Origination of Propagating Normal Domains in Large Composite Superconductors.

V.S. Kovner, Raz Kupferman and R.G. Mints.
School of Physics and Astronomy,
Raymond and Beverly Sackler Faculty of Exact Science
Tel–Aviv University,
69978 Ramat–Aviv, Israel.

Abstract– The origination of propagating normal domains in large superconducting composites is studied numerically by means of an effective circuit model. The initial perturbation is considered to be a thermal pulse. The minimum energy required to form a propagating normal domain is calculated as a function of the dimensionless transport current and three parameters characterizing the cooling conditions and the conductor. An analytical expression is proposed to determine this energy in the region of parameters of practical interest.

I. INTRODUCTION

Large composite superconductors have been recently tested for use in superconducting magnetic energy storage (SMES) systems [1]. These conductors are composed of superconducting multifilament strands embedded in a large normal metal matrix with high thermal and electrical conductivity to stabilize the conductor against superconducting to normal transition. Because of the large size of the stabilizer, if a normal zone nucleates, the current in this region redistributes into the stabilizer, followed by a significant decrease of the joule power and the recovery of superconductivity. Despite the above stabilizing mechanism, it was found experimentally that normal domains of finite size can propagate along the conductor for transport currents larger than a certain threshold current I_d [1].

The dynamics of a traveling normal domain was investigated in a number of theoretical studies. Huang and Eyssa [2,3] performed numerical simulations for the diffusion of heat and the redistribution of current in the conductor in the presence of a normal zone. Their simulations showed the formation of a stable traveling normal domain. Dresner [4] proposed an analytical method to calculate the propagation velocity of a traveling normal domain, assuming the time dependance of the joule power. In [5,6] we investigated both numerically and analytically the nucleation and propagation of a traveling normal domain in large composite superconductors using an effective circuit model. We proposed explicit equations for the velocity of the domain and for the threshold current I_d.

Manuscript received August 24,1992

We consider now the influence of an external perturbation of a total energy, Q_p, on a large composite superconductor in the cryostable regime. We suppose that this perturbation creates a normal nucleus. In case when transport current $I < I_d$ the superconducting state is stable with respect to such perturbations. For $I > I_d$ the superconducting state is metastable. This means that it is stable against perturbations with sufficiently small Q_p, so that the normal nucleus dissapears after the perturbation is over. If the value of Q_p exceeds a certain critical value Q_{in} (which we name as initiating energy) the final state is a state with traveling normal domains. In general Q_{in} depends not only on the parameters of the superconductor and the coolant, but also on the time dependence of the perturbation and on its spatial distribution. An important particular case is when the length of the pulse is much shorter than the characteristic thermal length of the system and the duration of the pulse is much shorter than the thermal relaxation time of the system. In this case the initiating energy depends only on the parameters of the composite and cooling conditions.

In this paper we consider the initiating energy for large composite superconductors. We treat the cryostable regime in case, when it is unstable against the perturbations resulting in traveling normal domains. The effective circuit model is used for numerical simulations [5,6].

II. THE MAIN EQUATIONS

In this section we review the effective circuit model [5,6]. This model describes the dynamics of the temperature field and the current density distribution in a composite superconductor in the presence of a normal zone.

Let us consider a rectangular conductor consisting of two ribbons of equal width, a superconductor of thickness, d_s, and a stabilizer (normal metal) of thickness, d_n . The conductor carries transport current I, and is kept in thermal contact with a heat reservoir of temperature T_0.

In order to obtain the initiating energy Q_{in}, the dynamics of the temperature and the current density distributions in the composite has to be considered. A complete treatment of this problem requires the solution of the heat diffusion equation for the temperature field coupled to the

1051-8223/93$03.00

set of Maxwell equations for the current density distribution. A simplified one–dimensional model was proposed in [5,6]. This model takes into account the main physical features of the problem, and can be described by the electrical circuit sketched in Figure 1.

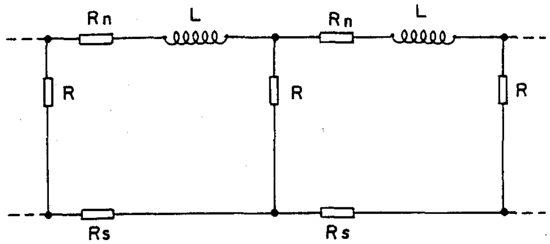

Figure 1: The effective circuit.

The upper chain of resistors represents the stabilizer, each resistor of resistance, $R_n = \rho_n \Delta x / d_n$, where ρ_n is the resistivity of the stabilizer and Δx is an arbitrary descretization length. Similary, the lower chain of resistors represents the superconductor each resistor of resistance, $R_s = \rho_s \Delta x / d_s$. Here ρ_s is the resistivity of the superconductor, which vanishes in the superconducting phase, and is finite in the normal phase. Both chains are linked through a chain of resistors $R = \gamma_R \rho_n d_n / \Delta x$, where γ_R is a numerical factor of the order of one, depending on the geometry of the conductor. Finally, the inclusion of a characteristic time scale in the electric current diffusion process is accomplished by taking into account the inductance of the stabilizer (the inductance of the superconductor is neglected) $\mathcal{L} = \gamma_l \mu_0 d_n \Delta x$. Here γ_l is another numerical factor. This model yields a set of two one–dimensional diffusion equations for the current density distribution in the superconductor $j_s(x,t)$ and for the temperature field $T(x,t)$

$$\left(\frac{\mathcal{L} d_n}{\rho_n}\right)\frac{\partial j_s}{\partial t} = \gamma_R d_n^2 \frac{\partial^2 j_s}{\partial x^2} - j_s\left(1 + \frac{\rho_s d_n}{\rho_n d_s}\right) + j, \qquad (2.1)$$

and

$$C\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) - W(T) + Q(T) + Q_p(x,t), \qquad (2.2)$$

where C is the heat capacity and k is the heat conductivity both taken to be constant. The parameter $j \equiv I/d_s$ is the current density in the superconductor far from a normal domain. The function $W(T)$ is the rate of heat transfer to the coolant per unit volume, which can be written in the form $W(T) = h(T)(T - T_0)/d$, where $d \equiv d_s + d_n$. The function $Q(T)$ is the rate of Joule heating per unit volume having three contributions: from the Joule heating in the superconductor when it is in the normal state, from the current in the stabilizer, and from the perpendicular current. As a result, $Q(T)$ is given by

$$Q(T) = \frac{1}{d}\Big[d_s \rho_s j_s^2 + \frac{d_s^2 \rho_n}{d_n}(j - j_s)^2 + \gamma_R d_n d_s^2 \rho_n \left(\frac{\partial j_s}{\partial x}\right)^2\Big]. \qquad (2.3)$$

The function $Q_p(x,t)$ is the power of the external heating per unit volume. The total energy of the pulse is given by

$$Q_p = A \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dt\, Q_p(x,t). \qquad (2.4)$$

where A is the cross–sectional area of the sample. For convenience, we use the following dimensionless variables, the temperature θ, and the current density in the superconductor i_s,

$$\theta \equiv \frac{T - T_0}{T_c - T_0}, \qquad i_s \equiv \frac{j_s}{j_c}. \qquad (2.5)$$

where T_c is the critical temperature of the superconductor. We define L_{th}, the characteristic thermal length and τ_{th}, the characteristic thermal relaxation time,

$${L_{th}}^2 \equiv \frac{(d_n + d_s)k}{h}, \qquad \tau_{th} \equiv \frac{(d_n + d_s)C}{h}, \qquad (2.6)$$

the characteristic length of the current redistribution L_m and the corresponding relaxation time τ_m

$${L_m}^2 \equiv \gamma_R {d_n}^2, \qquad \tau_m \equiv \frac{\mathcal{L} d_n}{\rho_n}. \qquad (2.7)$$

We assume here the "step model" for the resistivity of the superconductor [7].

$$\rho_s(j_s, T) = \rho_s \eta\Big[j_s - j_c(T)\Big], \qquad (2.8)$$

where η is the Heaviside step function ($\eta = 0$ if $x < 0$ and $\eta = 1$ if $x > 0$), and $j_c(T)$ is the critical current density in the superconductor given by

$$j_c(T) = j_c\Big[1 - \frac{(T - T_0)}{(T_c - T_0)}\Big] = j_c(1 - \theta). \qquad (2.9)$$

We treat perturbations with length $L_q << L_{th}$ and duration $\tau_q << \tau_{th}$. In this case the function $Q_p(x,t)$ is proportional to a product of two delta functions:

$$Q_p(x,t) = \frac{Q_p}{A}\delta(x)\delta(t), \qquad (2.10)$$

where Q_p is the total energy of the pulse.

Finally, we introduce three dimensionless parameters

$$\xi \equiv \frac{\rho_s d_n}{\rho_n d_s}, \qquad \alpha \equiv \frac{d_s^2 \rho_n j_c^2}{d_n h (T_c - T_0)}, \qquad q_p \equiv \frac{Q_p}{Q_h}, \quad (2.11)$$

where ξ is the ratio of the resistances of the superconductor and the stabilizer per unit length, α is the ratio of characteristic rates of Joule heating and heat flux to the coolant

(Stekly parameter), and q_p is the dimensionless total energy of the pulse, where

$$Q_h \equiv CAL_{th}(T_c - T_0). \tag{2.12}$$

Equations (2.1) and (2.2) in the dimensionless form are given by

$$\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial x^2} - \theta + \alpha(i - i_s)^2 + \xi\alpha i_s^2\eta(i_s + \theta - 1)$$

$$+\alpha\lambda^2\left(\frac{\partial i_s}{\partial x}\right)^2 + q_p\delta(z)\delta(t), \tag{2.13}$$

$$\tau\frac{\partial i_s}{\partial t} = \lambda^2\frac{\partial^2 i_s}{\partial x^2} - \left[1 + \xi\eta(i_s + \theta - 1)\right]i_s + i, \tag{2.14}$$

where time is measured in units of τ_{th} and length in units of L_{th}, the dimensionless parameters i, τ and λ are

$$i \equiv \frac{j}{j_c}, \qquad \tau \equiv \frac{\tau_m}{\tau_{th}}, \qquad \lambda \equiv \frac{L_m}{L_{th}}. \tag{2.15}$$

III. RESULTS AND DISCUSSION

The value of q_{in} for a given set of parameters i, α, ξ, τ and λ was obtained by means of numerical simulations of equations (2.13), (2.14) for different values of q_p.

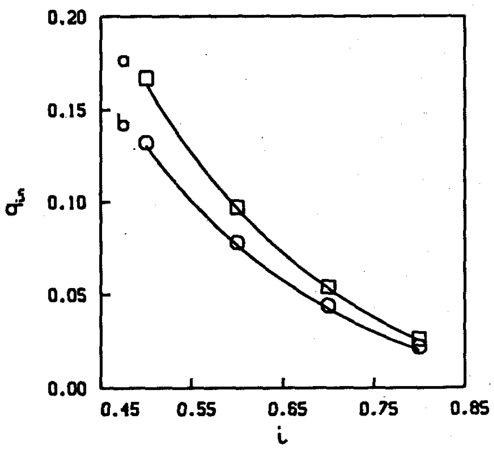

Figure 2: q_{in} *as a function of dimensionless current* i. *a)* $\alpha = .9$, $\xi = 120$, $\tau = 90$. *b)* $\alpha = .9$, $\xi = 190$, $\tau = 90$.

The initial conditions were taken as follows:

$$\theta(x, 0) = 0, \qquad i_s(x, 0) = i.$$

The large time behaviour of the system determines whether $q_p < q_{in}$ or $q_p > q_{in}$. Namely, for $q_p < q_{in}$ the system tends back to the initial superconducting state, whereas for $q_p > q_{in}$ a pair of travelling normal domains propagate in opposite directions along the system. The values of the parameters were taken from the References [1,4]. Typical values of ξ, τ and λ can be then estimated as $\xi = 100 - 200$, $\tau = 10 - 100$, and $\lambda = 0.1 - 1.0$. Specifically, as we

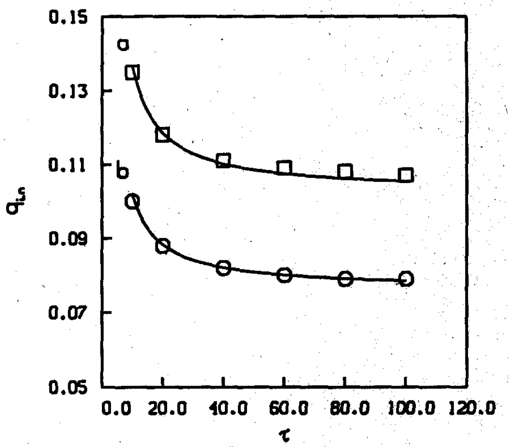

Figure 3: q_{in} *as a function of* τ. *a)* $i = .6$, $\alpha = .9$, $\xi = 100$. *b)* $i = .6$, $\alpha = .9$, $\xi = 180$.

are interested in cryostable conductors, the case $\alpha < 1$ is considered.

We plot the initiating energy, q_{in}, as a function of i, τ ξ and α in Fig. 2–5. The results of numerical calculations are presented by points. Solid lines present the results of following analytical estimation (see eq.(3.8) below). Note, that the dependence of q_{in} on λ was found to be practically negligible. It should be emphasized that the value of q_{in} does not depend on any of the parameters i, ξ, τ, α, when $i \to 1$.

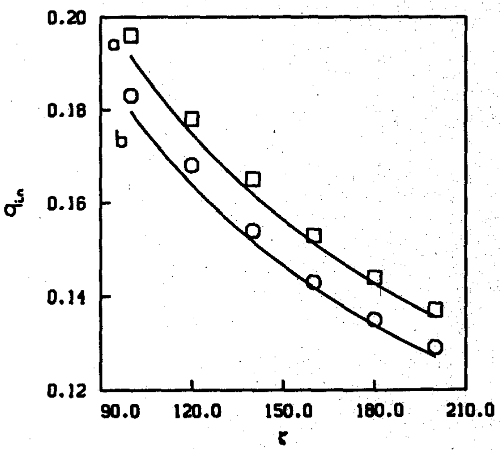

Figure 4: q_{in} *as a function of* ξ. *a)* $i = .5$, $\alpha = .8$, $\tau = 90$. *b)* $i = .5$, $\alpha = .9$, $\tau = 90$.

Let us estimate the value of the initiating energy q_{in} from the following qualitative considerations. When a part of the superconductor undergoes a normal trnsition, the current is confined in the superconductor during a time interval of the order of τ_m/ξ. During this interval the superconductor in the vicinity of the transition front is unstabilized as the value of the current i is higher than the

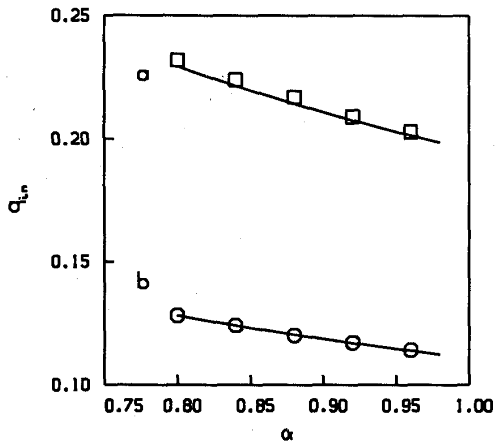

Figure 5: q_{in} *as a function of* α. *a)* $i = .5$, $\xi = 100$, $\tau = 20$. *b)* $i = .6$, $\xi = 100$, $\tau = 20$.

minimum propagation current for the superconductor itself. The normal zone boundary propagates with a certain velocity v. Thus, a region with the length of the order of $v\,\tau_m/\xi$, in front of the normal domain, becomes temporary unstabilized. The effective Stekly parameter α_{eff} associated with this unstabilized superconductor is equal to $\alpha_{eff} = \alpha\xi \gg 1$ [5,6].

To initiate a propagating normal zone by a thermal pulse it is necessary to heat up to a temperature of the order of $T_c(i)$ a region of a certain length l_{in}. In case, when the current in the superconductor is constant the value of $l_{in} = l_c$ can be estimated from the heat balance equation and for large α_{eff} it is equal to [7]

$$l_c \approx L_{th}\,\frac{\sqrt{1-i}}{i}\,\frac{1}{\sqrt{\alpha\xi}}. \quad (3.1)$$

The initiating energy in that case $q_{in} = q_c$ [7]

$$q_c \approx 2.3\frac{(1-i)^{3/2}}{i\sqrt{\alpha\xi}}. \quad (3.2)$$

At the same time the length of unstabilized segment in front of the traveling normal domain l_d is estimated as

$$l_d \approx v\,\frac{\tau_m}{\xi} = v\,t_{th}\,\frac{\tau}{\xi}. \quad (3.3)$$

For $\alpha_{eff} \gg 1$ the velocity v of the normal zone boundary propagation is equal to [5,6]:

$$v \approx \frac{L_{th}}{\tau_{th}}\,\sqrt{\alpha\xi}\,\frac{i}{\sqrt{1-i}}. \quad (3.4)$$

Substituting (3.4) in (3.3) we find that the value of l_d can be estimated as

$$l_d \approx L_{th}\,\tau\sqrt{\frac{\alpha}{\xi}}\,\frac{i}{\sqrt{1-i}}. \quad (3.5)$$

In case, when $l_c \ll l_d$, which corresponds to $\tau \gg 1$, formula (3.2) is a good approximation for the initiating energy q_{in}. To estimate q_{in} for a wider range of τ we have to take into consideration that current is not constant in the normal domain due to the redistribution into stabilizer. We account it by introducing the effective current i_{eff}, which is a function of the ratio l_c/l_d and $i_{eff} \to i$, when $l_c \ll l_d$. In the first (linear) approximation we obtain

$$i_{eff} = i(1 - \gamma\frac{l_c}{l_d}) \quad (3.6)$$

where γ is a numerical factor of the order of one. Substituting i_{eff} in (3.2) instead of i and using (3.1) and (3.5) we find the following expression for initiating energy

$$q_{in} \approx \frac{2.3}{\sqrt{\tau\xi}}\,\frac{(1-i)^{3/2}\left(\alpha\tau i^2 + 0.75\right)^{3/2}}{\alpha i^2\left[\alpha\tau i^2 - 0.75\,(1-i)\right]}. \quad (3.8)$$

The value $\gamma = 0.75$ we obtain by the best fitting to the numerical data. Expression (3.8) approximates the results of numerical simulations with a maximum deviation less than 4% for the values of parameters $100 < \xi < 200$, $40 < \tau < 100$, $0.8 < \alpha < 1.0$ and transport current $0.5 < i < 0.85$. The initiating energy q_{in} calculated by means of (3.7) is presented by solid lines in Fig. 3–6.

To summirize: we obtained numerically the initiating energy for a traveling normal domain in a large composite superconductor as a function of transport current and four dimensionless parameters characterising the composite and cooling conditions. The effective circuit model used for numerical simulations. An analytical expression for the initiating energy suggested. The values of the initiating energy obtained by means of this formula are in a good agreement with the results of numerical calculations.

References

[1] J.M. Pfotenhauer, M.K. Abdelsalam, F. Bodker, D. Huttelstone, Z. Jiang, O.D. Lokken, D. Scherbarth, B. Tao, and D. Yu, IEEE Trans. Magn. MAG **27**, 1704 (1991).

[2] X. Huang and Y.M. Eyssa, IEEE Trans. Magn. MAG **27**, 2304 (1991).

[3] X. Huang and Y.M. Eyssa, Cryogenics **32**, No 1, 28 (1992).

[4] L. Dresner, in proceedings of the Eleventh Conference on Magnetic Technology (MT–11), Tsukuba, Japan, August 28–September 1, 1989, edited by T. Sekiguchi and S. Shimamoto, Elsevier Applied Science, New–York, 1084 (1990).

[5] R. Kupferman, R.G. Mints, E. Ben–Jacob, J. Appl. Phys. **70**, 7484 (1991).

[6] R. Kupferman, R.G. Mints, E. Ben–Jacob, Cryogenics **32**, 485 (1992).

[7] A.Vl. Gurevich and R.G. Mints, Rev. Mod. Phys. **59**, 941 (1987).