

Oscillation Theory for Second Order Linear, Half-Linear, Superlinear and Sublinear Dynamic Equations

by

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