an incentive for connecting DG of 2.5 £/MW-year and a loss incentive of 48 £/MWh. The distribution network has 68 PQ nodes that are eligible for DG placement. The network was studied for locating and sizing three, five, seven and nine distributed generators.

4.1 Good enough solution in Θ^N

The OO procedure described in Section 3 was applied to obtain the best solution in the selected subset S, which is a good enough solution in Θ^N with high probability. The two extreme cases in Table 1 were considered:

Case 1: $\alpha = 0.1$ and $P \geq 99\%$.

Case 2: $\alpha = 0.01$ and $P \geq 99.999\%$.

With $\alpha = 0.1$ and $P \geq 99\%$ (Case 1), the size of Θ^N is required to be no less than 4603. The corresponding values of the search space size N, the number of candidate node locations N_C and the size s of the selected subset are given in Table 2 for the different numbers of DG N_{DG}. It is seen from Table 2 that the largest number of candidate node locations is $N_C = 32$ corresponding to $N_{DG} = 3$. The heuristic rule for ranking the DG locations therefore requires executing $32 \times (2 \times 68 - 32 + 1)/2 = 1680$ OPF solutions, which takes around 300 s. The ranked list is applicable not only to the problem involving three distributed generators, but also to the other instances that require installing a larger number. The good enough solutions for Case 1 are shown in Table 3 together with the DG sizes (obtained using MATPOWER) corresponding to the DG locations given by a GA–OPF approach and reported in [13]. These results demonstrate the possibility of spreading the capacity by connecting more but smaller distributed generators. Table 4 shows the corresponding values of the total DG capacity, total DG capacity over the minimum load ratio, the total incentive and its two components. It is clear that as far as the total incentive (objective function (2)) is concerned, the good enough solutions given by the OO approach are slightly better than those given by the GA approach for N_{DG} equal to 5, 7 and 9.

In an attempt to obtain still better solutions, Case 2, which requires sampling no less than 115 124 possible designs from Θ, was studied. The corresponding values of N, N_C and s are given for this case in Table 5. The good enough solutions for Case 2 are different than those in Table 3 only for $N_{DG} = 5$, where the good enough DG locations are found to be at

Table 2 OO parameters for $P = 99\%$ and $\alpha = 0.1$ (Case 1)

N_{DG}	N	N_C	s
3	4960	32	296
5	6188	17	370
7	6435	15	385
9	5005	15	299

nodes 4, 26, 40, 48 and 62 with corresponding capacities of 0.9418, 0.7601, 0.7723, 0.6433 and 0.8895 MW. The last block in Table 4 shows the total DG capacities and incentives for Case 2. Although increasing the number of samples in Θ^N did not yield appreciably better results in this particular instance, it does give more confidence in the quality of the obtained solutions by requiring Θ^N to include at least one design from the top 0.01% designs in Θ with a probability of more than 99.999%.

4.2 Best solution in Θ^N

The validation of the OO approach and its corresponding results requires executing a computationally intensive task in which all designs in Θ^N are accurately evaluated using the OPF programme. This task is referred to as brute force enumeration (BFE). The results of this BFE allow the determination of (i) the actual alignment level $|G \cap S|$ between the good enough and the selected subsets and (ii) the best solution in Θ^N. These results are shown in Table 6 for Cases 1 and 2, where it is seen that the total incentives of the best solutions in S and Θ^N are identical. This fact is supported by the very high value of $|G \cap S|$ for all cases, which is attributed to the suitability of the crude LP model to the problem at hand. The results also confirm that the sizes s in Tables 3 and 6 are indeed upper bounds on the sizes of the selected subsets that contain at least one good enough design ($k = 1$).

The heuristic rule for generating the search space Θ^N was also validated by comparing with the strategy of randomly picking the N DG locations. For each value of N_{DG}, Table 7 shows the best solution obtained from the heuristic rule as compared to the best solution obtained from random sampling for the search space sizes in Table 2 (Case 1) and Table 5 (Case 2). The results demonstrate that the search space generated by the heuristic rule contains a better solution as compared to the one obtained from randomly picking candidate locations. Indeed, this was observed in five independent runs of the random picking strategy. As seen in Table 4, the best solution in the search space generated by the heuristic rule is also slightly better than the one given by the GA–OPF approach in [13].

4.3 Computational requirements

The computational requirements for obtaining the good enough and best solutions in Θ^N are given in Table 8. The

Table 3 DG locations and capacities for Case 1

Node	$N_{DG} = 3$		$N_{DG} = 5$		$N_{DG} = 7$		$N_{DG} = 9$	
	DG capacities, MW		DG capacities, MW		DG capacities, MW		DG capacities, MW	
	[13]	OO	[13]	OO	[13]	OO	[13]	OO
4			0.9418	0.9418		0.9418	0.4678	0.7018
5					0.6330			
6							0.2311	
13					0.2676		0.2432	0.2433
17								0.5947
21							0.2719	
26	0.7378	0.7378	0.7601	0.7601		0.7601		0.6343
27					0.7295		0.6765	
30						1.1407		1.1405
35	1.0365	1.0365	0.7628		0.7631		0.7631	
40			0.7089	0.8073	0.7206	0.7199	0.7209	0.7200
49				0.5774		0.5458		0.5458
57					0.7945		0.7472	
58						0.7044		0.7044
62	0.8872	0.8872	0.8895	0.8895		0.7179	0.7066	0.7179
65					0.6523			
Total	2.6614	2.6614	4.0630	3.9761	4.5608	5.5305	4.8282	6.0027

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Table 4 Optimal capacity and incentives

Method	Parameter	Number of distributed generators			
		3	5	7	9
GA–OPF [13]	total capacity, MW	2.6614	4.0630	4.5608	4.8282
	capacity/min load, %	59.5665	90.9366	102.0776	108.0609
	DG incentive, £/h	0.7616	1.1627	1.3052	1.3817
	loss incentive, £/h	7.9670	9.5791	9.9790	10.1383
	total incentive, £/h	8.7286	10.7419	11.2842	11.5200
OO case 1	total capacity, MW	2.6614	3.9761	5.5305	6.0027
	capacity/min load, %	59.5665	88.9913	123.7808	134.3484
	DG incentive, £/h	0.7616	1.1379	1.5827	1.7178
	loss incentive, £/h	7.9670	9.6135	9.7790	9.9612
	total incentive, £/h	8.7286	10.7513	11.3617	11.6790
OO case 2	total capacity, MW	2.6614	4.0069	5.5305	6.0027
	capacity/min load, %	59.5665	89.6810	123.7808	134.3484
	DG incentive, £/h	0.7616	1.1467	1.5827	1.7178
	loss incentive, £/h	7.9670	9.6310	9.7790	9.9612
	total incentive, £/h	8.7286	10.7777	11.3617	11.6790

Table 5 OO parameters for $P = 99.999\%$ and $\alpha = 0.01$ (Case 2)

N_{DG}	N	N_C	s
3	50 116[a]	68	3002
5	118 755	29	7114
7	116 280	21	6966
9	167 960	20	10 062

[a]Maximum number of combinations

table compares the execution times for solving N LP and s OPF problems as required by OO theory against the BFE of N OPF solutions. The speed up factor (SUF = BFE execution time/OO total execution time) shows that the OO approach can speed up the search by a factor that ranges between 9.00 and 9.99 in comparison with the BFE approach. Table 8 also shows that performing the complete study for Case 1 on the four N_{DG} values takes around 6 min using the OO approach and 1 h by BFE. For Case 2, OO requires around 2 h whereas BFE takes more than 19 h to terminate. These results together with the comparative study in Table 6 support the adoption of OO, particularly the use of a computationally cheap crude model for ordering a large number of alternatives.

Table 6 Validation of OO results

N_{DG}	Case 1			Case 2		
	$\lvert G \cap S \rvert$	Total incentive, £/h, of best solution in		$\lvert G \cap S \rvert$	Total incentive, £/h, of best solution in	
		S	Θ^N		S	Θ^N
3	50	8.7286	8.7286	50	8.7286	8.7286
5	50	10.7513	10.7513	50	10.7777	10.7777
7	50	11.3617	11.3617	50	11.3617	11.3617
9	44	11.6790	11.6790	50	11.6790	11.6790

Table 7 Validation of the heuristic rule

N_{DG}	Case 1		Case 2	
	Total incentive, £/h, of best solution in Θ^N		Total incentive, £/h, of best solution in Θ^N	
	Heuristic rule	Random picking	Heuristic rule	Random picking
3	8.7286	8.5746	8.7286	8.7286
5	10.7513	10.5650	10.7777	10.7062
7	11.3617	10.9966	11.3617	11.2289
9	11.6790	11.4119	11.6790	11.5179

Table 8 Comparison of computational performance

	N_{DG}	3	5	7	9
case 1	LP–OO, s	28	38	44	37
	OPF- OO, s	50	64	66	51
	Total-OO, s	78	102	110	88
	OPF–BFE, s	779	989	1049	818
	SUF	9.99	9.70	9.54	9.30
case 2	LP-OO, s	309	761	841	1367
	OPF-OO, s	497	1175	1167	1572
	Total-OO, s	806	1936	2008	2939
	OPF–BFE, s	7745	18 443	18 268	26 451
	SUF	9.61	9.53	9.10	9.00

5 Conclusion

This paper presented an OO approach for locating and sizing of DG. OO is known to give good enough solutions to hard problems; the confidence in the OO solution is set mainly by two parameters: α that specifies the percentage of top designs and P that is the probability level that at least one of the sampled designs belongs to the top α percent. The OO approach is deterministic and can be easily implemented by making use of existing software packages for LP and OPF. This is unlike many of the stochastic search methods in which the results are not necessarily repeatable and the termination criteria do not reflect the top percentage of the search space to which the obtained solution belongs. In fact, the use of a stochastic method to solve a particular instance of a problem usually requires running the same method for several times to yield the best solution. The proposed OO implementation has been compared with the results of a recently published hybrid GA–OPF approach and was shown to give marginally better results. Its performance was also contrasted with BFE in which it was demonstrated that OO gives good enough solutions with more than 9-fold reduction in computational effort.

This research can be extended in several directions. One possibility is to account for the time variability of the load through the extension of the crude and exact models to cover several time periods. Another research direction would entail investigating additional constraints on DG connections resulting from short-circuit [4] and stability studies [29]. A third direction is to quantify the benefit of DG in deferring network upgrades [13] as a first step to include this benefit in the OO framework. Vector OO [17] can be also investigated for the optimisation of multi-objective formulations that include models of load and price uncertainties, which are, respectively, accounted for through technical and economic risk objective functions [30]. Vector OO is particularly promising for handling problems where the priority among the objective functions is not clearly known to the decision maker.

6 References

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7 Appendix 1

Ordinal optimisation is based on two tenets stating that the optimisation of complex problems can be made much easier by order comparison and goal softening [17]:

1. Order comparison: 'order' is much more robust against estimation noise as compared to 'value'.

2. Goal softening: for many practical problems, it is enough to settle for a 'good enough' solution instead of insisting on getting the 'best'.

Let Θ be the search space of optimisation variables, Θ^N the set of N chosen designs, N the number of designs uniformly chosen from Θ, G the good enough subset, usually the true top-g designs in Θ^N, S the selected subset, usually the estimated top-s designs in Θ^N, $G \cap S$ the set of truly good enough designs in S, AP the alignment probability $= \Pr[|G \cap S| \geq k]$, the probability that there are actually k truly good enough designs in S and k the alignment level.

The procedure for the practical application OO to complex optimisation problems is as follows:

Step 1: Uniformly sample N designs from Θ to form Θ^N.

Step 2: Estimate the performance of the designs in Θ^N using a crude and computationally fast model.

Step 3: Specify the size of the good enough subset, g, the required alignment level, k, and the corresponding alignment probability, AP.

Step 4: Use (15) to determine the size of the selected subset, s.

Step 5: Select the estimated top-s designs from Step 2 to form the selected subset, S.

Step 6: OO theory ensures that S contains at least k truly good enough designs with a probability level no less than AP.

Ho *et al.* [17] provides a theoretical foundation of the OO method by showing that the alignment probability converges exponentially with respect to the number of replications and with respect to the sizes of the good enough and selected subsets.

8 Appendix 2

This section presents the derivation of the approximation formula in (12) and (13). The derivation makes use of the quadratic equations of the power in terms of the real e_i and imaginary f_i node voltage components. Without loss of generality, let node 1 denote the slack node, nodes 2 to $(m+1)$ denote m PV nodes and nodes $(m+2)$ to n denote the remaining $(n-m-1)$ PQ nodes. Define the $(2n-1) \times 1$ state vector $\boldsymbol{x}$ and the $(2n-1) \times 1$ injection vector $\boldsymbol{z}$ as

$$\boldsymbol{x} = [e_1, e_2, \ldots, e_n, f_2, \ldots, f_n]^{\mathrm{T}} \tag{18}$$

$$\boldsymbol{z} = [V_1^2, V_2^2, \ldots, V_{m+1}^2, P_2, \ldots, P_n, Q_{m+2}, \ldots, Q_n]^{\mathrm{T}} \tag{19}$$

It follows from the load flow equations in rectangular format that any component of the injection vector can be expressed as

$$z_k(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}} \boldsymbol{J}_k \boldsymbol{x}, \quad k = 1, \ldots, (2n-1) \tag{20}$$

where $\boldsymbol{J}_k$ is a $(2n-1) \times (2n-1)$ real symmetric matrix defined by the type of the injection. If $\boldsymbol{x}_0$ denotes the initial state vector, then substituting $\boldsymbol{x} = \boldsymbol{x}_0 + \Delta\boldsymbol{x}$ in (20) and neglecting higher order terms yields

$$z_k(\boldsymbol{x}) + z_k(\boldsymbol{x}_0) = 2\boldsymbol{x}_0^{\mathrm{T}} \boldsymbol{J}_k \boldsymbol{x} \tag{21}$$

Equation (21) can be expressed in the vector form as

$$\boldsymbol{z}(\boldsymbol{x}) + \boldsymbol{z}(\boldsymbol{x}_0) = 2\boldsymbol{L}(\boldsymbol{x}_0)\boldsymbol{x} \tag{22}$$

where $\boldsymbol{L}(\boldsymbol{x}_0)$ is a $(2n-1) \times (2n-1)$ matrix defined by

$$\boldsymbol{L}(\boldsymbol{x}_0) = \begin{bmatrix} \boldsymbol{x}_0^{\mathrm{T}} \boldsymbol{J}_1 \\ \vdots \\ \boldsymbol{x}_0^{\mathrm{T}} \boldsymbol{J}_{2n-1} \end{bmatrix} \tag{23}$$

The state vector $\boldsymbol{x}$ can be solved from (22) as

$$\mathbf{x} = \frac{1}{2}[\boldsymbol{L}(\boldsymbol{x}_0)]^{-1}(\mathbf{z}(\boldsymbol{x}) + \mathbf{z}(\boldsymbol{x}_0)) \tag{24}$$

An arbitrary load flow variable $y(\boldsymbol{x})$ can be also expressed in a

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form similar to (20)

$$y(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{x} \tag{25}$$

where $\boldsymbol{R}$ is a $(2n-1)\times(2n-1)$ real symmetric matrix defined by the type of the variable. Therefore an approximation similar to (21) also holds

$$y(\boldsymbol{x}) + y(\boldsymbol{x}_0) = 2\boldsymbol{x}_0^{\mathrm{T}}\boldsymbol{R}\boldsymbol{x} \tag{26}$$

By substituting (24) into (25) and noting that $\boldsymbol{z}(\boldsymbol{x}_0) = \boldsymbol{L}(\boldsymbol{x}_0)\boldsymbol{x}_0$, it follows that

$$y(\boldsymbol{x}) = \boldsymbol{x}_o^{\mathrm{T}}\boldsymbol{R}[\boldsymbol{L}(\boldsymbol{x}_0)]^{-1}\boldsymbol{z}(\boldsymbol{x}) \tag{27}$$

Equation (27) can be written as

$$\boldsymbol{y} = \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{z} \tag{28}$$

where $\boldsymbol{\beta}$ is a $(2n-1)\times 1$ vector, which is the solution of

$$[\boldsymbol{L}(\boldsymbol{x}_0)^{\mathrm{T}}]\boldsymbol{\beta} = \boldsymbol{R}\boldsymbol{x}_0 \tag{29}$$

The same approximation formula appears originally in [31] but is derived differently herein.