Orientifolds of K3 and Calabi-Yau Manifolds with Intersecting D-branes

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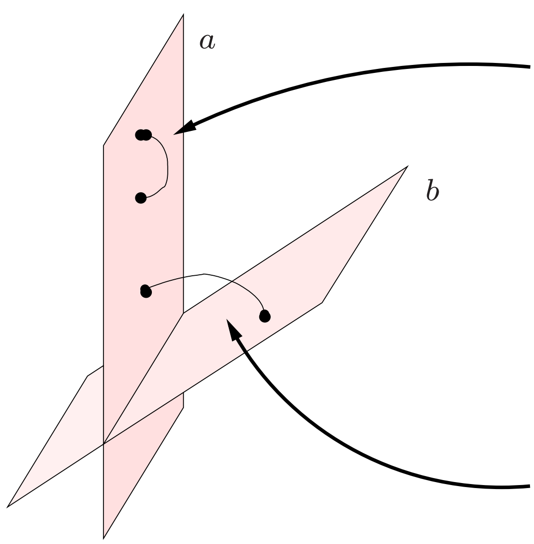
Motivation: Intersecting Brane Worlds

- Effective U(N) gauge theory on N Dp-branes

$$\begin{aligned}\mathcal{S}_{\text{eff}} &= \int_{\mathbb{R}^4\times\mathcal{M}^6} dx^4 d\xi^6 \ \mathcal{L}_{\text{gravity}}(g, B_2, \phi, C_p) \\ &\quad + \int_{\mathbb{R}^4\times\mathcal{W}^{p-3}} dx^4 d\zeta^{p-3} \ \big(\underbrace{\mathcal{L}_{\text{DBI}}(g,\mathcal{F},\phi)}_{\text{Tension}} + \underbrace{\mathcal{L}_{\text{CS}}(\mathcal{F},C_p)}_{\text{Charge}}\big)\end{aligned}$$

  Compact $\mathcal{M}^6 \overset{\text{SUSY}}{\Rightarrow}$ Orientifold Op-planes needed.

- Two D-branes a, b intersecting in a point:



$\mathcal{N} = 4$ VM in (adj$_a$)
- non-chiral -

χ in (N$_a$, N$_b$)
- chiral -

  χ: Chiral open string R-groundstate.

- Intersecting Brane Worlds (IBW):
  Space-time filling D-branes intersecting in points.

General set-up: IBW-Orientifolds

- Define **general IBW** (in $10-2d$ dimensions) by

$$\frac{\text{Type II on K3/CY}_3}{\Omega\bar{\sigma}} \quad + \quad \begin{array}{c}\text{D}q_a\text{-branes} \\ \text{on sLag cycles} \\ \pi_a \in H_d(\mathcal{M}^{2d};\mathbb{Z})\end{array}$$

- General Dq-brane defined geometrically by
  $\imath : \mathcal{W}^{q+1} \hookrightarrow \mathbb{R}^{10-2d} \times \mathcal{M}^{2d}$ + gauge bundle E on $\mathcal{W}^{q+1}$
  IBW: Want $q+1 = 10-2d+d$ and E flat.

- Consider background $(\mathcal{M}^{2d}, J, \Omega_d)$ with $\mathbb{Z}_2$-involution

$$\bar{\sigma}(J) = -J, \quad \bar{\sigma}(\Omega_d) = \bar{\Omega}_d$$

  Locally: $\bar{\sigma}$ complex conjugation.

- Combine with world sheet parity Ω into $\Omega\bar{\sigma}$

$$\text{Fix}(\Omega\bar{\sigma}) = \text{Orientifold O}q\text{−planes}$$

  and Fix($\Omega\bar{\sigma}$) is special Lagrangian:

$$\imath^* J = \imath^* \Im(\Omega_d) = 0, \quad \imath^* \Re(\Omega_d) = \imath^* d\text{vol}$$

- Cancellation of Oq-plane charge/tension by

$$\text{D}q_a\text{-branes on sLag cycles } \pi_a$$

  For π_a only require $\imath^* \Re(e^{i\theta_a}\Omega_d) = \imath^* d\text{vol}$

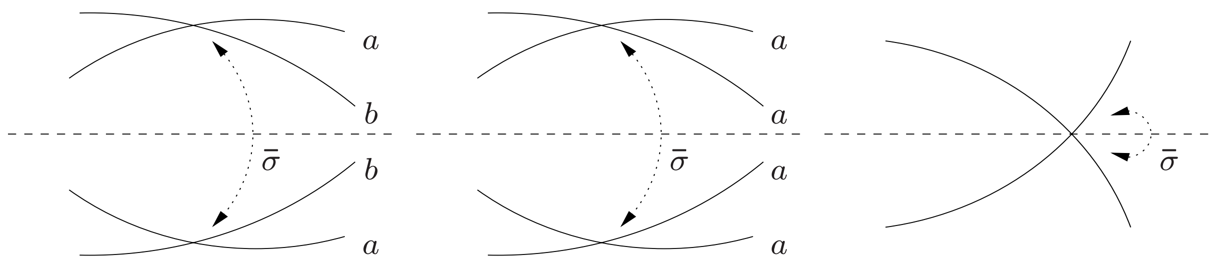
  Weak (individual) SUSY conservation.

General set-up: Chiral matter

- Chan-Paton indices permuted by $\bar\sigma$

  $\lambda^{ab}|\text{osc}, ab\rangle \xrightarrow{\Omega\bar\sigma} \gamma(\Omega)\lambda^{ab}\gamma(\Omega)^{-1}|\Omega\bar\sigma\cdot\text{osc}, \bar\sigma(a)\bar\sigma(b)\rangle$

  Chiral matter in representations



$(\mathsf{N}_a, \mathsf{N}_b)$ $\quad$ $(\mathbf{adj}_a)$ or $(\mathbf{S}_a \oplus \mathbf{A}_a)$ $\quad$ $(\mathbf{A}_a)$

  Multiplicities depend on the dimension.

- RR-charge cancellation: Simplification of $\mathcal{L}_{\text{CS}}$

$$\mathcal{L}_{\text{CS}}^{(p)} \sim \text{ch}(\mathcal{F}) \wedge \sqrt{\widehat{A}(R)} \wedge C_p \xrightarrow{\text{sLag}} \text{rk}(E)\ C_p$$

  leads to Bianchi-identity (δ : Poincaré duality)

$$d \star dC_{q+1} \sim \sum_a \underbrace{\mathsf{N}_a\delta(\pi_a)}_{\text{D}q_a-\text{brane}} + \underbrace{Q_q\delta(\pi_{\text{O}q})}_{\text{O}q-\text{plane}}$$

  Topological tadpole cancellation conditions

$$\sum_a \mathsf{N}_a\pi_a + Q_q\pi_{\text{O}q} = 0$$

  Maximal number of equations: $b_d(\mathcal{M}^{2d})$.

IBW on K3:

- Chiral fermion spectrum ($\pi'_a = \pi_{\bar{\sigma}(a)}$)

Representation	Multiplicity
(adj_a)	$\pi_a \circ \pi_a$
$\left(\mathsf{A}_a \oplus \bar{\mathsf{A}}_a\right)$	$\frac{1}{2}\left(\pi_a \circ \pi'_a + \pi_a \circ \pi_{\mathsf{O7}}\right)$
$\left(\mathsf{S}_a \oplus \bar{\mathsf{S}}_a\right)$	$\frac{1}{2}\left(\pi_a \circ \pi'_a - \pi_a \circ \pi_{\mathsf{O7}}\right)$
$(\mathsf{N}_a, \mathsf{N}_b) \oplus \left(\bar{\mathsf{N}}_a, \bar{\mathsf{N}}_b\right)$	$\pi_a \circ \pi_b$
$\left(\mathsf{N}_a, \bar{\mathsf{N}}_b\right) \oplus \left(\bar{\mathsf{N}}_a, \mathsf{N}_b\right)$	$\pi_a \circ \pi'_b$

induces gravitational R^4-anomaly coefficient

$$\text{anomaly} \sim 14\, \pi_{\mathsf{O7}} \circ \pi_{\mathsf{O7}}$$

Using $n_{\mathsf{H}} - n_{\mathsf{V}} + 29 n_{\mathsf{T}} = 273$ one can obtain

$$2(9 - n_{\mathsf{T}}) = \pi_{\mathsf{O7}} \circ \pi_{\mathsf{O7}} \overset{\mathsf{sLag}}{=} -\chi(\mathsf{Fix}(\bar{\sigma}))$$

Get n_{T} from action of $\bar{\sigma}$ on $H^*_{\mathsf{DR}}(\mathsf{K3})$.

- This relation was checked with

Blown-up K3-orbifolds $\mathbb{T}^4/\mathbb{Z}_N$
Algebraic model: Quartic in $\mathbb{CP}^3$
F-theory generalizations of type I′ vacua on CY_3

and mathematically proven.

Example: Blown-up $\mathbb{T}^4/\mathbb{Z}_2$ orbifold

- Define K3-orbifold on rectangular $\mathbb{T}^4 = \mathbb{T}^2_1 \times \mathbb{T}^2_2$ by
$$\Theta : z_i \mapsto -z_i, \quad i = 1, 2$$
Θ has $16 = 4 \times 4$ fixed points $P_{\alpha\beta}$, $\mathcal{A}_1$-singularities.

- Blowing-up the singularities adds 16 $\mathbb{CP}^1$, $e_{\alpha\beta}$, with
$$e_{\alpha\beta} \circ e_{\gamma\epsilon} = -2\delta_{\alpha\gamma}\delta_{\beta\epsilon} \quad \text{and} \quad \Omega\bar{\sigma} : e_{\alpha\beta} \mapsto -e_{\alpha\beta}$$

- Now find
$$\pi_{O7} = 2(\Re(z_1) \otimes \Re(z_2)) + 2(\Im(z_1) \otimes \Im(z_2))$$
Add two stacks of $N_1 = N_2 = 16$ D7-branes on
$$\pi_1 = \frac{1}{2}(\Re(z_1) \otimes \Re(z_2)) + \frac{1}{2}(e_{11} + e_{12} + e_{21} + e_{22})$$
$$\pi_2 = \frac{1}{2}(\Im(z_1) \otimes \Im(z_2)) + \frac{1}{2}(e_{11} + e_{13} + e_{31} + e_{33})$$

From intersection numbers get chiral matter

Representation $U(16) \times U(16)$	Multiplicity
$(\text{adj}, 1) \oplus (1, \text{adj})$	2
$(\text{A}, 1) \oplus (1, \text{A}) \oplus$ *c.c.*	2
$(16, 16) \oplus$ *c.c.*	1

Identical to "famous" BS or GP orientifold.
But: Much more solutions available for $\mathbb{T}^4/\mathbb{Z}_2$.

Example on CY_3: The SM on the quintic

- Chiral fourdimensional spectrum:

Representation	Multiplicity
$(\mathsf{A}_a)_L$	$\frac{1}{2}\left(\pi_a \circ \pi'_a + \pi_a \circ \pi_{O6}\right)$
$(\mathsf{S}_a)_L$	$\frac{1}{2}\left(\pi_a \circ \pi'_a - \pi_a \circ \pi_{O6}\right)$
$\left(\bar{\mathsf{N}}_a, \mathsf{N}_b\right)_L$	$\pi_a \circ \pi_b$
$(\mathsf{N}_a, \mathsf{N}_b)_L$	$\pi_a \circ \pi'_b$

- Fermat quintic CY_3

$$P(z_i) = z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0 \subset \mathbb{CP}^4$$

with sLag $\mathbb{RP}^3$ $\bar{\sigma}$-fixed set

$$P(x_i) = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 = 0 \subset \mathbb{RP}^4$$

- Use $\mathbb{Z}_5^5$, $z_i \mapsto \omega^{k_i} z_i$ with $\omega^5 = 1$, $k_i \in \mathbb{Z}_5$, to get

$$|k_2, k_3, k_4, k_5\rangle = \{[x_1 : \omega^{k_2} x_2 : \ \ldots \ : \omega^{k_5} x_5] | P(x_i) = 0\},$$

$5^4 = 625$ sLag $\mathbb{RP}^3$, calibrated with $\Re(\prod_i \omega^{k_i} \Omega_3)$.

- Intersection number with $|1, 1, 1, 1\rangle$ from

$$\prod_{i=1}^{5} \left(g_i + g_i^2 - g_i^3 - g_i^4\right) \bmod \langle g_i^5 = 1, \prod_{i=1}^{5} g_i = 1\rangle$$

Example on CY_3: The SM on the quintic

- Overall Supersymmetry $\Rightarrow$ trivial solution $\pi_a = \pi_{O6}$.
- Non-supersymmetric SM by using

$$\begin{aligned}\pi_a &= |0,0,3,1\rangle,\\ \pi_b &= |4,3,0,3\rangle,\\ \pi_c &= |3,0,1,1\rangle - 2|4,3,0,3\rangle,\\ \pi_d &= |4,2,4,4\rangle - 2|0,0,3,1\rangle\end{aligned}$$

  with $N_a = 3$, $N_b = 2$ and $N_c = N_d = 1$ produces

  3 generation SM fermion spectrum

  with right-handed neutrinos.
- Anomaly-free hypercharge is

$$U(1)_Y = \frac{1}{3}U(1)_a - U(1)_c + U(1)_d$$

- GS couplings to cancel $U(1) - SU(\mathrm{N})^2$ anomalies.
- An invisible sector needed for tadpole cancellation.
- SUSY breaking at M_s: large transverse volume

$$\mathrm{vol}_\perp^{9-q} \gg \mathrm{vol}_{\mathrm{int}}^{q+1}(\mathrm{D}q\mathrm{-brane})$$

SUSY breaking: Scalar potential

- Perturbing SUSY vacua (with "A-type" branes):

$$\text{F-terms} \longleftrightarrow \text{Kähler moduli}$$
$$\text{D-terms} \longleftrightarrow \text{Complex structure moduli}$$

- F-terms vanish perturbatively $\longrightarrow$ disc instantons

$$W(t) \sim \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-nt}$$

Examples with non-vanishing $W(t)$ known.

- D-terms induced by brane tension

$$\int_{\mathcal{W}^6} d\zeta^3 \ \mathcal{L}_{\mathrm{DBI}} \ \sim \ e^{-\phi_4} \left(\sum_a \mathrm{N}_a \left| \int_{\pi_a} \Omega_3 \right| + \int_{\pi_{\mathrm{O6}}} \Re(\Omega_3) \right)$$

of type

$$\mathcal{V}_{\mathrm{D-term}} = \sum_a \frac{1}{2g_a^2} \left(\sum_i q_a^i |X_i|^2 + \xi_a \right)^2 = \sum_a \frac{\xi_a^2}{2g_a^2} + \cdots$$

with

$$\xi_a^2 \sim \left| \int_{\pi_a} \Omega_3 \right| - \int_{\pi_a} \Re(\Omega_3)$$