Outage Analysis of Coded Cooperation

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***Abstract*—Cooperative communication is an emerging paradigm where multiple mobiles share their resources (bandwidth and power) to achieve better overall performance. *Coded cooperation* is a mechanism where cooperation is combined with—and operates through—channel coding, as opposed to the repetition-based methods. This work develops expressions for outage probability of coded cooperation. In this work, each node acts as both a data source as well as a relay, i.e., only active (transmitting) nodes are available to assist other nodes, and each node operates under overall (source + relay) power and bandwidth constraints. Outage expressions confirm that full diversity is achieved by coded cooperation. This shows that despite superficial similarities, coded cooperation is distinct from decode-and-forward, which has been shown to have diversity one. The outage probability expressions developed in this work characterize coded performance at various rates. Furthermore, outage probabilities yield bounds that are arguably more insightful than the bit-error rate (BER) results previously available for coded cooperation. Numerical comparisons shed light on the relative merits of coded cooperation and various repetition-based methods, under various inter-user and uplink channel conditions.**

***Index Terms*—Channel coding, coded cooperation, diversity, outage probability, space–time coding, transmit diversity, user cooperation, wireless communications.**

I. Introduction

In many wireless applications, wireless transmitters (which we shall call *users* throughout this paper), may not be able to support multiple antennas due to size, complexity, power, or other constraints. Cooperation between pairs of users has been suggested [1]–[11] as a means to provide transmit diversity in the face of this limitation. In these methods, diversity is achieved by a signaling scheme that allows two single-antenna users to each send their information using both of their antennas.

The genesis of this idea is arguably in the work of van der Muelen [12], [13] and Cover and El Gamal [14] on the relay channel. The recent surge of interest in cooperative communication, however, was subsequent to the work of Sendonaris *et al.* [1], [2], which uses a generalized interference channel model to develop achievable rate regions and outage probabilities, and also proposes practical algorithms for cooperation in a code-division multiple-access (CDMA) framework, where each mobile decodes and relays certain bits received from its partner. Shortly afterwards, Laneman [3]–[5] proposed several cooperative protocols and analyzed their performance in both ergodic as well as quasi-static channels. Among the methods he proposed and analyzed are decode-and-forward, amplify-and-forward, and adaptive methods that switch between the two. It was shown that both amplify-and-forward and adaptive methods achieve diversity order of two for two-user cooperation.

In parallel to the work of Laneman, an alternative framework was proposed, called *coded cooperation* [6], in which cooperative signaling is integrated with channel coding. The basic idea is that each user, instead of repeating the received bits (either via amplification or decoding) tries to transmit incremental redundancy for its partner. The motivation for this development is that repetition of bits by the relay, as used in previous methods, is an inefficient code. Instead, coded cooperation in essence splits each codeword into two partitions, each of them transmitted by one of the cooperating partners. In addition to the coding advantage, coded cooperation is based on incremental redundancy and thus allows a more flexible distribution of channel symbols between the source and relay, compared to repetition. The operation of coded cooperation is discussed in [6]–[11] and will be reviewed in the sequel. The interested reader is referred to [15] for a summary of the work to date on cooperative communication.

In this work, we study the outage capacity of coded cooperation. A key result arising from this work is that, despite superficial similarities between decode-and-forward and coded cooperation, the former (as shown in [3]–[5]) has diversity one while the latter enjoys diversity in the number of cooperating partners. The outage analysis and resulting expressions in this paper are unfortunately somewhat cumbersome at times, partially due to the nature of coded cooperation, and partially because in addition to asymptotic results, we also provide general outage expressions for arbitrary signal-to-noise ratio (SNR). The numerical evaluation of outage shows that, although several cooperation protocols have the same (full) diversity order, coded cooperation enjoys improved performance compared to many cooperation protocols across a wide range of SNR.

After a brief review of the coded cooperation framework in Section II, we derive outage probability expressions for coded cooperation in Section III. We consider both the case where the channels between the two users are mutually independent (independent inter-user channels), and the case where the two users see an identical instantaneous SNR between them (reciprocal inter-user channels).[1] In addition, we demonstrate that coded co-

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[1]The nature of the inter-user channel, i.e., independent, reciprocal, or correlated to some degree, depends in part on the multiple-access scheme employed. This issue is discussed in more detail in [9], [10].

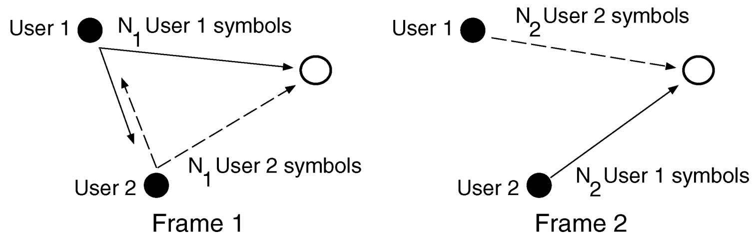


Fig. 1. Illustration of the coded cooperation framework.

operation achieves full diversity (order two for two users) in the asymptote of user transmit power. In Section IV, we consider the more general case of $n \geq 2$ cooperating users, and show via asymptotic analysis that coded cooperation achieves full diversity of order n. Finally, in Section V, we present numerical results which illustrate the outage probability behavior of coded cooperation for various channel conditions between the partners and to their destination. These results show that coded cooperation provides substantial gains in performance compared to noncooperative transmission. In addition, we compare coded cooperation with repetition-based protocols, and show that coded cooperation provides better performance, primarily as a result of the relative inefficiency of repetition coding. In fact, the repetition-based schemes become worse than no cooperation at low SNR or higher rates. In contrast, coded cooperation does not exhibit this behavior.

A. Other Related Work

Interestingly, the recent work in user cooperation seems to have sparked renewed interest in the relay channel. Høst-Madsen [16] develops general capacity results for both the relay channel and user cooperation using a combination of ideas from [14], [17]. In this work, as in [14], it is assumed that the partner, or relay, can transmit coherently with the source to achieve a beamforming effect. Zhao and Valenti [18], [19] give capacity and outage probability expressions for various protocols for the wireless relay channel, with the added constraint that the source and relay transmit on orthogonal channels (i.e., in time, frequency, or spreading code). This condition is more realistic for practical wireless systems, and has typically been assumed for user cooperation protocols (i.e., [4], [9]). The relay in the relay channel model typically does not have data of its own to transmit, while in user cooperation, cooperating users are all data sources. Our outage probability analysis considers the fully cooperative case where both users are transmitting their own data while simultaneously attempting to cooperate with each other, under a coded cooperation framework.

II. Review of the Coded Cooperation Framework

Coded cooperation works by sending different portions of each user's codewords via two independent fading paths. The basic idea is that each user tries to transmit incremental redundancy for his partner. Whenever that is not possible, the users automatically revert back to a noncooperative mode. The key to the efficiency of coded cooperation is that all this is managed automatically through code design, with no feedback between users.

Fig. 1 illustrates the general coded cooperation framework. The users segment their source data into blocks which are encoded with an error detection code, e.g., a cyclic redundancy check (CRC) code. These codes have exceptional error coverage and minimal loss of rate. Each block is then encoded with a forward error-correcting (FEC) code, so that, for an overall rate R, we have N total coded symbols allocated for each source block. The two users cooperate by dividing the transmission of their coded source blocks into two successive time segments, which we call *frames*. In the first frame, each user transmits a rate-R_1 codeword ($R_1 > R$) with N_1 symbols. This itself is a valid (albeit weaker) codeword which can be decoded to obtain the original information. If the user successfully decodes the partner's first-frame transmission (determined from the CRC code), the user computes and transmits N_2 additional parity symbols for the partner's data in the second frame, where $N_1 + N_2 = N$. These additional parities are selected such that they can be combined with the first-frame codeword to produce a more powerful rate-R codeword. If the user does not successfully decode the partner, N_2 additional parity bits for the user's own data are transmitted. Each user always transmits a total of N bits per source block over the two frames, and the users only transmit in their own orthogonal multiple-access channels. We can quantify the level of cooperation with the parameter

$$\alpha = N_1/N = R/R_1 \quad (1)$$

which denotes the portion each user's N total channel symbols allocated for the first frame.

Coded cooperation can use a wide variety of channel coding methods. For example, the overall code may be a block or convolutional code, or a combination of both. The code symbols for the two frames may be partitioned through puncturing, product codes, or other forms of concatenation. The examples in [6]–[9] employ rate-compatible punctured convolutional (RCPC) codes [20]. In [11], turbo codes are applied to this framework, while in [19], turbo codes are applied to a similar framework for the relay channel.

In coded cooperation, the two users act independently in the second frame, with no knowledge of whether their own first frame was correctly decoded by their partner. In other words, no feedback is assumed between the cooperating partners. As a result, there are four possible cooperative cases for the transmission of the second frame, illustrated in Fig. 2. For correct decoding, the receiver must know which cooperative case has happened; this can come either via negligible rate overhead from the two partners, or via repeated decoding and error checking at

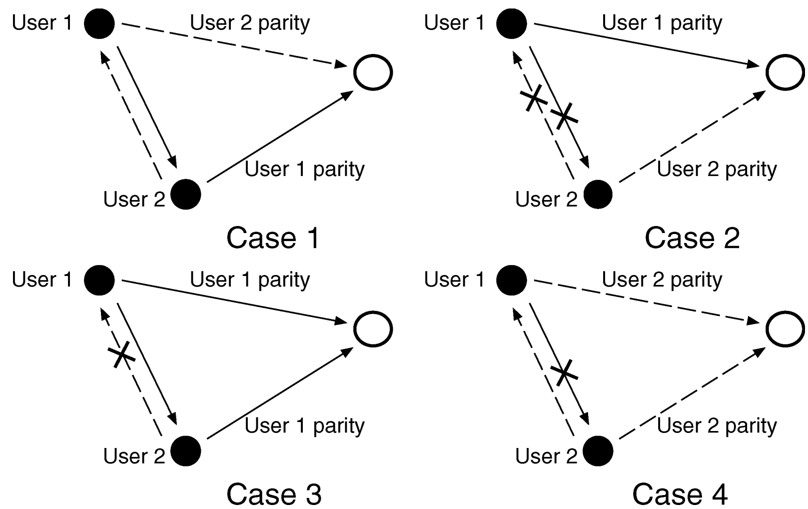


Fig. 2. Four cooperative cases for second frame transmission based on the first-frame decoding results.

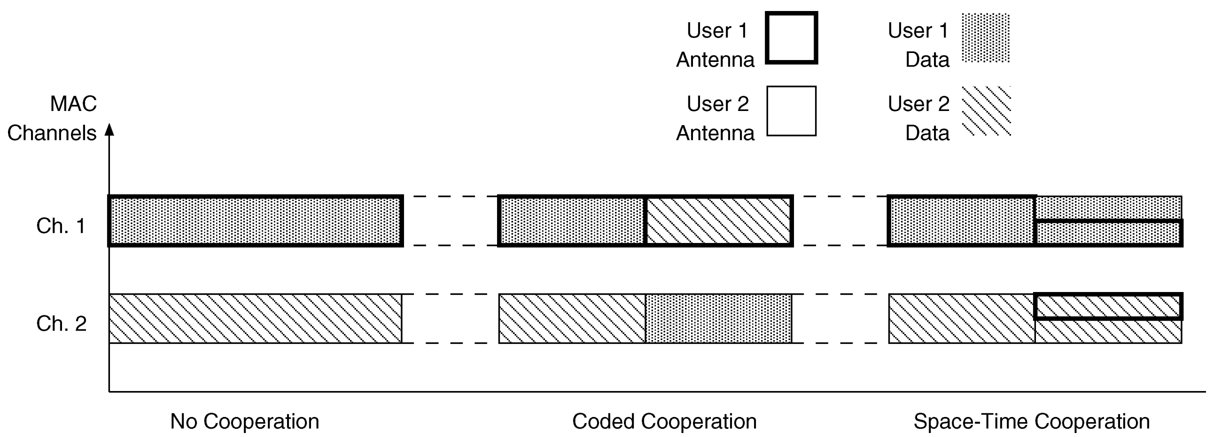


Fig. 3. Space–time cooperation compared to coded cooperation and no cooperation.

the receiver, as described in [10], [9]. We now briefly describe the different cooperative cases.

In Case 1, both users successfully decode each other, so that they each transmit for their partner in the second frame, resulting in the fully cooperative scenario depicted in Fig. 1. In Case 2, neither user successfully decodes their partner's first frame, so each will use the available channel in the second frame to transmit incremental redundancy for its *own data*. Thus, in this case the system automatically reverts to the noncooperative case. In Case 3, User 2 successfully decodes User 1, but User 1 does not successfully decode User 2. Consequently, neither user transmits additional parity for User 2 in the second frame, but instead both transmit additional parity for User 1. These two independent copies of User 1's parities are optimally combined at the destination prior to decoding. Case 4 is identical to Case 3 with the roles of User 1 and User 2 reversed. Decoding given the four cases may be accomplished without any additional knowledge at the destination; the details of reception and decoding can be found in [9], [10].

A. Space–Time Cooperation

An extension to coded cooperation, known as *space–time cooperation*, was proposed in [11]. Similar ideas have been developed independently in several other works, including [21]–[23]. In space–time cooperation (as defined in [11]) each user splits the power of the second frame between his own bits and the partner's bits, thus creating a hedge against the unbalanced cases (Cases 3 and 4 above). Specifically, in the second frame, instead of allocating all power to the partner, the user splits the power according to the ratio β_i ($i \in \{1, 2\}$ denotes User i). The user's own additional parity symbols are transmitted with power $\beta_i P_T$ and additional parity for the partner is transmitted with power $(1-\beta_i)P_T$, where P_T denotes the user's total average transmit power. Obviously, if the partner is not successfully decoded all the power is allocated to the user's own parity, just as with coded cooperation. Fig. 3 compares space–time cooperation with coded cooperation and no cooperation. It is assumed that the two coincident transmissions (in the same multiple-access channel) of User i's parities in the second frame can be optimally combined at the destination. This may be accomplished by concatenation with a space–time code in the second frame (e.g., for time-division multiple access (TDMA)), or may be possible simply by exploiting the nature of the multiple-access scheme (e.g., asynchronous CDMA). See [11] for a more detailed discussion of this issue.

The original motivation for space–time cooperation was to provide improved performance over coded cooperation in a fast-fading environment; i.e., when the fading coefficients are independent and identically distributed (i.i.d.) for each transmitted symbol. The advantage of space–time cooperation over coded cooperation in fast fading is demonstrated in [11]. Our interest in space–time cooperation for this work stems from the fact that in a sense it represents a generalization of the original framework. Coded cooperation can be viewed as a special case for $\beta_i = 0$, $i = 1, 2$, and thus, we would like to see if this power splitting (e.g., $\beta_i \neq 0$) provides any advantages in a quasi-static fading environment.

III. Outage Probability Analysis

As a baseline, we consider noncooperative direct transmission between source and destination. With quasi-static fading, the capacity conditioned on the channel realization, characterized by the instantaneous SNR γ, can be expressed by the familiar Shannon formula $C(\gamma) = \log_2(1+\gamma)$ bits per second per hertz (b/s/Hz). The channel is *in outage* if the conditional capacity falls below a selected threshold rate R, and the corresponding *outage event* is $\{C(\gamma) < R\}$, or equivalently, $\{\gamma < 2^R - 1\}$. The *outage probability* is thus defined as

$$P_{\text{out}} = \Pr\{\gamma < 2^R - 1\} = \int_0^{2^R-1} p_\gamma(\gamma) d\gamma \tag{2}$$

where, with an abuse of notation, $p_\gamma(\gamma)$ denotes the probability density function (pdf) of random variable γ. For the case of Rayleigh fading, γ has an exponential pdf with parameter $1/\Gamma$, where Γ denotes the mean value of SNR over the fading and accounts for the combination of transmit power and large-scale path loss and shadowing effects. The outage probability for Rayleigh fading can thus be evaluated as

$$P_{\text{out}} = \int_0^{2^R-1} \frac{1}{\Gamma} \exp\left(-\frac{\gamma}{\Gamma}\right) d\gamma = 1 - \exp\left(-\frac{2^R-1}{\Gamma}\right). \tag{3}$$

A. Coded Cooperation

As discussed in the previous section, in coded cooperation the user codewords are transmitted over two successive frames. In the first frame, each user transmits a rate $R_1 = R/\alpha$ codeword. There are four possible cases for second-frame transmission based on whether each user successfully decodes the partner's first-frame codeword. We parameterize the four cases by $\Theta \in \{1, 2, 3, 4\}$ and express the corresponding conditional capacities and outage events for each case as follows.

- **Case 1** ($\mathbf{\Theta = 1}$): In this case, both partners correctly decode each other. In an information-theoretic sense, correct decoding corresponds to the following events:

$$\begin{aligned} C_{1,2}(\gamma_{1,2}) &= \log_2(1+\gamma_{1,2}) > R/\alpha \\ C_{2,1}(\gamma_{2,1}) &= \log_2(1+\gamma_{2,1}) > R/\alpha \end{aligned} \tag{4}$$

where the subscript form i, j denotes transmission from User i to User j. In the second frame, both users transmit additional parity for each other. For a given user, the destination will receive a transmission from both the user (first frame) and the partner (second frame). The first frame uses a fraction α of the total N allocated bits, while the second frame uses $1-\alpha$. These two transmissions can thus be viewed as parallel (conditionally) Gaussian channels, whose capacities add together [24, Sec. 10.4]. Equivalently, the two transmissions can be viewed as time sharing between two independent channels, where the first channel is used a fraction α of the time. We can thus write the outage events for Users 1 and 2 as

$$\begin{aligned} C_{1,d}(\gamma_{1,d}, \gamma_{2,d}|\Theta = 1) &= \alpha \log_2(1+\gamma_{1,d}) \\ &\quad + (1-\alpha)\log_2(1+\gamma_{2,d}) < R \\ C_{2,d}(\gamma_{1,d}, \gamma_{2,d}|\Theta = 1) &= \alpha \log_2(1+\gamma_{2,d}) \\ &\quad + (1-\alpha)\log_2(1+\gamma_{1,d}) < R \end{aligned} \tag{5}$$

where the subscript d denotes the destination.

- **Case 2** ($\mathbf{\Theta = 2}$): In this case, neither user correctly decodes their partner. This corresponds to both users being in outage with respect to their partner

$$\begin{aligned} C_{1,2}(\gamma_{1,2}) &= \log_2(1+\gamma_{1,2}) < R/\alpha \\ C_{2,1}(\gamma_{2,1}) &= \log_2(1+\gamma_{2,1}) < R/\alpha. \end{aligned} \tag{6}$$

In the second frame both users transmit additional parity for their own data. The corresponding outage events are

$$\begin{aligned} C_{1,d}(\gamma_{1,d}|\Theta = 2) &= \log_2(1+\gamma_{1,d}) < R \\ C_{2,d}(\gamma_{2,d}|\Theta = 2) &= \log_2(1+\gamma_{2,d}) < R. \end{aligned} \tag{7}$$

- **Case 3** ($\mathbf{\Theta = 3}$): In this case, User 2 correctly decodes User 1, but User 1 does not correctly decode User 2. This corresponds to the events

$$\begin{aligned} C_{1,2}(\gamma_{1,2}) &= \log_2(1+\gamma_{1,2}) > R/\alpha \\ C_{2,1}(\gamma_{2,1}) &= \log_2(1+\gamma_{2,1}) < R/\alpha. \end{aligned} \tag{8}$$

In the second frame, User 1 and User 2 both transmit the same additional parity for User 1, while no additional parity is transmitted for User 2. The corresponding outage events are

$$\begin{aligned} C_{1,d}(\gamma_{1,d}, \gamma_{2,d}|\Theta = 3) &= \alpha \log_2(1+\gamma_{1,d}) \\ &\quad + (1-\alpha)\log_2(1+\gamma_{1,d}+\gamma_{2,d}) < R \\ C_{2,d}(\gamma_{2,d}|\Theta = 3) &= \log_2(1+\gamma_{2,d}) < R/\alpha. \end{aligned} \tag{9}$$

- **Case 4** ($\mathbf{\Theta = 4}$): This case is identical to Case 3 with the roles of Users 1 and 2 reversed. Thus, for the first frame we have the events

$$\begin{aligned} C_{1,2}(\gamma_{1,2}) &= \log_2(1+\gamma_{1,2}) < R/\alpha \\ C_{2,1}(\gamma_{2,1}) &= \log_2(1+\gamma_{2,1}) > R/\alpha \end{aligned} \tag{10}$$

and the outage events for Users 1 and 2 are

$$\begin{aligned} C_{1,d}(\gamma_{1,d}|\Theta=4) &= \log_2(1+\gamma_{1,d}) < R/\alpha \\ C_{2,d}(\gamma_{1,d},\gamma_{2,d}|\Theta=4) &= \alpha\log_2(1+\gamma_{2,d}) \\ &\quad + (1-\alpha)\log_2(1+\gamma_{1,d}+\gamma_{2,d}) < R. \end{aligned} \tag{11}$$

Note that the above assumes an independent inter-user channel ($\gamma_{1,2}$ and $\gamma_{2,1}$ independent), the most general condition. Since the four cases are disjoint, and assuming that $\{\gamma_{1,2},\gamma_{2,1},\gamma_{1,d},\gamma_{2,d}\}$ are all mutually independent, we can write the overall outage probability for User 1 as

$$\begin{aligned} P_{\text{out},1} &= \Pr\{\gamma_{1,2} > 2^{R/\alpha}-1\}\cdot\Pr\{\gamma_{2,1} > 2^{R/\alpha}-1\} \\ &\quad \cdot\Pr\{(1+\gamma_{1,d})^{\alpha}(1+\gamma_{2,d})^{1-\alpha} < 2^R\} \\ &\quad + \Pr\{\gamma_{1,2} < 2^{R/\alpha}-1\}\cdot\Pr\{\gamma_{2,1} < 2^{R/\alpha}-1\} \\ &\quad \cdot\Pr\{\gamma_{1,d} < 2^R-1\} \\ &\quad + \Pr\{\gamma_{1,2} > 2^{R/\alpha}-1\}\cdot\Pr\{\gamma_{2,1} < 2^{R/\alpha}-1\} \\ &\quad \cdot\Pr\{(1+\gamma_{1,d})^{\alpha}(1+\gamma_{1,d}+\gamma_{2,d})^{1-\alpha} < 2^R\} \\ &\quad + \Pr\{\gamma_{1,2} < 2^{R/\alpha}-1\}\cdot\Pr\{\gamma_{2,1} > 2^{R/\alpha}-1\} \\ &\quad \cdot\Pr\{\gamma_{1,d} < 2^{R/\alpha}-1\}. \end{aligned} \tag{12}$$

Due to symmetry, we can obtain an identical expression for User 2 by simply reversing the roles of Users 1 and 2. In the remainder of this paper, we derive various outage probability expressions for User 1 only, with the understanding that the corresponding expressions for User 2 are identical.

For the case of Rayleigh fading, we can evaluate (12) as

$$\begin{aligned} P_{\text{out},1} &= \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\cdot\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right) \\ &\quad \cdot\iint_A \Phi(\gamma_{1,d},\gamma_{2,d},\Gamma_{1,d},\Gamma_{2,d})d\gamma_{1,d}d\gamma_{2,d} \\ &\quad + \left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\right]\cdot\left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right)\right] \\ &\quad \cdot\left[1-\exp\left(\frac{1-2^{R}}{\Gamma_{1,d}}\right)\right] \\ &\quad + \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\cdot\left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right)\right] \\ &\quad \cdot\iint_B \Phi(\gamma_{1,d},\gamma_{2,d},\Gamma_{1,d},\Gamma_{2,d})d\gamma_{1,d}d\gamma_{2,d} \\ &\quad + \left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\right]\cdot\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right) \\ &\quad \cdot\left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,d}}\right)\right] \end{aligned} \tag{13}$$

where

$$\begin{aligned} A &\equiv \{(\gamma_{1,d},\gamma_{2,d}) : (1+\gamma_{1,d})^{\alpha}(1+\gamma_{2,d})^{1-\alpha} < 2^R\} \\ B &\equiv \{(\gamma_{1,d},\gamma_{2,d}) : (1+\gamma_{1,d})^{\alpha}(1+\gamma_{1,d}+\gamma_{2,d})^{1-\alpha} < 2^R\} \\ &\Phi(\gamma_{1,d},\gamma_{2,d},\Gamma_{1,d},\Gamma_{2,d}) \\ &\quad = \frac{1}{\Gamma_{1,d}}\exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}\right)\frac{1}{\Gamma_{2,d}}\exp\left(-\frac{\gamma_{2,d}}{\Gamma_{2,d}}\right). \end{aligned} \tag{14}$$

Using the results of Appendix A, we can simplify (13) to obtain

$$\begin{aligned} P_{\text{out},1} &= \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right)\left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,d}}\right)\right. \\ &\quad \left. -\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\cdot\Psi_1(\Gamma_{1,d},\Gamma_{2,d},R,\alpha)\right] \\ &\quad + \left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right)\right]\left[1-\exp\left(\frac{1-2^{R}}{\Gamma_{1,d}}\right)\right. \\ &\quad \left. -\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\cdot\Psi_2(\Gamma_{1,d},\Gamma_{2,d},R,\alpha)\right] \end{aligned} \tag{15}$$

where

$$\begin{aligned} &\Psi_1(\Gamma_{1,d},\Gamma_{2,d},R,\alpha) \\ &\quad = \int_0^{2^{R/\alpha}-1}\frac{1}{\Gamma_{1,d}}\exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}-\frac{a}{\Gamma_{2,d}}\right)d\gamma_{1,d} \\ &\Psi_2(\Gamma_{1,d},\Gamma_{2,d},R,\alpha) \\ &\quad = \int_0^{2^{R}-1}\frac{1}{\Gamma_{1,d}}\exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}-\frac{b}{\Gamma_{2,d}}\right)d\gamma_{1,d} \\ &a = \frac{2^{R/(1-\alpha)}}{(1+\gamma_{1,d})^{\alpha/(1-\alpha)}}-1 \\ &b = \frac{2^{R/(1-\alpha)}}{(1+\gamma_{1,d})^{\alpha/(1-\alpha)}}-1-\gamma_{1,d}. \end{aligned} \tag{16}$$

In the case of reciprocal inter-user channels ($\gamma_{1,2}=\gamma_{2,1}$), the events (8) and (10) (Cases 3 and 4) do not occur. As a result, (12) simplifies to

$$\begin{aligned} P_{\text{out},1} &= \Pr\{\gamma_{1,2} > 2^{R/\alpha}-1\}\cdot\Pr\{(1+\gamma_{1,d})^{\alpha}(1+\gamma_{2,d})^{1-\alpha} < 2^R\} \\ &\quad + \Pr\{\gamma_{1,2} < 2^{R/\alpha}-1\}\cdot\Pr\{\gamma_{1,d} < 2^R-1\}. \end{aligned} \tag{17}$$

Using results from Appendix A, we can evaluate (17) for Rayleigh fading as

$$\begin{aligned} &P_{\text{out},1} \\ &= \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,d}}\right)-\Psi_1(\Gamma_{1,d},\Gamma_{2,d},R,\alpha)\right] \\ &\quad + \left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\right]\left[1-\exp\left(\frac{1-2^{R}}{\Gamma_{1,d}}\right)\right] \end{aligned} \tag{18}$$

where $\Psi_1(\Gamma_{1,d},\Gamma_{2,d},R,\alpha)$ is the same as in (16).

We see from (15) and (18) that the outage probability for coded cooperation is a function of the mean channel SNR values $\{\Gamma_{1,2},\Gamma_{2,1},\Gamma_{1,d},\Gamma_{2,d}\}$, the allocated rate R, and the cooperation level α. While the channel SNR and allocated rate may often be set by environmental or system constraints, α is a free parameter that can be varied to optimize performance. Obtaining a general expression for an optimal value of α as a function of the other parameters is complicated by the fact that α appears in the limits of integrals in (15) and (18). Nevertheless, for any given parameter set, an optimal α may be determined through iteration. We illustrate this in Section V.

B. Asymptotic Analysis and Diversity Order

We would like to examine the behavior of the outage probability in the high-SNR regime to determine the diversity order achieved by coded cooperation. To facilitate this, we re-parameterize the mean SNR $\Gamma_{i,j}$ as follows:

$$\Gamma_{i,j} \Longrightarrow \Gamma_T \cdot \Gamma_{i,j} \tag{19}$$

where now Γ_T is the ratio of the user transmit power to the received noise, and $\Gamma_{i,j}$ is a finite constant accounting for large-scale path loss and shadowing effects. For the purposes of this work, we assume that Γ_T is the same for both users. Relative differences in quality between the various channels are still captured by the $\Gamma_{i,j}$ values. This re-parameterization decouples the user transmit power from the physical impairments of the channel itself. Thus, by expressing outage probability as a function of $1/\Gamma_T$, and then letting $\Gamma_T \to \infty$ (e.g., the high-SNR regime), the diversity order is given by the smallest exponent of $1/\Gamma_T$.

To obtain the outage probability as a function of $1/\Gamma_T$ for the case of independent inter-user channels, we expand each exponential term in (15) using the equivalent Taylor's series representation (e.g., [25, p. 299]) and collect like-order terms. This results in the following expression for User 1:

$$P_{\text{out},1} = \frac{1}{\Gamma_T^2} \cdot \left[\frac{(2^{R/\alpha}-1)^2}{\Gamma_{1,d}\Gamma_{1,2}} + \frac{\Lambda(R,\alpha)}{\Gamma_{1,d}\Gamma_{2,d}}\right] + O\left(\frac{1}{\Gamma_T^3}\right) \tag{20}$$

where

$$\Lambda(R,\alpha) = \begin{cases} \left(\frac{\alpha}{1-2\alpha}\right) 2^{R/\alpha} - \left(\frac{1-\alpha}{1-2\alpha}\right) 2^{R/(1-\alpha)} + 1 & \alpha \neq 1/2 \\ R \cdot 2^{2R+1} \cdot \ln 2 - 2^{2R} + 1 & \alpha = 1/2 \end{cases} \tag{21}$$

and $O\left(\frac{1}{\Gamma_T^3}\right)$ denotes the higher order terms from the Taylor's series expansion.[2] Appendix B provides details of how (20) is obtained. It is interesting to note that, in the high-SNR regime, the dependence of outage probability for User 1 on $\Gamma_{2,1}$ appears only in the terms of third-order and higher. Appendix B demonstrates why this occurs.

For the case of reciprocal inter-user channels, we can obtain a similar expression using the results in Appendix B

$$P_{\text{out},1} = \frac{1}{\Gamma_T^2} \cdot \left[\frac{(2^R-1)(2^{R/\alpha}-1)}{\Gamma_{1,d}\Gamma_{1,2}} + \frac{\Lambda(R,\alpha)}{\Gamma_{1,d}\Gamma_{2,d}}\right] + O\left(\frac{1}{\Gamma_T^3}\right) \tag{22}$$

where $\Lambda(R,\alpha)$ is given in (21).

We see from (20) and (22) that, as $\Gamma_T \to \infty$, the outage probability is a function of $1/\Gamma_T^2$. This shows that coded cooperation achieves full diversity, in this case diversity order two for two cooperating users.

C. Space–Time Cooperation

For space–time cooperation, we have the same four cases, resulting from the first frame transmissions, as with coded cooperation. The outage events for User 1 become

$$\begin{aligned} &C_{1,d}(\gamma_{1,d},\gamma_{2,d}|\Theta=1) \\ &\quad = \alpha\log_2(1+\gamma_{1,d}) + (1-\alpha)\log_2[1+\beta_1\gamma_{1,d}+(1-\beta_2)\gamma_{2,d}] < R \\ &C_{1,d}(\gamma_{1,d}|\Theta=2) \\ &\quad = \log_2(1+\gamma_{1,d}) < R \\ &C_{1,d}(\gamma_{1,d},\gamma_{2,d}|\Theta=3) \\ &\quad = \alpha\log_2(1+\gamma_{1,d}) + (1-\alpha)\log_2[1+\gamma_{1,d}+(1-\beta_2)\gamma_{2,d}] < R \\ &C_{1,d}(\gamma_{1,d}|\Theta=4) \\ &\quad = \alpha\log_2(1+\gamma_{1,d}) + (1-\alpha)\log_2(1+\beta_1\gamma_{1,d}) < R. \end{aligned} \tag{23}$$

For the case of independent inter-user channels, the outage probability for User 1 is

$$\begin{aligned} P_{\text{out},1} = &\Pr\{\gamma_{1,2} > 2^{R/\alpha}-1\} \cdot \Pr\{\gamma_{2,1} > 2^{R/\alpha}-1\} \\ &\cdot \Pr\{(1+\gamma_{1,d})^\alpha(1+\beta_1\gamma_{1,d}+(1-\beta_2)\gamma_{2,d})^{1-\alpha} < 2^R\} \\ &+ \Pr\{\gamma_{1,2} < 2^{R/\alpha}-1\} \cdot \Pr\{\gamma_{2,1} < 2^{R/\alpha}-1\} \\ &\cdot \Pr\{\gamma_{1,d} < 2^R-1\} \\ &+ \Pr\{\gamma_{1,2} > 2^{R/\alpha}-1\} \cdot \Pr\{\gamma_{2,1} < 2^{R/\alpha}-1\} \\ &\cdot \Pr\{(1+\gamma_{1,d})^\alpha(1+\gamma_{1,d}+(1-\beta_2)\gamma_{2,d})^{1-\alpha} < 2^R\} \\ &+ \Pr\{\gamma_{1,2} < 2^{R/\alpha}-1\} \cdot \Pr\{\gamma_{2,1} > 2^{R/\alpha}-1\} \\ &\cdot \Pr\{(1+\gamma_{1,d})^\alpha(1+\beta_1\gamma_{1,d})^{1-\alpha} < 2^R\}. \end{aligned} \tag{24}$$

For Rayleigh fading we can evaluate (24) as

$$\begin{aligned} P_{\text{out},1} = &\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right) \cdot \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right) \\ &\cdot \iint_C \Phi(\gamma_{1,d},\gamma_{2,d},\Gamma_{1,d},\Gamma_{2,d})\,d\gamma_{1,d}d\gamma_{2,d} \\ &+ \left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\right] \cdot \left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right)\right] \\ &\cdot \left[1-\exp\left(\frac{1-2^R}{\Gamma_{1,d}}\right)\right] \\ &+ \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right) \cdot \left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right)\right] \\ &\cdot \iint_D \Phi(\gamma_{1,d},\gamma_{2,d},\Gamma_{1,d},\Gamma_{2,d})\,d\gamma_{1,d}d\gamma_{2,d} \\ &+ \left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\right] \cdot \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right) \\ &\cdot \int_E \frac{1}{\Gamma_{1,d}} \exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}\right) d\gamma_{1,d} \end{aligned} \tag{25}$$

where we get (26) at the bottom of the page, and $\Phi(\gamma_{1,d},\gamma_{2,d},\Gamma_{1,d},\Gamma_{2,d})$ is the same as (14). We can simplify (25) in a manner similar to that described in Appendix A for (13) to obtain

$$\begin{aligned} P_{\text{out},1} = &\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right) \left[\int_E \frac{1}{\Gamma_{1,d}} \exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}\right) d\gamma_{1,d} \right. \\ &\left. - \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right) \cdot \Psi_5(\Gamma_{1,d},\Gamma_{2,d},R,\alpha)\right] \\ &+ \left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{2,1}}\right)\right] \left[1-\exp\left(\frac{1-2^R}{\Gamma_{1,d}}\right) \right. \\ &\left. - \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right) \cdot \Psi_6(\Gamma_{1,d},\Gamma_{2,d},R,\alpha)\right] \end{aligned} \tag{27}$$

[2]Throughout this paper, $O(\cdot)$ denotes the familiar order notation; see for example [26, pp. 2–3].

$$\begin{aligned} C &\equiv \{(\gamma_{1,d},\gamma_{2,d}) : (1+\gamma_{1,d})^\alpha(1+\beta_1\gamma_{1,d}+(1-\beta_2)\gamma_{2,d})^{1-\alpha} < 2^R\} \\ D &\equiv \{(\gamma_{1,d},\gamma_{2,d}) : (1+\gamma_{1,d})^\alpha(1+\gamma_{1,d}+(1-\beta_2)\gamma_{2,d})^{1-\alpha} < 2^R\} \\ E &\equiv \{(\gamma_{1,d},\gamma_{2,d}) : (1+\gamma_{1,d})^\alpha(1+\beta_1\gamma_{1,d})^{1-\alpha} < 2^R\} \end{aligned} \tag{26}$$

where

$$\Psi_5(\Gamma_{1,d},\Gamma_{2,d},R,\alpha) = \int_E \frac{1}{\Gamma_{1,d}} \exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}} - \frac{c}{(1-\beta_2)\Gamma_{2,d}}\right) d\gamma_{1,d}$$

$$\Psi_6(\Gamma_{1,d},\Gamma_{2,d},R,\alpha) = \int_0^{2^R-1} \frac{1}{\Gamma_{1,d}} \exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}} - \frac{b}{(1-\beta_2)\Gamma_{2,d}}\right) d\gamma_{1,d}$$

$$c = \frac{2^{R/(1-\alpha)}}{(1+\gamma_{1,d})^{\alpha/(1-\alpha)}} - 1 - \beta_1\gamma_{1,d} \quad (28)$$

and b is the same as (16).

For reciprocal inter-user channels, (24) simplifies to

$$P_{\text{out},1} = \Pr\{\gamma_{1,2} > 2^{R/\alpha}-1\} \cdot \Pr\{(1+\gamma_{1,d})^\alpha(1+\beta_1\gamma_{1,d}+(1-\beta_2)\gamma_{2,d})^{1-\alpha} < 2^R\} + \Pr\{\gamma_{1,2} < 2^{R/\alpha}-1\} \cdot \Pr\{\gamma_{1,d} < 2^R - 1\} \quad (29)$$

and using the above results we obtain for Rayleigh fading

$$P_{\text{out},1} = \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\left[\int_E \frac{1}{\Gamma_{1,d}}\exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}\right)d\gamma_{1,d} - \Psi_5(\Gamma_{1,d},\Gamma_{2,d},R,\alpha)\right] + \left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\right]\left[1-\exp\left(\frac{1-2^R}{\Gamma_{1,d}}\right)\right] \quad (30)$$

where $\Psi_5(\Gamma_{1,d},\Gamma_{2,d},R,\alpha)$ is the same as in (28).

In addition to the cooperation level α, with space–time cooperation we have two additional free parameters, the power splitting ratios β_1 and β_2. Since in general the optimal values of each of these parameters may not be independent, all three must be optimized simultaneously. In addition, since all three appear in the limits of integrals in the outage probability expressions, obtaining a general solution analytically is difficult. Nevertheless, as with α for the coded cooperation case, for any given set of channel conditions $\{\Gamma_{1,2},\Gamma_{2,1},\Gamma_{1,d},\Gamma_{2,d}\}$, the optimal parameter set $\{\alpha,\beta_1,\beta_2\}$ can be determined numerically. We examine this further in Section V.

To determine the diversity achieved by space–time cooperation, we reparameterize $\Gamma_{i,j}$ and expand the exponential terms using Taylor's series as we did for coded cooperation (Section III-B and Appendix B). For the case of independent inter-user channels, we obtain

$$P_{\text{out},1} = \frac{1}{\Gamma_T^2}\cdot\left[\int_E \left(\frac{2^{R/\alpha}-1}{\Gamma_{1,d}\Gamma_{1,2}} + \frac{c}{\Gamma_{1,d}\cdot(1-\beta_2)\Gamma_{2,d}}\right)d\gamma_{1,d}\right] + O\left(\frac{1}{\Gamma_T^3}\right) \quad (31)$$

and for the case of reciprocal inter-user channels we have

$$P_{\text{out},1} = \frac{1}{\Gamma_T^2}\cdot\left[\frac{(2^R-1)(2^{R/\alpha}-1)}{\Gamma_{1,d}\Gamma_{1,2}} + \int_E \frac{c}{\Gamma_{1,d}\cdot(1-\beta_2)\Gamma_{2,d}}d\gamma_{1,d}\right] + O\left(\frac{1}{\Gamma_T^3}\right) \quad (32)$$

where c is the same as in (28). We see from (31) and (32) that space–time cooperation also achieves full diversity order.

IV. Multiple Partners

In this section, we consider the case where a cluster of n users cooperate in the uplink transmission. The transmission protocol is similar to the two-user case; when user i transmits, all the other users in the cluster first listen to the transmitted packet of the user i. In the cooperation phase, only those users who correctly decode the user i's signal, send extra information about user i's packet (e.g., more parity bits) to the base station.

Let us assume that when user i transmits a packet and users with indices belonging to the set

$$\Omega_i = \{j_1, j_2, \ldots, j_k\} \subset \{1, 2, \ldots, i-1, i+1, \ldots, n\}$$

can correctly decode user i's packet. Thus, conditioned on Ω_i, the probability of outage is

$$P_{\text{out}}(\Omega_i) = Pr[C_{i,d}(\Omega_i) < R]$$

where $C_{i,d}(\Omega_i)$ is the mutual information between user i and the destination, assuming that all users in Ω_i cooperate with user i

$$C_{i,d}(\Omega_i) = \alpha\log_2(1+\gamma_{i,d}) + (1-\alpha)\log_2(1+\gamma_{i,d}+\sum_{j\in\Omega_i}\gamma_{j,d}).$$

If P_{ij} accounts for the outage probability of the interuser channel between user i and user j, then the total outage probability when n users selectively cooperate with user i is

$$P_{\text{out},i} = \sum_{\Omega_i}\left(\prod_{j\notin\Omega_i} P_{ij}\right)\left(\prod_{j\in\Omega_i} 1-P_{ij}\right) Pr[C_{i,d}(\Omega_i) < R]. \quad (33)$$

In order to understand the asymptotic behavior of the outage probability at high SNR, similar to the two-user case, we assume the following reparameterization for the average SNR values among users, or between the users and the base station:

$$\Gamma_{i,j} \Rightarrow \Gamma_{i,j}\cdot\Gamma_T.$$

In this formulation, $\Gamma_{i,j}$ captures the large-scale path-loss and shadowing effects and represents the relative differences between different users' channel. Γ_T accounts for the ratio of the transmitted power to noise variance at the receiver.

Theorem: The diversity order of every user in the above scheme is n (the number cooperating users in the cluster).

Proof: Let $k = |\Omega_i|$, we first prove that

$$P_{\text{out}}(\Omega_i) = O(\frac{1}{\Gamma_T^{k+1}}).$$

Under Rayleigh fading assumption we have (34) at the top of the following page. where we have the second equation at the top of the following page, and m_j denotes the total number of users (including User i) that User j cooperates with. By re-arranging

$$P_{\rm out}(\Omega_i) = Pr\left[\alpha\log_2(1+\gamma_{i,d}) + (1-\alpha)\log_2\left(1+\gamma_{i,d}+(1-\alpha)\sum_{j\in\Omega_i}\gamma_{j,d}\right) < R\right]$$
$$= \int\cdots\int_{\mathcal{A}_i}\frac{1}{\Gamma_T\Gamma_{i,d}}\exp\left(-\frac{\gamma_{i,d}}{\Gamma_T\Gamma_{i,d}}\right)d\gamma_{i,d}\cdot\prod_{j\in\Omega_i}\frac{m_j}{\Gamma_T\Gamma_{j,d}}\exp\left(-\frac{m_j\gamma_{j,d}}{\Gamma_T\Gamma_{j,d}}\right)d\gamma_{j,d} \quad (34)$$

$$\mathcal{A}_i \triangleq \left\{(\gamma_{i,d},\gamma_{j_1,d},\ldots,\gamma_{j_k,d})\mid \gamma_{i,d}\geq 0, \gamma_{j,d}\geq 0, j\in\Omega_i, (1+\gamma_{i,d})^{\alpha}\left(1+\sum_{j\in\Omega_i}\gamma_{j,d}\right)^{1-\alpha} < R\right\}$$

$$P_{\rm out}(\Omega_i) = \frac{1}{\Gamma_T^{k+1}}\frac{1}{\Gamma_{i,d}}\prod_{j\in\Omega_i}\frac{m_j}{\Gamma_{j,d}}\int\cdots\int_{\mathcal{A}_i}\exp\left(-\frac{\gamma_{i,d}}{\Gamma_T\Gamma_{i,d}}-\sum_{j\in\Omega_i}\frac{m_j\gamma_{j,d}}{\Gamma_T\Gamma_{j,d}}\right)\cdot d\gamma_{i,d}\prod_{j\in\Omega_i}d\gamma_{j,d}. \quad (35)$$

$$\mathcal{C}_i = \left\{(\gamma_{i,d},\gamma_{j_1,d},\ldots,\gamma_{j_k,d})\,\middle|\, 0\leq\gamma_{i,d}\leq 2^{R/\alpha}-1,\ 0\leq\gamma_{j,d}\leq 2^{R/(1-\alpha)}-1,\ j\in\Omega_i\right\}$$

(34) we have (35) also at the top of the page. We notice that the set $\mathcal{A}_i$ is a compact subset of the hyper-cube at the top of the page, hence using the fact that $\exp(-x)\leq 1$ for all $x\geq 0$, we have

$$P_{\rm out}(\Omega_i) \leq \frac{1}{\Gamma_T^{k+1}}\frac{1}{\Gamma_{i,d}}\prod_{j\in\Omega_i}\frac{m_j}{\Gamma_{j,d}}\int\cdots\int_{\mathcal{A}_i}d\gamma_{i,d}\prod_{j\in\Omega_i}d\gamma_{j,d}$$
$$= \frac{1}{\Gamma_T^{k+1}}\frac{\mathrm{vol}(\mathcal{A}_i)}{\Gamma_{i,d}}\prod_{j\in\Omega_i}\frac{m_j}{\Gamma_{j,d}}$$
$$\leq \left(\frac{\mathrm{vol}(\mathcal{C}_i)}{\Gamma_{i,d}}\prod_{j\in\Omega_i}\frac{m_j}{\Gamma_{j,d}}\right)\left(\frac{1}{\Gamma_T^{k+1}}\right).$$

Therefore,[3] $P_{\rm out}(\Omega_i) = O\left(\frac{1}{\Gamma_T^{k+1}}\right)$. On the other hand, the inter-user channels all have exponentially distributed instantaneous SNR thus, the inter-user outage probability for $n-k-1$ noncooperating users in the asymptote of large Γ_T behaves as

$$\prod_{j\notin\Omega_i}P_{ij} = \prod_{j\notin\Omega_i}\left(1-\exp(-\frac{2^{R/\alpha}-1}{\Gamma_{ij}\Gamma_T})\right) = O\left(\frac{1}{\Gamma_T^{n-k-1}}\right).$$

This means that each term contributing to the sum in (33) behaves as $O\left(\frac{1}{\Gamma_T{}^n}\right)$ thus, $P_{{\rm out},i}$ (the outage probability of the user i) also behaves as $O\left(\frac{1}{\Gamma_T{}^n}\right)$. Q.E.D.

V. Numerical Results

In this section, we present outage probability results for coded cooperation. For each of the curves shown, the outage probabilities for coded cooperation and space–time cooperation at each point correspond to the cooperation level α which minimizes the average outage probability over both the users. This value of α is determined iteratively as discussed in Section III-A, and we examine the optimal α values in more detail in Section V-B. To highlight the difference in performance between coded cooperation and space–time cooperation, we use $\beta_1=\beta_2=0.5$ for space–time cooperation in all of the plots below. We examine the optimization of β values and the implications for space–time cooperation in Section V-B.

For ease of exposition, we set $\Gamma_{1,2}=\Gamma_{2,1}$ for all cases (obviously true for reciprocal inter-user channels, and reasonable for independent inter-user channels since path loss is a reciprocal phenomenon, and large-scale shadowing, i.e., from buildings or other large obstructions, is also in many cases). As a result of this, we note that the outage probabilities for both users are equal if their channels to the destination (uplink channels) have equal mean SNR ($\Gamma_{1,d}=\Gamma_{2,d}$).

The first set of plots focuses on the low-rate regime. Specifically, we consider as an example rate $R=1/2$ b/s/Hz. Fig. 4 shows outage probability versus mean uplink SNR ($\Gamma_{1,d}=\Gamma_{2,d}$) for various conditions of the inter-user channel. The different sets of curves correspond to the inter-user mean SNR equal to the mean uplink SNR, 10 dB less than the mean uplink SNR, and equal to ∞; i.e., a noiseless inter-user channel. This latter case represents a lower bound on the achievable outage probability for the coded cooperation framework.

All of the cases in Fig. 4 clearly show that coded cooperation achieves diversity order two (compared with diversity order one for noncooperative transmission). The outage probability with reciprocal inter-user channels is always less than that with independent inter-user channels, since the latter admits Cases 3 and 4, where no additional parities are transmitted for one of the users. Nevertheless, the difference between reciprocal and independent inter-user channels is quite small compared to the overall gain relative to the noncooperative system. Coded cooperation generally maintains an advantage over space–time cooperation in the slow fading environment, although again the difference is small. It is interesting to note, however, that the difference in outage probability for reciprocal versus independent inter-user channels is actually smaller with space–time co-

[3] In fact one can use the monotone convergence theorem to show that (35) leads to $P_{\rm out}(\Omega_i)=\Theta\left(\frac{1}{\Gamma_T^{k+1}}\right)$.

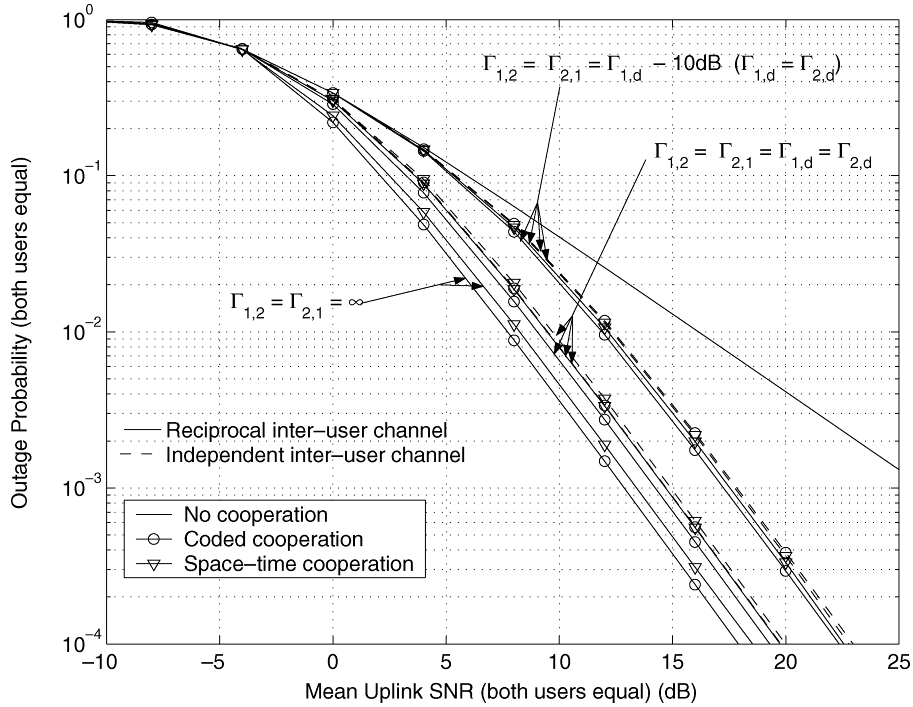


Fig. 4. Outage probability versus SNR for rate $R = 1/2$ b/s/Hz. Various sets of curves correspond to the inter-user channel mean SNR equal to the mean uplink SNR, 10 dB less than the mean uplink SNR, and equal to ∞; e.g., a noiseless inter-user channel.

operation. This indicates that a user always transmitting its own parities with some power in the second frame provides a degree of robustness against the deleterious effects of Cases 3 and 4. Finally, we note two interesting points about the effect of the inter-user channel quality. First, that coded cooperation can provide significant gains even if the inter-user mean SNR is less than that of the uplink channels. Second, as the inter-user channel quality improves relative to the uplink channels, we observe diminishing returns. Having a mean inter-user SNR equal to that of the uplink channels provides most of the achievable gain; increasing the mean inter-user SNR further provides minimal additional improvement.

Fig. 5 shows rate versus mean uplink SNR (both users equal) with fixed outage probability for various conditions of the inter-user channel (the same as those in Fig. 4). These results indicate the improvement in throughput that coded cooperation provides over the noncooperative system. Again we see that, even when the inter-user channel quality is poor relative to the uplink channels, coded cooperation still provides a significant improvement.

Fig. 6 shows the effect of multiple-user cooperation. In particular, we compare the performance of two-user cooperation with that of six-user cooperation (computed numerically). It is observed that diversity on the order of number of cooperating partners is obtained, as shown in Section IV.

Fig. 7 shows outage probability versus uplink SNR for unequal mean uplink SNR ($\Gamma_{1,d} \neq \Gamma_{2,d}$). The x-axis is the mean uplink SNR for User 2, $\Gamma_{2,d}$. In this case, User 1 has a better uplink channel, with mean SNR $\Gamma_{1,d} = \Gamma_{2,d}$+10 dB. The mean inter-user SNR is equal to $\Gamma_{2,d}$. As one might expect, User 2 improves significantly relative to the noncooperative system by cooperating with a partner that has a better quality uplink channel. More interesting is the fact that User 1 also achieves significant gain, despite cooperating with User 2 that has a poorer uplink channel. While the gain of User 1 is slightly less than in the comparable equal uplink SNR case from Fig. 4, the diversity improvement provided by cooperation nevertheless results in a substantial performance improvement, despite the fact that User 2's uplink channel is poorer. This illustrates that even a user with a very good uplink channel has a strong motivation to cooperate, which is an important practical result. In addition, we observe that the difference in outage probability between Users 1 and 2 is noticeably reduced in the cooperative system. This shows that cooperation inherently reallocates the system resources in a more effective manner.

A. Comparison With Repetition-Based Methods

Next, we compare coded cooperation with two repetition-based methods; namely, the amplify-and-forward and selection decode-and-forward protocols introduced in [4]. For both of these schemes, the cooperation level α is $1/2$ by definition. The outage probability of amplify-and-forward (with respect to User 1; the outage probability for User 2 is similar) is given by [4]

$$P_{\text{out},1} = \Pr\left\{\gamma_{1,d} + \frac{\gamma_{1,2}\gamma_{2,d}}{\gamma_{1,2} + \gamma_{2,d} + 1} < 2^{2R} - 1\right\}. \quad (36)$$

Though obtaining an expression for Rayleigh fading is difficult, outage probability can be estimated via Monte Carlo simulation of (36). Note also that (36) applies for both reciprocal and independent inter-user channels; i.e., for given $\Gamma_{1,2}$ and $\Gamma_{2,1}$ the performance of amplify-and-forward does not depend on the correlation between $\gamma_{1,2}$ and $\gamma_{2,1}$. This is because a user always amplifies and forwards the partner's signal regardless of the channel conditions. The users are effectively decoupled, in

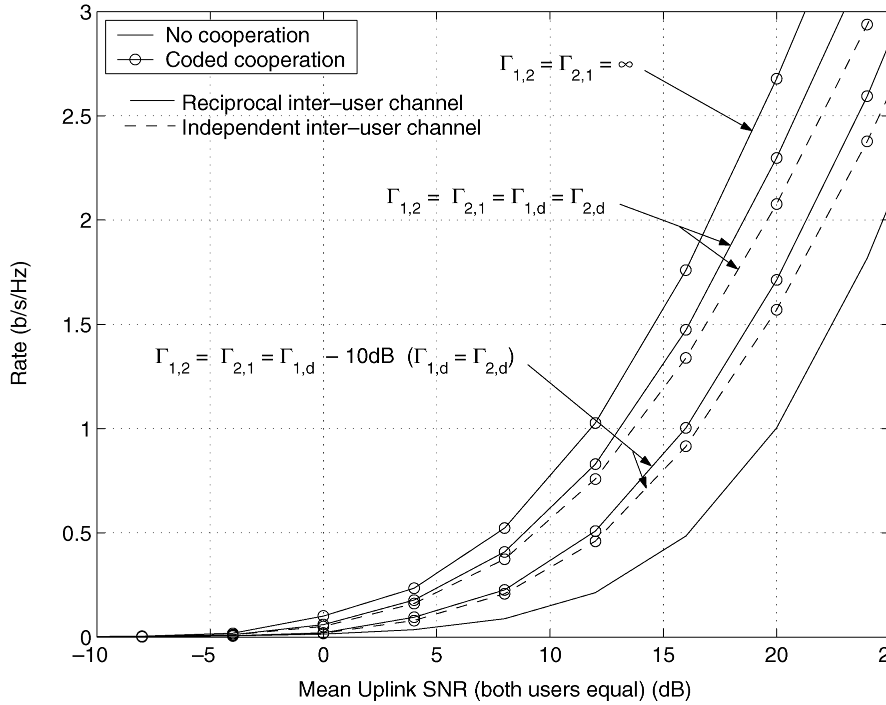


Fig. 5. Rate versus SNR for outage probability 10^{-2}. Various sets of curves correspond to the inter-user channel mean SNR equal to the mean uplink SNR, 10 dB less than the mean uplink SNR, and equal to ∞; e.g., a noiseless inter-user channel. For simplicity, only results for coded cooperation are shown. Results for space–time cooperation are similar.

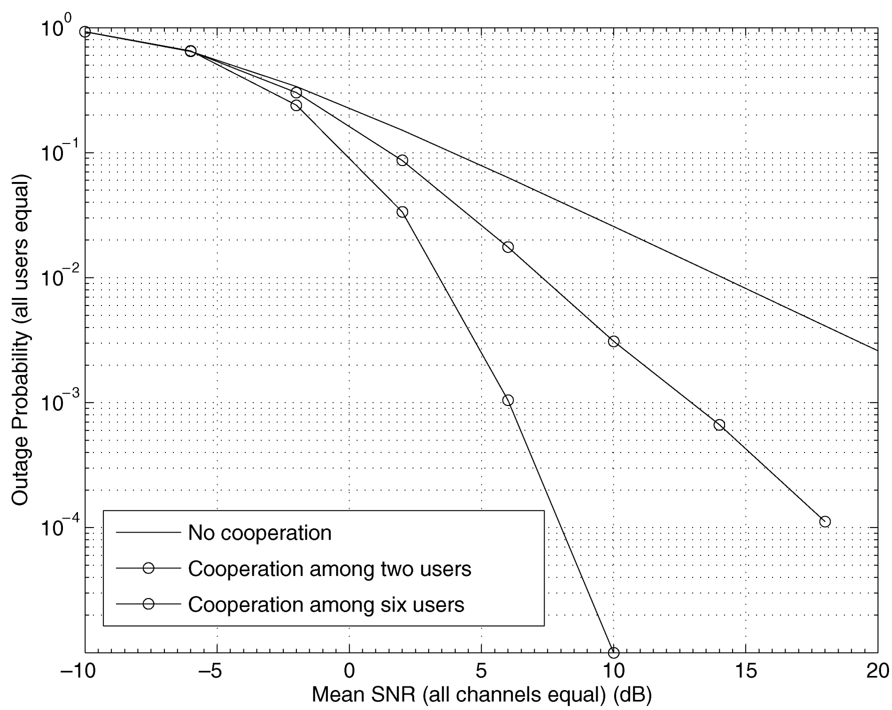


Fig. 6. Comparison of two-user and six-user cooperation. $R = 1/3$ b/s/Hz, $\alpha = 0.25$, all channels have the same (average) SNR, and fading is independent for each channel.

that we do not have the four cases that characterize coded cooperation.

Selection decode-and-forward is similar to coded cooperation, except that the user repeats the first-frame symbols in the second frame (the partner's if correctly decoded, otherwise the user's own) instead of transmitting additional parity. Based on [4], we can generalize selection decode-and-forward for independent inter-user channels and derive outage probability expressions in a manner similar to that which we have used for coded cooperation. We refer the interested reader to Appendix C for the details of this derivation.

Fig. 8 compares outage probability versus mean uplink SNR for coded cooperation, amplify-and-forward, and selection decode-and-forward (for simplicity, we do not show space–time

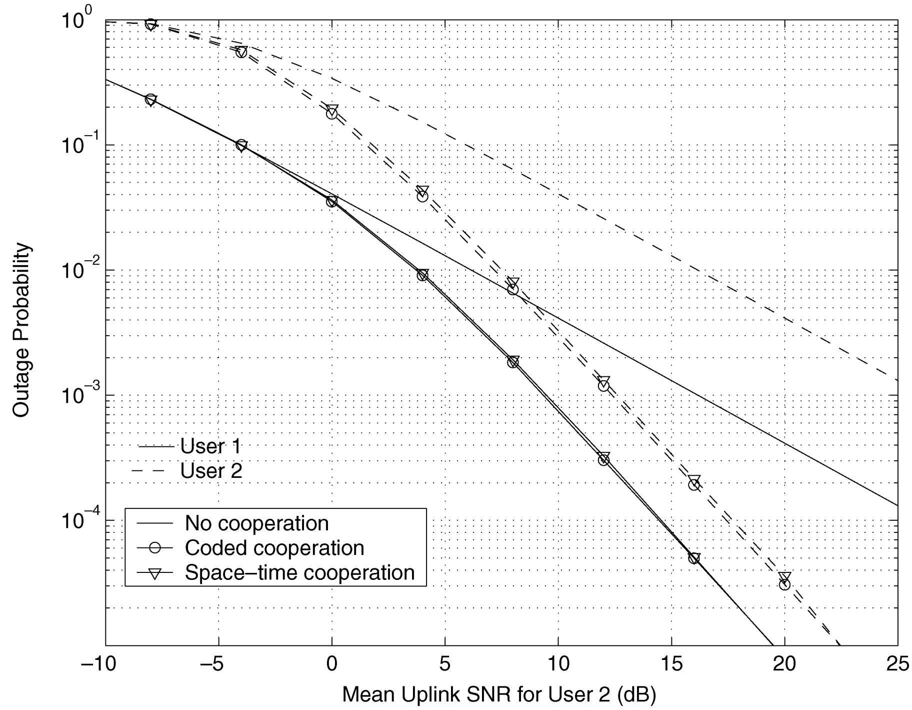


Fig. 7. Outage probability versus SNR for rate $R = 1/2$ b/s/Hz and unequal mean SNR for the uplink channels. The mean SNR of the inter-user channel is equal to that of User 2. The mean SNR of User 1's uplink channel is 10 dB higher. For simplicity, only results for reciprocal inter-user channels are shown. Results for independent inter-user channels are similar.

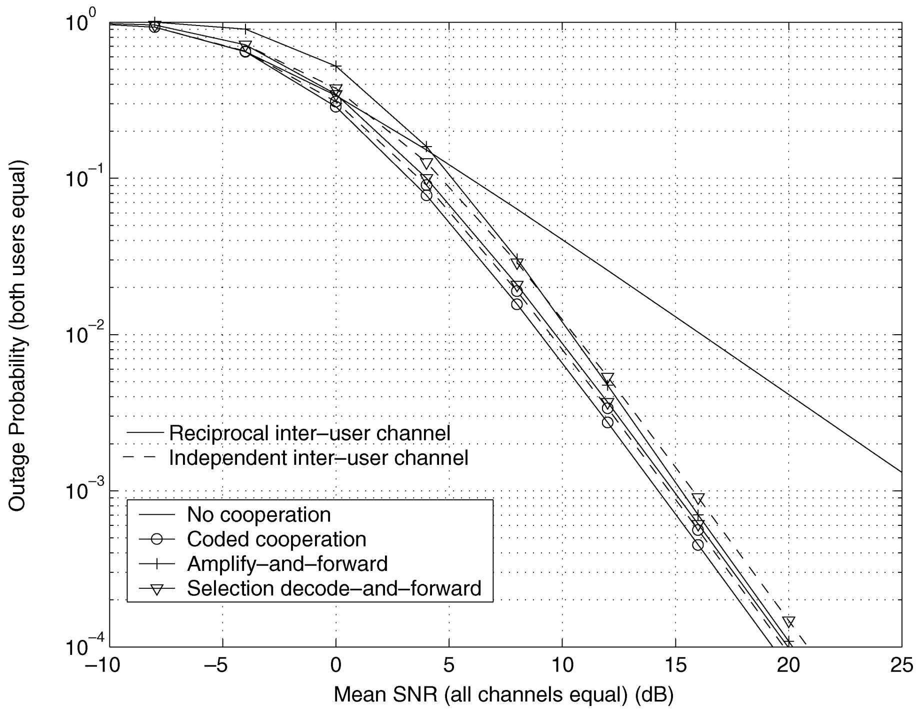
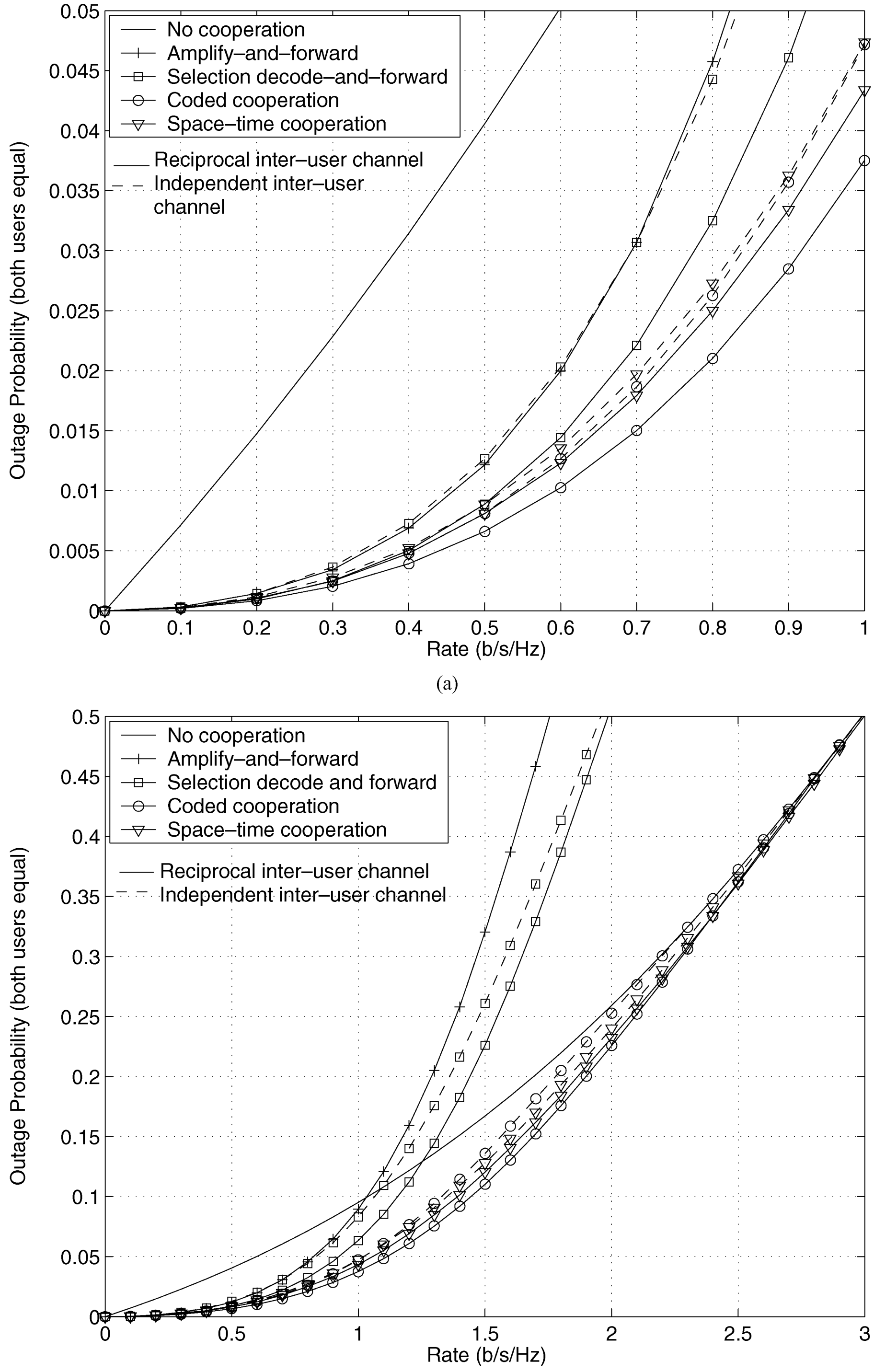


Fig. 8. Outage probability versus SNR for rate $R = 1/2$ b/s/Hz. All channels have equal mean SNR. Comparison of coded cooperation, amplify-and-forward, and selection decode-and-forward.

cooperation). Again, we consider the low-rate regime with rate $R = 1/2$, with all channels having equal mean SNR ($\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d} = \Gamma_{2,d}$). Coded cooperation maintains a slight advantage over the two repetition-based methods, regardless of whether the inter-user channels are independent or reciprocal. At low mean uplink SNR, amplify-and-forward and selection decode-and-forward are worse than no cooperation, which is a result of the inefficiency of repetition coding in this region (noise amplification by the partner adds to this effect in the case of amplify-and-forward). An inherent property of coded cooperation is that is never performs worse than no cooperation. Another interesting observation for the repetition-based schemes is that selection decode-and-forward becomes worse than amplify-and-forward with increasing uplink SNR for the case of



(a)

(b)

Fig. 9. Outage probability versus rate. All channels have mean SNR of 10 dB. In (a) the focus is on the low-rate regime, while (b) shows a broader range for rate.

independent inter-user channels. In this scenario, selection decode-and-forward experiences Cases 3 and 4 occasionally even for high SNR. This detracts slightly from its performance compared to amplify-and-forward, which exhibits comparable performance to selection decode-and-forward at high SNR, but by nature is not subject to Cases 3 and 4.

Fig. 9 compares outage probability versus rate for the various cooperative schemes. In the low-rate regime, shown in Fig. 9(a), all methods provide significant improvement, with coded cooperation generally performing slightly better overall. As the rate increases, the outage probability of the repetition-based methods exceeds that of the noncooperative system. This is again a manifestation of the inefficiency of repetition coding. The higher rate region also highlights the main strength of space–time cooperation, namely, its robustness in the case of independent inter-user channels. The outage probability for coded cooperation exceeds that of space–time cooperation at higher rates when the inter-user channels are independent, a result of Cases 3 and 4 having a significant impact in this region. The power splitting that occurs in space–time cooperation

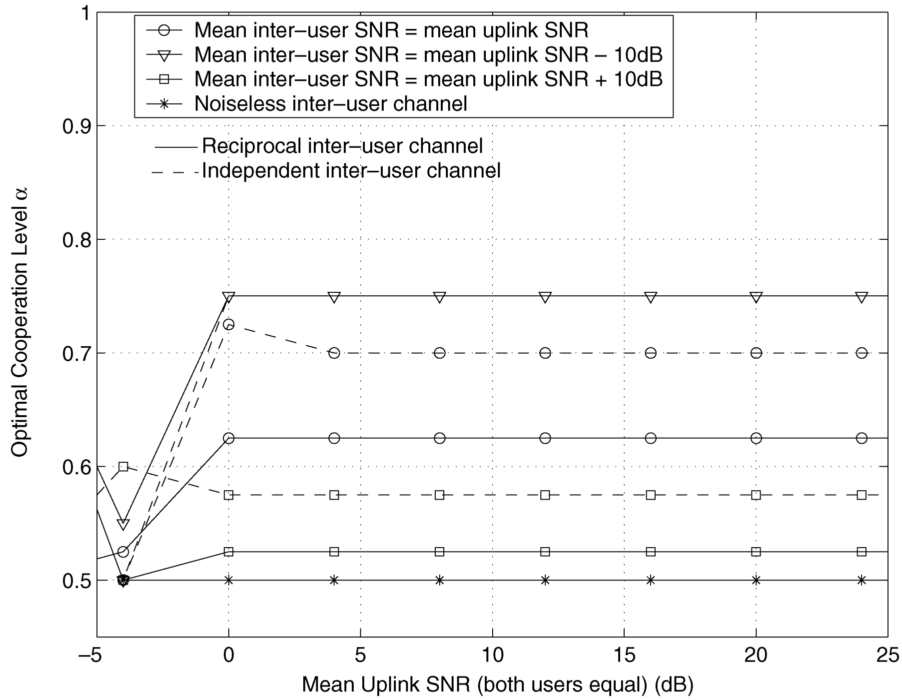


Fig. 10. Optimal cooperation level α versus SNR for coded cooperation, rate $R = 1/2$ b/s/Hz. The different curves represent various inter-user channel qualities. For all cases $\Gamma_{1,d} = \Gamma_{2,d}$.

noticeably retards this effect. As in Fig. 8, we note that both coded cooperation and space–time cooperation always perform at least as well as no cooperation in the worst case.

B. Optimization of Parameters α, β_1, and β_2

Fig. 10 shows optimal values of cooperation level α for coded cooperation corresponding to various degrees of inter-user channel quality. Again we consider rate $R = 1/2$ and $\Gamma_{1,d} = \Gamma_{2,d}$. With the exception of extreme conditions (e.g., very high mean inter-user SNR or very low mean uplink SNR), the optimal α value is almost always greater than $1/2$. This indicates that it is generally better to allocate a higher portion of the total rate R to the first frame, which improves the chances of successful detection of a user by its partner. As the inter-user channel worsens relative to the uplink channels, the optimal α value increases correspondingly, which allocates even more rate to the first frame. In addition, we note that, for a given set inter-user and uplink channel SNR, the optimal α value for independent inter-user channels is greater than for reciprocal inter-user channels. This difference corresponds to trying to reduce the probability of occurrence for Cases 3 and 4 for independent inter-user channels. We have observed that optimizing α can reduce the uplink average SNR required to achieve a given outage probability by as much as 2 dB in some cases, which is a significant improvement in performance.

For space–time cooperation, Table I shows the optimal parameter set $\{\alpha, \beta_1, \beta_2\}$ for various scenarios. The upper portion of the table highlights the low-rate regime with $R = 1/2$ b/s/Hz and corresponds to the scenarios shown in Figs. 4 and 10. The lower portion of the table focuses on the high-rate regime, with various rates above 1 b/s/Hz corresponding to the scenario in Fig. 9. Note that in all cases, we are considering equal mean uplink SNR, which implies that $\beta_1 = \beta_2$. We see that, for reciprocal inter-user channels (regardless of rate), the optimum β values are $\beta_1 = \beta_2 = 0$, which is equivalent to coded cooperation (and thus, the optimal α values are equal to those shown in Fig. 4 for coded cooperation). In other words, space–time cooperation provides no advantage under these conditions. For $R = 1/2$ b/s/Hz and independent inter-user channels, when the mean inter-user SNR is equal to or greater than the mean uplink SNR, we again see that coded cooperation is the optimum choice. When the mean inter-user SNR is worse than the mean uplink SNR, space–time cooperation with optimized β_1 and β_2 provides better performance, but the improvement over coded cooperation is really not significant (a small fraction of a decibel gain at most). With higher rates ($R > 1$ b/s/Hz) and independent inter-user channels, the optimum β values are actually $\beta_1 = \beta_2 = 0.5$ (what we have used throughout this section), with a significant improvement over coded cooperation as shown in Fig. 9 and discussed above. Overall these results suggest that, for a quasi-static fading environment, coded cooperation is generally superior to space–time cooperation; the one exception being independent inter-user channels in the high-rate regime. Nevertheless, space–time cooperation remains desirable due to its performance in fast fading, as shown in [11].

In the case of unequal uplink SNR, assuming that in general we allow $\beta_1 \neq \beta_2$, there are a variety of possibilities depending on the objective of the optimization (i.e., minimize the average outage probability of the two users, minimize the maximum outage probability between the two users, etc.). For example, a user with a very good uplink channel may sacrifice some performance to assist a user with a poor uplink channel such that both achieve a minimum quality of service. An example of optimization of β values for the case of fast fading is given in [11].

TABLE I
SPACE–TIME COOPERATION OPTIMAL $\{\alpha, \beta_1, \beta_2\}$ FOR VARIOUS CASES

Low Rate Regime ($R = 1/2$ b/s/Hz), $\Gamma_{1,d} = \Gamma_{2,d} =$ 0dB to 20dB				
	Reciprocal inter-user channels		Independent inter-user channels	
	α	$\beta_1 = \beta_2$	α	$\beta_1 = \beta_2$
$\Gamma_{1,2} = \Gamma_{2,1} = \infty$	0.5	0	0.5	0
$\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d} + 10\text{dB}$	0.525	0	0.575	0
$\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d}$	0.625	0	0.7	0
$\Gamma_{1,2} = \Gamma_{2,1} = \Gamma_{1,d} - 10\text{dB}$	0.75	0	0.75	0.4
High Rate Regime ($R > 1$ b/s/Hz), $\Gamma_{1,d} = \Gamma_{2,d} = \Gamma_{1,2} = \Gamma_{2,1}$ 10dB				
$R = 1.5$ b/s/Hz	0.625	0	0.6	0.5
$R = 1.75$ b/s/Hz	0.625	0	0.625	0.5
$R = 2.0$ b/s/Hz	0.625	0	0.7	0.5
$R = 2.25$ b/s/Hz	0.625	0	0.7	0.5

VI. CONCLUSION

Cooperation between wireless users has been proposed as a mechanism by which the users can achieve spatial diversity in applications where the users can only support a single transmit antenna. Recently, we have introduced a new framework, called coded cooperation, where cooperation is integrated with channel coding. In this framework, a user transmits addition parity symbols for its partner according to some overall coding scheme, instead of repeating the symbols initially transmitted by the partner. Bit- and block-error rate analysis has been performed for coded cooperation, and examples with specific coding schemes show significant improvement over noncooperative transmission.

To understand coded cooperation in a more general context that is independent of any particular coding scheme, we examine the outage probability of coded cooperation in this work. We derive expressions for outage probability in the case of quasi-static Rayleigh fading and two cooperating users, and then using these results we show that coded cooperation achieves full diversity (diversity order two for two cooperating users) in the high-SNR regime; i.e., as the transmit power of the users becomes large. We perform a similar analysis for space–time cooperation, which is an extension of the original coded cooperation framework where each user splits its power in the second frame and transmits for both itself and the partner. In addition, we generalize these results for more than two users and demonstrate via asymptotic analysis that full diversity in the order of the number of cooperating users is obtained.

Numerical outage probability results demonstrate that both coded cooperation and space–time cooperation provide significant gains over noncooperation transmission, especially in the low-rate regime ($R \leq 1$ b/s/Hz). Coded cooperation generally performs slightly better than space–time cooperation, except at higher rates when the inter-user channels are independent. Under these conditions, space–time cooperation provides increased robustness against the effects of independent inter-user channels. The results also show that coded cooperation performs better than repetition-based cooperative protocols. Due to the inefficiency of repetition coding, the repetition-based schemes are worse than no cooperation for low SNR or higher rates. In contrast, we see that coded cooperation and space–time cooperation in the worst case always perform at least as well as noncooperative transmission.

APPENDIX A
SIMPLIFICATION OF (13)

Consider the first integral in (13). We can rewrite the constraint A as

$$\gamma_{2,d} < \frac{2^{R/(1-\alpha)}}{(1+\gamma_{1,d})^{\alpha/(1-\alpha)}} - 1 = a. \tag{A1}$$

Since $\gamma_{2,d}$ is always greater than zero, this in turn leads to

$$\begin{aligned}(1+\gamma_{1,d})^{\alpha/(1-\alpha)} &< 2^{R/(1-\alpha)} \\ \gamma_{1,d} &< 2^{R/\alpha} - 1.\end{aligned} \tag{A2}$$

We can now rewrite the first integral in (13) as (A3) at the top of the following page.

Next, consider the second integral in (13). We can rewrite the constraint B as

$$\gamma_{2,d} < \frac{2^{R/(1-\alpha)}}{(1+\gamma_{1,d})^{\alpha/(1-\alpha)}} - 1 - \gamma_{1,d} = b. \tag{A4}$$

Since $\gamma_{2,d}$ is always greater than zero, this in turn leads to

$$\begin{aligned}\frac{2^{R/(1-\alpha)}}{(1+\gamma_{1,d})^{\alpha/(1-\alpha)}} &> (1+\gamma_{1,d}) \\ \gamma_{1,d} &< 2^{R} - 1.\end{aligned} \tag{A5}$$

We can now rewrite the second integral in (13) as (A6) also at the top of the following page.

We see that the second terms of (A3) and (A6) are Ψ_1 and Ψ_2, respectively, defined in (16). Equation (15) is thus obtained by substituting (A3) and (A6) into (13) and factoring like terms. We can apply similar techniques to obtain (18), the corresponding expression for reciprocal inter-user channels.

$$\iint_A \Phi(\gamma_{1,d},\gamma_{2,d},\Gamma_{1,d},\Gamma_{2,d})d\gamma_{1,d}d\gamma_{2,d} = \int_0^{2^{R/\alpha}-1} \frac{1}{\Gamma_{1,d}}\exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}\right)\left[\int_0^a \frac{1}{\Gamma_{2,d}}\exp\left(-\frac{\gamma_{2,d}}{\Gamma_{2,d}}\right)d\gamma_{2,d}\right]d\gamma_{1,d}$$
$$= \int_0^{2^{R/\alpha}-1} \frac{1}{\Gamma_{1,d}}\exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}\right)\left[1-\exp\left(-\frac{a}{\Gamma_{2,d}}\right)\right]d\gamma_{1,d}$$
$$= \left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,d}}\right)\right] - \int_0^{2^{R/\alpha}-1}\frac{1}{\Gamma_{1,d}}\exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}-\frac{a}{\Gamma_{2,d}}\right)d\gamma_{1,d}. \quad \text{(A3)}$$

$$\iint_B \Phi(\gamma_{1,d},\gamma_{2,d},\Gamma_{1,d},\Gamma_{2,d})d\gamma_{1,d}d\gamma_{2,d} = \int_0^{2^{R}-1} \frac{1}{\Gamma_{1,d}}\exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}\right)\left[\int_0^b \frac{1}{\Gamma_{2,d}}\exp\left(-\frac{\gamma_{2,d}}{\Gamma_{2,d}}\right)d\gamma_{2,d}\right]d\gamma_{1,d}$$
$$= \int_0^{2^{R}-1} \frac{1}{\Gamma_{1,d}}\exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}\right)\left[1-\exp\left(-\frac{b}{\Gamma_{2,d}}\right)\right]d\gamma_{1,d}$$
$$= \left[1-\exp\left(\frac{1-2^{R}}{\Gamma_{1,d}}\right)\right] - \int_0^{2^{R}-1}\frac{1}{\Gamma_{1,d}}\exp\left(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}-\frac{b}{\Gamma_{2,d}}\right)d\gamma_{1,d}. \quad \text{(A6)}$$

APPENDIX B
ASYMPTOTIC ANALYSIS OF (15)

The Taylor's series representation of $\exp(x)$ is given by [25, p. 299]

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots. \quad \text{(B1)}$$

Thus, for terms of the form

$$\exp\left(\frac{1-2^r}{\Gamma_T\Gamma_{i,j}}\right) \quad \text{and} \quad 1-\exp\left(\frac{1-2^r}{\Gamma_T\Gamma_{i,j}}\right)$$

($r = R$ or R/α), we can write

$$\exp\left(\frac{1-2^r}{\Gamma_T\Gamma_{i,j}}\right) = 1 - \frac{2^r-1}{\Gamma_T\Gamma_{i,j}} + \frac{(2^r-1)^2}{2\Gamma_T^2\Gamma_{i,j}^2} + O\left(\frac{1}{\Gamma_T^3}\right)$$
$$1-\exp\left(\frac{1-2^r}{\Gamma_T\Gamma_{i,j}}\right) = \frac{2^r-1}{\Gamma_T\Gamma_{i,j}} - \frac{(2^r-1)^2}{2\Gamma_T^2\Gamma_{i,j}^2} + O\left(\frac{1}{\Gamma_T^3}\right). \quad \text{(B2)}$$

Note that for this analysis, since we expect to have diversity order two for coded cooperation, we are primarily interested in the terms of first and second orders in $1/\Gamma_T$. For the integral term Ψ_1 (15), (16), applying (B1) gives

$$\Psi_1(\gamma_{1,d},\Gamma_{1,d},\Gamma_{2,d},R,\alpha) = \frac{1}{\Gamma_T\Gamma_{1,d}}\int_0^{2^{R/\alpha}-1}\left[1-\frac{\gamma_{1,d}}{\Gamma_T\Gamma_{1,d}}-\frac{a}{\Gamma_T\Gamma_{2,d}}\right]d\gamma_{1,d}+O\left(\frac{1}{\Gamma_T^3}\right) \quad \text{(B3)}$$

where the argument of the integral results from the first two terms of the Taylor's series expansion of $\exp(\cdot)$ (B1). The remaining terms are represented by $O\left(\frac{1}{\Gamma_T^3}\right)$. We evaluate the integral term to obtain

$$\Psi_1(\gamma_{1,d},\Gamma_{1,d},\Gamma_{2,d},R,\alpha) = \frac{2^{R/\alpha}-1}{\Gamma_T\Gamma_{1,d}} - \frac{(2^{R/\alpha}-1)^2}{2\Gamma_T^2\Gamma_{1,d}^2} - \frac{\Lambda(R,\alpha)}{\Gamma_T^2\Gamma_{1,d}\Gamma_{2,d}} + O\left(\frac{1}{\Gamma_T^3}\right) \quad \text{(B4)}$$

where $\Lambda(R,\alpha)$, defined in (21), is the integral of a (16) from 0 to $2^{R/\alpha}-1$. Similarly, for Ψ_2 (15),(16) we obtain

$$\Psi_2(\gamma_{1,d},\Gamma_{1,d},\Gamma_{2,d},R,\alpha) = \frac{1}{\Gamma_T\Gamma_{1,d}}\int_0^{2^{R}-1}\left[1-\frac{\gamma_{1,d}}{\Gamma_T\Gamma_{1,d}}-\frac{b}{\Gamma_T\Gamma_{2,d}}\right]d\gamma_{1,d}+O\left(\frac{1}{\Gamma_T^3}\right)$$
$$= \frac{2^{R}-1}{\Gamma_T\Gamma_{1,d}} - \frac{(2^{R}-1)^2}{2\Gamma_T^2\Gamma_{1,d}^2} - \frac{\Delta(R,\alpha)}{\Gamma_T^2\Gamma_{1,d}\Gamma_{2,d}} + O\left(\frac{1}{\Gamma_T^3}\right) \quad \text{(B5)}$$

where

$$\Delta(R,\alpha) = \begin{cases} \left(\frac{1-\alpha}{1-2\alpha}\right)(2^{R/\alpha}-2^{R/(1-\alpha)})-2^{2R-1}+1/2, & \alpha\neq 1/2 \\ R\cdot 2^{2R+1}\cdot\ln 2-2^{2R-1}+1/2, & \alpha=1/2 \end{cases} \quad \text{(B6)}$$

is the integral of b (16) from 0 to 2^R-1. Now, using (B2) and (B3), we can express the two bracketed terms in (15) involving Ψ_1 and Ψ_2 as

$$\left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,d}}\right) - \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\cdot\Psi_1(\gamma_{1,d},\Gamma_{1,d},\Gamma_{2,d},R,\alpha)\right]$$
$$= \frac{(2^{R/\alpha}-1)^2}{\Gamma_T^2\Gamma_{1,d}\Gamma_{1,2}} - \frac{\Lambda(R,\alpha)}{\Gamma_T^2\Gamma_{1,d}\Gamma_{2,d}} + O\left(\frac{1}{\Gamma_T^3}\right) \quad \text{(B7)}$$

$$\left[1-\exp\left(\frac{1-2^{R}}{\Gamma_{1,d}}\right) - \exp\left(\frac{1-2^{R/\alpha}}{\Gamma_{1,2}}\right)\cdot\Psi_2(\gamma_{1,d},\Gamma_{1,d},\Gamma_{2,d},R,\alpha)\right]$$
$$= \frac{(2^{R}-1)(2^{R/\alpha}-1)}{\Gamma_T^2\Gamma_{1,d}\Gamma_{1,2}} - \frac{\Delta(R,\alpha)}{\Gamma_T^2\Gamma_{1,d}\Gamma_{2,d}} + O\left(\frac{1}{\Gamma_T^3}\right). \quad \text{(B8)}$$

Note that all the first-order terms have canceled, leaving only second-order and higher order terms. Finally, from (B2) we see that

$$\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_T\Gamma_{2,1}}\right)\cdot\text{(B7)} = \frac{(2^{R/\alpha}-1)^2}{\Gamma_T^2\Gamma_{1,d}\Gamma_{1,2}} - \frac{\Lambda(R,\alpha)}{\Gamma_T^2\Gamma_{1,d}\Gamma_{2,d}} + O\left(\frac{1}{\Gamma_T^3}\right) \quad \text{(B9)}$$

$$\left[1-\exp\left(\frac{1-2^{R/\alpha}}{\Gamma_T\Gamma_{2,1}}\right)\right]\cdot\text{(B8)} = O\left(\frac{1}{\Gamma_T^3}\right) \quad \text{(B10)}$$

which, when added, gives the result (20). The dependence on $\gamma_{2,1}$ occurs only in the terms of third-order and higher in (B9). Note also that, as a result of (B10), $\Delta(R,\alpha)$ does not appear in the second-order terms of (20). Using (B2) and (B3) in a similar fashion results in (22). In addition, we can use the same techniques to obtain the asymptotic results for space–time cooperation (Section III-C) and the selection decode-and-forward protocol of [4] (Appendix C).

APPENDIX C
OUTAGE PROBABILITY FOR SELECTION DECODE-AND-FORWARD

We can generalize the selection decode-and-forward scheme of [4] for independent inter-user channels, and obtain outage probability expressions as shown below. In this scenario the cooperation level α is $1/2$ by definition. Again we have the same four cases, resulting from the first frame transmissions, as with coded cooperation. The corresponding outage events for User 1 become

$$\begin{aligned} C_{1,d}(\gamma_{1,d},\gamma_{2,d}|\Theta=1) &= \log_2(1+\gamma_{1,d}+\gamma_{2,d}) < 2R\\ C_{1,d}(\gamma_{1,d},\gamma_{2,d}|\Theta=2) &= \log_2(1+2\gamma_{1,d}) < 2R\\ C_{1,d}(\gamma_{1,d},\gamma_{2,d}|\Theta=3) &= \log_2(1+2\gamma_{1,d}+\gamma_{2,d}) < 2R\\ C_{1,d}(\gamma_{1,d},\gamma_{2,d}|\Theta=4) &= \log_2(1+\gamma_{1,d}) < 2R. \end{aligned} \tag{C1}$$

For the case of independent inter-user channels, the outage probability for User 1 is

$$\begin{aligned} P_{\text{out},1} = \Pr\{\gamma_{1,2} > 2^{2R}-1\}&\\ \cdot \Pr\{\gamma_{2,1} > 2^{2R}-1\} \cdot \Pr\{\gamma_{1,d}+\gamma_{2,d} < 2^{2R}-1\}&\\ + \Pr\{\gamma_{1,2} < 2^{2R}-1\}&\\ \cdot \Pr\{\gamma_{2,1} < 2^{2R}-1\} \cdot \Pr\left\{\gamma_{1,d} < \frac{2^{2R}-1}{2}\right\}&\\ + \Pr\{\gamma_{1,2} > 2^{2R}-1\}&\\ \cdot \Pr\{\gamma_{2,1} < 2^{2R}-1\} \cdot \Pr\{2\gamma_{1,d}+\gamma_{2,d} < 2^{2R}-1\}&\\ + \Pr\{\gamma_{1,2} < 2^{2R}-1\}&\\ \cdot \Pr\{\gamma_{2,1} > 2^{2R}-1\} \cdot \Pr\{\gamma_{1,d} < 2^{2R}-1\}.& \end{aligned} \tag{C2}$$

For Rayleigh fading, we have

$$\begin{aligned} P_{\text{out},1} = \exp\left(\frac{1-2^{2R}}{\Gamma_{1,2}}\right) \cdot \exp\left(\frac{1-2^{2R}}{\Gamma_{2,1}}\right)&\\ \cdot \iint_F \Phi(\gamma_{1,d},\gamma_{2,d},\Gamma_{1,d},\Gamma_{2,d}) d\gamma_{1,d} d\gamma_{2,d}&\\ + \left[1-\exp\left(\frac{1-2^{2R}}{\Gamma_{1,2}}\right)\right] \cdot \left[1-\exp\left(\frac{1-2^{2R}}{\Gamma_{2,1}}\right)\right]&\\ \cdot \left[1-\exp\left(\frac{1-2^{2R}}{2\Gamma_{1,d}}\right)\right]&\\ + \exp\left(\frac{1-2^{2R}}{\Gamma_{1,2}}\right) \cdot \left[1-\exp\left(\frac{1-2^{2R}}{\Gamma_{2,1}}\right)\right]&\\ \cdot \iint_G \Phi(\gamma_{1,d},\gamma_{2,d},\Gamma_{1,d},\Gamma_{2,d}) d\gamma_{1,d} d\gamma_{2,d}&\\ + \left[1-\exp\left(\frac{1-2^{2R}}{\Gamma_{1,2}}\right)\right] \cdot \exp\left(\frac{1-2^{2R}}{\Gamma_{2,1}}\right)&\\ \cdot \left[1-\exp\left(\frac{1-2^{2R}}{\Gamma_{1,d}}\right)\right]& \end{aligned} \tag{C3}$$

where

$$\begin{aligned} F &\equiv \{(\gamma_{1,d},\gamma_{2,d}) : (\gamma_{1,d}+\gamma_{2,d}) < 2^{2R}-1\}\\ G &\equiv \{(\gamma_{1,d},\gamma_{2,d}) : (2\gamma_{1,d}+\gamma_{2,d}) < 2^{2R}-1\} \end{aligned} \tag{C4}$$

and $\Phi(\gamma_{1,d},\gamma_{2,d},\Gamma_{1,d},\Gamma_{2,d})$ is the same as (14). Again, we can simplify (C3) in a manner similar to that described in Appendix A for (13) to obtain

$$\begin{aligned} P_{\text{out},1} = \exp\left(\frac{1-2^{2R}}{\Gamma_{2,1}}\right)\Bigg[1-\exp\left(\frac{1-2^{2R}}{\Gamma_{1,d}}\right)&\\ - \exp\left(\frac{1-2^{2R}}{\Gamma_{1,2}}\right) \cdot \Psi_3(\Gamma_{1,d},\Gamma_{2,d},R)\Bigg]&\\ + \left[1-\exp\left(\frac{1-2^{2R}}{\Gamma_{2,1}}\right)\right]\Bigg[1-\exp\left(\frac{1-2^{2R}}{2\Gamma_{1,d}}\right)&\\ - \exp\left(\frac{1-2^{2R}}{\Gamma_{1,2}}\right) \cdot \Psi_4(\Gamma_{1,d},\Gamma_{2,d},R)\Bigg]& \end{aligned} \tag{C5}$$

where we get (C6) at the bottom of the page. For reciprocal inter-user channels, (C2) simplifies to

$$\begin{aligned} P_{\text{out},1} = \Pr\{\gamma_{1,2} > 2^{2R}-1\} \cdot \Pr\{\gamma_{1,d}+\gamma_{2,d} < 2^{2R}-1\}&\\ + \Pr\{\gamma_{1,2} < 2^{2R}-1\} \cdot \Pr\left\{\gamma_{1,d} < \frac{2^{2R}-1}{2}\right\}.& \end{aligned} \tag{C7}$$

Using the above results we can obtain for Rayleigh fading

$$\begin{aligned} P_{\text{out},1} = \exp\left(\frac{1-2^{2R}}{\Gamma_{1,2}}\right)\Bigg[1-\exp\left(\frac{1-2^{2R}}{\Gamma_{1,d}}\right)&\\ -\Psi_3(\Gamma_{1,d},\Gamma_{2,d},R)\Bigg]&\\ + \left[1-\exp\left(\frac{1-2^{2R}}{\Gamma_{1,2}}\right)\right]\left[1-\exp\left(\frac{1-2^{2R}}{2\Gamma_{1,d}}\right)\right]& \end{aligned} \tag{C8}$$

where $\Psi_3(\Gamma_{1,d},\Gamma_{2,d},R)$ is the same as in (C6).

To determine the diversity achieved by selection decode-and-forward, we again reparameterize $\Gamma_{i,j}$ and expand the exponen-

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$$\begin{aligned} \Psi_3(\Gamma_{1,d},\Gamma_{2,d},R) &= \frac{1}{\Gamma_{1,d}}\exp\left(\frac{1-2^{2R}}{\Gamma_{2,d}}\right)\int_0^{2^{2R}-1}\exp\left[-\gamma_{1,d}\left(\frac{1}{\Gamma_{1,d}}-\frac{1}{\Gamma_{2,d}}\right)\right]d\gamma_{1,d}\\ &= \begin{cases} \left(\frac{2^{2R}-1}{\Gamma_{1,d}}\right)\exp\left(\frac{1-2^{2R}}{\Gamma_{2,d}}\right), & \Gamma_{1,d}=\Gamma_{2,d}\\ \left(\frac{\Gamma_{2,d}}{\Gamma_{1,d}-\Gamma_{2,d}}\right)\left[\exp\left(\frac{1-2^{2R}}{\Gamma_{1,d}}\right)-\exp\left(\frac{1-2^{2R}}{\Gamma_{2,d}}\right)\right], & \Gamma_{1,d}\neq\Gamma_{2,d} \end{cases}\\ \Psi_4(\Gamma_{1,d},\Gamma_{2,d},R) &= \frac{1}{\Gamma_{1,d}}\exp\left(\frac{1-2^{2R}}{\Gamma_{2,d}}\right)\int_0^{(2^{2R}-1)/2}\exp\left[-\gamma_{1,d}\left(\frac{1}{\Gamma_{1,d}}-\frac{2}{\Gamma_{2,d}}\right)\right]d\gamma_{1,d}\\ &= \begin{cases} \left(\frac{2^{2R}-1}{2\Gamma_{1,d}}\right)\exp\left(\frac{1-2^{2R}}{\Gamma_{2,d}}\right), & 2\Gamma_{1,d}=\Gamma_{2,d}\\ \left(\frac{\Gamma_{2,d}}{2\Gamma_{1,d}-\Gamma_{2,d}}\right)\left[\exp\left(\frac{1-2^{2R}}{2\Gamma_{1,d}}\right)-\exp\left(\frac{1-2^{2R}}{\Gamma_{2,d}}\right)\right], & 2\Gamma_{1,d}\neq\Gamma_{2,d}. \end{cases} \end{aligned} \tag{C6}$$

tial terms using Taylor's series as we did for coded cooperation (Section III-B and Appendix B). For the case of independent inter-user channels, we obtain

$$P_{\text{out},1} = \frac{1}{\Gamma_T^2} \cdot \left[\frac{3(2^{2R}-1)^2}{2\Gamma_{1,d}} \left(\frac{\Gamma_{1,2}+\Gamma_{2,d}}{\Gamma_{1,2}\Gamma_{2,d}} \right) \right] + O\left(\frac{1}{\Gamma_T^3} \right) \tag{C9}$$

and for the case of reciprocal inter-user channels we have

$$P_{\text{out},1} = \frac{1}{\Gamma_T^2} \cdot \left[\frac{(2^{2R}-1)^2}{2\Gamma_{1,d}} \left(\frac{\Gamma_{1,2}+\Gamma_{2,d}}{\Gamma_{1,2}\Gamma_{2,d}} \right) \right] + O\left(\frac{1}{\Gamma_T^3} \right). \tag{C10}$$

We see from (C9) and (C10) that selection decode-and-forward achieves full diversity order. We also note that, for reciprocal inter-user channels, (C10) matches the asymptotic outage probability results given in [4].

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