OSCILLATORY MOTION OF AN ELECTRICALLY CONDUCTING VISCOELASTIC FLUID OVER A STRETCHING SHEET IN A SATURATED POROUS MEDIUM WITH SUCTION/BLOWING

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The effect of an oscillatory motion of a viscoelastic fluid over an infinite stretching sheet through porous media in the presence of magnetic field with applied suction has been studied. The surface absorbs the fluid in a porous medium in the presence of magnetic field and the velocity oscillates depending on the stretching rate (b). Analytical expressions for the velocity and the coefficient of skin friction have been studied, first by the perturbation method and then by power series method. The effect of viscoelastic parameter k_1, porous parameter k_2, magnetic parameter Mn, and the vertical distance x in the presence of suction/blowing on the velocity and the flow characteristics are discussed. The velocity of the viscoelastic fluid is found to decrease in the presence of magnetic field and porous media, as compared to the study of viscous fluid. It is also found that the effect of unsteadiness in the wall velocity and skin friction are found to be appreciable in the presence of suction/blowing parameter.

1. Introduction

Boundary layer flow on continuous moving accelerating surfaces is an important type of flow occurring in a number of technical processes which have applications in fuel industries, in an aerodynamic extrusion of plastic sheets and boundary layer along liquid film in condensation processes. Drag, heat, and mass transfer are governed by the structure of the layer. Flows due to a continuously moving surface involve continuous pulling of a sheet through a reaction zone as in metallurgy, in textile and in paper industries, and in the manufacture of polymer sheets, sheet glass, and crystalline materials.

In a series of three articles entitled "Boundary layer behaviour on continuous solid surfaces" Sakiadis [20] pointed out the differences in boundary conditions between a moving flat plate of finite length and a continuous surface. The governing equations for both two-dimensional and axisymmetric flows were determined. In a subsequent paper

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he gave exact and approximate solutions for the flat sheet and the cylinder. This was followed by the work of Crane [2], P. S. Gupta and A. S. Gupta [7] and Rajagopal et al. [18].

A theoretical and experimental treatment for the moving flat plate was made by Tsou et al. [27]. They determined heat transfer rates for certain values of the Prandtl numbers.

Siddappa and Abel [22] have extended Crane's flow problem to the viscoelastic fluid of Walters' liquid B' model, and obtained the solutions of equations of motion for boundary layer flow past a stretching sheet.

One of the most popular models for non-Newtonian fluids is the model that is called the second-order fluid or fluid of second grade. It is reasonable to use the second-order fluid model to do numerical calculations. The constitutive assumption for the fluids of second grade is in the following form:

$$T = -pI + \mu A - \alpha_1 A_2 + \alpha_2 A_1^2, \tag{1.1}$$

where T is the Cauchy stress, $-pI$ the spherical stress due to the constraint of incompressibility, μ the coefficient of viscosity, α_1 and α_2 material moduli and A_1 and A_2 are the first two Rivlin-Ericksen tensors which are discussed in detail by Fosdick and Rajagopal [6]. Further, a comprehensive discussion on the restrictions for μ, α_1, and α_2 can be found in the work by Dunn and Rajagopal [4]. The sign of the material moduli α_1 and α_2 is the subject of much controversy which was discussed by Rajagopal [14]. In the experiments on several non-Newtonian fluids, the experimentalists have not confirmed these restrictions on α_1 and α_2. Thus, the conclusion is that the fluids that have been tested are not fluids of second grade and they are characterised by a different constitutive structure. The equation of motion of incompressible second-grade fluid, in general, is of higher order than of the Navier-Stokes equations. The Navier-Stokes equation is a second-order partial differential equation, but the equation of motion of a second-order fluid is a third-order partial differential equation. But in the present article we considered momentum equation for a Viscoelastic incompressible fluid of Walters' liquid B' model which is of fourth-order highly nonlinear differential equation having four boundary conditions which is solved analytically first by perturbation method and then by power series solution. A marked difference between the case of the Navier-Stoke's theory and that for fluids of second grade is that ignoring the nonlinearity in the Navier-Stoke's equation does not lower the order of the equation, however ignoring the higher-order nonlinearities in the case of the second-grade fluid reduces the order of the equation. The no-slip boundary condition is sufficient for a Newtonian fluid but for a second-order fluid it may not be sufficient and therefore one needs an additional condition at boundary. Therefore a critical review on the boundary conditions and the existence and uniqueness of the solution has been given by Rajagopal [15]. In order to clarify these points, Rajagopal and Gupta [16] have presented a paper on the flow of a second-order fluid past an infinite porous plate with velocity component along the x-axis tending to U as y approaches infinity. The augumentation of the boundary conditions has also been discussed by Rajagopal and Gupta [16].

Subhas and Veena [25] have studied about the viscoelastic fluid flow and heat transfer characteristics in a saturated porous medium over a stretching surface with frictional heating and internal heat generation or absorption for both PST and PHF cases considering steady-state boundary layer equation.

A similar situation is obtained when the porous plate is bounded by another porous plate. A similar work has been provided by Rajagopal and Kaloni [17].

Unsteady flows of a second-order fluid in a bounded region have been studied by Ting [26]. Some unsteady uni-directional flows of second order fluids have been considered by Rajagopal [13]. These works showed that the no-slip condition at the boundary for this type of flow suffices. A recent work of Rajagopal et al. [1] showed that the solutions for unsteady flows of a second-order fluid occupying the space above a plate are bounded if the coefficient of higher-order derivative is positive. However, this is not necessary for steady flows of a second-order fluid.

Erdogan [5] has studied the unsteady motions of a second-order fluid over a plane wall and he has shown that a Newtonian fluid induced by a flat plate that applies a constant stress to the fluid flows faster than a second-order fluid.

Besides these excellent reviews of the literature dealing with nonsteady flows presented by Lighthill [10] and Stuart [24], the behavior of viscoelastic fluids in laminar flow through porous media has been the subject by numerous investigators, including Pilitisis and Beirs [12], H. Pascal and F. Pascal [11], Jones and Walters' [8], Rudraiah et al. [19], and Vafai and Kim [28].

Siddappa et al. [23] have investigated the oscillatory motion of a viscoelastic fluid past a stretching sheet and have shown that the unsteadiness in the wall velocity and skin friction are found to be appreciable. Devi and Nath [3] have discussed the similar solutions of the unsteady boundary layer equations for a moving wall and they predicted their results as the Prandtl number strongly affects the heat transfer, but the skin friction is unaffected by it.

Nonlinear streaming due to the oscillatory stretching of a sheet in a viscous fluid was discussed by Wang [29]. He considered an elastic sheet which was stretched back and forth in a viscous fluid. His problem was governed by a nondimensional parameter S which represents the relative magnitude of frequency to stretching rate.

As unsteady flows of viscoelastic fluids through porous media in the presence of magnetic field are of great interest and have several applications such as in electromagnetic propulsion and in the flow of nuclear fuel slurries, flow of fluid metals, and alloys, Sarpakaya [21] was the first who has studied the MHD flows in non-Newtonian fluids. He studied about the MHD flow in Bingham plastic and Ostwald fluids and presented his results with the conclusion that as the intensity of the magnetic field increases, the distribution of velocity is increasingly more uniform.

The nonuniqueness of MHD flow of a second-order fluid past a stretching sheet was presented by Lawrence and Rao [9].

Motivated by all the above analyses, in the present paper we therefore investigate the oscillatory motion of a viscoelastic Walters' liquid B' model over a stretching sheet through porous media in the presence of magnetic field considering the nonsteady boundary layer equations with suction/injection. The effects of velocity profiles u on different parameters such as viscoelastic parameter k_1, permeability permeter k_2, and magnetic parameter Mn are discussed.

2. Formulation of the problem and mathematical solution

We consider the unsteady flow of an incompressible electrically conducting Viscoelastic fluid (Walters' liquid B') through a porous medium over a stretching sheet with suction/injection that issues from a thin slit in which the flow approaches the sheet with zero angle coincidence. Two equal and opposite forces are introduced along the sheet so that the wall is stretched, keeping the origin fixed. The x-axis is taken along the sheet, the y-axis perpendicular to it, and the origin is the slit. The speed of a point on the sheet is assumed to be proportional to its distance from the slit.

The constitutive equations for the incompressible Viscoelastic (Walters' liquid B') fluid are

$$P_{ik} = -Pg_{ik} + P'_{ik},$$
$$P'^{ik}(x,t) = 2\int_{-\infty}^{t} \phi(t-t')\frac{\partial x^i}{\partial x'^m}\frac{\partial x^k}{\partial x'^r}e^{(1)mr}(x',t')dt, \tag{2.1}$$

where

$$\phi(t-t') = \int_0^{\infty} \frac{N(\tau)}{\tau}\cdot \exp\left[-(t-t') * \tau\right]d\tau, \tag{2.2}$$

where $N(\tau)$ is the distribution function of relaxation times, P_{ik} the stress tensor, P is an isotropic pressure, $g_{ik}(x)$ is the metric tensor of fixed co-ordinate system x', and P'_{ik} is the rate of strain tensor. In the case of fluids with short memories, that is, short relaxation times, the above equation of state can be written in the following simplified form:

$$P'_{ik} = 2\mu e^{(1)ik} - 2k_0\frac{\delta}{\delta t}e^{(1)ik} \tag{2.3}$$

in which $\mu = \int_0^{\infty} N(\tau)d\tau$ is the limiting viscosity at small rates of shear, $k_0 = \int_0^{\infty}\tau N(\tau)d\tau$, and $\delta/\delta t$ denotes the convected time derivative introduced by Oldroyd (1958).

The governing boundary layer equation for Walters' liquid B' through porous media in the presence of magnetic field with suction or injection is of the form

$$\begin{aligned}\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= \nu\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{\partial p}{\partial x}\\ &\quad - k_0\left\{\frac{\partial^3 u}{\partial t\partial y^2} + u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\cdot\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}\\ &\quad - \frac{\nu}{k'}u - \frac{\sigma B_0^2 u}{\rho}\end{aligned} \tag{2.4}$$

and the equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.5}$$

where ν is the coefficient of viscosity, k' the coefficient of porosity, σ electrical conductivity, B_0 the magnetic field strength, and ρ the density of the fluid.

The corresponding boundary conditions are

$$\begin{gathered} u = bx(1+\varepsilon \operatorname{Cos}\omega t), \qquad v = v_w \quad \text{at } y = 0, \\ u = 0, \qquad u_y = 0 \quad \text{as } y \longrightarrow \infty. \end{gathered} \tag{2.6}$$

Here b denotes the maximum rate of stretch with dimension $(\text{time})^{-1}$.

We assume $S = \omega/b \equiv 1/\varepsilon \gg 1$, where ε is very small. S implies the small amplitude of oscillations. The boundary conditions suggest the following transformation:

$$\begin{gathered} u = bxf_\eta(\eta,\tau), \qquad v = -(b\nu)^{1/2} f(\eta,\tau), \qquad p = p(\eta,\tau), \\ \tau = \omega t, \quad \eta = \sqrt{\frac{b}{\nu}}y. \end{gathered} \tag{2.7}$$

Substitution of (2.7) in (2.4) leads to

$$\begin{aligned} &S_1 f_{\eta\tau}(\eta,\tau) + S_2\left[f_\eta^2(\eta,\tau) - f(\eta,\tau) f_{\eta\eta}(\eta,\tau)\right] \\ &\quad = S_2 f_{\eta\eta\eta}(\eta,\tau) - \frac{k_0}{\nu}\left[S_3 f_{\eta\eta\eta\tau}(\eta,\tau) + 2 f_\eta(\eta,\tau) f_{\eta\eta\eta}(\eta,\tau)\right. \\ &\qquad\qquad \left. - f(\eta,\tau) f_{\eta\eta\eta\eta}(\eta,\tau) - f_{\eta\eta}^2(\eta,\tau)\right] - (S_4+S_5) f_\eta(\eta,\tau), \end{aligned} \tag{2.8}$$

where

$$\begin{gathered} S_1 = \frac{\omega}{b^2}, \qquad S_2 = \frac{S}{\omega} = \frac{1}{b}, \qquad S_3 = \frac{\omega}{b}, \\ S_4 = \frac{\nu}{k' b^2}, \qquad S_5 = \frac{\sigma B_0^2}{\rho b^2}. \end{gathered} \tag{2.9}$$

The corresponding boundary conditions are

$$\begin{gathered} f_\eta(\eta,\tau) = l + \varepsilon \operatorname{Cos}\omega t; \qquad f(\eta,\tau) = -\frac{v_w}{\sqrt{b\nu}} \quad \text{at } \eta = 0, \\ f_\eta(\eta,\tau) = 0; \qquad f_{\eta\eta}(\eta,\tau) = 0 \quad \text{as } \eta \longrightarrow \infty. \end{gathered} \tag{2.10}$$

Here v_w is the suction velocity across the stretching sheet when $v_w < 0$ and it is blowing velocity when $v_w > 0$.

To solve (2.8) we employ perturbation analysis of Wang [29] by setting

$$f(\eta,\tau) = g_1(\eta) + \varepsilon \operatorname{Re}\{e^{i\omega t} g_2(\eta)\}. \tag{2.11}$$

Substituting (2.11) in (2.8) and (2.10) and equating harmonic and nonharmonic terms to zero, we obtain fourth-order nonlinear ordinary differential equation for $g_1(\eta)$ and fourth-order nonlinear differential equation for $g_2(\eta)$ with variable coefficients

$$\begin{aligned} &S_2\{g_{1\eta\eta}^2(\eta) - g_{1\eta\eta}(\eta)g_1(\eta)\} \\ &\quad = S_2 g_{1\eta\eta\eta}(\eta) - \frac{k_0}{\nu}\left[2g_{1\eta\eta\eta}(\eta)g_{1\eta}(\eta) - g_{1\eta\eta\eta\eta}(\eta)g_1(\eta) - g_{1\eta\eta}^2(\eta)\right] - (S_4+S_5)g_{1\eta}(\eta), \end{aligned} \tag{2.12}$$

$$\begin{aligned} &S_6 g_{2\eta}(\eta) + S_2\{2g_{1\eta}(\eta)g_{2\eta}(\eta) - g_{1\eta\eta}(\eta)g_2(\eta) - g_{2\eta\eta}(\eta)g_1(\eta)\} \\ &\quad = S_2 g_{2\eta\eta\eta}(\eta) - k_1\left[S_3 iw g_{2\eta\eta\eta}(\eta) + 2g_{2\eta\eta\eta}(\eta)g_{1\eta}(\eta) + 2g_{1\eta\eta\eta}(\eta)g_{2\eta}(\eta)\right. \\ &\qquad \left. - g_{2\eta\eta\eta\eta}(\eta)g_1(\eta) - g_{1\eta\eta\eta\eta}(\eta)g_2(\eta) - 2g_{1\eta\eta}(\eta)g_{2\eta\eta}(\eta)\right] - (S_4+S_5)g_{2\eta}(\eta), \end{aligned} \tag{2.13}$$

where

$$\begin{gathered} k_1 = \frac{k_0}{\nu}, \qquad S_4 = \frac{\nu}{k' b^2} = \frac{k_2}{b^2}, \qquad S_5 = \frac{\sigma B_0^2}{\rho b^2} = \frac{\mathrm{Mn}}{b^2}, \\ S_6 = S_1 i\omega, \qquad S_7 = S_3 i\omega. \end{gathered} \tag{2.14}$$

The corresponding boundary conditions are

$$\begin{gathered} g_1(\eta) = -\frac{v_w}{\sqrt{b\nu}}, \qquad g_2(\eta) = 0, \qquad g_{1\eta}(\eta) = g_{2\eta}(\eta) = 1 \quad \text{at } \eta = 0, \\ g_{1\eta}(\eta) = g_{2\eta}(\eta) = g_{1\eta\eta}(\eta) = g_{2\eta\eta}(\eta) = 0 \quad \text{as } \eta \longrightarrow \infty, \end{gathered} \tag{2.15}$$

suffix denotes differentiation with respect to η. Making use of (2.15), we derive the exact analytical solution of (2.12) in the form

$$\begin{gathered} g_{1\eta}(\eta) = e^{-\alpha\eta}, \\ g_1(\eta) = \frac{1 - e^{-\alpha\eta}}{\alpha} - \frac{v_w}{\sqrt{b\nu}}. \end{gathered} \tag{2.16}$$

α is the positive root of the cubic equation

$$\alpha^3 - \frac{(k_1 - S_2)}{(v_w/\sqrt{b\nu})k_1}\alpha^2 + \frac{S_2}{k_1}\alpha - \frac{(S_2+S_4+S_5)}{(v_w/\sqrt{b\nu})k_1} = 0. \tag{2.17}$$

Further, we apply series method to obtain the solution for (2.13). Hence the solution of (2.13) subjected to the boundary conditions (2.15) is obtained as

$$g_2(\eta) = \frac{[1 + (A/B)\exp(-\alpha\eta) - (A/B)\cdot(C/D)\exp(-2\alpha\eta) + \cdots]}{[(A/B) - 2(A/B)\cdot(C/D) + \cdots]}, \tag{2.18}$$

where

$$\begin{gathered} A = S_2 + k_1\alpha^2, \\ B = k_1\alpha^2 - S_2\alpha^2 + k_1S_7\alpha^2 + S_{56} - S_2, \\ C = S_2 + 2k_1\alpha^2, \\ D = 8k_1\alpha^2 - 4S_2\alpha^2 + 4k_1S_7\alpha^2 + S_6 - 2S_2. \end{gathered} \tag{2.19}$$

3. Skin friction

Shearing stress at a point on the plate for the Viscoelastic fluid is

$$\tau_0 = u_0\left[-\mu\frac{\partial u}{\partial y} - \nu k_0\frac{\partial^2 u}{\partial y^2}\right]_{y=0}. \tag{3.1}$$

Substituting similarity transforms in the above equation, the form of the equation will be

$$\tau_0 = u_0\lfloor -\mu f_{\eta\eta} + k_0 f f_{\eta\eta\eta}\rfloor. \tag{3.2}$$

$u_0 = b^{3/2}x/\sqrt{\nu}$ and the calculated skin friction is

$$\begin{gathered} \tau_0 = \frac{u_0\alpha}{AX}[\mu AX + \varepsilon\exp[i\omega t](AY - k_0TB)], \\ X = 1 - 2\frac{C}{D} + \cdots, \quad Y = 1 + \frac{A}{B} - \frac{A}{B}\frac{C}{D} + \cdots, \quad T = 1 - 4\frac{C}{D} + \cdots. \end{gathered} \tag{3.3}$$

4. Results and discussion

In Figure 4.1(a) a graph of $g_1(\eta)$ versus η is drawn for various values of the elastic parameter $k_1 = 0.4, 0.6, 0.8$ in both cases of suction and blowing. It is observed from the figure that g_1 decreases with increasing values of k_1.

In Figure 4.1(b) a graph of $g_1(\eta)$ versus η is drawn for various values of the permeability parameter $k_2 = 1.0, 100, 1000$ and it is seen from the figure that velocity increases as the permeability parameter k_2 increases owing to the inhibitive influence of the permeability parameter k_2 in both the cases of suction and blowing.

In Figure 4.1(c) a graph of $g_1(\eta)$ versus η is drawn for various values of the magnetic parameter Mn and it is observed from the figure that g_1 increases with increasing values

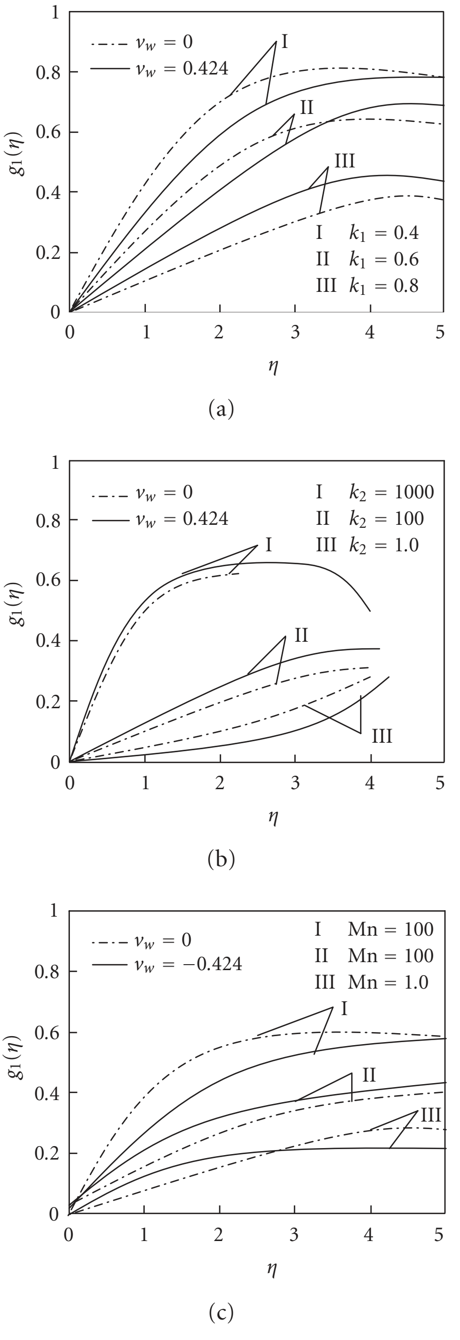


Figure 4.1. (a) Velocity profiles $g_1(\eta)$ for various values of the elastic parameter $k_1 = 0.4, 0.6, 0.8$ with $b = 0.5$, $\nu = 0.04$, $k_2 = 1.0$, Mn $= 0.5$. (b) Velocity profiles $g_1(\eta)$ for various values of the permeability parameter k_2 with $b = 0.5$, $\nu = 0.04$, $k_1 = 0.4$, and Mn $= 0.5$. (c) Velocity profiles $g_1(\eta)$ for various values of the magnetic parameter Mn keeping $b = 0.5$, $k_1 = 0.4$, $\nu = 0.04$, and $k_2 = 1.0$ fixed.

of Mn in the presence of porous medium, that is, velocity increases with small values of elastic parameter k_1, which might be regarded as a manifestation of the presence of normal stresses inside the boundary layer. Speaking in physical terms, the thickening of the boundary layer may be attributed to the tensile stress in the layer which causes an axial contraction. These results are well in agreement with the results of Rajagopal et al. [18].

In Figure 4.2(a) velocity profiles for $g_2(\eta)$ versus η are drawn for various values of viscoelastic parameter k_1 and it is noticed from the figure that g_2 decreases with increasing values of k_1 in the presence of suction or blowing.

In Figure 4.2(b) velocity profiles for $g_2(\eta)$ versus η are drawn for different values of permeability parameter k_2 and it is noticed from the figure that g_2 increases as k_2 increases in the presence of both suction and blowing.

In Figure 4.2(c) a graph of $g_2(\eta)$ versus η is drawn for various values of magnetic parameter Mn in the presence of suction or blowing and it is seen from the figure that velocity increases as Mn increases.

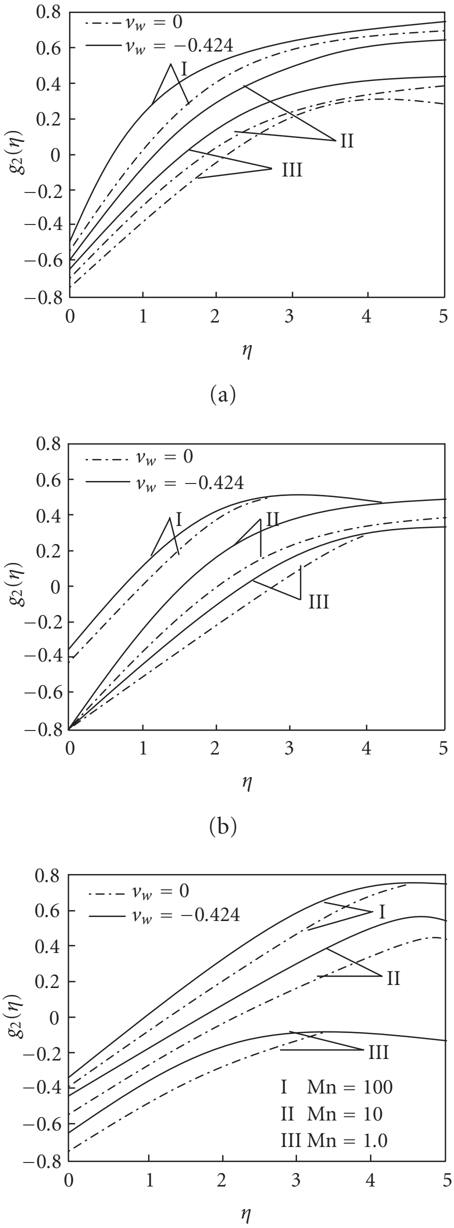
In Figure 4.3(a), that is, from the graph of $g_1'(\eta)$ versus η, it is found that the effects of impermeability of the boundary wall velocity increase as Mn increases in the case of suction and blowing on the horizontal velocity profiles in the boundary layer with permeability $k_2 = 100$ and the viscoelastic effect of $k_1 = 0.2$ being drawn in the absence of magnetic field and it is observed from the figure that velocity decreases with increase of distance. The effect of suction is to decrease the velocity and that of blowing is to increase the velocity. These results are consistent with the physical situation.

Figure 4.3(b) is plotted for the same set of parameters except in the absence of porous medium and with the magnetic effect of Mn = 100 and it is observed that g_1' decreases with increase in the values of magnetic parameter Mn. Comparison of these two graphs, 4.3(a) and 4.3(b), reveals the fact that the effect of porosity and magnetic parameter is to decrease the velocity for all cases of suction, blowing, and impermeability of the wall. This is because of porous medium's and magnetic field's obstruction to the flow over the oscillatory motion of a stretching sheet.

In Figure 4.4 velocity profiles for $g_2'(\eta)$ versus η for a set of parameters like $k_1 = 0.4$, Mn $= 10.0$, $b = 0.4$, $\nu = 0.04$ in the absence of porous medium are drawn and it is noticed from the figure that velocity decreases within the boundary layer of an oscillatory stretching sheet with the increase of distance from the boundary. The effect of suction is to decrease the velocity and injection increases the velocity. we also notice that velocity decreases as the distance increases from the boundary sheet and also observe that the oscillatory motion of the stretching sheet produces oscillating flow in the fluid.

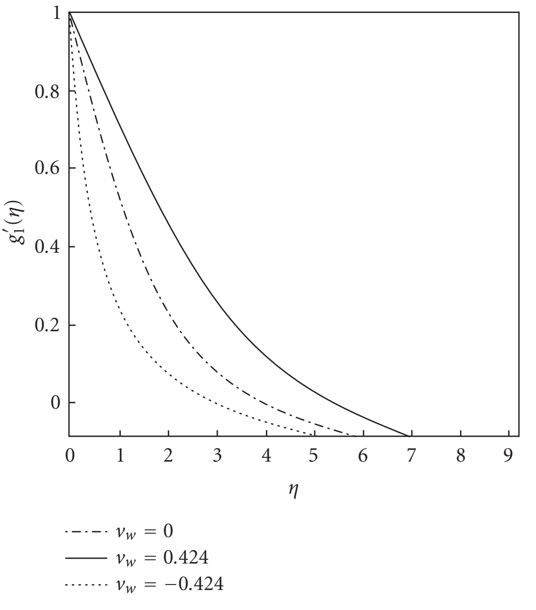
We reveal the fact that for increasing values of stretching rate parameter (b) the oscillation also increases. In other words, that the mode of oscillation depends on stretching rate.

It is observed that the magnitude of skin friction decreases initially and later increases as the viscoelastic parameter k_1 increases. Similarly, for various values of magnetic parameter Mn and for fixed value of $k_2 = 1.0$, we observed that the effect of magnetic parameter decreases the magnitude of skin friction up to certain level and suddenly increases to the top level.

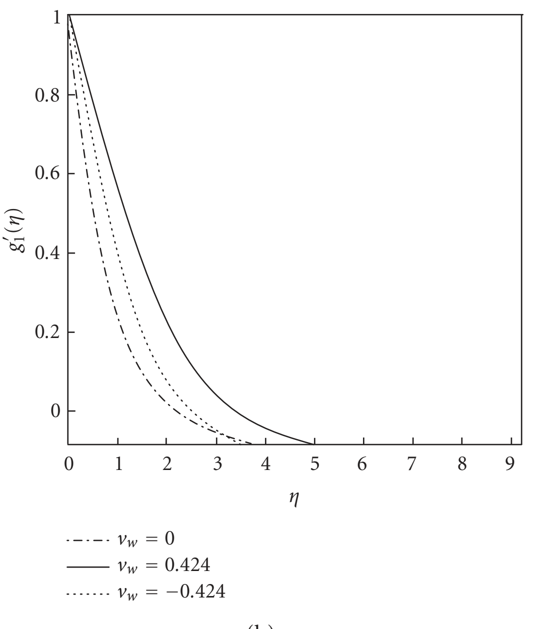


(c)

Figure 4.2. (a) Velocity profiles $g_2(\eta)$ for various values of viscoelastic parameter k_1 for fixed values of $b = 0.5$, $\nu = 0.04$, $k_2 = 0.5$, and Mn $= 1.0$. (b) Velocity profiles $g_2(\eta)$ versus η for various values of permeability parameter k_2 keeping $k_1 = 0.4$, $b = 0.5$, $\nu = 0.04$, and Mn $= 1.0$ fixed. (c) Velocity profiles $g_2(\eta)$ versus η for various values of magnetic parameter Mn for fixed values of $k_1 = 0.4$, $b = 0.5$, $\nu = 0.04$, and $k_2 = 0.5$.



(a)



(b)

Figure 4.3. (a) Velocity profiles for $g_1'(\eta)$ fixed values of permeability parameter $k_2 = 100$, $k_1 = 0.2$, $b = 1.0$, Mn = 0.5. (b) Velocity profiles for $g_1'(\eta)$ fixed values of magnetic parameter Mn = 10, $k_1 = 0.2$, $b = 1.0$, and $k_2 = 0.0$.

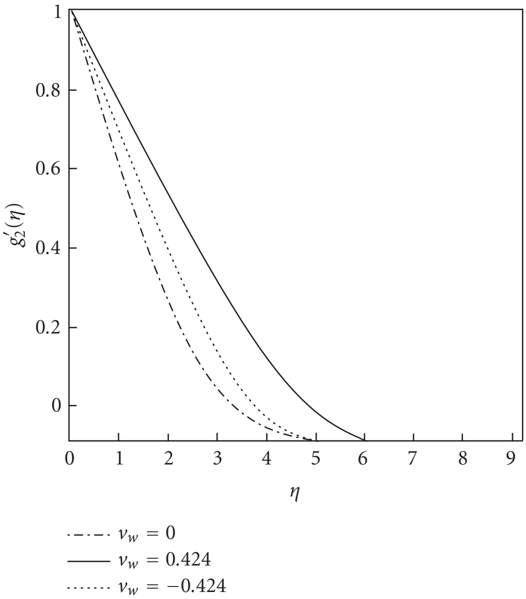


Figure 4.4. Velocity profiles for $g_2'(\eta)$ versus η for $k_1 = 0.125$, $k_2 = 0.0$, $\mathrm{Mn} = 100$, $b = 0.4$, $\nu = 0.04$.

Table 4.1. Values of τ.

k_1	0.005	0.01	0.02	0.05
τ	-38.4438	-18.2708	-8.1637	-4.7763

Table 4.1 represents the values of shear stress for different values of viscoelastic parameter and it is observed that the magnitude of skin friction coefficient decreases with increasing values of viscoelastic parameter k_1. For industrial applications, this result is of some importance, since the power expenditure in stretching the sheet decreases with increasing values of k_1. The same idea has already been investigated by Rajagopal et al. [18] in the case of steady flows.

References

[1] R. Bandelli, K. R. Rajagopal, and G. P. Galdi, *On some unsteady motions of fluids of second grade*, (Archiwum Mechaniki Stosowanej) **47** (1995), no. 4, 661–676.

[2] L. J. Crane, *Flow past a stretching plate*, Zeitschrift für Angewandte Mathematik und Physik **21** (1970), 645–647.

[3] C. D. S. Devi and G. Nath, *Similarity solutions of the unsteady boundary layer equations for a moving wall*, Indian Journal of Pure and Applied Mathematics **17** (1986), 1405–1411.

[4] J. E. Dunn and K. R. Rajagopal, *Fluids of differential type: critical review and thermodynamic analysis*, International Journal of Engineering Science **33** (1995), no. 5, 689–729.

[5] M. E. Erdogan, *On unsteady motions of a second order fluid over a plane wall*, International Journal of Non-Linear Mechanics **38** (2003), no. 7, 1045–1051.

[6] R. L. Fosdick and K. R. Rajagopal, *Uniqueness and drag for fluids of second grade in steady motion*, International Journal of Non-Linear Mechanics **13** (1978), no. 3, 131–137.

[7] P. S. Gupta and A. S. Gupta, *Heat and mass transfer on a stretching sheet with suction or blowing*, The Canadian Journal of Chemical Engineering **55** (1977), 744–746.

[8] D. M. Jones and K. Walters, *The behaviour of polymer solutions in extension dominated flows, with applications to enhanced oil recovery*, Journal of Rheology **28** (1989), no. 6, 482–498.

[9] P. S. Lawrence and B. N. Rao, *The non-uniqueness of MHD flow of a visco-elastic fluid past a stretching sheet*, Acta Mechanica Sinica **112** (1985), 223.

[10] M. J. Lighthill, *The response of laminar skin friction and heat transfer to fluctuations in the stream velocity*, Proceedings of the Royal Society. London. Series A. Mathematical, Physical and Engineering Sciences **224** (1954), 1–23.

[11] H. Pascal and F. Pascal, *On visco-elastic effects in non-Newtonian steady flows through porous media*, Transport in Porous Media **A** (1989), 17–35.

[12] S. Pilitisis and A Beirs, *Calculations of steady state visco-elastic flow in an undulating tube*, Journal of Non-Newtonian Fluid Mechanics **31** (1989), 231–287.

[13] K. R. Rajagopal, *A note on unsteady unidirectional flows of a non-Newtonian fluid*, International Journal of Non-Linear Mechanics **17** (1982), no. 5-6, 369–373.

[14] ———, *Flow of visco-elastic fluids between rotating disks*, Theoretical and Computational Fluid Dynamics **3** (1992), 185–206.

[15] ———, *On boundary conditions for fluids of the differential type*, Navier-Stokes Equations and Related Nonlinear Problems (Funchal, 1994) (A. Sequeira, ed.), Plenum, New York, 1995, pp. 273–278.

[16] K. R. Rajagopal and A. S. Gupta, *An exact solution for the flow of a non-Newtonian fluid past an infinite porous plate* , Meccanica. Journal of the Italian Association of Theoretical and Applied Mechanics **19** (1984), no. 2, 158–160.

[17] K. R. Rajagopal and P. N. Kaloni, *Some remarks on boundary conditions for flows of fluids of the differential type*, Continuum Mechanics and Its Applications (Burnaby, BC, 1988) (G. A. C. Gram and S. K. Malik, eds.), Hemisphere, New York, 1989, pp. 935–942.

[18] K. R. Rajagopal, T. Y. Na, and A. S. Gupta, *Flow of a visco-elastic fluid over a stretching sheet*, Journal of Rheology **23** (1984), 213–215.

[19] N. Rudraiah, P. N. Kaloni, and P. V. Radhadevi, *Oscillatory convection in a visco-elastic fluid through a porous layer heated from below*, Journal of Rheology **28** (1989), no. 1, 48–53.

[20] B. C. Sakiadis, *Boundary layer behaviour on continuous solid surfaces I. Boundary-layer equations for two-dimensional and axisymmetric flow*, AIChE Journal **7** (1961), no. 1, 26–28.

[21] T. Sarpkaya, *Flow of non-Newtonian fluids in a magnetic field*, AIChE Journal **7** (1961), 324–328.

[22] B. Siddappa and S. Abel, *Non-Newtonian flow past a stretching sheet*, Zeitschrift für Angewandte Mathematik und Physik **36** (1985), 890–892.

[23] B. Siddappa, A. Subhas, and P. H. Veena, *Oscillatory motion of a visco-elastic fluid past a stretching sheet*, Il Nuovo Cimento **17D** (1995), no. 1, 53–60.

[24] J. T. Stuart, *A solution of the Navier-Stokes and energy equations illustrating the response of skin friction and temperature of an infinite plate thermometer to fluctations in the stream velocity*, Proceedings of the Royal Society. London. Series A. Mathematical, Physical and Engineering Sciences **231** (1955), 116–130.

[25] A. Subhas and P. H. Veena, *Visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet* , International Journal of Non-Linear Mechanics **33** (1998), no. 3, 531–540.

[26] T. W. Ting, *Certain non-steady flows of second-order fluids*, Archive for Rational Mechanics and Analysis **14** (1963), 1–26.

[27] F. K. Tsou, E. M. sparrow, and R. J. Goldstein, *Flow and heat transfer in the boundary layer on a continuous moving surface*, International Journal of Heat and Mass Transfer **10** (1967), 219–235.

[28] K. Vafai and S. Kim, *Fluid mechanics of the interface region between a porous medium and a fluid layer: an exact solution*, International Journal of Heat and Fluid Flow **11** (1990), 254–256.
[29] C. Y. Wang, *Non-linear streaming due to the oscillatory stretching of a sheet in a viscous fluid*, Acta Mechanica Sinica **72** (1988), 261–268.

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