Outage Performance of Decode-and-Forward in Two-Way Relaying with Outdated Channel State Information

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Abstract

In this paper, we analyze the outage behavior of decode-and-forward relaying in the context of selective two-way cooperative systems. First, a new relay selection metric is proposed to take into consideration both transmission rates and instantaneous link conditions between cooperating nodes. Afterwards, the outage probability of the proposed system is derived for Nakagami-m fading channels in the case when perfect channel state information is available and then extended to the more realistic scenario where the available channel state information (CSI) is outdated due to fast fading. New expressions for the outage probability are obtained, and the impact of imperfect CSI on the performance is evaluated. Illustrative numerical results, Monte Carlo simulations, and comparisons with similar approaches are presented to assess the accuracy of our analytical derivations and confirm the performance gain of the proposed scheme.

Index Terms

Channel State Information, Decode-and Forward, Fading Channels, Outage Probability, Relay Selection, Sum Rate, Two-Way Relaying.

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I. Introduction

The concept of two-way channels was first introduced in [1], but with the recent emergence of cooperative communications, *two-way relaying* is attracting considerable interest. It allows for simultaneous communications over half-duplex links between two source terminals through relays. Spectral efficiency could be, in general, considerably enhanced [2]. In addition, for many practical reasons, a *relay selection* is usually adopted to reduce the complexity of the two-way relaying schemes. However, unlike conventional one-way relaying [3]–[5], the selection process is not straightforward since it involves two end nodes, with different resources and interests, that should agree on a single relay. Hence, a common selection metric should be carefully elaborated.

A number of works in the literature have considered relay selection in the context of two-way relaying for both Decode-and-Forward (DF) [6], [7] and Amplify-and-Forward (AF) relaying strategies. For the case of DF, the authors in [8] investigate two relay selection metrics: the conventional *Max-Min* criterion [9], and the *Max-Sum* approach [10] to maximize the instantaneous sum rate of the cooperating nodes. Another relay selection scheme is proposed in [11], where the relay is selected to maximize the weighted sum rate of the bidirectional rate pair on the boundary of the achievable rate region. Other examples include [12] where a two-way relaying technique based on modular network coding and opportunistic relay selection [13] is proposed, and [14] where the authors present a simple *Double-Max* criterion based on which a "best" relay is selected for each user. In all the cited works, the relay selection is based on perfect channel state information (CSI). The impact of imperfect CSI on the performance of relay systems is considered in [15].

For the case of AF relaying, the authors in [16], present a Max-Min two-way relay selection technique based on outdated CSI and analyze the outage performance of the considered system. Another work in [17], also investigates the Max-Min criterion with outdated CSI and analyzes the system's performance in terms of the end-to-end symbol error rate (SER).

In this work, we focus on the outage performance of two-way selective relaying with DF. First, we propose a new "constrained" relay selection approach based on the maximization of the "weighted sum rate", combining the knowledge of end users transmission rates with the available information on links quality. This is not to be confused with other works (e.g., [11]) aiming also to maximize linear combinations of two-way transmission rates[1]. Second, we derive closed-form expressions for the outage probability of

[1]In [11], authors adopt a different approach to maximize the achievable rate of each link. The coefficients in the linear combination, therein, are predetermined multiplication factors reflecting the most constraining link.

the analyzed system over independently but not necessarily identically distributed Nakagami-m fading channels. Then, we extend the analysis to the more realistic scenario where the available CSI for relay selection is outdated due to fast fading. We obtain new expressions for the outage probability and we evaluate the impact of imperfect CSI on the performance.

The contributions of this paper vis-a-vis our conference paper [18] are summarized as follows:

- We develop a general framework for the proposed relay selection technique when the available CSI at the central controller is imperfect and hence the selection is based on outdated channel estimates. Then, we compare the corresponding performance with the perfect CSI case. Moreover, we consider both scenarios when the transmission from the end node terminals to the selected relay is achieved over two different subcarriers, in the orthogonal case, or over the same subcarrier, in the non-orthogonal case.
- We investigate the outage performance of the system with the proposed relay selection technique over Nakagami-m fading channels and we present closed-form expressions for the outage probability.
- We compare the outage probability performance of the system when using the proposed relay selection techniques with other approaches from the literature, namely the Max-Min technique, the Max-Sum technique, and random relay selection.

The rest of the paper is organized as follows. In Section II, we describe the analyzed system model and present a new relay selection metric in the specific context of two-way relaying. The outage analysis over Nakagami-m fading channels is presented in Section III. Numerical examples illustrating our analysis and comparisons with previous approaches are presented in Section IV, and Section V concludes the paper.

II. New Metric for Relay Selection

A. System Model

We consider a two-phase two-way relay network consisting of two end terminals (T_1 and T_2) and K intermediate DF relays (r_1, $\ldots$, r_K), as depicted in Fig. 1. All nodes are equipped with a single antenna for both transmission and reception and they all operate in the half-duplex mode (i.e., a node can not transmit and receive at the same time).

At the beginning of each transmission phase, a *relay selection* scheme is adopted to select one intermediate node r_k among the K available relays (the same for both end terminals) to assist the communication between T_1 and T_2. The criterion for relay selection as well as the adopted metric are introduced in the following subsection.

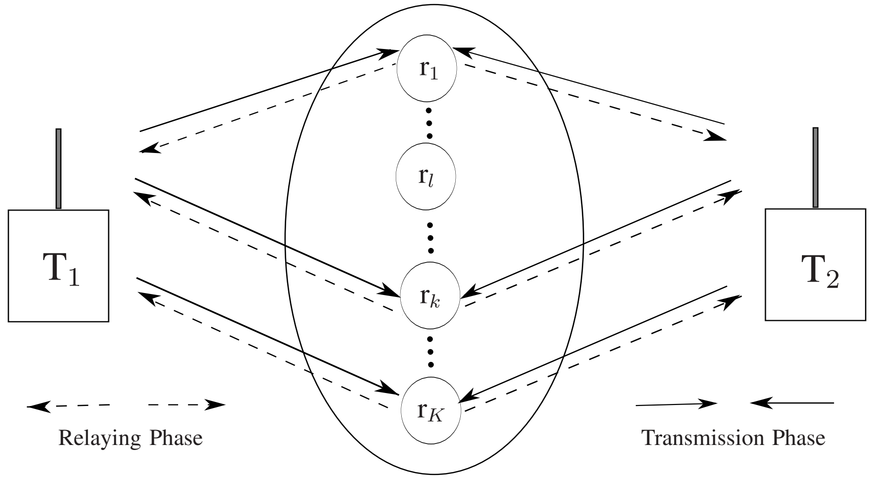

Fig. 1. Block diagram of a two-way relaying system with K parallel relays.

The fading coefficients between T_1 and r_k, r_k and T_1, T_2 and r_k, and r_k and T_2 are respectively denoted as h_{1k}, h_{k1}, h_{2k} and h_{k2}. All channels are assumed to be independent but not necessarily identically distributed and not necessarily reciprocal. The relay selection is based on outdated estimates of the fading coefficients denoted as $\hat{h}_{1k}$, $\hat{h}_{k1}$, $\hat{h}_{2k}$ and $\hat{h}_{k2}$. The noise over all channels is zero-mean additive white Gaussian (AWGN) with the same variance N_0. An average transmit power constraint P is imposed on every transmission block, at both terminals and at each relay. The average signal-to-noise ratio (SNR) is given by $\mathsf{SNR} = P/N_0$. Thus, the instantaneous SNR over the channel between nodes i and j is given by $\gamma_{\mathrm{ij}} = |h_{\mathrm{ij}}|^2\mathsf{SNR}$. We denote by x_1 and x_2 the symbols transmitted by T_1 and T_2 respectively, and we assume that $\mathrm{E}\{|\ x_1\ |^2\} = \mathrm{E}\{|\ x_2\ |^2\} = 1$. The mutual information between nodes i and j , denoted by I_{ij}, can then be given by :

$$\mathrm{I}_{\mathrm{ij}}\left(\mathsf{SNR}\right) = \frac{1}{2}\log_2\bigl(1 + \mathsf{SNR}|h_{\mathrm{ij}}|^2\bigr)\,.$$

The communication between the two end terminals takes place in two successive phases: the transmission phase and the relaying phase. In the transmission phase, terminals T_1 and T_2 transmit, simultaneously, information x_1 and x_2 to relay r_k in the same time slot. The received signal at r_k can be written as

$$y_k = \sqrt{P}h_{1k}x_1 + \sqrt{P}h_{2k}x_2 + n_k,$$

where n_k is the noise component at relay r_k. In this transmission phase, two scenarios are considered [19]:

- In the first scenario, both terminals T_1 and T_2 transmit orthogonally to the relay. This orthogonality could be achieved using two different subcarriers to convey the data of each end terminal to the relay [20]. The relaying node will then decode the information coming from each terminal without experiencing any interference. Given the target transmission rates $\tilde{R}_{\text{th}_1}$ and $\tilde{R}_{\text{th}_2}$ between $\mathrm{T}_1 \rightarrow \mathrm{r}_k$ and $\mathrm{T}_2 \rightarrow \mathrm{r}_k$, respectively, an outage occurs at the relay if $\mathrm{I}_{1k} < \tilde{R}_{\text{th}_1}$ and $\mathrm{I}_{2k} < \tilde{R}_{\text{th}_2}$. In the rest of this paper, we refer to this first case as the "orthogonal" case.

- In the second scenario, no orthogonality is assumed and the terminals transmit in the same frequency band. In this case, the relaying node will be able to decode the information sent by terminals T_1 and T_2 if $\mathrm{I}_{1k} > \tilde{R}_{\text{th}_1}$, $\mathrm{I}_{2k} > \tilde{R}_{\text{th}_2}$ and $\frac{1}{2}\log_2(1+\gamma_{1k}+\gamma_{2k}) > \tilde{R}_{\text{th}_1}+\tilde{R}_{\text{th}_2}$ as proven in [21], otherwise an outage event is declared at the relay. This scenario will be referred to as the "non-orthogonal" case.

In the relaying phase, if the decoding is successful at the relay, r_k will broadcast a bitwise XOR version of the two received and decoded signals in the the same time slot and the same subcarrier [22], [23]. The received signal at terminal $\mathrm{T}_i/i \in \{1,2\}$ can be written as

$$y_i = \sqrt{P}h_{ik}(x_1 \oplus x_2) + n_i$$

where n_i is the noise component at T_i. Each terminal will then decode the received bitwise XOR signal and then perform self interference cancellation to eliminate its own signal. Note that for the orthogonal case, if the relay is only able to decode the information sent by one terminal, only this information will be broadcasted in the relaying phase. Details on the transmission scenario and the outage event occurrence are described in Fig. 2 for both the orthogonal and the non-orthogonal cases.

B. Selection Metric

At the beginning of each block, a relay selection process is conducted to choose one relay to assist the communication between T_1 and T_2. We consider a centralized selection process where all communicating nodes feedback their channel state information to a central controller that is responsible for the selection. The feedback information from the communicating nodes to the controller may be delayed in time and hence the CSI available for relay selection is assumed to be outdated.

The selection is performed in two steps. At first, the central controller starts by determining the set of relays satisfying $|\hat{h}_{k1}|^2 \geq \mu_1$ and $|\hat{h}_{k2}|^2 \geq \mu_2$, where μ_1 and μ_2 are given selection thresholds. This will limit the selection to potentially "good" relays and hence reduces the complexity in terms of computations for the selection process. We denote the set of selected relays by K_{th}, and its cardinality by $\overline{K}_{\text{th}}$.

In the second step, the central controller selects one relay among the $\overline{K}_{\text{th}}$ pre-selected ones. The selection metric is based on maximizing the weighted sum of the achievable rates during the transmission phase, i.e., $q_{k2}\log_2\left(1+\mathsf{SNR}|\hat{h}_{1k}|^2\right)+q_{k1}\log_2\left(1+\mathsf{SNR}|\hat{h}_{2k}|^2\right)$ where q_{k1} and q_{k2} represent the weighting coefficients

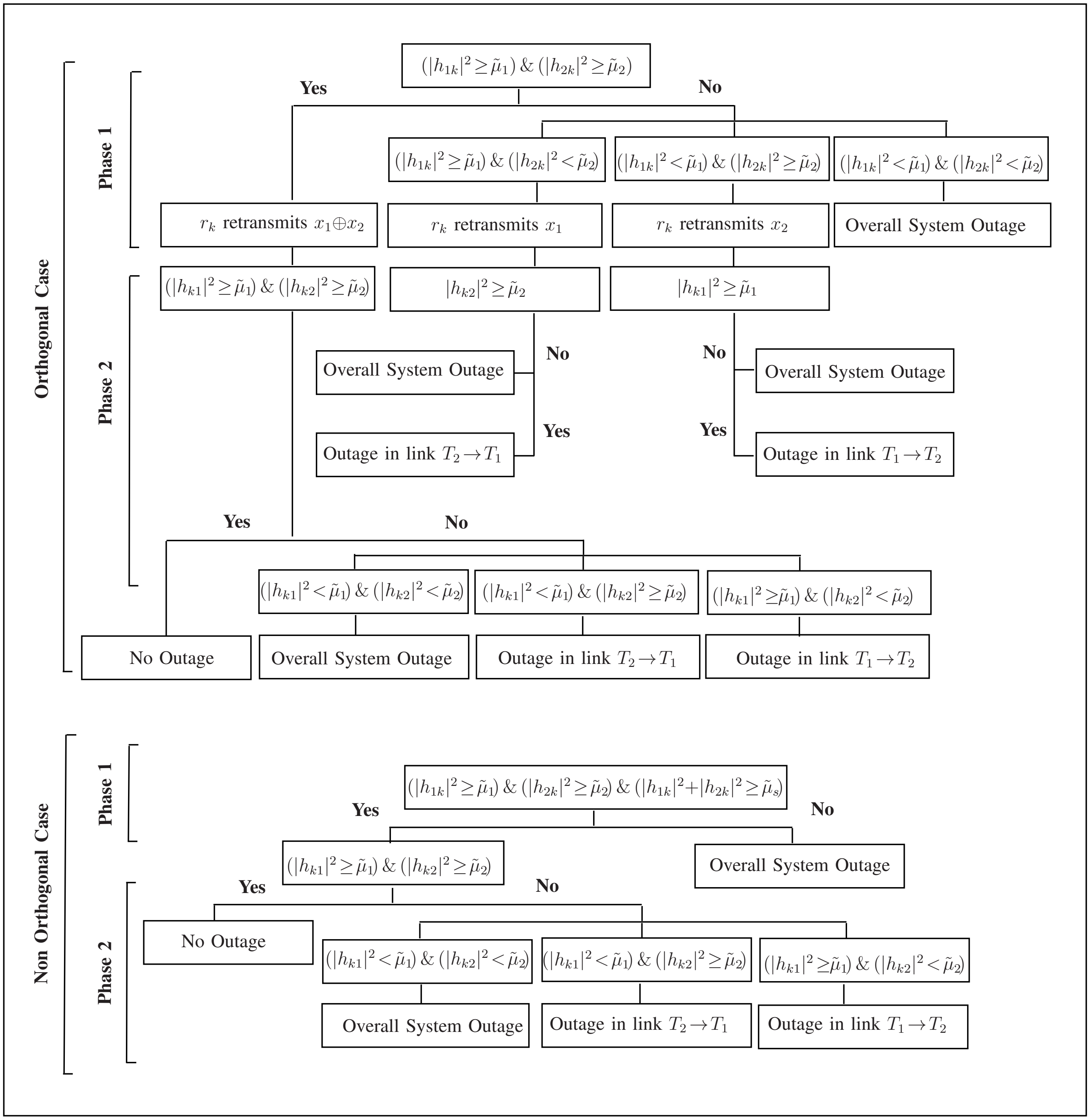

Fig. 2. Block diagram of outage events for both the orthogonal and the non-orthogonal cases.

and $k \in \{1, \cdots, \overline{K}_{\text{th}}\}$. Since the two end terminals experience different channel fading conditions during the transmission and the relaying phases, we define the weighting coefficients as a function of the channel's quality in the relaying phase, i.e., $q_{k2} = |\hat{h}_{k2}|^2$ and $q_{k1} = |\hat{h}_{k1}|^2$. The weighted sum rate is then given in terms of two elements: the transmission rates by each end terminal as well as the instantaneous channel quality from the relay to the other terminal. The selected relay r_k, $k \in \{1, ..., \overline{K}_{\text{th}}\}$, should then satisfy

$$k = \arg \max_{k \in \{1,...,\overline{K}_{\text{th}}\}} \left\{ |\hat{h}_{k2}|^2 \log_2 \left(1 + \mathsf{SNR}|\hat{h}_{1k}|^2\right) + |\hat{h}_{k1}|^2 \log_2 \left(1 + \mathsf{SNR}|\hat{h}_{2k}|^2\right) \right\}. \tag{1}$$

The proposed relay selection technique guarantees a better outage performance compared to other selection techniques. This is shown through simulations in the numerical results section. However the analytical derivations for the outage probability with this technique are challenging. For this reason, and based on the proposed metric, we consider a modified relay selection technique where the instantaneous transmission rates are substituted by the averaged ones. The selected relay r_k, $k \in \{1, ..., \overline{K}_{\text{th}}\}$, with this modified relay selection technique should then satisfy

$$k = \arg \max_{k \in \{1,...,\overline{K}_{\text{th}}\}} \left\{ |\hat{h}_{k2}|^2 R_1 + |\hat{h}_{k1}|^2 R_2 \right\}, \tag{2}$$

where $R_1 = \mathbb{E}\left[\log_2\left(1+\mathsf{SNR}|\hat{h}_{1k}|^2\right)\right]$ and $R_2 = \mathbb{E}\left[\log_2\left(1+\mathsf{SNR}|\hat{h}_{2k}|^2\right)\right]$ are, respectively, the average transmission rates by terminal T_1 and terminal T_2.

Note that, the selection metric is, implicitly, equivalent to a sort of "fairness" where the end terminal with higher rate and/or better channel conditions during both the transmission and the relaying phases will be privileged during the selection. The objective is to ensure that the total rate received by both end terminals T_1 and T_2, at the end of the communication, is the maximum possible rate.

C. Notation

A summary of some notation used in the rest of this paper is presented here. First, we denote by $\Pr[\mathrm{X}]$ the probability of event X, by $f_X(x)$ the probability density function of variable X at point x and by $\overline{\mathrm{X}}$ the cardinality of the set X. In addition, we denote by μ_i the selection threshold in the link $\mathrm{r}_k \to \mathrm{T}_i$ and we set $\mu = \mu_2 R_1 + \mu_1 R_2$ with R_i representing the transmission rate of terminal T_i as defined in the selection metric subsection. Given the target transmission rate $\tilde{R}_{\text{th}_i}$ between $\mathrm{T}_i \to \mathrm{r}_k$, we denote by $\tilde{\mu}_i$ the SNR-normalized outage threshold given by: $\tilde{\mu}_i = \dfrac{2^{2\tilde{R}_{\text{th}_i}} - 1}{\mathsf{SNR}}$.

III. Outage Probability Analysis

In this section, we investigate the outage performance of the system when using the modified two-way relay selection scheme presented in Section II. We start by considering a general case where an arbitrary fading channel is considered. Then, we derive the outage probability expressions in the particular case of Nakagami-m fading channels.

A. General Case

Given the transmission scenario and the adopted relay selection technique presented in Section II, a total outage event occurs if:

- the set of pre-selected relays $\overline{K}_{\text{th}}$ is empty, i.e. $\overline{K}_{\text{th}}$=0. This happens when no relay satisfies the thresholding selection and hence no relay can be chosen to convey the information of the end terminals.
- the set of pre-selected relays $\overline{K}_{\text{th}}$ is non-empty, but the channels between the end terminals and the selected relay are on outage.

The total outage probability can then be written, with a simple rearrangement of indices, as

$$\mathrm{P}_{\text{out}} = \Pr\left[\overline{K}_{\text{th}} = 0\right] + \sum_{\overline{K}_{\text{th}}=1}^{K} \Pr\left[\overline{K}_{\text{th}}\right] \sum_{k=1}^{\overline{K}_{\text{th}}} \Pr\left[k\right] \mathrm{P}_{\text{out}|k}, \tag{3}$$

where $\Pr\left[\overline{K}_{\text{th}}\right]$ is the threshold selection probability representing the probability that the pre-selected set contains $\overline{K}_{\text{th}}$ relays, $\Pr\left[k\right]$ is the relay selection probability, i.e. the probability that relay r_k among the $\overline{K}_{\text{th}}$ available relays is selected, and $\mathrm{P}_{\text{out}|k}$ is the conditional outage probability given that relay r_k has been selected. In what follows, we develop the expressions for each of these probabilities.

- *Threshold Selection Probability*

The central controller starts by determining the set K_{th} of relays having the magnitude square of their second hop channel coefficients $\hat{h}_{k1}$ and $\hat{h}_{k2}$ above thresholds μ_1 and μ_2 respectively, i.e., $K_{\text{th}} = \left\{\mathrm{r}_k \in \{1, ..., K\} : \hat{h}_{k1} \geq \mu_1 \quad \text{and} \quad \hat{h}_{k2} \geq \mu_2\right\}$.

Let $\mathcal{P}$ be the power set of $\{1, ..., K\}$, we denote the set of subsets of cardinality $\overline{K}_{\text{th}}$ by

$$\mathcal{P}_{\overline{K}_{\text{th}}} = \{A \in \mathcal{P} : \overline{A} = \overline{K}_{\text{th}}\}.$$

The threshold selection probability can be written as

$$\begin{aligned}\Pr\left[\overline{K}_{\text{th}}\right] &= \sum_{A \in \mathcal{P}_{\overline{K}_{\text{th}}}} \prod_{k \in A} \Big(1 - \Pr\left[|h_{k2}|^2 < \mu_2\right]\Big)\Big(1 - \Pr\left[|h_{k1}|^2 < \mu_1\right]\Big) \\ &\times \prod_{k \notin A} \Big(\Pr\left[|h_{k2}|^2 < \mu_2\right] + \Pr\left[|h_{k1}|^2 < \mu_1\right] - \Pr\left[|h_{k2}|^2 < \mu_2\right] \cdot \Pr\left[|h_{k1}|^2 < \mu_1\right]\Big).\end{aligned} \tag{4}$$

- *Relay Selection Probability*

After determining the set K_{th}, the central controller selects relay r_k, $k \in \{1, ..., \overline{K}_{\text{th}}\}$, that will assist the two end terminals T_1 and T_2 to exchange their blocks of information. The selection is based on the maximization of the weighted sum rate. We denote that sum by z_k, i.e., $z_k = q_{k2}R_1 + q_{k1}R_2$. Since r_k is selected from the set K_{th}, we have $q_{k2} = |\hat{h}_{k2}|^2 \geq \mu_2$ and $q_{k1} = |\hat{h}_{k1}|^2 \geq \mu_1$. Thus, $z_k \geq \mu$ with $\mu = \mu_2 R_1 + \mu_1 R_2$.

Using [24] and denoting the probability density function (PDF) of the truncated variable z_k from a threshold μ by f_{Z_k}, we can derive the probability to select relay r_k as

$$\begin{aligned} \Pr[k] &= \Pr\left[k = \arg \max_{i \in \{1,\ldots,K_{\text{th}}\}} z_i\right] \\ &= \int_{\mu}^{+\infty} f_{Z_k}(z_k) \prod_{l \neq k} \left(\int_{\mu}^{+\infty} f_{Z_l}(z_l) \cdot \Pr[z_l < z_k] \, \mathrm{d}z_l \right) \mathrm{d}z_k \end{aligned} \tag{5}$$

- *Conditional Outage Probability*

Once the central controller has selected relay r_k which maximizes the weighted sum rate, the communication can start between T_1 and T_2. An overall outage event occurs when each end node cannot receive the initial block of information sent by the other node.

Orthogonal Case: The two-way communication between T_1 and T_2 can be seen, in this case, as two simultaneous communications: $\mathrm{T}_1 \to \mathrm{T}_2$ and $\mathrm{T}_2 \to \mathrm{T}_1$. We denote by $\mathrm{P}_{\text{out}|k}(\mathrm{T}_1 \to \mathrm{T}_2)$, the outage probability in the direction $\mathrm{T}_1 \to \mathrm{T}_2$, and by $\mathrm{P}_{\text{out}|k}(\mathrm{T}_2 \to \mathrm{T}_1)$ the outage probability in the other direction. We have for $(i,j) \in \{(1,2);(2,1)\}$

$$\mathrm{P}_{\text{out}|k}(\mathrm{T}_i \to \mathrm{T}_j) = \Pr\left[|h_{ik}|^2 < \tilde{\mu}_i\right] + \Pr\left[|h_{ik}|^2 > \tilde{\mu}_i\right] . \Pr\left[\left(|h_{kj}|^2 < \tilde{\mu}_j\right) \mid \left(|\hat{h}_{kj}|^2 \geq \mu_j\right)\right]. \tag{6}$$

The overall outage probability when the relay is already selected can be expressed as the product of $\mathrm{P}_{\text{out}|k}(\mathrm{T}_1 \to \mathrm{T}_2)$ and $\mathrm{P}_{\text{out}|k}(\mathrm{T}_2 \to \mathrm{T}_1)$. Using (6), the final expression of the outage probability when the relay is already selected can be written as

$$\begin{aligned} \mathrm{P}_{\text{out}|k} = &\left[\Pr\left[|h_{1k}|^2 < \tilde{\mu}_1\right] + \left(1 - \Pr\left[|h_{1k}|^2 < \tilde{\mu}_1\right]\right) . \Pr\left[\left(|h_{k2}|^2 < \tilde{\mu}_2\right) \mid \left(|\hat{h}_{k2}|^2 \geq \mu_2\right)\right]\right] \\ &\times \left[\Pr\left[|h_{2k}|^2 < \tilde{\mu}_2\right] + \left(1 - \Pr\left[|h_{2k}|^2 < \tilde{\mu}_2\right]\right) \cdot \Pr\left[\left(|h_{k1}|^2 < \tilde{\mu}_1\right) \mid \left(|\hat{h}_{k1}|^2 \geq \mu_1\right)\right]\right]. \end{aligned} \tag{7}$$

Non-Orthogonal Case: The two-way communication between T_1 and T_2 cannot be seen, in this case, as two simultaneous communications. The relaying node will be able to decode the information sent by T_1 and T_2 if $\mathrm{I}_{1k} > \tilde{R}_{\text{th}_1}$, $\mathrm{I}_{2k} > \tilde{R}_{\text{th}_2}$ and $1/2 \cdot \log_2(1 + \gamma_{1k} + \gamma_{2k}) > \tilde{R}_{\text{th}_1} + \tilde{R}_{\text{th}_2}$ [21] [25]. Taking this into consideration, the expression in (6) will be written as

$$\begin{aligned} &\mathrm{P}_{\text{out}|k}(\mathrm{T}_i \to \mathrm{T}_j) = 1 - \Pr\left[|h_{1k}|^2 \geq \tilde{\mu}_1\right] \cdot \Pr\left[|h_{2k}|^2 \geq \tilde{\mu}_2\right] \\ &\times \Pr\left[|h_{1k}|^2 + |h_{2k}|^2 \geq \tilde{\mu}_{\text{th}_s} \mid \left(|h_{1k}|^2 \geq \tilde{\mu}_1\right), \left(|h_{2k}|^2 \geq \tilde{\mu}_2\right)\right] \cdot \left(1 - \Pr\left[\left(|h_{kj}|^2 < \tilde{\mu}_j\right) \mid \left(|\hat{h}_{kj}|^2 \geq \mu_j\right)\right]\right), \end{aligned} \tag{8}$$

with $\tilde{\mu}_{\text{th}_s} = \dfrac{2^{2\left(\tilde{R}_{\text{th}_1} + \tilde{R}_{\text{th}_2}\right)} - 1}{\mathsf{SNR}}$.

B. Nakagami-m fading Channels

In this case, the channel parameters h_{1k}, h_{k1}, h_{2k} and h_{k2} are from Nakagami-m profiles with respective shape parameters m_{1k}, m_{k1}, m_{2k} and m_{k2}, and respective scale parameters Ω_{1k}, Ω_{k1}, Ω_{2k} and Ω_{k2}. Hence, $|h_{\mathrm{ij}}|^2$ follows a gamma distribution $\mathcal{G}(m_{\mathrm{ij}}, \Omega_{\mathrm{ij}}/m_{\mathrm{ij}})$ with a shape parameter m_{ij} and a scale parameter $\Omega_{\mathrm{ij}}/m_{\mathrm{ij}}$, $(\mathrm{i},\mathrm{j}) \in \{(\mathrm{r}_k, 1), (\mathrm{r}_k, 2), (1, \mathrm{r}_k), (2, \mathrm{r}_k)\}$. It is assumed that all fading parameters m_{ij} are integers.

We denote the variable of interest $|h_{\mathrm{ij}}|^2$ by x_{ij}. The PDF of x_{ij} is given by

$$f_{X_{\mathrm{ij}}}(x_{\mathrm{ij}}) = \frac{x_{\mathrm{ij}}^{m_{\mathrm{ij}}-1}}{\Gamma\left(m_{\mathrm{ij}}\right)} \left(\frac{m_{\mathrm{ij}}}{\Omega_{\mathrm{ij}}}\right)^{m_{\mathrm{ij}}} \exp\left(-\frac{m_{\mathrm{ij}} x_{\mathrm{ij}}}{\Omega_{\mathrm{ij}}}\right). \tag{9}$$

where $\Gamma\left(.\right)$ is the Gamma function.

In what follows, we will start by considering the case where perfect CSI is available for relay selection at the central controller, then, we extend the result to the outdated case.

1) Perfect CSI Case:

When perfect CSI is available at the central controller, we have $\hat{h}_{ij}{=}h_{ij}$, where (i,j) represents the different links between the end terminals and the selected relay. For this case, we investigate the different probabilities involved in the outage probability expression in (3).

- *Threshold Selection Probability*

We have

$$\Pr\left[|h_{ki}|^2 < \mu_i\right] = \gamma\left(m_{ki}, \frac{m_{ki}\mu_i}{\Omega_{ki}}\right), \qquad i = \{1, 2\}, \tag{10}$$

where $\gamma\left(.,.\right)$ is the regularized lower incomplete gamma function. Substituting (10) in (4), the threshold selection probability in the Nakagami-m case can be written as

$$\begin{aligned} \Pr\left[\overline{K}_{\mathrm{th}}\right] = &\sum_{A\in\mathcal{P}_{\overline{K}_{\mathrm{th}}}} \prod_{k\in A} \left[1-\gamma\left(m_{k2}, \frac{m_{k2}\mu_2}{\Omega_{k2}}\right)\right] \cdot \left[1-\gamma\left(m_{k1}, \frac{m_{k1}\mu_1}{\Omega_{k1}}\right)\right] \\ &\times \prod_{k\notin A} \left[\gamma\left(m_{k2}, \frac{m_{k2}\mu_2}{\Omega_{k2}}\right) + \gamma\left(m_{k1}, \frac{m_{k1}\mu_1}{\Omega_{k1}}\right) \cdot \left[1-\gamma\left(m_{k2}, \frac{m_{k2}\mu_2}{\Omega_{k2}}\right)\right]\right]. \end{aligned} \tag{11}$$

For the special case of Rayleigh fading channels (i.e., $m_{ij} = 1$ and $\lambda_{ij} = 1/\Omega_{ij}$), the threshold selection probability can be written as

$$\Pr\left[\overline{K}_{\mathrm{th}}\right] = \sum_{A\in\mathcal{P}_{\overline{K}_{\mathrm{th}}}} \left[\prod_{k\in A} \mathrm{e}^{-\lambda_{k2}\mu_2-\lambda_{k1}\mu_1}\right] \cdot \left[\prod_{k\notin A} \left(1-\mathrm{e}^{-\lambda_{k2}\mu_2-\lambda_{k1}\mu_1}\right)\right]. \tag{12}$$

- *Relay Selection Probability*

In the Nakagami-m case, the weighted sum rate, defined as $z_l = q_{l2}R_1 + q_{l1}R_2$, is a sum of two truncated gamma variables:

$$q_{l2}R_1 \sim \mathcal{G}\left(m_{l2}, \frac{\Omega_{l2}R_1}{m_{l2}}; \mu_2 R_1\right) \quad \text{and} \quad q_{l1}R_2 \sim \mathcal{G}\left(m_{l1}, \frac{\Omega_{l1}R_2}{m_{l1}}; \mu_1 R_2\right)$$

Proposition 1. *The CDF of z_l is given by :*

$$F_{z_l}(z_l) = \frac{1}{1-\gamma\left(m_{l1}, \frac{m_{l1}\mu_1 R_2}{\Omega_{l1} R_2}\right)} \cdot \frac{1}{1-\gamma\left(m_{l2}, \frac{m_{l2}\mu_2 R_1}{\Omega_{l2} R_1}\right)} \cdot \Bigg\{\gamma\left(m_{l1}, \frac{m_{l1}\left(z_l-\mu_2 R_1\right)}{\Omega_{l1} R_2}\right) - \gamma\left(m_{l1}, \frac{m_{l1}\mu_1 R_2}{\Omega_{l1} R_2}\right)$$
$$-\gamma\left(m_{l2}, \frac{m_{l2}\mu_2 R_1}{\Omega_{l2} R_1}\right)\left[\gamma\left(m_{l1}, \frac{m_{l1}\left(z_l-\mu_2 R_1\right)}{\Omega_{l1} R_2}\right) - \gamma\left(m_{l1}, \frac{m_{l1}\mu_1 R_2}{\Omega_{l1} R_2}\right)\right] - \vartheta\left(z_l, x_{th_1}, x_{th_2}; m_{l1}, m_{l2}, \overline{\Omega}_{12}, \overline{\Omega}_{21}\right)\Bigg\}, \tag{13}$$

where we have set $x_{\text{th}_1} = \mu_1 R_2$, $x_{\text{th}_2} = \mu_2 R_1$, $\overline{\Omega}_{12} = \Omega_{l1} R_2$, $\overline{\Omega}_{21} = \Omega_{l2} R_1$, and

$$\vartheta\left(z, x_{\text{th}_1}, x_{\text{th}_2}; m_1, m_2, \overline{\Omega}_1, \overline{\Omega}_2\right) = \left(\frac{m_1}{\overline{\Omega}_1}\right)^{m_1} \frac{1}{\Gamma\left(m_1\right)} \exp\left(-\frac{m_2 z}{\overline{\Omega}_2}\right) \sum_{k=0}^{m_2-1} \frac{1}{k!}\left(\frac{m_2}{\overline{\Omega}_2}\right)^k \sum_{l=0}^{k} \Gamma\left(m_1+l\right)\binom{k}{l}(-1)^l$$
$$\times z^{k-l}\left(\frac{m_1}{\overline{\Omega}_1} - \frac{m_2}{\overline{\Omega}_2}\right)^{-m_1-l} \cdot \left[\gamma\left(m_1+l, \left(\frac{m_1}{\overline{\Omega}_1} - \frac{m_2}{\overline{\Omega}_2}\right)\left(z - x_{\text{th}_2}\right)\right) - \gamma\left(m_1+l, \left(\frac{m_1}{\overline{\Omega}_1} - \frac{m_2}{\overline{\Omega}_2}\right) x_{\text{th}_1}\right)\right]. \tag{14}$$

Proof: The proof is given in the Appendix. ■

Proposition 2. *The PDF of z_l is given by :*

$$f_{z_l}\left(z_l\right) = \frac{1}{1-\gamma\left(m_{l1}, \frac{m_{l1}\mu_1 R_2}{\Omega_{l1} R_2}\right)} \cdot \frac{1}{1-\gamma\left(m_{l2}, \frac{m_{l2}\mu_2 R_1}{\Omega_{l2} R_1}\right)}$$
$$\times\Bigg\{\frac{\left(z_l-\mu_2 R_1\right)^{m_{l1}-1}}{\Gamma\left(m_{l1}\right)} \cdot \exp\left(-\frac{m_{l1}\left(z_l-\mu_2 R_1\right)}{\Omega_{l1} R_2}\right) \cdot \left[\left(\frac{m_{l1}}{\Omega_{l2} R_1}\right)^{m_{l1}} - \left(\frac{m_{l1}}{\Omega_{l1} R_2}\right)^{m_{l1}} \cdot \gamma\left(m_{l2}, \frac{m_{l2}\mu_2 R_1}{\Omega_{l2} R_1}\right)\right]$$
$$+\left(\frac{m_{l2}}{\Omega_{l2} R_1}\right) \cdot \vartheta\left(z_l, x_{th_1}, x_{th_2}; m_{l1}, m_{l2}, \overline{\Omega}_{12}, \overline{\Omega}_{21}\right) - \xi\left(z_l, x_{th_1}, x_{th_2}; m_{l1}, m_{l2}, \overline{\Omega}_{12}, \overline{\Omega}_{21}\right)$$
$$-\psi\left(z_l, x_{th_1}, x_{th_2}; m_{l1}, m_{l2}, \overline{\Omega}_{12}, \overline{\Omega}_{21}\right)\Bigg\}. \tag{15}$$

where

$$\xi\left(z, x_{\text{th}_1}, x_{\text{th}_2}; m_1, m_2, \overline{\Omega}_1, \overline{\Omega}_2\right) = \left(\frac{m_1}{\overline{\Omega}_1}\right)^{m_1}\left(\frac{1}{\Gamma\left(m_1\right)}\right) \exp\left(-\frac{m_2 z_l}{\overline{\Omega}_2}\right) \sum_{i=0}^{m_2-1} \frac{1}{i!}\left(\frac{m_2}{\overline{\Omega}_2}\right)^i$$
$$\times \sum_{l=0}^{i}\binom{i}{l}(-1)^l z_l^{i-l}\left(z_l - x_{\text{th}_2}\right)^{m_1+l-1} \exp\left(-\left(\frac{m_1}{\overline{\Omega}_1} - \frac{m_2}{\overline{\Omega}_2}\right)\left(z_l - x_{\text{th}_2}\right)\right), \tag{16}$$

and

$$\psi\left(z, x_{\text{th}_1}, x_{\text{th}_2}; m_1, m_2, \overline{\Omega}_1, \overline{\Omega}_2\right) = \frac{1}{\Gamma\left(m_1\right)}\left(\frac{m_1}{\overline{\Omega}_1}\right)^{m_1} \exp\left(-\frac{m_2 z_l}{\overline{\Omega}_2}\right) \sum_{i=1}^{m_2-1} \frac{1}{i!}\left(\frac{m_2}{\overline{\Omega}_2}\right)^i \sum_{l=0}^{i-1}\binom{i}{l}(-1)^l (i-l) z_l^{i-l}$$
$$\times \Gamma(m_1+l)\left(\frac{m_1}{\overline{\Omega}_1} - \frac{m_2}{\overline{\Omega}_2}\right)^{-m_1-l}\left(\gamma\left(m_1+l, \left(\frac{m_1}{\overline{\Omega}_1} - \frac{m_2}{\overline{\Omega}_2}\right)\left(z_l - x_{\text{th}_2}\right)\right) - \gamma\left(m_1+l, \left(\frac{m_1}{\overline{\Omega}_1} - \frac{m_2}{\overline{\Omega}_2}\right) x_{\text{th}_1}\right)\right). \tag{17}$$

Proof: The proof is given in the Appendix. ■

The relay selection probability $\Pr[k]$ in the Nakagami-m case is obtained according to (5). The resulting expression is evaluated numerically.

For the special case of Rayleigh fading channels (i.e., $m_{ij} = 1$ and $\lambda_{ij} = 1/\Omega_{ij}$), and adopting the following notation

$$\begin{cases} \vartheta_i = \dfrac{\lambda_{i1}\lambda_{i2}}{\lambda_{i1}R_1 - \lambda_{i2}R_2}\exp\left(\lambda_{i1}\mu_1 + \lambda_{i2}\mu_2\right); & \mu = R_1\mu_2 + R_2\mu_1; \\ \alpha_i = \exp\left(\left(\dfrac{\lambda_{i2}}{R_1} - \dfrac{\lambda_{i1}}{R_2}\right)R_2\mu_1\right); & \theta_i = \dfrac{\lambda_{i2}}{R_1}; \\ \beta_i = \exp\left(\left(\dfrac{\lambda_{i1}}{R_2} - \dfrac{\lambda_{i2}}{R_1}\right)R_1\mu_2\right); & \delta_i = \dfrac{\lambda_{i1}}{R_2}, \end{cases} \tag{18}$$

the relay selection probability can be written as

$$\begin{aligned} \Pr[k] = \prod_{l \neq k} \Bigg[1 - \vartheta_k\vartheta_l \Bigg(& \frac{\alpha_l}{\theta_l}\left(\frac{\alpha_k}{\theta_k}\mathrm{e}^{-\theta_k\mu} - \frac{\beta_k}{\delta_k}\mathrm{e}^{-\delta_k\mu}\right)\mathrm{e}^{-\theta_l\mu} - \frac{\beta_l}{\delta_l}\left(\frac{\alpha_k}{\theta_k}\mathrm{e}^{-\theta_k\mu} - \frac{\beta_k}{\delta_k}\mathrm{e}^{-\delta_k\mu}\right)\mathrm{e}^{-\delta_l\mu} \\ & + \frac{\alpha_l\beta_k}{\delta_k(\theta_l+\delta_k)}\mathrm{e}^{-(\theta_l+\delta_k)\mu} - \frac{\beta_l\beta_k}{\delta_k\left(\delta_l+\delta_k\right)}\mathrm{e}^{-(\delta_l+\delta_k)\mu} - \frac{\alpha_l\alpha_k}{\theta_k(\theta_l+\theta_k)}\mathrm{e}^{-(\theta_l+\theta_k)\mu} + \frac{\beta_l\alpha_k}{\theta_k\left(\delta_l+\theta_k\right)}\mathrm{e}^{-(\delta_l+\theta_k)\mu}\Bigg)\Bigg]. \end{aligned} \tag{19}$$

- *Conditional Outage Probability*

Orthogonal Case: For the Nakagami-m case, we have

$$\Pr\left[\left(|h_{ki}|^2 < \tilde{\mu}_i\right) \mid \left(|h_{ki}|^2 \geq \mu_i\right)\right] = 1 - \frac{1 - \gamma\left(m_{ki}, \dfrac{m_{ki}\max\left(\tilde{\mu}_i, \mu_i\right)}{\Omega_{ki}}\right)}{1 - \gamma\left(m_{ki}, \dfrac{m_{ki}\mu_i}{\Omega_{ki}}\right)}. \tag{20}$$

The conditional outage probability is then given by

$$\begin{aligned} \mathrm{P}_{\mathrm{out}|k} = & \left[\gamma\left(m_{1k}, \frac{m_{1k}\tilde{\mu}_1}{\Omega_{1k}}\right) + \left(1 - \gamma\left(m_{1k}, \frac{m_{1k}\tilde{\mu}_1}{\Omega_{1k}}\right)\right)\left(1 - \frac{1 - \gamma\left(m_{k2}, \dfrac{m_{k2}\cdot\max\left(\tilde{\mu}_2, \mu_2\right)}{\Omega_{k2}}\right)}{1 - \gamma\left(m_{k2}, \dfrac{m_{k2}\mu_2}{\Omega_{k2}}\right)}\right)\right] \\ & \times \left[\gamma\left(m_{2k}, \frac{m_{2k}\tilde{\mu}_2}{\Omega_{2k}}\right) + \left(1 - \gamma\left(m_{2k}, \frac{m_{2k}\tilde{\mu}_2}{\Omega_{2k}}\right)\right)\left(1 - \frac{1 - \gamma\left(m_{k1}, \dfrac{m_{k1}\cdot\max\left(\tilde{\mu}_1, \mu_1\right)}{\Omega_{k1}}\right)}{1 - \gamma\left(m_{k1}, \dfrac{m_{k1}\mu_1}{\Omega_{k1}}\right)}\right)\right]. \end{aligned} \tag{21}$$

For the special case of Rayleigh fading channels (i.e., $m_{ij} = 1$ and $\lambda_{ij} = 1/\Omega_{ij}$), the conditional outage probability can be written as

$$\mathrm{P}_{\mathrm{out}|k} = \left[1 - \exp\left(\lambda_{k2}\mu_2 - \lambda_{1k}\tilde{\mu}_1 - \lambda_{k2}\max\left(\tilde{\mu}_2, \mu_2\right)\right)\right] \cdot \left[1 - \exp\left(\lambda_{k1}\mu_1 - \lambda_{2k}\tilde{\mu}_1 - \lambda_{k1}\max\left(\tilde{\mu}_1, \mu_1\right)\right)\right]. \tag{22}$$

The total outage probability can be finally written in the Nakagami-m case as

$$\begin{aligned}\mathrm{P}_{\text{out}} =& \prod_{k=1}^{K}\left[\gamma\left(m_{k2},\frac{m_{k2}\mu_2}{\Omega_{k2}}\right)+\gamma\left(m_{k1},\frac{m_{k1}\mu_1}{\Omega_{k1}}\right)\left(1-\gamma\left(m_{k2},\frac{m_{k2}\mu_2}{\Omega_{k2}}\right)\right)\right] \\ &+\sum_{\overline{K}_{\text{th}}=1}^{K}\sum_{A\in\mathcal{P}_{\overline{K}_{\text{th}}}}\left[\prod_{j\in A}\left(1-\gamma\left(m_{j2},\frac{m_{j2}\mu_2}{\Omega_{j2}}\right)\right)\left(1-\gamma\left(m_{j1},\frac{m_{j1}\mu_1}{\Omega_{j1}}\right)\right)\right] \\ &\times\left[\prod_{j\notin A}\left(\gamma\left(m_{j2},\frac{m_{j2}\mu_2}{\Omega_{j2}}\right)+\gamma\left(m_{j1},\frac{m_{j1}\mu_1}{\Omega_{j1}}\right)\left(1-\gamma\left(m_{j2},\frac{m_{j2}\mu_2}{\Omega_{j2}}\right)\right)\right)\right]\sum_{k=1}^{\overline{K}_{\text{th}}}\Pr[k]\cdot\mathrm{P}_{\text{out}|k}. \end{aligned} \tag{23}$$

Non-Orthogonal Case: The derivations are similar to the orthogonal case. However, for this scenario, we should also derive the expression of the probability $\Pr\left[|h_{1k}|^2+|h_{2k}|^2 \geq \tilde{\mu}_{\text{th}_s} \middle| (|h_{1k}|^2 \geq \tilde{\mu}_1),(|h_{2k}|^2 \geq \tilde{\mu}_2)\right]$ in (8). Using (37), with $x_{\text{th}_1} = \tilde{\mu}_1$ and $x_{\text{th}_2} = \tilde{\mu}_2$, this probability can be written as

$$\begin{aligned}&\Pr\left[|h_{1k}|^2+|h_{2k}|^2 \geq \tilde{\mu}_{\text{th}_s} \middle| \left(|h_{1k}|^2 \geq \tilde{\mu}_2\right),\left(|h_{1k}|^2 \geq \tilde{\mu}_1\right)\right] = \frac{1}{1-\gamma\left(m_1,\frac{m_1\tilde{\mu}_1}{\Omega_1}\right)}\cdot\frac{1}{1-\gamma\left(m_2,\frac{m_2\tilde{\mu}_2}{\Omega_2}\right)} \\ &\times\Bigg\{\gamma\left(m_1,\frac{m_1(z-\tilde{\mu}_2)}{\Omega_1}\right)-\gamma\left(m_1,\frac{m_1\tilde{\mu}_1}{\Omega_1}\right)-\gamma\left(m_2,\frac{m_2\tilde{\mu}_2}{\Omega_2}\right)\left(\gamma\left(m_1,\frac{m_1(z-\tilde{\mu}_2)}{\Omega_1}\right)-\gamma\left(m_1,\frac{m_1\tilde{\mu}_1}{\Omega_1}\right)\right) \\ &-\vartheta\left(z,\tilde{\mu}_1,\tilde{\mu}_2;m_{1k},m_{2k},\Omega_1,\Omega_2\right)\Bigg\},\end{aligned} \tag{24}$$

where the function $\vartheta(\cdot)$ is defined in (14).

2) Outdated CSI Case:

In practical scenarios, the available CSI at the central controller may be outdated. In this subsection, we investigate the impact of imperfect CSI on the performance of the previously analyzed perfect CSI case. We have $\widehat{h}_{k1}$ and $\widehat{h}_{k2}$ follow similar distributions as h_{k1} and h_{k2}, and are from Nakagami-m profiles with shape parameters m_{k1} and m_{k2}, and scale parameters $\widehat{\Omega}_{k1}$ and $\widehat{\Omega}_{k2}$. We consider the general case where the scale parameters of the outdated channel distributions are different from those of the actual channel distributions. This is for example the case when the delay is so considerable compared to the fading rapidity. The variables $|\widehat{h}_{ki}|^2$ and $|h_{ki}|^2$ are then two correlated gamma variates ($i \in \{1,2\}$) with a joint PDF given by [26]

$$\begin{aligned}f_{|\widehat{h}_{ki}|^2,|h_{ki}|^2}(x_1,x_2) =& \sum_{j=0}^{+\infty}\frac{\rho^j(m_{ki})_j}{j!\,(1-\rho)^{m_{ki}+2j}}\,\frac{m_{ki}^{2(m_{ki}+j)}(x_1x_2)^{m_{ki}+j-1}}{\Gamma(m_{ki}+j)^2\left(\Omega_{ki}\widehat{\Omega}_{ki}\right)^{m_{ki}+j}} \\ &\times\exp\left(-\frac{m_{ki}x_1}{\Omega_{ki}(1-\rho)}\right)\exp\left(-\frac{m_{ki}x_2}{\widehat{\Omega}_{ki}(1-\rho)}\right);\qquad x_1,x_2\geq 0\end{aligned} \tag{25}$$

where $(x)_y$ denotes the Pochhammer symbol and ρ represents the correlation coefficient between $|\widehat{h}_{ki}|^2$ and $|h_{ki}|^2$.

In this case, and since the estimated channel coefficients are similarly distributed as the perfect coefficients, the derivations of the threshold selection probability and relay selection probability obtained in (11), (13) and (15) remain unchanged where $\widehat{h}_{ki}$ and $\widehat{\Omega}_{ki}$ should be considered instead of h_{ki} and Ω_{ki}. However, since the selection is based on outdated channel estimates, the conditional outage probability has to be rederived.

First, the conditional outage probability could be expressed as

$$\Pr\left[\left(|h_{ki}|^2 < \tilde{\mu}_i\right) \mid \left(|\widehat{h}_{ki}|^2 \geq \mu_i\right)\right] \;=\; \frac{\displaystyle\int_0^{\tilde{\mu}_i}\int_{\mu_i}^{+\infty} f_{|\widehat{h}_{ki}|^2,|h_{ki}|^2}\left(x_1, x_2\right) \mathrm{d}x_2 \;\; \mathrm{d}x_1}{\displaystyle\int_{\mu_i}^{+\infty} f_{|\widehat{h}_{ki}|^2}\left(x_2\right) \mathrm{d}x_2}. \tag{26}$$

We denote the numerator of the expression in (26) by I_1 and the denominator by I_2. Using the expression of the joint PDF in (25), we can write

$$\begin{aligned} I_1 = & \sum_{j=0}^{+\infty} \frac{\rho^j}{j!\left(1-\rho\right)^{m_{ki}+2j}} \frac{m_{ki}^{2(m_{ki}+j)}}{\Gamma\left(m_{ki}\right)\Gamma\left(m_{ki}+j\right)\left(\Omega_{ki}\widehat{\Omega}_{ki}\right)^{m_{ki}+j}} \\ & \times \int_0^{\tilde{\mu}_i} x_1^{m_{ki}+j-1} \exp\left(-\frac{x_1 m_{ki}}{\Omega_{ki}\left(1-\rho\right)}\right) \mathrm{d}x_1 \cdot \int_{\mu_i}^{+\infty} x_2^{m_{ki}+j-1} \exp\left(-\frac{x_2 m_{ki}}{\widehat{\Omega}_{ki}\left(1-\rho\right)}\right) \mathrm{d}x_2, \end{aligned} \tag{27}$$

where the integrals in the expression of I_1 are given by

$$\begin{aligned} \int_0^{\tilde{\mu}_i} x_1^{m_{ki}+j-1} \exp\!\left(-\frac{x_1 m_{ki}}{\Omega_{ki}\left(1-\rho\right)}\right) \mathrm{d}x_1 &= \Gamma\left(m_{ki}+j\right)\cdot\left(\frac{\left(1-\rho\right)\Omega_{ki}}{m_{ki}}\right)^{m_{ki}+j} \cdot\gamma\!\left(m_{ki}+j, \frac{m_{ki}\tilde{\mu}_i}{\left(1-\rho\right)\Omega_{ki}}\right) \\ \int_{\mu_i}^{+\infty} x_2^{m_{ki}+j-1} \exp\!\left(-\frac{x_2 m_{ki}}{\widehat{\Omega}_{ki}\left(1-\rho\right)}\right) \mathrm{d}x_2 &= 1-\Gamma\left(m_{ki}+j\right)\cdot\left(\frac{\left(1-\rho\right)\widehat{\Omega}_{ki}}{m_{ki}}\right)^{m_{ki}+j} \cdot\gamma\!\left(m_{ki}+j, \frac{m_{ki}\mu_i}{\left(1-\rho\right)\widehat{\Omega}_{ki}}\right) \end{aligned} \tag{28}$$

which yields

$$\begin{aligned} I_1 = & \sum_{j=0}^{+\infty} \frac{\rho^j}{j!\left(1-\rho\right)^{m_{ki}+2j}} \frac{m_{ki}^{2(m_{ki}+j)}}{\Gamma\left(m_{ki}\right)\left(\Omega_{ki}\widehat{\Omega}_{ki}\right)^{m_{ki}+j}} \cdot\gamma\left(m_{ki}+j, \frac{m_{ki}\tilde{\mu}_i}{\left(1-\rho\right)\Omega_{ki}}\right)\left(\frac{\left(1-\rho\right)\Omega_{ki}}{m_{ki}}\right)^{m_{ki}+j} \\ & \times \left(1-\Gamma\left(m_{ki}+j\right)\gamma\left(m_{ki}+j, \frac{m_{ki}\mu_i}{\left(1-\rho\right)\widehat{\Omega}_{ki}}\right)\left(\frac{\left(1-\rho\right)\widehat{\Omega}_{ki}}{m_{ki}}\right)^{m_{ki}+j}\right). \end{aligned} \tag{29}$$

On the other hand, the denominator I_2 is given by

$$I_2 = 1-\gamma\left(m_{ki}, \frac{m_{ki}\mu_i}{\widehat{\Omega}_{ki}}\right), \tag{30}$$

and we get

$$\Pr\left[\left(|h_{ki}|^2 < \tilde{\mu}_i\right) \mid \left(|\widehat{h}_{ki}|^2 \geq \mu_i\right)\right] = \frac{1}{1-\gamma\left(m_{ki}, \frac{m_{ki}\mu_i}{\widehat{\Omega}_{ki}}\right)} \sum_{j=0}^{+\infty} \frac{\rho^j}{j!\,(1-\rho)^j} \frac{m_{ki}^{(m_{ki}+j)}}{\Gamma(m_{ki})\,\widehat{\Omega}_{ki}^{m_{ki}+j}}$$
$$\times\, \gamma\left(m_{ki}+j, \frac{m_{ki}\tilde{\mu}_i}{(1-\rho)\,\Omega_{ki}}\right) \cdot \left(1-\Gamma(m_{ki}+j)\,\gamma\left(m_{ki}+j, \frac{m_{ki}\mu_i}{(1-\rho)\,\widehat{\Omega}_{ki}}\right)\left(\frac{(1-\rho)\,\widehat{\Omega}_{ki}}{m_{ki}}\right)^{m_{ki}+j}\right). \tag{31}$$

Substituting (31) in (7), the conditional outage probability could be written for the Nakagami-m case with outdated CSI as

$$\begin{aligned}
\mathrm{P}_{\mathrm{out}|k} = &\Bigg[\gamma\left(m_{1k}, \frac{m_{1k}\tilde{\mu}_1}{\Omega_{1k}}\right) + \frac{1-\gamma\left(m_{1k}, \frac{m_{1k}\tilde{\mu}_1}{\Omega_{1k}}\right)}{1-\gamma\left(m_{k2}, \frac{m_{k2}\mu_2}{\widehat{\Omega}_{k2}}\right)} \sum_{j=0}^{+\infty} \frac{\rho^j m_{k2}^{(m_{k2}+j)}}{j!\,(1-\rho)^j\,\Gamma(m_{k2})\,\widehat{\Omega}_{k2}^{m_{k2}+j}} \\
&\times \left(1-\Gamma(m_{k2}+j)\,\gamma\left(m_{k2}+j, \frac{m_{k2}\mu_2}{(1-\rho)\,\widehat{\Omega}_{k2}}\right)\left(\frac{(1-\rho)\,\widehat{\Omega}_{k2}}{m_{k2}}\right)^{m_{k2}+j}\right) \\
&\times \gamma\left(m_{k2}+j, \frac{m_{k2}\tilde{\mu}_2}{(1-\rho)\,\Omega_{k2}}\right)\Bigg] \times \Bigg[\gamma\left(m_{2k}, \frac{m_{2k}\tilde{\mu}_2}{\Omega_{2k}}\right) + \frac{1-\gamma\left(m_{2k}, \frac{m_{2k}\tilde{\mu}_2}{\Omega_{2k}}\right)}{1-\gamma\left(m_{k1}, \frac{m_{k1}\mu_1}{\widehat{\Omega}_{k1}}\right)} \\
&\times \sum_{j=0}^{+\infty} \frac{\rho^j m_{k1}^{(m_{k1}+j)}}{j!\,(1-\rho)^j\,\Gamma(m_{k1})\,\widehat{\Omega}_{k1}^{m_{k1}+j}}\;\gamma\left(m_{k1}+j, \frac{m_{k1}\tilde{\mu}_1}{(1-\rho)\,\Omega_{k1}}\right) \\
&\times \left(1-\Gamma(m_{k1}+j)\,\gamma\left(m_{k1}+j, \frac{m_{k1}\mu_1}{(1-\rho)\,\widehat{\Omega}_{k1}}\right)\left(\frac{(1-\rho)\,\widehat{\Omega}_{k1}}{m_{k1}}\right)^{m_{k1}+j}\right)\Bigg].
\end{aligned} \tag{32}$$

Note that the particular case of $\rho = 1$ is equivalent to the Nakagami-m case with perfect CSI. The outage probability is given in this case by (23). For the case of $\rho = 0$, the outage performance of the system is similar to a system without relay selection.

IV. Numerical Examples

In this section, we present the results obtained based on our analytical expressions in the Nakagami-m case along with the results of numerical simulations.

The simulated system consists of three relays with the following parameters : $\{\Omega_{1k}\}_{k=1}^{3} = \{2, 3, 1\}$ dB, $\{\Omega_{2k}\}_{k=1}^{3} = \{3, 4, 5\}$ dB, $\{\Omega_{k1}\}_{k=1}^{3} = \{4, 3, 1\}$ dB and $\{\Omega_{k2}\}_{k=1}^{3} = \{5, 4, 2\}$ dB. We recall that all fading parameters m_{ij} are set to integer values.

First, we confirm the closed-form characterization of the distribution of the sum of two truncated gamma variates, which is given in (13) and (15) and derived in details in the Appendix. In Fig. 3, a comparison

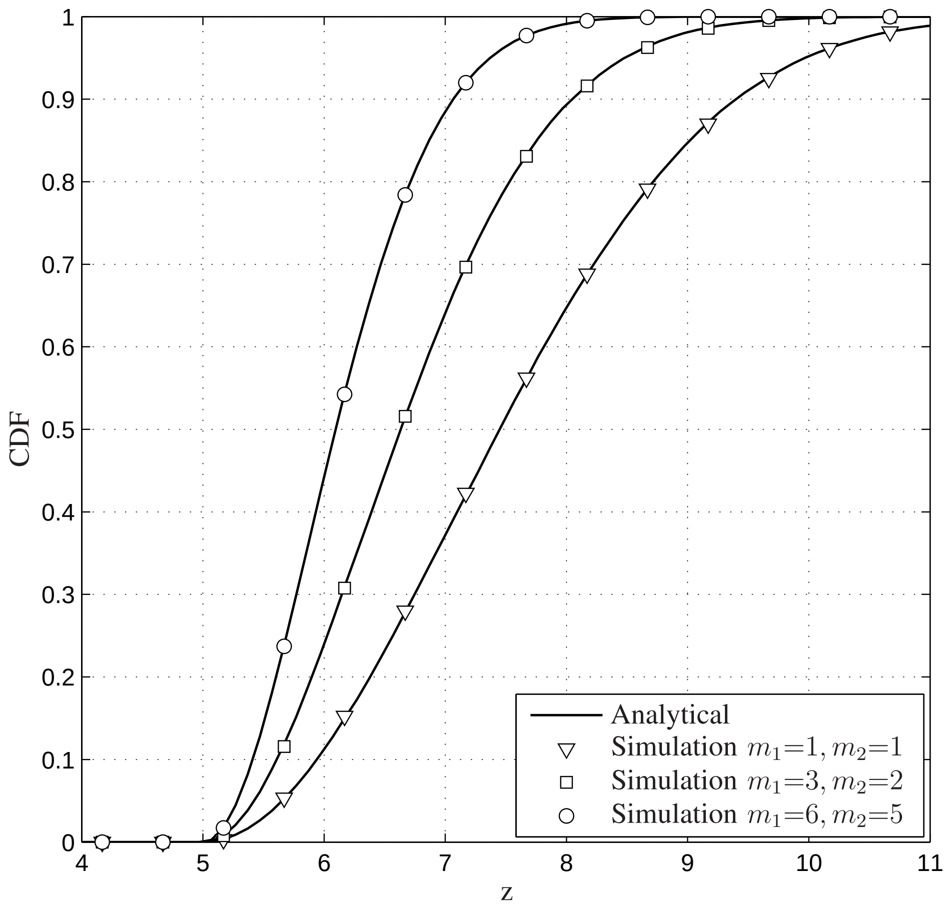

Fig. 3. CDF of the sum of two truncated Gamma variates obtained analytically in (37) and via Monte Carlo simulation for different values of m_1 and m_2.

between the analytical expression in (37) and the CDF obtained via Monte Carlo simulations is shown for different values of the fading parameters $m_1 = m_{1k} = m_{k1}$ and $m_2 = m_{2k} = m_{k2} =$ with $\mu_1 = 1$ dB and $\mu_2 = 2$ dB. Also, a comparison between the analytical expression in (38) and the PDF obtained via Monte Carlo simulation is shown in Fig. 4 for the special case of Rayleigh fading channels (m_1=m_2=1). Both figures confirm the accuracy of our closed-form expressions.

In Fig. 5, the total outage probability obtained in the Nakagami-m case is shown as a function of the SNR for both orthogonal and non-orthogonal cases. Different values of the fading parameters m_1 and m_2 are considered. Once again, our analytical results are in perfect agreement with simulations. From a different point of view, in Fig. 6, the total outage probability is presented as a function of the threshold γ_{th_1} for $m_1 = 2$, $m_2 = 1$ and $\mu_2 = 2$ dB. The results are shown for three illustrative values of the average SNR: 10, 15 and 20 dB. The figure shows the existence of a local optimum which is attained when γ_{th_1} is relatively close to $\tilde{\gamma}_{\text{th}_1}$. Note that beyond that point the set K_{th} becomes smaller and the probability

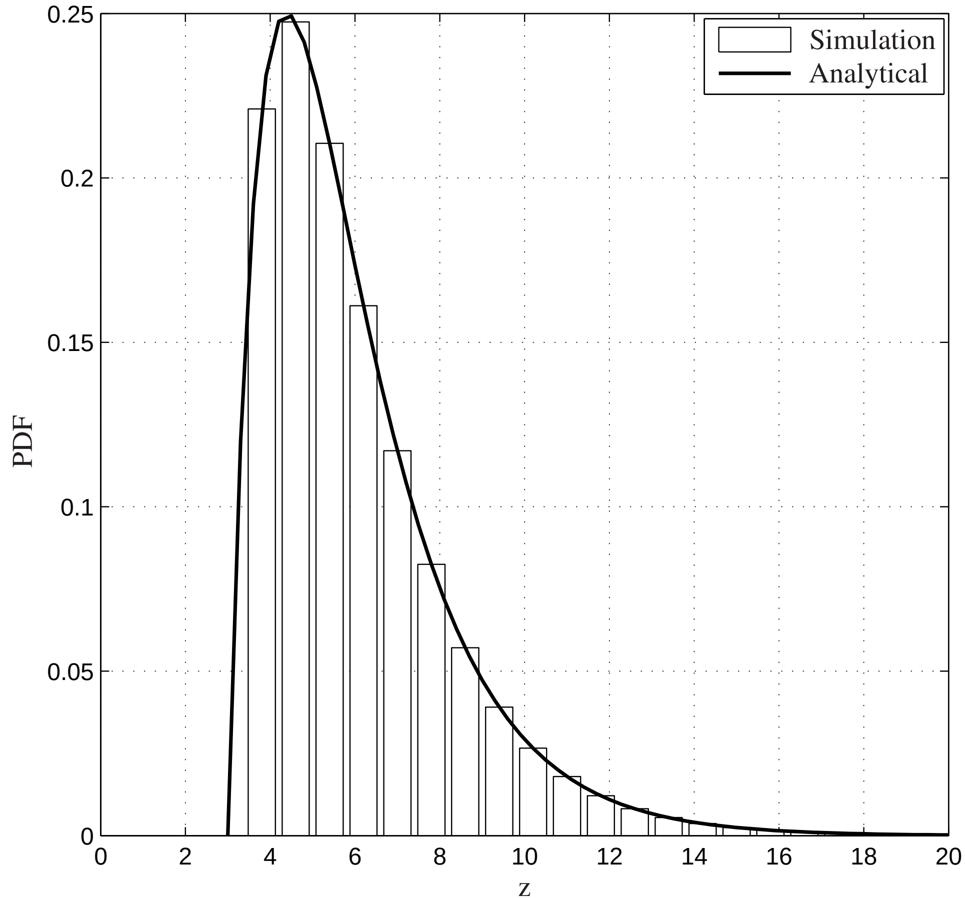

Fig. 4. PDF of the sum of two truncated exponential variates obtained analytically and via Monte Carlo simulation.

of relay selection will decrease, i.e., it becomes very hard to find a satisfying relay with the imposed constraints. Hence, the outage probability will predictably increase as shown by the figure.

In Fig. 7, we show the effect of outdated CSI—captured by the correlation coefficient ρ between actual and outdated channel coefficients—on the outage performance in the case of Nakagami-m fading channels. The total outage probability is presented as a function of the SNR for $\rho = 0, 0.5$ and 0.99, and for different values of the fading parameters m_1 and m_2. Note that when the correlation coefficient ρ approaches 1, we get similar result as those obtained with perfect CSI in Fig. 6. This confirms the accuracy of our model and derivations[2]. Also, from the figure, we can see that practical considerations (that may result in outdated CSI at the central controller) may cause a loss of 1–2 dB compared to the perfect scenario with exact CSI.

Finally, Fig. 8 and Fig. 9 present a comparison between the proposed relay selection technique and other approaches from the literature, namely the Max-Min technique analyzed in [8], the Max-Sum technique

[2]To compute the infinite sum in (32), less than 50 terms are actually necessary to converge to the final value.

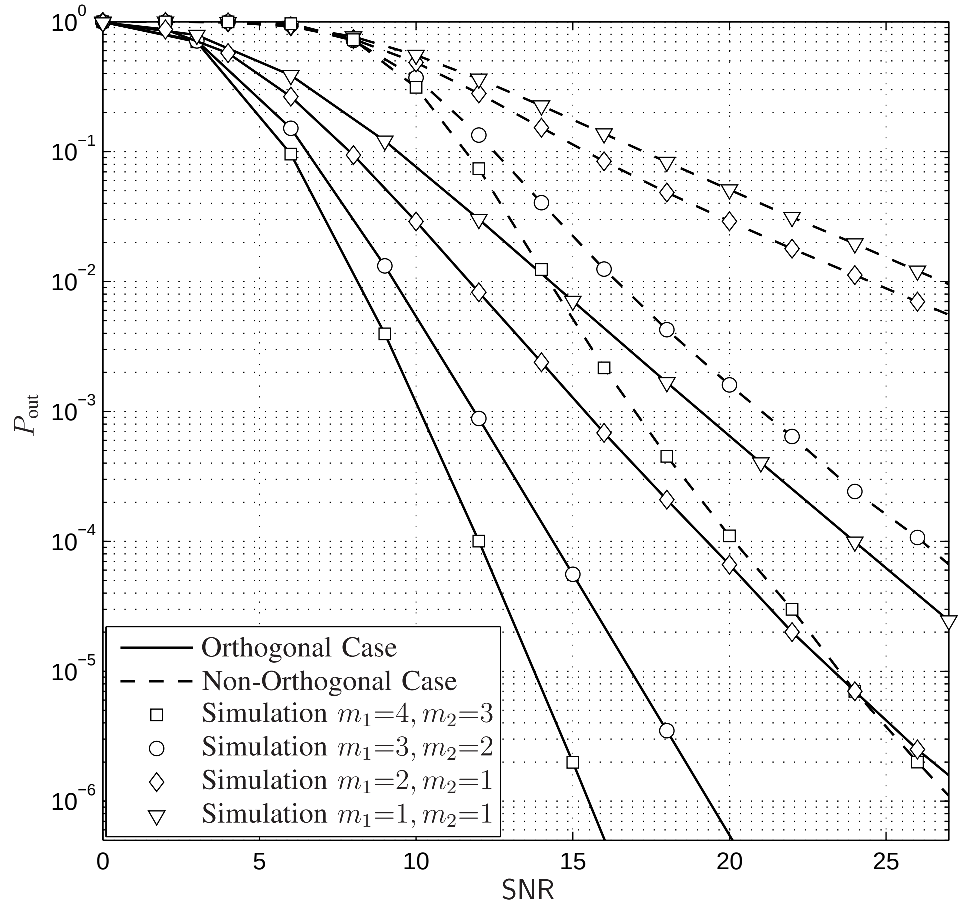

Fig. 5. Outage Probability as a function of the average SNR for different values of m_1 and m_2. Lines correspond to analytical results and simulation results are represented with markers.

considered in [10], random selection over the relays in the pre-selected set K_{th}, and random selection over all K relays. In Fig. 8, we present the outage probability of the system in terms of the SNR, while in Fig. 9, the outage probability is ploted versus the number of relays K. Both figures show a clear performance gain using the proposed metric in (1). This is expected since our metric takes into consideration the achievable rates during the transmission phase as well as the channel quality experienced by each of the rates during the relaying phase. On the other hand, the outage performance of the system with the modified relay selection technique in (2) are close to the performance of the Max-Min and the Max-Sum techniques. From Fig. 9, we can see that as the number of relays K grows, the proposed relay selection technique clearely outperforms the other approaches.

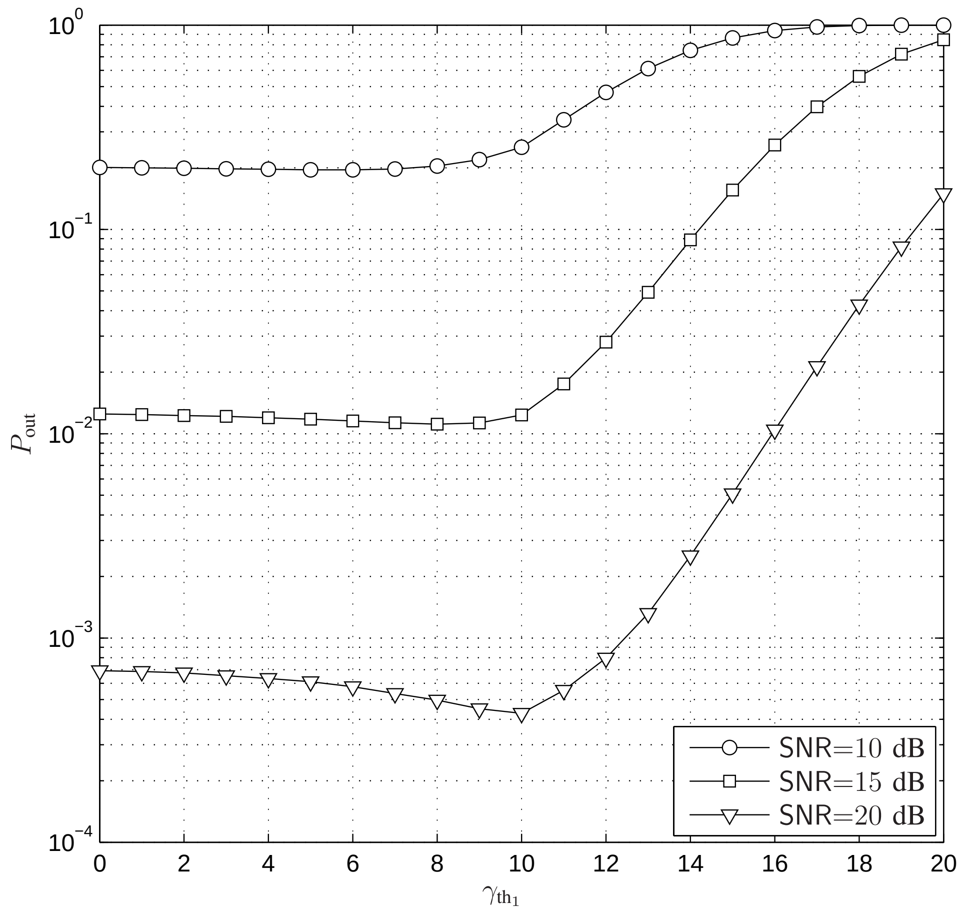

Fig. 6. Outage Probability as a function of the selection threshold γ_{th_1} for m_1=2, m_2=1, and different values of SNR.

V. Conclusion

We analyzed the outage performance of decode-and-forward relaying in the context of selective two-way cooperative systems. A new relay selection metric, under the form of a weighted sum-rate, was proposed, and we derived the expressions of the outage probability of the system over Nakagami-m fading channels. The impact of imperfect channel state information on the outage behavior was investigated and analytically quantified. The obtained results show that the proposed scheme outperforms conventional approaches, and give a realistic insight into the design of practical two-way relaying systems.

Appendix

CDF and PDF of the sum of two truncated Gamma variates

The CDF and the PDF of the sum of two Gamma variates have been derived in [27]. In a similar fashion, we derive here the CDF and the PDF of the sum of two truncated Gamma variates.

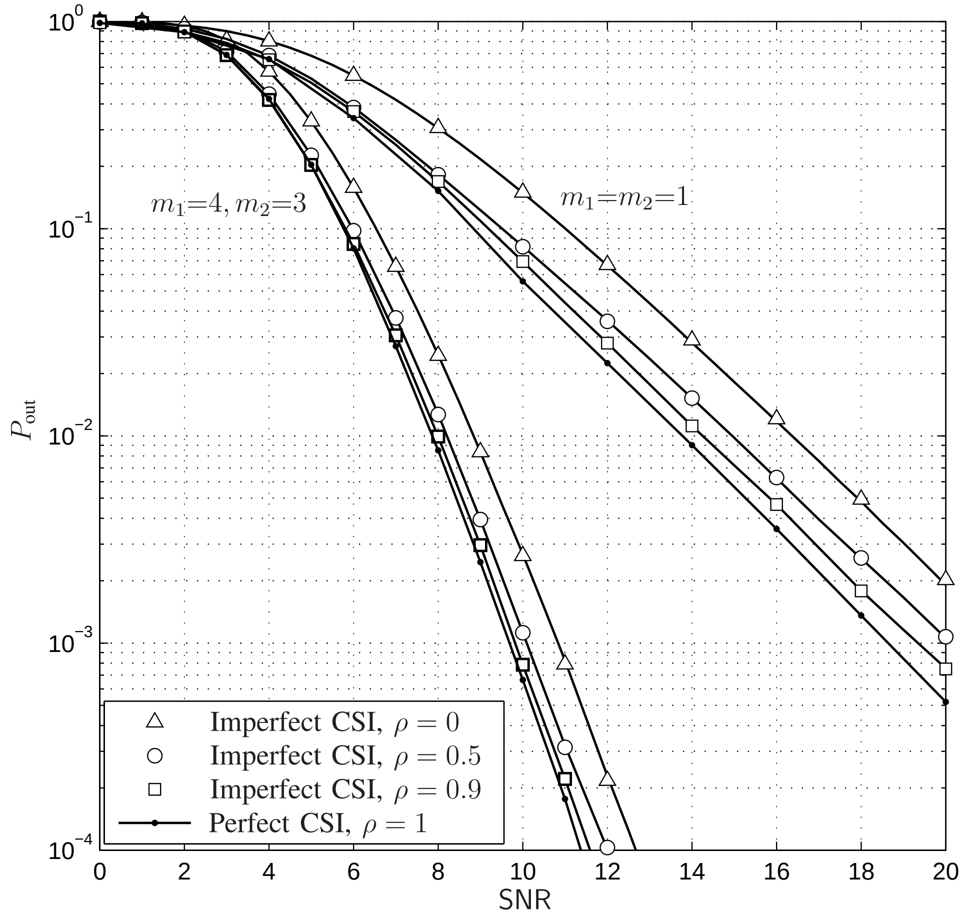

Fig. 7. Outage Probability as a function of the SNR, for the orthogonal transmission case, with three different values of the correlation coefficient ρ and different values of m_1 and m_2. Lines correspond to analytical results and simulation results are represented with markers.

Let $X_1 \sim \mathcal{G}\left(m_1, \Omega_1/m_1, x_{\text{th}_1}\right)$ and $X_2 \sim \mathcal{G}\left(m_2, \Omega_2/m_2, x_{\text{th}_2}\right)$ be two truncated gamma variates with respective realizations x_1 and x_2. Their PDFs, denoted by $f_{X_i}\left(x_i\right) i \in \{1,2\}$, are thus given by

$$\begin{cases} \dfrac{1}{1-\gamma\left(m_i, \dfrac{m_i x_{\text{th}_i}}{\Omega_i}\right)} \dfrac{x_i^{m_i-1}}{\Gamma(m_i)} \left(\dfrac{m_i}{\Omega_i}\right)^{m_i} \exp\left(-\dfrac{m_i x_i}{\Omega_i}\right) & \text{if } x \geq x_{\text{th}_i} \\ 0 & \text{otherwise} \end{cases} \tag{33}$$

We assume without any loss of generality that $\Omega_2/m_2 \geq \Omega_1/m_1$. Let Z be the random variable corresponding to the sum of the two truncated gamma variables X_1 and X_2. We denote by z the realization of Z and by x_{th} the sum of x_{th_1} and x_{th_2}.

In this Appendix, we derive the CDF of Z as

$$F_Z\left(z\right) = \Pr\left[X_1 + X_2 \leq z\right] = \begin{cases} \displaystyle\int_{x_{\text{th}_1}}^{z-x_{\text{th}_2}} F_{X_2}\left(z-x\right) f_{X_1}\left(x\right) \mathrm{d}x & \text{if } z \geq x_{\text{th}} \\ 0 & \text{otherwise} \end{cases}, \tag{34}$$

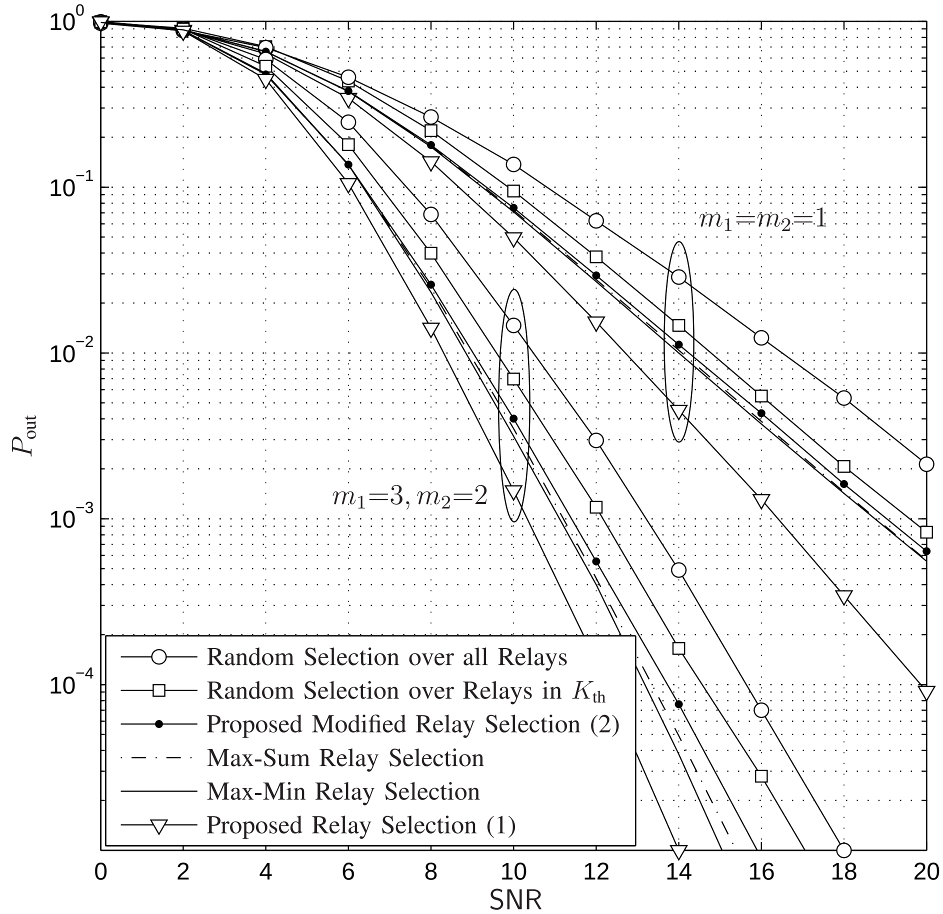

Fig. 8. Outage Probability as a function of the SNR, for the orthogonal transmission case, with different relay selection schemes.

where

$$F_{X_2}(z-x) = \frac{1}{1-\gamma\left(m_2, \frac{m_2 x_{\text{th}_2}}{\Omega_2}\right)} \times \left\{1-\exp\left(-\frac{m_2 z}{\Omega_2}\right)\sum_{k=0}^{m_2-1}\frac{1}{k!}\left(\frac{m_2}{\Omega_2}\right)^k \sum_{l=0}^{k}\binom{k}{l}(-1)^l z^{k-l}x^l \exp\left(\frac{m_2 x}{\Omega_2}\right) - \gamma\left(m_2, \frac{m_2 x_{\text{th}_2}}{\Omega_2}\right)\right\}. \quad (35)$$

Substituting (35) in (34) for $z \geq x_{th}$, we obtain

$$\begin{aligned} F_Z(z) = & \frac{1}{1-\gamma\left(m_2, \frac{m_2 x_{\text{th}_2}}{\Omega_2}\right)} \cdot \left\{ F_{X_1}(z - x_{\text{th}_2}) - F_{X_1}(x_{\text{th}_1}) - \frac{1}{1-\gamma\left(m_1, \frac{m_1 x_{\text{th}_1}}{\Omega_1}\right)} \right. \\ & \times \left(\frac{m_1}{\Omega_1}\right)^{m_1} \frac{1}{\Gamma(m_1)} \exp\left(-\frac{m_2 z}{\Omega_2}\right) \sum_{k=0}^{m_2-1} \frac{1}{k!}\left(\frac{m_2}{\Omega_2}\right)^k \sum_{l=0}^{k}\binom{k}{l}(-1)^l z^{k-l} \\ & \left. \times \int_{x_{\text{th}_1}}^{z-x_{\text{th}_2}} x^{\alpha} \mathrm{e}^{-\beta x} \mathrm{d}x - \gamma\left(m_2, \frac{m_2 x_{\text{th}_2}}{\Omega_2}\right)\left(F_{X_1}(z - x_{\text{th}_2}) - F_{X_1}(x_{\text{th}_1})\right) \right\}, \end{aligned} \quad (36)$$

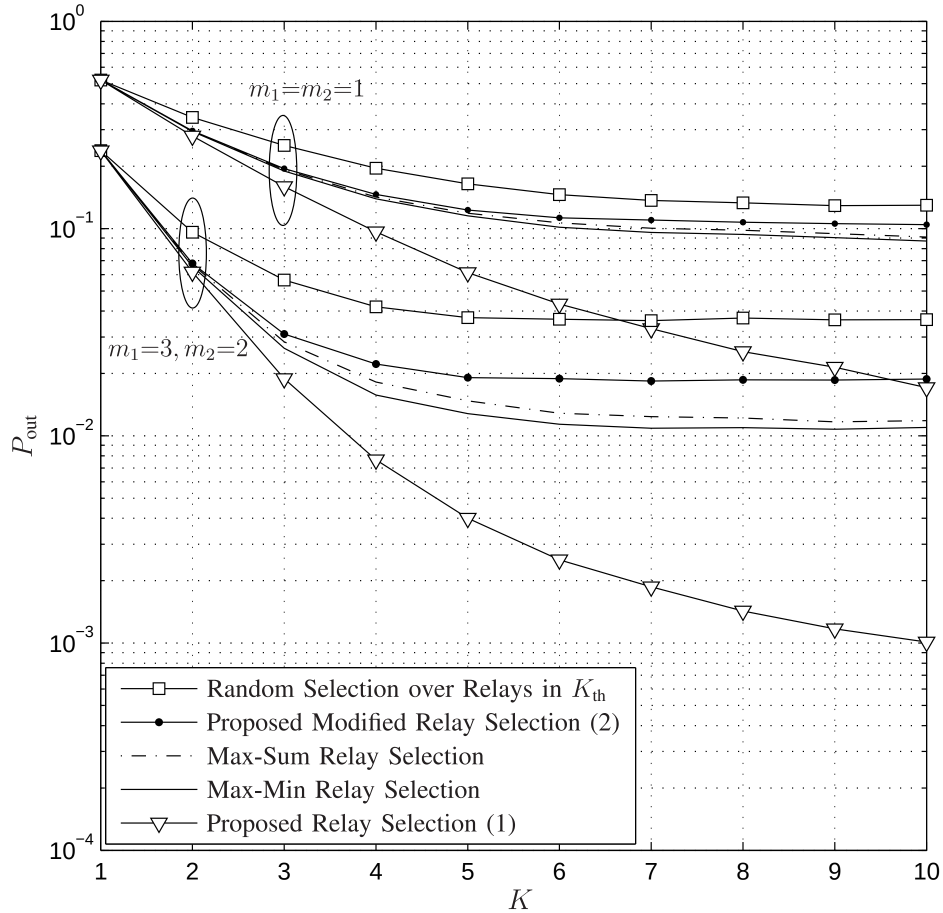

Fig. 9. Outage Probability as a function of the number of relays K, for the orthogonal transmission case, with different relay selection schemes.

with $\alpha = m_1 + l - 1$ and $\beta = \dfrac{m_1}{\Omega_1} - \dfrac{m_2}{\Omega_2}$.

The expression of the CDF when $z \geq x_{th}$ can be obtained in closed-form as

$$F_Z(z) = \frac{1}{1-\gamma\left(m_1, \frac{m_1 x_{\text{th}_1}}{\Omega_1}\right)} \cdot \frac{1}{1-\gamma\left(m_2, \frac{m_2 x_{\text{th}_2}}{\Omega_2}\right)} \cdot \left\{ \gamma\left(m_1, \frac{m_1\left(z - x_{\text{th}_2}\right)}{\Omega_1}\right) - \gamma\left(m_1, \frac{m_1 x_{\text{th}_1}}{\Omega_1}\right) \right.$$
$$\left. - \gamma\left(m_2, \frac{m_2 x_{\text{th}_2}}{\Omega_2}\right)\left(\gamma\left(m_1, \frac{m_1\left(z - x_{\text{th}_2}\right)}{\Omega_1}\right) - \gamma\left(m_1, \frac{m_1 x_{\text{th}_1}}{\Omega_1}\right)\right) - \vartheta\left(z, x_{\text{th}_1}, x_{\text{th}_2}; m_1, m_2, \Omega_1, \Omega_2\right) \right\}, \tag{37}$$

where the function $\vartheta(\cdot)$ is defined in (14).

By deriving the expression in (37) with respect to the variable z, we obtain the PDF of the sum of two truncated gamma variates as

$$f_Z(z) = \frac{1}{1-\gamma\left(m_1, \frac{m_1 x_{\text{th}_1}}{\Omega_1}\right)} \cdot \frac{1}{1-\gamma\left(m_2, \frac{m_2 x_{\text{th}_2}}{\Omega_2}\right)} \cdot \left\{ \frac{(z-x_{\text{th}_2})^{m_1-1}}{\Gamma(m_1)} \left(\frac{m_1}{\Omega_2}\right)^{m_1} \exp\left(-\frac{m_1(z-x_{\text{th}_2})}{\Omega_1}\right) \right.$$
$$- \gamma\left(m_2, \frac{m_2 x_{\text{th}_2}}{\Omega_2}\right) \left(\frac{(z-x_{\text{th}_2})^{m_1-1}}{\Gamma(m_1)}\right) \left(\frac{m_1}{\Omega_1}\right)^{m_1} \exp\left(-\frac{m_1(z-x_{\text{th}_2})}{\Omega_1}\right)$$
$$+ \left(\frac{m_2}{\mu_2}\right) \cdot \vartheta\left(z, x_{\text{th}_1}, x_{\text{th}_2}; m_1, m_2, \Omega_1, \Omega_2\right) - \xi\left(z, x_{\text{th}_1}, x_{\text{th}_2}; m_1, m_2, \Omega_1, \Omega_2\right)$$
$$\left. - \psi\left(z, x_{\text{th}_1}, x_{\text{th}_2}; m_1, m_2, \Omega_1, \Omega_2\right) \right\}, \tag{38}$$

where the functions $\xi(\cdot)$ and $\psi(\cdot)$ are defined, respectively in (16) and (17).

Note that, by setting $x_{\text{th}_1} = 0$ and $x_{\text{th}_2} = 0$, we obtain the CDF and PDF of the sum of two gamma variates [27] [28].

References

[1] C. E. Shannon, "Two-way communication channels," *4th Berkeley Symposium on Mathematical Statistics and Probability*, pp. 611–644, 1961.

[2] B. Rankov and A. Wittneben, "Spectral efficient signaling for half-duplex relay channels," *in Proc. Asilomar Conf. on Signals, Systems, and Computers, Pacific Grove, CA*, pp. 1066–1071, Nov. 2005.

[3] A. Müller and J. Speidel, "Relay selection in dual-hop transmission systems: selection strategies and performance results," *in Proc. IEEE ICC, Beijing, China*, pp. 4998–5003, May 2008.

[4] A. Bletsas, H. Shin, and M. Z. Win, "A simple cooperative diversity method based on network path selection," *IEEE J. Select. Areas Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.

[5] I. Krikidis, J. Thompson, S. McLaughlin, and N. Goertz, "Amplify-and-forward with partial relay selection," *IEEE Commun. Lett.*, vol. 12, no. 4, pp. 235–237, Apr. 2008.

[6] T. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, pp. 572–584, Sep. 1979.

[7] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[8] I. Krikidis, "Relay selection for two-way relay channels with MABC DF: A diversity perspective," *IEEE Trans. Veh. Technol.*, vol. 59, no. 9, pp. 4620–4628, Nov. 2010.

[9] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Wireless Commun.*, vol. 6, no. 9, pp. 3450–3460, Sep. 2007.

[10] K.-S. Hwang, Y.-C. Ko, and M.-S. Alouini, "Performance bounds for two-way amplify-and-forward relaying based on relay path selection," *in Proc. IEEE VTC-Spring, Barcelona, Spain*, pp. 1–5, Apr. 2009.

[11] T. Oechtering and H. Boche, "Bidirectional regenerative half-duplex relaying using relay selection," *IEEE Trans. Wireless Commun.*, vol. 7, no.5, pp. 1879–1888, May 2008.

[12] Q. F. Zhou, L. Yonghui, F. C. M. Lau, and B. Vucetic, "Decode-and-forward two-way relaying with network coding and opportunistic relay selection," *IEEE Trans. Commun.*, vol. 58, no. 11, pp. 3070–3076, Nov. 2010.

[13] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, “A simple cooperative diversity method based on network path selection,” *IEEE J. Select. Areas Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.

[14] L. Yonghui, R. H. Y. Louie, and B. Vucetic, “Relay selection with network coding in two-way relay channels,” *IEEE Trans. Veh. Technol.*, vol. 59, no. 9, pp. 4489–4499, Nov. 2010.

[15] Y. Huang, F. Al-Qahtani, C. Zhong, Q. Wu, J. Wang, and H. Alnuweiri, “Performance analysis of multiuser multiple antenna relaying networks with co-channel interference and feedback delay,” *IEEE Trans. Commun.*, vol. 62, no. 1, pp. 59–73, Jan. 2014.

[16] K.-S. Hwang, M. Ju, , and M.-S. Alouini, “Outage performance of opportunistic two-way amplify-and-forward relaying with outdated channel state information,” *IEEE Trans. Commun.*, vol. 61, no. 9, pp. 3635–3643, 2013.

[17] H. Cui, R. Zhang, L. Song, and B. Jiao, “Relay selection for bidirectional af relay network with outdated CSI,” *IEEE Trans. Veh. Technol.*, vol. 62, no. 9, pp. 4357–4365, 2013.

[18] A. Hyadi, M. Benjillali, and M.-S. Alouini, “Outage performance of two-way DF relaying systems with a new relay selection metric,” *in Proc. IEEE WCNC, Paris, France*, pp. 570–574, Apr. 2012.

[19] M. Benjillali, L. Szczecinski, and M.-S. Alouini, “A spectrally efficient detect-and-forward scheme with two-tier adaptive cooperation,” *IEEE Trans. Wireless Commun.*, vol. 59, no. 9, pp. 2367–2372, Sep. 2011.

[20] S. J. Kim, N. Devroye, P. Mitran, and V. Tarokh, “Achievable rate regions and performancevcomparison of half duplex bi-directional relaying protocols,” *IEEE Trans. Inform. Theory*, vol. 57, no. 10, pp. 6405–6418, Oct. 2011.

[21] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. 2nd ed. Wiley & Sons, 2006.

[22] P. Larsson, N. Johansson, and K.-E. Sunell, “Coded bi-direction relaying,” *in Proc. IEEE VTC-Spring, Montreal, Quebec, Canada*, pp. 851–855, May 2006.

[23] P. Popovski and H. Yomo, “Physical network coding in two-way wireless relay channel,” *in Proc. IEEE ICC, Glasgow, Scotland*, pp. 707–712, Jun. 2007.

[24] M. Benjillali and M.-S. Alouini, “Outage performance of reactive cooperation in Nakagami-m fading channels,” *in Proc. IEEE SPAWC, Marrakech, Morocco*, pp. 1–5, June 2010.

[25] X. Ji, B. Zheng, J. Cui, S. Tang, and X. Gao, “A study of half-duplex two-way decode-and-forward relay transmission with asymmetric traffic using superposition coding,” *in Proc IEEE 23rd PIMRC, Sydney, Australia*, pp. 1857–1861, Sept. 2012.

[26] J. Reig, L. Rubio, and N. Cardona, “Bivariate Nakagami-m with arbitrary fading parameters,” *Electronics Letters*, vol. 38, no. 25, pp. 1715–1717, Dec. 2002.

[27] G. K. Karagiannidis, N. C. Sagias, and T. A. Tsiftsis, “Exact closed-form statistics for the sum of squared Nakagami-m variates and its applications,” *IEEE Trans. Commun.*, vol. 54, no. 8, pp. 1353–1359, Aug. 2006.

[28] M. Benjillali and M.-S. Alouini, “Outage performance of decode-and-forward partial selection in Nakagami-m fading channels,” *in Proc. IEEE Int. Conf. on Telecommun. (ICT'10), Doha, Qatar*, pp. 71–76, Apr. 2010.