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Ordinal Welfare Comparisons with Multiple Discrete Indicators:

A First Order Dominance Approach and Application to Child Poverty

Channing Arndt[1], Roberta Distante[1], M. Azhar Hussain[1], Lars Peter Østerdal[2], Pham Lan Huong[3], Maimuna Ibraimo[4]

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Abstract

We develop an approach for making welfare comparisons between populations with multidimensional discrete well-being indicators observed at the micro level. The approach is rooted in the concept of multidimensional first order dominance. It assumes that, for each indicator, the levels can be ranked ordinally from worse to better, however no assumptions are made about relative importance of any dimension nor about complementarity/substitutability relationships between dimensions. We also introduce an efficient algorithm for determining dominance and employ a bootstrap approach that permits cardinal rankings of populations. These approaches are applied to household survey data from Vietnam and Mozambique.

Keywords: ordinal, welfare, multi-dimensional poverty measurement, first order dominance, Mozambique, Vietnam

JEL classification: I32, D63, O10

[1] Department of Economics, University of Copenhagen, [2] Department of Business and Economics, University of Southern Denmark, Odense. [3] Central Institute for Economic Management, Hanoi, [4] National Directorate of Studies and Policy Analysis, Ministry of Planning and Development, Maputo. E-mail of the corresponding author: channing.arndt@econ.ku.dk.

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UNU World Institute for Development Economics Research (UNU-WIDER)
Katajanokanlaituri 6 B, 00160 Helsinki, Finland

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1 Introduction

Appropriate poverty measurement remains an active area of research. Traditional models of social welfare and inequality assume one-dimensional indicators, usually based on monetary variables (e.g. Sen 1973). Nevertheless, poverty (or welfare) has long been recognized as a multi-dimensional phenomenon. Motivated by the desire to consider more dimensions in analyzing social welfare, poverty and inequality (e.g. UNDP 1990; Sen 2006), recent literature has frequently focused on multidimensional measures of poverty. For example, Alkire and Foster (2011), Roelen and Gassmann (2008) and Rippin (2010) discuss weighting schemes to aggregate across multiple indicators of poverty and well-being. Application of a weighting scheme is very convenient and can be easily justified when a reasonably high degree of consensus exists on the appropriate values for weights. Absent such a consensus, application of methods that require weighting schemes can quickly become problematic as alternative weighting schemes may alter conclusions with respect to the welfare rankings of populations. In these cases, it is useful to consider what can be said concerning the welfare status of two populations without making recourse to a weighting scheme.

In response to the challenge of limiting the imposition of subjective assumptions, other contributions have focused on development of 'robust' methods for comparing population welfare, poverty and/or inequality with multidimensional data. These methods allow for valid comparisons across broad classes of underlying social welfare functions. Following the seminal work by Atkinson and Bourguignon (1982, 1987) and Bourguignon (1989), recent contributions include Duclos et al. (2006, 2007), Bourguignon and Chakravarty (2003), Crawford (2005), Grab and Grimm (2007), Gravel et al. (2009), Batana and Duclos (2010), Gravel and Mukhopadhyay (2010), and Muller and Trannoy (2011) among others. Still, these contributions apply conditions that are typically formulated in terms of specified signs on the second or higher order cross-derivatives of the underlying social welfare functions.

In this paper, we consider the problem of making welfare comparisons between populations in a situation where only ordinal information is available at the micro level in terms of multidimensional (discrete) well-being indicators. The term 'ordinal' here means that, for each well-being indicator, the levels can be ranked from worse to better. However, no assumptions are made about the strength of preference for each dimension, nor about the relative desirability of changes between levels within or between dimensions or the complementarity/substitutability between the dimensions.

To accomplish this, we draw upon a concept known in the literature as multidimensional first order dominance (henceforth, FOD). This concept allows us to make welfare comparisons between two populations on the basis of a series of (binary or multileveled) ordinal welfare indicators.[1] In addition, we introduce a rapid and reliable algorithm for empirically determining whether one population dominates another on the basis of available binary indicators by drawing upon linear programming theory.

[1] Note that the analysis conducted here focuses on relative welfare/poverty. There is no attempt to define a threshold below which some share of the population is considered poor.

The FOD approach obviates the need for the analyst to apply an (arbitrary) weighting scheme across multiple criteria or to impose conditions on the social welfare function, which can be a considerable advantage. However, as with any other ‘robust’ method, this gain comes at some cost. First, the procedure may be unable to determine any difference between two populations. In other words, it can happen that population A does not dominate population B and population B does not dominate population A. Hence, the welfare ranking, based on FOD, is indeterminate. Second, as a pure binary indicator, the FOD procedure provides no sense as to the degree of dominance (or similarity) between two populations. Assume population A dominates population B. Without additional information, one does not know whether population A dominates population B by a considerable degree, such that the conclusion of dominance remains even if ‘large’ declines in the individual welfare indicators of population A occur, or whether the conclusion of dominance rests on a knife's edge such that even a ‘small’ decline in any one welfare indicator for population A would lead to an indeterminate outcome.

We mitigate these costs through the application of a bootstrap approach (technical details are presented in Appendix B). In short, repeated bootstrap samples are drawn from the comparator populations, which are often themselves samples of larger populations. When these repeated bootstrap samples are compared, the final output can be interpreted as an empirical probability that population A dominates population B. These probabilities yield significantly more information than the static application of FOD. For example, we may find that occasionally population A dominates population B and occasionally the inverse occurs but most of the time the results are indeterminate. Or, we might find that A dominates B almost always. Or, we may find that the probability that A dominates B is fairly high while the probability that B dominates A is very low or zero. These cases correspond with the conclusions of rough equality between A and B, solid dominance of A over B, and likely dominance of A over B respectively. Finally, if one is willing to accept the probability that A dominates B as a cardinal measure of welfare, one can then easily derive measures that yield cardinal welfare rankings across multiple populations (e.g. all provinces in a country or all countries in a region). Hence, without imposing weights on the various chosen binary welfare indicators that determine all results, one can cardinally rank populations by welfare status.

These approaches are applied to data from Vietnam and Mozambique with a focus on the distribution and evolution of child poverty through space and time. These countries were chosen because they are surprisingly close Asian and African analogs. In addition, they both exhibited rapid rates of economic growth over the periods considered. The focus on child poverty relates to the strong preference for a multi-dimensional view when evaluating the welfare of children (Roelen et al. 2010). It also permits comparison with existing studies in Mozambique and Vietnam that have employed multi-dimensional indicators.

In Vietnam, it is well accepted that most objective welfare indicators have been improving on average, including multi-dimensional child poverty measures (Roelen 2010); however, the distribution of gains is increasingly in focus. As will be shown, the FOD approach is particularly well-suited to considering whether gains are broad-based and to making comparisons across sub-groups (such as regions). In Mozambique, current debate centres on the recent stagnation in measured consumption poverty (DNEAP 2010). While Arndt et al. (forthcoming) show that this stagnation is consistent with an array of economic indicators, the multi-dimensional analysis conducted here provides a valuable additional perspective and complements existing deprivation-based studies of child poverty (UNICEF 2006, 2011).

The remainder of this article is laid out as follows. Section 2 provides a technical review of the multidimensional FOD methodology. Section 3 introduces our case countries, Vietnam and Mozambique, and presents the binary welfare indicators employed to measure child welfare. Section 4 presents results, and section 5 presents concluding remarks and directions for future research.

2 Multidimensional FOD

FOD comparisons provide a way of comparing multidimensional well-being without relying on *ad hoc* assumptions about individual well-being or social welfare. FOD can be characterized in several equivalent ways, as reviewed in the following paragraphs. [2]

2.1 Definitions

Much research into the nature of distributional dominance concepts has been conducted, and the theory is by now well developed (see e.g. Marshall and Olkin 1979; Müller and Stoyan 2002; Shaked and Shanthikumar 2007 for general treatments). The traditional criterion for one distribution being unambiguously 'better' than another is that of FOD, also known as the *usual (stochastic) order* in the stochastic dominance literature.

We start by reviewing the classical theory of one-dimensional first order dominance. For our purpose, we can focus on a model with only a finite set of possible outcomes for each individual in the population. Assume, therefore, that the distribution of well-being of some population is described by probability mass function f over a finite set of real-valued outcomes X (i.e. $\sum f(x) = 1$ and $f(x) \geq 0$ for all x in X), and another population is described by the probability mass function g. In this one-dimensional case, f first order dominates g if any of the following (equivalent) conditions (a), (b) and (c) hold:

(a) g can be obtained from f by a finite sequence of bilateral transfers of density to less desirable outcomes.

(b) Social welfare is at least as high for f than for g for any non-decreasing additively separable social welfare function; i.e., $\sum_{x \in X} f(x)w(x) \geq \sum_{x \in X} g(x)w(x)$ for any non-decreasing real function w.

(c) $F(x) \leq G(x)$ for all x in X, where $F(\cdot)$ and $G(\cdot)$ are the cumulative distribution functions corresponding to f and g.

Intuitively, we could think of condition (a) as one distribution first order dominates another if one could hypothetically move from one population distribution to the other by iteratively shifting population mass in the direction from a better outcome to a worse

[2] See also Østerdal (2010) for a discussion.

outcome. Thus, whenever we are able to observe FOD between two population distributions, the dominating population is unambiguously 'better off'.

This fundamental characterization can be extended to a multidimensional setting (e.g. Lehmann 1955; Strassen 1965; Levhari et al. 1975; Grant 1995). Suppose now that f and g denote multidimensional probability mass functions over a finite subset X of R^n. Then, f first order dominates g if one of the following three equivalent properties (A)-(C) hold.[3]

(A) g can be obtained from f by a finite number of shifts of density from one outcome to another that is worse.

(B) $\sum_{x\in X} f(x)w(x) \geq \sum_{x\in X} g(x)w(x)$ for every non-decreasing real-valued function w.

(C) $\sum_{x\in Y} g(x) \geq \sum_{x\in Y} f(x)$ for any comprehensive set $Y \subseteq X$.[4]

Again, notice that (A) provides perhaps the most intuitively appealing definition.

2.2 Checking FOD in practice

For empirical work, it is important to be able to determine in an 'efficient' way whether one distribution dominates another. In principle, one can check for FOD by directly checking all the inequalities in (C). This is a simple but generally inefficient method, as the number of inequalities to be checked is very large if you have many dimensions and levels. Algorithms dealing with FOD have been invented, though most of them are only built for the one-dimensional case (e.g. Bawa et al. 1979; Fishburn and Lavalle 1995). Preston (1974) and Hansel and Troallic (1978) assert that an algorithm for finding the maximum flow in a properly defined network can be used to determine dominance. More usefully, for the multivariate discrete case, Mosler and Scarsini (1991) and Dyckerhoff and Mosler (1997) show from definition (A) that FOD corresponds to a linear program that has a feasible solution. Hence, FOD can be verified using a linear programming package. We operationalize the linear programming technique in GAMS (GAMS Development Corporation 2008). In our experience, FOD is rapidly and robustly verified using the CONOPT solver (Drud 2008). We provide an example linear program for the three dimensional case in Appendix A. Extension to higher dimensions is straightforward.

2.3 Illustration of FOD with binary indicators

To illustrate the concept, let us consider a hypothetical example of two binary 0-1 variables (dimensions) A and B, i.e. $n = 2$ and $X = \{(0,0), (0,1), (1,0), (1,1)\}$.[5] In every dimension, it is useful to think of the outcome 1 as the good outcome (non-

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[3] The equivalence between (B) and (C) was proved by Lehmann (1955) and re-discovered in economics by Levhari et al. (1975). The equivalence between (A) and (C) has been obtained as a corollary of Strassen's Theorem (Strassen 1965), cf. e.g. Kamae et al. (1977). Østerdal (2010) provides a constructive direct proof of the equivalence between (A) and (C).

[4] Y is comprehensive if $x \in Y$, $y \in X$ and $y \leq x$ implies $y \in Y$.

[5] An empirical illustration to the 2×2 case is presented in Sonne-Schmidt et al. (2011).

deprived) and 0 as the bad outcome (deprived). Thus, the outcome $(0,0)$ for a person means she is deprived in both dimensions; $(0,1)$ means she is deprived in the first dimension and non-deprived in the second dimension, and so forth.

Let f and g be two probability mass functions on X, defined as indicated in Table 1. (The percentages in bold at the right side and bottom of the table indicate the marginal distributions).

Table 1: A hypothetical example

The distribution for f.

f			**Dimension B**		**Total**
			0 (Deprived)	1 (Not deprived)	
Dimension A	0	(Deprived)	25%	25%	**50%**
	1	(Not deprived)	25%	25%	**50%**
Total			**50%**	**50%**	**100%**

The distribution for g.

g			**Dimension B**		**Total**
			0 (Deprived)	1 (Not deprived)	
Dimension A	0	(Deprived)	30%	10%	**40%**
	1	(Not deprived)	10%	50%	**60%**
Total			**40%**	**60%**	**100%**

The distribution for h.

h			**Dimension B**		**Total**
			0 (Deprived)	1 (Not deprived)	
Dimension A	0	(Deprived)	15%	25%	**40%**
	1	(Not deprived)	25%	35%	**60%**
Total			**40%**	**60%**	**100%**

Source: Hypothetical examples developed by authors.

When analyzing each dimension separately, distribution g will appear to be better than distribution f because, for each dimension, a higher share of the population is not deprived (60 versus 50 per cent). However, the welfare ranking of distributions g and f is indeterminate according to the FOD criterion. Formally, this can be seen with reference to (C). We have that f first order dominates g if and only if the following four inequalities (i)-(iv) are jointly satisfied:

(i) $g(0,0) \geq f(0,0)$, (ii) $g(0,0) + g(0,1) \geq f(0,0) + f(0,1)$, (iii) $g(0,0) + g(1,0) \geq f(0,0) + f(1,0)$, and (iv) $g(0,0) + g(1,0) + g(0,1) \geq f(0,0) + f(1,0) + f(0,1)$.

Here, f does not FOD g, nor does g FOD f, since we have $g(0,0) > f(0,0)$ but, for example, $f(0,0) + f(1,0) > g(0,0) + g(1,0) + g(0,1)$. Intuitively, no distribution is dominant, since f would be better in the case where what matters most is to minimize the share of population deprived in both dimensions, while g would be better in the case where what matters most is, for example, the share of population not deprived in dimension A.

Let us now consider the probability mass function, h, given in Table 1. Comparing distributions h and g, h does not FOD g, nor does g FOD h, since we have $g(0,0) > h(0,0)$ but $h(0,0) + h(1,0) + h(0,1) > g(0,0) + g(1,0) + g(0,1)$. Intuitively, FOD does not occur since h would be better if what matters most is minimization of the share of population deprived in both dimensions, while g would be better if what matters most is maximization of the share of the population not deprived in either dimension.

However, h FOD f. This is immediately verified from checking the four inequalities in (C) listed above. An intuitive way of seeing this (by reference to condition (A)), is to observe that we can obtain f from h by moving some probability mass (10 per cent) from the outcome $(1,1)$ to $(0,0)$.

From the examples, it can also be seen that the FOD criterion differs from the criteria for robust welfare comparisons of the Atkinson-Bourguignon type as invoked by Atkinson and Bourguignon (1982, 1987), Bourguignon (1989), Bourguignon and Chakravarty (2003), Duclos et al. (2006, 2007) and others (see the Introduction for further references). The latter are instances of what is also known as orthant stochastic orderings (Dyckerhoff and Mosler 1997). Orthant orderings make stronger assumptions about the underlying social welfare function than the FOD criterion.[6] In its primary variant (assuming 'substitutability' between dimensions), f orthant dominates g if and only if $\sum_{y \leq x} g(y) \geq \sum_{y \leq x} f(y)$ for any $x \in X$. Note that this criterion is less restrictive than (C) and hence orthant dominance may appear when condition (A) (or (B)) is violated. For the 2×2 case, conditions (i)-(iii) (without condition (iv)) are necessary and sufficient for orthant dominance. In our example, h orthant dominates g. Hence, the FOD criterion differs from orthant dominance orderings even in the 2×2 case.

[6] A possible source of confusion is that in the multidimensional context the term 'first order dominance' has been used with different meanings. In particular, in the economics literature, orthant stochastic orderings of the Atkinson and Bourguignon type for welfare comparisons are often referred to as first order dominance criteria. (Second– and higher order dominance criteria are then derived from further assumptions on the underlying social welfare function.)

3 Case countries and choice of welfare indicators

Vietnam and Mozambique are in focus for the empirical analysis. Arndt et al. (2011) describe a number of similarities between Vietnam and Mozambique. In terms of geography, they are both long relatively thin countries with substantial coastline. In terms of recent history, both have conducted socialist experiments and endured brutal and extended periods of warfare. In addition, both formally adopted a much more market-oriented economic approach in the same year, 1986. Since the early 1990s, both Vietnam and Mozambique have been among the fastest growing economies in the world. There are structural similarities as well. In both countries, about 70 per cent of the population is rural. Also, the composition of value added across sectors is surprisingly similar (Arndt et al. 2011). Finally, both Vietnam and Mozambique receive significant external resources. Mozambique has been, since the early 1990s, one of the largest aid recipients in the world on a per capita basis. At the same time, Vietnam has been one of the largest aid recipients in absolute terms. When aid to Vietnam is combined with offshore oil revenues, the per capita value of these resources is roughly similar between the two countries.

There are also important differences. Economic takeoff began in earnest earlier and from a more developed base in Vietnam. As a result, Vietnam is richer. Population size differs dramatically with the Vietnamese population being about four times larger than the population of Mozambique. At the same time, land area is smaller in Vietnam. Vietnam is one of the most densely populated countries in the world while population density in Mozambique is relatively sparse. Finally, while both countries are investing heavily in education, Vietnam began its economic takeoff with much higher levels of educational attainment and these differences persist. Other social indicators, such as the infant mortality rate and access to health care services, are generally much better in Vietnam for similar reasons.

As highlighted in the Introduction, current debate in Vietnam tends to center around the distribution of gains. In Mozambique, there is considerable interest in determining whether the recent stagnation in consumption poverty is being accompanied by a slowdown or stagnation in other measures of welfare. The FOD analysis here contributes to both of these debates.

3.1 Multidimensional child poverty in Vietnam

As indicated, following the Doi Moi reforms, the country experienced rapid economic growth that allowed a reduction in monetary poverty. However, little was known about the specific situation of Vietnamese children until the works by MOLISA, University of Maastricht, and UNICEF (2008) (UNICEF (2008), henceforth), Roelen (2010), and Roelen et al. (2009, 2010). These studies apply an outcome- and deprivation-based approach to estimating child poverty in Vietnam, make use of the same data, and obtain broadly comparable results.

The UNICEF (2008) report employs MICS and Vietnam Households Living Standard Survey (VHLLS) survey data from 2006 to produce two Vietnam-specific outcome measures, namely the Child Poverty Rate (CPR) and the Child Poverty Index (CPI). The former is a headcount measure referred to the proportion of poor children, the latter is an index calculated at the regional level. To this end, the authors select six domains (with one or more indicators each) for their child poverty approach along the lines of the works by Biggeri (2007) and Alkire (2008): education poverty, health poverty, shelter poverty, water and sanitation poverty, social inclusion and protection poverty, and child work. Thus, the indicators are aggregated over attributes per individual into the two outcome measures, first at the domain level and then at the overall level.[7]

According to the CPR, 31 to 37 per cent of all children below 16 years of age are poor, with a marked difference in poverty incidence between rural and urban areas and vast heterogeneity—therefore inequality—among different geographic regions. No gender differences are found. The CPI, for its part, indicates that the best performing region is Red River Delta, while the worst performing one is the North West region.[8]

Roelen et al. (2010) also estimate child poverty incidence, depth and severity in Vietnam. The authors calculate the poverty headcount based on a dual cut-off identification strategy, a child poverty gap measure (percentage of deprivations over the maximum number of observable deprivations) and a child poverty severity measure. Findings show that 37 per cent of children are poor, with an average poverty gap of 21 per cent. High poverty incidence is correlated with deeper and more severe poverty. Rural areas are poorer than urban areas, and poverty incidence varies among geographic regions. The results also suggest variability in poverty depth and severity by age group: youngest and oldest children are subject to deeper and more severe poverty.[9]

3.2 Multidimensional childhood poverty in Mozambique

UNICEF (2006, 2011) takes a human rights-based approach to childhood poverty in Mozambique. These studies examine deprivation-based child poverty in the dimensions of water, sanitation, shelter, education, health, nutrition, and information (the Bristol Indicators, cf. Gordon et al. 2003a, 2003b). A number of data sets were used to capture a broader picture of childhood poverty: MICS 2008 (also used here), IOF 2008/09 and the National Child Mortality Study 2009. Overall, Mozambique experienced reductions in child and maternal mortality, stunting, and increase in school enrolment. At the same time, child poverty and deprivation remains high with improvements threatened by the AIDS pandemic. In both studies, absolute poverty is defined as having two or more severe deprivations in any of the mentioned deprivation domains. The studies lend some support to the view that deprivation-based poverty (at least two deprivations) and consumption-based poverty (being below the poverty line) are not necessarily highly correlated. Deprivation-based poverty (at least two severe deprivations) decreased from 59 per cent in 2003 to 48 per cent in 2008, which is in contrast to consumption-based

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[7] The poverty criterion used to compute the CPR is deprivation in at least two domains, where the deprivation thresholds partly match those defined in the Bristol Study (Gordon et al. 2003a, 2003b). The overall proportion of poor children is determined both at the regional and national level. As to the CPI, instead, average indicator poverty rates per domain are aggregated by dividing the sum of squared domain scores by the number of domains. Indeed, such index can be considered a squared domain severity index.

[8] Roelen et al. (2009) develop a similar analysis with MICS 2006 data. They design two Vietnam-specific outcome measures, namely the Child Vulnerability to Poverty Rate (CVPR) and the Child Vulnerability to Poverty Index (CVPI), computed in the same way as the CPR and CPI, respectively. The only difference with the UNICEF (2008) study is the inclusion of a leisure poverty domain and their findings are very similar.

[9] However, this might be the byproduct of differences in age group-specific indicators.

poverty that shows stagnation at around 55 per cent of the population from 2002/03 to 2008/09.

3.3 Welfare indicators

To consider the living standards of Vietnamese and Mozambican children through time and across space, we choose five main indicators of welfare in the spirit of the severe deprivation notion of the Bristol Indicators (cf. Gordon et al. 2003a, 2003b). They are defined as follows:

Severe water deprivation. Children who only have access to surface water for drinking or for whom the nearest source of water is not within 15 minutes from their dwelling.

Severe sanitation facilities deprivation. Children who have no access to any kind of improved latrine or toilet.

Severe shelter deprivation. Children living in dwellings with more than five people per room (severe overcrowding) or with no flooring material (e.g. a mud floor).

Severe education deprivation. Children who had never been to school and were not currently attending school.

Severe information deprivation. Children who belong to a household where there is not access to a TV set nor to a radio.

For Vietnam, we use the Multiple Indicator Cluster Surveys (MICS) from 2000 and 2006. For Mozambique we use the Demographic and Health Survey (DHS) from 2003 and the MICS from 2008.[10]

4 Results

4.1 Descriptive statistics

For the purposes of the analysis presented here, we focus on children aged 7-17.[11] For this age group, we consider the five indicators of well-being presented above. The percentage of children not deprived in each dimension is presented in Table 2.

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[10] The definitional and operational differences between the DHS and MICS surveys are (relatively) small. To assure comparability, consultations were conducted between the MICS and DHS teams (Gordon et al. 2010). Both surveys were conducted by the National Institute of Statistics.

[11] We also conducted the analysis for children aged 0-4 years. Four of the five indicators are the same. The education indicator is not relevant for children aged 0-4 years. We substitute an indicator of health deprivation (vaccinations received). Results for the 0-4 age group are qualitatively similar to the results for the 7-17 age group in both countries.

Table 2: Children not deprived by welfare indicator, 7-17 years old (%)

	Vietnam		Mozambique	
	2000	2006	2003	2008
Water	75.7	87.8	37.6	33.3
Sanitation	37.1	70.9	52.7	60.0
Shelter	60.4	78.4	30.3	46.0
Education	96.0	98.2	76.0	88.4
Information	76.9	87.1	61.8	63.5

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

With five binary indicators, the number of possible welfare indicator combinations is $2^5=32$. Hence, in the analysis, each comparator population is divided into 32 sub-groups. Due to the properties of the approach, a population may fail to register improvement through time due to stagnation or regress in only a small number of sub-groups. This demanding characteristic renders the approach particularly well-suited to determining whether gains are broad-based. Similar logic applies to regional comparisons at the same point in time.

The share of children falling into each combination of welfare indicators is presented in Table 3. The first row of the table shows the share of the population characterized by severe deprivation in all dimensions. This result has a very small probability in Vietnam. In Mozambique, it is about 7 per cent in 2003 with substantial improvement by 2008. The bottom row of the table illustrates the probability of a child not being deprived in any of the five dimensions. Here, the gain in Vietnam is impressive registering an absolute increase of about 26 percentage points, corresponding to a relative change of 100 per cent between the two waves. Mozambique also registers improvement in the final row (child not deprived in all dimensions) though the improvement is marginal.

Table 3: Children by combination of welfare indicators, 7-17 years (%)

Welfare indicator combination					Vietnam			Mozambique		
Water	Sanita.	Shelter	Educa.	Inform.	2000	2006	Change	2003	2008	Change
0	0	0	0	0	1.33	0.22	-1.11	6.92	2.09	-4.83
0	0	0	0	1	1.13	0.08	-1.05	5.88	2.03	-3.86
0	0	0	1	0	5.44	1.24	-4.20	10.80	9.07	-1.73
0	0	0	1	1	8.30	1.68	-6.63	11.17	10.24	-0.94
0	0	1	0	0	0.13	0.02	-0.11	0.50	0.75	0.26
0	0	1	0	1	0.18	0.05	-0.12	0.61	1.03	0.42
0	0	1	1	0	1.97	0.85	-1.12	0.80	2.93	2.13
0	0	1	1	1	4.59	1.79	-2.80	1.33	5.61	4.28
0	1	0	0	0	0.00	0.03	0.03	2.60	1.27	-1.33
0	1	0	0	1	0.00	0.06	0.06	2.41	1.02	-1.39
0	1	0	1	0	0.07	0.40	0.33	5.19	6.83	1.65
0	1	0	1	1	0.22	1.26	1.04	8.43	8.96	0.53
0	1	1	0	0	0.00	0.00	0.00	0.14	0.24	0.10
0	1	1	1	0	0.00	0.05	0.05	0.30	0.55	0.24
0	1	1	0	0	0.12	0.51	0.39	1.35	3.36	2.01
0	1	1	1	1	0.85	3.98	3.13	3.99	10.74	6.75
1	0	0	0	0	0.35	0.35	0.00	0.88	0.76	-0.12
1	0	0	0	1	0.41	0.19	-0.22	0.98	0.25	-0.73
1	0	0	1	0	5.34	3.49	-1.85	2.05	1.72	-0.33
1	0	0	1	1	12.58	4.89	-7.69	2.44	1.57	-0.87
1	0	1	0	0	0.08	0.05	-0.03	0.06	0.15	0.09
1	0	1	0	1	0.23	0.23	-0.01	0.17	0.14	-0.03
1	0	1	1	0	4.19	2.25	-1.94	0.47	0.67	0.20
1	0	1	1	1	16.69	11.72	-4.97	2.19	0.94	-1.25
1	1	0	0	0	0.02	0.01	-0.01	0.77	0.28	-0.49
1	1	0	0	1	0.03	0.13	0.10	0.95	0.39	-0.56
1	1	0	1	0	1.04	1.08	0.03	2.89	3.08	0.19
1	1	0	1	1	3.39	6.51	3.11	5.34	4.39	-0.96
1	1	1	0	0	0.01	0.03	0.02	0.13	0.18	0.05
1	1	1	0	1	0.15	0.29	0.14	0.66	0.41	-0.25
1	1	1	1	0	3.00	2.36	-0.64	2.68	3.13	0.45
1	1	1	1	1	28.16	54.21	26.05	14.88	15.18	0.31
Total					100.00	100.00	0.00	100.00	100.00	0.00

Note: In the first five columns, a ‘0’ means that the child is deprived and a ‘1’ means that the child is not deprived with respect to a given of the five presented welfare indicators.

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

4.2 FOD comparisons

Tables 4 and 5 illustrate the temporal FOD comparisons for Vietnam and Mozambique respectively. In Vietnam, advance in well-being is registered at the national level, in rural zones, and in two regions using the static approach. The bootstrap confirms that the advances at the national level and in rural zones are robust. Advance in the Mekong River Delta is also robust while advance in the South East region is somewhat more likely than an indeterminate outcome. Positive (empirical) probability of advance is also registered in urban zones and three additional regions. There is essentially no probability of regression through time in any region. These results provide evidence that, at the national level and frequently on a region by region basis, gains over 2000 to 2006 period were reasonably broad-based.

Table 4: Temporal FOD comparisons for Vietnam (probabilities)

	Static case	Bootstrap			
		2006 FOD		2000 FOD	
		2000	Undecided	2006	Total
National	1	1.00	0.00		1
Rural	1	1.00	0.00		1
Urban		0.30	0.70		1
Red River Delta			1		1
North East		0.14	0.86	0.00	1
North West		0.04	0.96	0.00	1
North Central Coast			1		1
South Central Coast			1		1
Central Highlands		0.30	0.70		1
South East	1	0.54	0.46		1
Mekong River Delta	1	0.98	0.02		1

Note: A ‘1’ in the static case indicates that the region's last year welfare level FOD the first year welfare level, while an empty cell indicates no domination. In the bootstrap case a ‘1’ indicates that all 1,000 bootstrap replications resulted in the mentioned domination, while a ‘1.00’ indicates that there were between 995 and 999 dominations, an empty cell indicate that there were no dominations and finally a ‘0.00’ indicates that there were between 1-4 dominations out of a total of 1,000 bootstrap replications.

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

Table 5: Temporal FOD comparisons for Mozambique (probabilities)

	Static case	Bootstrap			
		2008 FOD		2003 FOD	
		2003	Undecided	2008	Total
National		0.01	0.99		1
Rural		0.08	0.92		1
Urban		0.00	1.00		1
Niassa	1	0.53	0.47		1
Cabo Delgado		0.01	0.99		1
Nampula		0.01	0.99		1
Zambezia		0.24	0.76		1
Tete			1		1
Manica			1		1
Sofala		0.01	1.00		1
Inhambane			1		1
Gaza		0.01	0.99		1
Maputo Province		0.00	1.00		1
Maputo City			1		1

Note: A ‘1’ in the static case indicates that the region's last year welfare level FOD the first year welfare level, while an empty cell indicates no domination. In the bootstrap case a ‘1’ indicates that all 1,000 bootstrap replications resulted in the mentioned domination, while a ‘1.00’ indicates that there were between 995 and 999 dominations, an empty cell indicate that there were no dominations and finally a ‘0.00’ indicates that there were between 1-4 dominations out of a total of 1,000 bootstrap replications.

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

Mozambique registers fewer gains through time. As in Vietnam, there is essentially no evidence of regression through time. Nevertheless, only one province, Niassa, exhibits gains through time using the static approach over the 2003 to 2008 period. Niassa was also one of the best performing provinces in terms of poverty headcount using consumption as a metric over the same period (DNEAP 2010). Nevertheless, the bootstrap indicates that this gain is only somewhat more likely than an indeterminate outcome. There is positive probability of advance at the national level, in rural zones, and in six of eleven provinces. However, these probabilities tend to be quite small. Zambézia province registers about a one in four chance of advance through time. These results point in broadly the same directions as the consumption poverty measures with the exception of Zambézia, which exhibited an increase in consumption-based poverty.

FOD comparisons are also possible across regions for a given point in time. Tables 6-9 show, for Vietnam, regional comparisons for the cases: static 2000, bootstrap 2000, static 2006, and bootstrap 2006 respectively. Tables 10-13 show analogous results for Mozambique. In each case, the row average and the column average is provided. The row (column) average provides the probability that the region dominates (is dominated by) another region.

Table 6: Spatial FOD comparisons for Vietnam, 2000

	National	Rural	Urban	RRD	NE	NW	NCC	SCC	CH	SE	MRD	Avg.
National		1			1	1					1	0.40
Rural						1						0.10
Urban	1	1			1	1			1	1	1	0.70
Red River Delta	1	1			1	1	1		1		1	0.70
North East						1						0.10
North West												0.00
North Central Coast									1			0.10
South Central Coast		1				1					1	0.30
Central Highlands												0.00
South East	1	1			1	1			1		1	0.60
Mekong River Delta												0.00
Average	0.30	0.50	0.00	0.00	0.40	0.70	0.10	0.00	0.40	0.10	0.50	0.30

Note: A ‘1’ in the static case indicates that the region's last year welfare level FOD the first year welfare level, while an empty cell indicates no domination. In the bootstrap case a ‘1’ indicates that all 1,000 bootstrap replications resulted in the mentioned domination, while a ‘1.00’ indicates that there were between 995 and 999 dominations, an empty cell indicate that there were no dominations and finally a ‘0.00’ indicates that there were between 1-4 dominations out of a total of 1,000 bootstrap replications.

Averages: The row averages are the fraction of times the row region dominates other regions. The column averages indicates the fraction of times the column region is dominated by other regions.

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

Table 7: Bootstrap spatial FOD comparisons for Vietnam, 2000 (probabilities)

	National	Rural	Urban	RRD	NE	NW	NCC	SCC	CH	SE	MRD	Avg.
National		1			0.47	0.96			0		0.66	0.31
Rural					0.06	0.78					0.26	0.11
Urban	1	1			0.98	1	0.09	0.23	0.91	0.47	1	0.67
Red River Delta	0.99	1			0.94	1	0.39	0.4	0.99	0.02	0.96	0.67
North East	0.01	0.04				0.67	0		0.01		0.15	0.09
North West											0.00	0.00
North Central Coast	0.01	0.07			0.08	0.28		0.01	0.31		0.09	0.08
South Central Coast	0.12	0.52			0.31	0.93			0.18		0.72	0.28
Central Highlands	0.00	0.01			0.02	0.22					0.10	0.03
South East	0.92	0.98			0.88	1			0.62		0.98	0.54
Mekong River Delta						0.04						0.00
Average	0.31	0.46	0.00	0.00	0.37	0.69	0.05	0.06	0.30	0.05	0.49	0.28

Note: A ‘1’ in the static case indicates that the region's last year welfare level FOD the first year welfare level, while an empty cell indicates no domination. In the bootstrap case a ‘1’ indicates that all 1,000 bootstrap replications resulted in the mentioned domination, while a ‘1.00’ indicates that there were between 995 and 999 dominations, an empty cell indicate that there were no dominations and finally a ‘0.00’ indicates that there were between 1-4 dominations out of a total of 1,000 bootstrap replications.

Averages: The row averages are the fraction of times the row region dominates other regions. The column averages indicates the fraction of times the column region is dominated by other regions.

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

Table 8: Spatial FOD comparisons for Vietnam, 2006

	National	Rural	Urban	RRD	NE	NW	NCC	SCC	CH	SE	MRD	Avg.
National		1			1	1						0.30
Rural						1						0.10
Urban	1	1			1	1	1	1	1		1	0.80
Red River Delta	1	1			1	1	1	1	1			0.70
North East						1						0.10
North West												0.00
North Central Coast		1			1	1			1			0.40
South Central Coast					1	1						0.20
Central Highlands						1						0.10
South East	1	1			1	1			1			0.50
Mekong River Delta						1						0.10
Average	0.30	0.50	0.00	0.00	0.60	1.00	0.20	0.20	0.40	0.00	0.10	0.33

Note: A '1' in the static case indicates that the region's last year welfare level FOD the first year welfare level, while an empty cell indicates no domination. In the bootstrap case a '1' indicates that all 1,000 bootstrap replications resulted in the mentioned domination, while a '1.00' indicates that there were between 995 and 999 dominations, an empty cell indicate that there were no dominations and finally a '0.00' indicates that there were between 1-4 dominations out of a total of 1,000 bootstrap replications.

Averages: The row averages are the fraction of times the row region dominates other regions. The column averages indicates the fraction of times the column region is dominated by other regions.

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

Table 9: Bootstrap spatial FOD comparisons for Vietnam, 2006 (probabilities)

	National	Rural	Urban	RRD	NE	NW	NCC	SCC	CH	SE	MRD	Avg.
National		1			0.46	1			0.02			0.25
Rural					0.27	0.99			0.01			0.13
Urban	1	1			0.98	1	0.72	0.48	0.98	0.01	0.51	0.67
Red River Delta	1	1			0.98	1	0.82	0.87	1	0.02	0.13	0.68
North East	0.00	0.00				0.59	0		0.01			0.06
North West												0.00
North Central Coast	0.08	0.41			0.73	1		0	0.3			0.25
South Central Coast	0.01	0.07			0.36	0.97	0		0.35			0.18
Central Highlands					0.03	0.92						0.10
South East	0.69	0.90			0.84	1	0.06	0.24	0.78		0.00	0.45
Mekong River Delta					0.05	0.87			0.02			0.09
Average	0.28	0.44	0.00	0.00	0.47	0.93	0.16	0.16	0.35	0.00	0.06	0.29

Note: A ‘1’ in the static case indicates that the region's last year welfare level FOD the first year welfare level, while an empty cell indicates no domination. In the bootstrap case a ‘1’ indicates that all 1,000 bootstrap replications resulted in the mentioned domination, while a ‘1.00’ indicates that there were between 995 and 999 dominations, an empty cell indicate that there were no dominations and finally a ‘0.00’ indicates that there were between 1-4 dominations out of a total of 1,000 bootstrap replications.

Averages: The row averages are the fraction of times the row region dominates other regions. The column averages indicates the fraction of times the column region is dominated by other regions.

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

Using these metrics, relatively well-off regions should have relatively large row averages while relatively poor regions should have relatively large column averages. In Vietnam, urban zones, the Red River Delta and the South East are shown to be relatively well off. On the other hand, the rural zones, North East, the North West and the Central Highlands are shown to be relatively poor. Consistent with the temporal analysis, the Mekong River Delta is shown to be relatively poor in 2000 but improves to being neither particularly poor nor particularly well-off in 2006.

For Mozambique, the relatively well off regions are the urban zones and Maputo Province and City. Relatively disfavoured provinces include rural zones, Tete, Zambézia, Nampula, and Cabo Delgado. In the temporal analysis, Niassa registers improvement through time. This does not show up in the changes in the static spatial row/column averages through time as Niassa becomes dominated by the urban zone (Niassa is 77 per cent rural) while remaining dominated by Maputo Province and Maputo City. Some progress is evident in the bootstrap where Niassa registers a small gain. Zambézia also exhibits a reasonable chance of temporal gain. This is more evident in the inter-regional comparisons. Zambézia province is dominated by other provinces less frequently and less decidedly in 2008 compared with 2003. Despite these gains, Zambézia is the poorest province in both 2003 and 2008 using the column average as the metric.

Table 10: Spatial FOD comparisons for Mozambique, 2003

	Nat	Rur	Urb	NI	CD	NA	ZA	TE	MA	SO	IN	GA	MP	MC	Avg.
National		1					1								0.15
Rural							1								0.08
Urban	1	1			1	1	1	1	1	1					0.62
Niassa															0.00
Cabo Delgado							1								0.08
Nampula															0.00
Zambezia															0.00
Tete							1								0.08
Manica		1					1								0.15
Sofala							1								0.08
Inhambane		1					1								0.15
Gaza		1			1		1								0.23
Maputo Province	1	1		1	1	1	1	1		1	1	1			0.77
Maputo City	1	1	1	1	1	1	1	1	1	1	1	1			0.92
Average	0.23	0.54	0.08	0.15	0.31	0.23	0.85	0.23	0.15	0.23	0.15	0.15	0.00	0.00	0.25

Note: A ‘1’ in the static case indicates that the region's last year welfare level FOD the first year welfare level, while an empty cell indicates no domination. In the bootstrap case a ‘1’ indicates that all 1,000 bootstrap replications resulted in the mentioned domination, while a ‘1.00’ indicates that there were between 995 and 999 dominations, an empty cell indicate that there were no dominations and finally a ‘0.00’ indicates that there were between 1-4 dominations out of a total of 1,000 bootstrap replications.

Averages: The row averages are the fraction of times the row region dominates other regions. The column averages indicates the fraction of times the column region is dominated by other regions.

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

Table 11: Bootstrap spatial FOD comparisons for Mozambique, 2003 (probabilities)

	Nat	Rur	Urb	NI	CD	NA	ZA	TE	MA	SO	IN	GA	MP	MC	Avg.
National		1				0.02	1	0.01							0.16
Rural							0.47								0.04
Urban	1	1		0.47	0.93	1	1	1.00	0.43	0.96	0.39	0.12			0.64
Niassa							0.18								0.01
Cabo Delgado		0.06		0.00			0.65	0.00							0.05
Nampula		0.04					0.48								0.04
Zambezia															0.00
Tete		0.19					0.87								0.08
Manica		0.95			0.02	0.01	0.99	0.11							0.16
Sofala		0.03				0.01	0.94								0.07
Inhambane	0.02	0.96		0.03	0.14	0.04	1.00	0.07							0.17
Gaza	0.03	0.97		0.05	0.37	0.16	1	0.20			0.10				0.22
Maputo Province	1	1	0.19	0.96	1	1	1	1	0.40	0.80	1	0.99			0.80
Maputo City	1	1	0.98	0.87	1	1	1	1	1	1	1.00	0.99	0.19		0.93
Average	0.23	0.55	0.09	0.18	0.27	0.25	0.81	0.26	0.14	0.21	0.19	0.16	0.01	0.00	0.26

Note: A ‘1’ in the static case indicates that the region's last year welfare level FOD the first year welfare level, while an empty cell indicates no domination. In the bootstrap case a ‘1’ indicates that all 1,000 bootstrap replications resulted in the mentioned domination, while a ‘1.00’ indicates that there were between 995 and 999 dominations, an empty cell indicate that there were no dominations and finally a ‘0.00’ indicates that there were between 1-4 dominations out of a total of 1,000 bootstrap replications.

Averages: The row averages are the fraction of times the row region dominates other regions. The column averages indicates the fraction of times the column region is dominated by other regions.

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

Table 12: Spatial FOD comparisons for Mozambique, 2008

	Nat	Rur	Urb	NI	CD	NA	ZA	TE	MA	SO	IN	GA	MP	MC	Avg.
National		1													0.08
Rural															0.00
Urban	1	1		1	1	1	1	1							0.54
Niassa					1										0.08
Cabo Delgado															0.00
Nampula															0.00
Zambezia															0.00
Tete															0.00
Manica		1					1								0.15
Sofala															0.00
Inhambane															0.00
Gaza		1			1		1		1		1				0.38
Maputo Province	1	1		1	1	1	1	1	1		1				0.69
Maputo City	1	1	1	1	1	1	1	1	1	1	1	1			0.92
Average	0.23	0.46	0.08	0.23	0.38	0.23	0.38	0.23	0.23	0.08	0.23	0.08	0.00	0.00	0.22

Note: A ‘1’ in the static case indicates that the region's last year welfare level FOD the first year welfare level, while an empty cell indicates no domination. In the bootstrap case a ‘1’ indicates that all 1,000 bootstrap replications resulted in the mentioned domination, while a ‘1.00’ indicates that there were between 995 and 999 dominations, an empty cell indicate that there were no dominations and finally a ‘0.00’ indicates that there were between 1-4 dominations out of a total of 1,000 bootstrap replications.

Averages: The row averages are the fraction of times the row region dominates other regions. The column averages indicates the fraction of times the column region is dominated by other regions.

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

Table 13: Bootstrap spatial FOD comparisons for Mozambique, 2008 (probabilities)

	Nat	Rur	Urb	NI	CD	NA	ZA	TE	MA	SO	IN	GA	MP	MC	Avg.
National		1				0.09	0.07								0.09
Rural															0.00
Urban	1	1		0.72	0.94	1	1.00	0.93	0.33	0.20	0.02				0.55
Niassa					0.27		0.05								0.02
Cabo Delgado							0.00								0.00
Nampula		0.00													0.00
Zambezia															0.00
Tete															0.00
Manica		0.42				0.00	0.40	0.01							0.06
Sofala		0.04				0.00	0.13	0.01							0.01
Inhambane		0.30			0.04	0.00	0.31								0.05
Gaza	0.04	0.98		0.01	0.44	0.09	0.90	0.28	0.52	0.02	0.26				0.27
Maputo Province	0.99	1	0.00	0.38	0.95	1.00	1.00	0.97	0.65	0.07	0.59	0.03			0.59
Maputo City	1	1	1	1	1	1	1	1	1	1	1.00	0.55			0.89
Average	0.23	0.44	0.08	0.16	0.28	0.24	0.37	0.25	0.19	0.10	0.14	0.04	0.00	0.00	0.20

Note: A ‘1’ in the static case indicates that the region's last year welfare level FOD the first year welfare level, while an empty cell indicates no domination. In the bootstrap case a ‘1’ indicates that all 1,000 bootstrap replications resulted in the mentioned domination, while a ‘1.00’ indicates that there were between 995 and 999 dominations, an empty cell indicate that there were no dominations and finally a ‘0.00’ indicates that there were between 1-4 dominations out of a total of 1,000 bootstrap replications.

Averages: The row averages are the fraction of times the row region dominates other regions. The column averages indicates the fraction of times the column region is dominated by other regions.

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

Finally, the results provide some indication of trends in spatial inequality. If all regions were nearly equal, then it is likely that no region would dominate any other (at least it would not do so with a very high probability for the bootstrapping case). The values provided in the matrices shown in Tables 6-13 would then tend to be small and with roughly equivalent values for the row and column averages. The average of the row averages (which equals the average of the column averages) provides a measure of total overall registered probability of dominance in the static and in the bootstrap case. An increase (decrease) in this value over time could be taken as an indication of an increase (decrease) in inequality across regions.

By this measure, spatial inequality in Vietnam has remained roughly the same (a small increase is found in both the static and in the bootstrap case). Overall, while the temporal FOD analysis indicates that the strong gains in average objective indicators are being reasonably shared across the population from a national perspective and in many regions, the pattern of improvement does not appear to be reducing existing (relatively large) spatial inequalities.

Turning to Mozambique and focusing on trends in spatial inequality, Mozambique registers a fairly dramatic decline in the probabilities of dominance in both the static and bootstrap cases. At the extremes, Maputo Province and City, while still performing relatively well, are less overwhelmingly dominant and Zambézia, while still performing relatively poorly, is less overwhelmingly dominated. In addition, the reductions in dominance do not occur uniquely at the extremes. Of the 14 regions considered (three aggregates and 11 provinces) and focusing on the bootstrap results, 11 exhibit reduced row averages and 11 (not the same 11) exhibit reduced column averages between 2003 and 2008. As a result, in many ways, the opposite conclusion to Vietnam pertains. While improvements in objective indicators are insufficiently strong or insufficiently broad-based to register robust improvements in the temporal analysis, the pattern of gains on a regional basis tends to point towards a reduction in spatial inequalities.

5 Conclusions

The FOD criterion is a demanding test for the dominance of a population distribution relative to another in that only a minimum of highly plausible assumptions on underlying social welfare criteria are made. Despite the generality of the criterion, the empirical analysis illustrates that this criterion delivers useful comparisons of populations. This is particularly true in the context of child poverty where the application of one-dimensional (household) income-based welfare/inequality/poverty measures tend to provide too narrow a view given the importance of other indicators such as access to publicly provided goods and services.

While the theoretical underpinnings of the multidimensional FOD criterion have been known and appreciated in the stochastic dominance literature for around half a century, to our knowledge, an empirical implementation of the multidimensional FOD criterion for comparisons of actual population distributions has never been conducted prior to this study. Our findings provide strong evidence for broad-based advance in the welfare of 7-17 year olds in Vietnam using the five chosen indicator variables. Because these findings are based on the barest minimum of underlying assumptions, they lend strong support to the similar conclusions obtained by existing studies in Vietnam. Evidence for advance in Mozambique, on the other hand, is much more muted. This result is consistent with recent evidence on consumption-based poverty. As pointed out by UNICEF (2011), some improvements have been registered. Nevertheless, these gains were insufficiently generalized across indicators and insufficiently broad-based across the population to register as unambiguous improvement. Importantly, in neither country is there any evidence of regression through time. Finally, the FOD analysis provides a useful and novel perspective on inequality. In Vietnam, regional differences remain relatively constant. In Mozambique, evidence exists for a reduction in regional disparities between 2003 and 2008. In absolute terms, spatial inequalities remain pronounced in both countries.

Future research may take several directions. We shall only mention two:

First, in our analysis, we have focused on the Bristol indicators for severe child deprivation (adapted to the context of the case countries and available data). While the welfare comparisons are robust for given indicators, changing the indicators themselves may, of course, change conclusions. Our empirical implementation strategy may be adapted to deal with additional (binary or multileveled) indicators. The number of inequalities to be tested for each pairwise comparison of distributions, however, increases dramatically with the addition of further levels or dimensions to the existing indicators and fewer FOD are to be expected. Future research may explore the value of expanding dimensions and levels in the FOD approach.

Second, in the present paper, we focused on a single age group (children aged 7-17) and welfare comparisons within a single country. With the widespread availability of data from Demographic and Health Surveys (potentially supplemented by MICS), the possibility exists to compare target populations across countries. If children remained in focus, it would be possible to consider the evolution of the living conditions of children and develop indicators of the degree of inequality in important indicators of welfare across a broad array of countries.

Appendix A: Example linear programming formulation of FOD test

We illustrate the linear program formulation of the FOD test by way of an example with three dimensions.

Define binary indices i, j, k, which each can take the value 0 or 1. The value 0 refers to deprived and the value 1 not deprived for the three dimensions.

Define binary indices i', j', k', which are aliases of i, j and k respectively.

For two populations A and B, let a_{ijk} and b_{ijk} be the share of the respective populations corresponding to the state of deprived and not deprived for the three indicators. So, for example, the value of a_{111} is the share of population A not deprived in any dimension while the value of b_{100} is the share of population B that is not deprived in the first dimension while deprived in all other dimensions.

Define the variable $x_{ijk,i'j'k'}$ which represents transfer of probability mass from outcome (ijk) to outcome $(i'j'k')$.

Define Z as the set of source-destination pairs $(ijk, i'j'k')$ that move probability from preferred to less preferred outcomes. That is, if outcome (ijk) is the source of the transfer and outcome $(i'j'k')$ is the destination, a legal transfer is where $i' \leq i$, $j' \leq j$ and $k' \leq k$. All three conditions must hold. For example, $(111{,}011)$ is an element of Z while $(001{,}011)$ is not an element of Z.

Under these conditions, population A FOD population B if and only if the following linear program is feasible.

Min $y = 1$

subject to:

$$a_{ijk} + \sum_{(i'j'k',ijk)\in Z} x_{i'j'k',ijk} - \sum_{(ijk,i'j'k')\in Z} x_{ijk,i'j'k'} = b_{ijk} \quad \forall\, i, j, k$$

$$x_{ijk,i'j'k'} \geq 0, \qquad x_{ijk,ijk} = 0.$$

Extension to a higher dimension involves defining a new index l, its alias l', and appropriately expanding the dimensions of all parameters, variables, and equations. The

GAMS code for operationalization of the FOD test with up to seven binary indicators is available from the authors upon request.

Appendix B: The bootstrapping

Bootstrapping is a general means of generating consistent estimates of an estimator's sampling distribution when an analytical solution cannot be derived or requires unreasonable assumptions (Efron 1979; Efron and Tibshirani 1993). It is based on repeated (J times) samples, drawn with replacement, of size K from the original sample data, of size N, where $K \leq N$. As the original sample size, N, increases, the bootstrap approach converges to Monte Carlo for fixed K. The primary assumption behind the bootstrap is that the distribution of the observed sample is a good approximation of the distribution of the population.

In our application, the bootstrap samples are drawn in a manner that mimics the stratified cluster sample design of the DHS and MICS surveys. That is, within each stratum, K clusters are randomly drawn, with replacement, where K is also the number of primary sampling units in the stratum (i.e., $K = N$). When a cluster is drawn, all of the households in that cluster are drawn. Because the bootstrap sampling is done with replacement, each cluster (and household) may appear one or more times in a given bootstrap sample, or not at all. The FOD analysis using the linear programming techniques discussed in the previous section is conducted for each bootstrap sample. The process is repeated $J = 1{,}000$ times. The share of times where temporal and/or spatial dominance is discovered over the $1{,}000$ bootstrap replications is then calculated for each result.

For the cases, like the ones considered in this article, where the populations being considered are in fact samples from larger populations (say A and B), the results of the bootstrap can be interpreted as probabilities for three possible outcomes: (i) A FOD B; (ii) B FOD A and (iii) an indeterminate outcome. It is, in this sense, a form of statistical inference analysis with respect to the static case. Development of more formal inference procedures is a potential topic for future research.

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