Outage Probability of Decode-and-Forward Cognitive Relay in Presence of Primary User's Interference

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Abstract—In the presence of the primary user's interference, the outage probability of a dual-hop decode-and-forward (DF) cognitive relay network (CRN) over Rayleigh fading channels is investigated. We first derive the exact expression for the outage probability, using which the impact of different system parameters on the outage performance is presented in the asymptotic regimes. In addition, the asymptotic outage probability is also derived in the high signal-to-noise ratio (SNR) regime.

Index Terms—Spectrum sharing, cognitive relay, decode-and-forward, outage probability

I. INTRODUCTION

C OGNITIVE relay networks (CRNs) improve the spectrum efficiency of wireless networks. Among the existing spectrum access paradigms, namely, *interleave*, *overlay* and *underlay* [1], the underlay scheme offers several practical advantages, *i.e.*, spectrum sharing approach, allowing the secondary user to share the spectrum of primary user provided the interference on the primary user is below a threshold. Performance analysis of spectrum-sharing CRNs has thus gained significant research interest in the literature.

For the amplify-and-forward (AF) CRNs, the outage performance over Rayleigh fading channels has been studied in [2]. In [3], three different relay selection strategies are considered, and the asymptotic outage probability in the high signal-tonoise ratio (SNR) regime is derived. For the decode-andforward (DF) CRNs, the closed-form outage performance is derived over Rayleigh fading channels in [4], proving that full selection diversity is realizable. However, [4] did not investigate the asymptotic outage probability. In [5], the outage probability analysis is extended to the Nakagami-*m* fading case with the same system set-up as [4]. The relay selection problem in multiple DF CRNs is studied in [6]–[8]. In [6], three relay selection schemes have been proposed for the DF CRNs, and their outage probabilities are also derived. In [7], the authors consider the DF CRN with direct link,

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and an approximate outage probability for the relay selection is obtained. In [8], Luo *et al.* investigate the same system set-up as [7], and an accurate approximation for the outage probability is derived.

However, the aforementioned works [2]-[8] ignore the interference from the primary transmitter to secondary receivers. In a practical spectrum-sharing CRN, since the secondary and primary users coexist in the same spectral band, the interference at secondary receiver, generated by the nearby primary transmitters, is not negligible and must be considered in the performance analysis [9]. Recently, the effect of primary user interference on the secondary AF CRN was analyzed in [10], which did not consider the maximum transmit power constraint at the secondary users. Motivated by these observations, this letter derives the exact and asymptotic outage probability for DF CRNs. The analysis considers Rayleigh fading channels and reveals the detrimental effect of the primary user's interference on the secondary system performance. For more insightful results, the impact of key system parameters on the outage performance is also investigated. Monte-Carlo simulations are presented to validate the theoretical analysis.

II. SYSTEM AND CHANNEL MODEL

Consider a dual-hop spectrum-sharing DF CRN (Fig.1). In the primary system, the primary transmitter (PU_T) , communicates with the primary receiver (PU_D) . In the secondary system, the secondary source $(S U_S)$ communicates with the secondary destination $(S U_D)$ with the help of a secondary DF relay (SU_R) . The direct link between SU_S and SU_D is assumed not available due to channel impairments such as shadowing, other macro impairments, or the receiver limitations. As mentioned before, the interference at SU_R and $S U_D$, which is generated by PU_T , is considered in this letter. In particular, SU_S and SU_R are allowed to use the same frequency as the primary system if the interference generated on PU_D remains below the interference threshold \overline{I} , which is the maximum interference powers tolerable at PU_D [1]. Thus, the transmit power of SU_S and SU_R must satisfy $P_s \leq \min(\frac{\overline{I}}{|h_{sp}|^2}, P), P_r \leq \min(\frac{\overline{I}}{|h_{rp}|^2}, P)$, respectively [5], where P is the maximum transmit power constraint of SU_S and SU_R . $|h_{ij}|^2$ $(i \in \{s, r, p\}, j \in \{d, r, p\}, i \neq j)$ are the channel gains as shown in Fig.1, which are exponentially distributed with mean value $E[|h_{ij}|^2] = \frac{1}{\lambda_{ij}}$. $E[\cdot]$ is the statistical expectation.

III. EXACT OUTAGE PERFORMANCE ANALYSIS

Outage probability is the probability that the instantaneous mutual information falls below a threshold rate R. Therefore,

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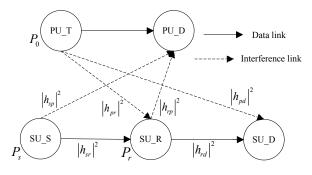


Fig. 1. The system model for a dual-hop CRN, where h_{ij} and $|h_{ij}|^2$ $(i \in \{s, r, p\}, j \in \{d, r, p\}, i \neq j)$ are the channel fading and channel powers, respectively.

for the dual-hop DF CRN, it can be formulated as [11]

$$P_{out}(\gamma_{th}) = P_r \left[\min(\frac{P_s |h_{sr}|^2}{P_0 |h_{pr}|^2 + N_0}, \frac{P_r |h_{rd}|^2}{P_0 |h_{pd}|^2 + N_0}) < \gamma_{th} \right], \quad (1)$$

where $\gamma_{th} = 2^{2R} - 1$ the corresponding threshold signal-tointerference-and-noise ratio (SINR), P_0 is the transmit power of the PU_T , and N_0 is noise variance at the SU_R and SU_D . From the theory of order statistics [12], we find

$$P_{out}(\gamma_{th}) = 1 - (1 - F_{\gamma_1}(\gamma_{th}))(1 - F_{\gamma_2}(\gamma_{th})),$$
(2)

where $\gamma_1 = \frac{\min(\frac{1}{|h_{sp}|^2}, P)|h_{sr}|^2}{P_0|h_{pr}|^2 + N_0}$ and $\gamma_2 = \frac{\min(\frac{1}{|h_{rp}|^2}, P)|h_{rd}|^2}{P_0|h_{pd}|^2 + N_0}$ are SINRs of the first and the second hop, respectively. $F_{\gamma_i}(x), i = 1, 2$ are the cumulative distribution functions (CDF) of $\gamma_i, i = 1, 2$.

Next, we derive the CDF of $\gamma_1 = \frac{U_1}{P_0 V_1 + N_0}$, where $U_1 = \min(\frac{\bar{I}}{|h_{sr}|^2}, P) |h_{sr}|^2$ and $V_1 = |h_{pr}|^2$. By using the definition of CDF of γ_1 , we find

$$F_{\gamma_1}(x) = \int_0^\infty \Pr(U_1 < (P_0 y + N_0) x) f_{V_1}(y) dy.$$
(3)

With the help of [4, Eq. (8)], and substituting the probability density function (PDF) of the exponential random variable V_1 , we find the CDF to be

$$F_{\gamma_{1}}(x) = 1 + \int_{0}^{\infty} \left(\frac{x(P_{0}y + N_{0})e^{-\lambda_{sp}\bar{I}/P}}{\lambda_{sp}\bar{I}/\lambda_{sr} + x(P_{0}y + N_{0})} - 1 \right) \\ \times e^{-(P_{0}y + N_{0})x\lambda_{sr}/P}\lambda_{pr}e^{-\lambda_{pr}y}dy.$$
(4)

After several algebraic manipulations, the integral (4) can be equivalently expressed as

$$F_{\gamma_1}(x) = 1 + \lambda_{pr} e^{-\lambda_{sr} x N_0/P} \int_0^\infty e^{-(xP_0 \lambda_{sr}/P + \lambda_{pr})y} \\ \times \left[(e^{-\lambda_{sp} \overline{I}/P} - 1) - \frac{e^{-\lambda_{sp} \overline{I}/P} \lambda_{sp} \overline{I}/\lambda_{sr}}{xP_0 y + \lambda_{sp} \overline{I}/\lambda_{sr} + xN_0} \right] dy.(5)$$

With the help of [13, Eq.(3.352.4)], and after some simplification, the CDF of γ_1 can be derived as (6), where $Ei(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt$ is the exponential integral function [13, Eq.(8.211.1)]. Using the same approach, the CDF of γ_2 can be derived as (7). Finally, substituting (6) and (7) into (2), the exact outage probability of the above DF CRN can be obtained.

To get insights about the impact of different system parameters on the outage probability, we investigate the following asymptotic regimes.

Scenario 1: The primary user's interference at secondary system is ignored ($P_0 \rightarrow 0$). In this case, since there is no interference from the primary transmitter, the corresponding outage analysis reduces to the previous work [4].

Scenario 2: The primary user can tolerate an unlimited interference from the secondary user $(\bar{I} \rightarrow \infty)$. According to [14, Eq.(5.1.7) and (5.1.20)], we have $\lim_{x \rightarrow \infty} e^x Ei(-x) = 0$. It is clear that $\lim_{x \rightarrow \infty} x/e^x = 0$. Using these formulas, for $\bar{I} \rightarrow \infty$, the CDFs in (6)-(7) can be approximated as

$$F_{\gamma_1}(x) \approx 1 - \frac{\lambda_{pr} e^{-\lambda_{sr} x N_0/P}}{\lambda_{pr} + \lambda_{sr} x P_0/P},$$
(8)

$$F_{\gamma_2}(x) \approx 1 - \frac{\lambda_{pd} e^{-\lambda_{rd} x N_0/P}}{\lambda_{pd} + \lambda_{rd} x P_0/P}.$$
(9)

In this case, $\min(\frac{\overline{I}}{|h_{ij}|^2}, P) \approx P$ with probability 1, suggesting that the secondary transmitters can transmit with any maximum power without an interference constraint.

IV. Asymptotic Outage Performance Analysis

Since the exact analysis is too complicated to render insight on the impact of primary user's interference, the asymptotic outage probability is investigated in the high SNR regime ¹.

For given $|h_{sp}|^2$ and $|h_{pr}|^2$, γ_1 is exponentially distributed with $E[\gamma_1] = \frac{\alpha}{\lambda_{sr}}, \alpha \stackrel{\Delta}{=} \frac{\min(\frac{1}{|h_{sp}|^2}, P)}{P_0|h_{pr}|^2 + N_0}$. Similarly, for given $|h_{rp}|^2$ and $|h_{pd}|^2$, γ_2 is exponentially distributed with $E[\gamma_2] = \frac{\beta}{\lambda_{rd}}, \beta \stackrel{\Delta}{=} \frac{\min(\frac{1}{|h_{rp}|^2}, P)}{P_0|h_{pd}|^2 + N_0}$. Therefore, for arbitrary α and β , the outage probability can be obtained as follows:

$$P_{out}(\gamma_{th} | \alpha, \beta) = 1 - e^{-\frac{\lambda_{sr}}{\alpha} \gamma_{th}} e^{-\frac{\lambda_{rd}}{\beta} \gamma_{th}}.$$
 (10)

With the help of the Taylor series $e^x = 1 + x + o(x)$ [13, Eq. (1.211.1)], the outage probability can be approximated as

$$P_{out}(\gamma_{th} | \alpha, \beta) = \left(\frac{\lambda_{sr}}{\alpha} + \frac{\lambda_{rd}}{\beta}\right) \gamma_{th} + o(\gamma_{th}), \tag{11}$$

where $\lim_{x \to 0} o(x)/x = 0$.

Averaging over the random variables α and β in (11), the asymptotic outage probability can be obtained as follows:

$$P_{out}(\gamma_{th}) \approx \left(\lambda_{sr} E\left[\frac{1}{\alpha}\right] + \lambda_{rd} E\left[\frac{1}{\beta}\right]\right) \gamma_{th}.$$
 (12)

Lemma 1. The mean of $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ can be expressed as

$$E\left[\frac{1}{\alpha}\right] = \left(\frac{P_0}{\lambda_{pr}} + N_0\right) \left[\frac{1 - \exp(-\lambda_{sp}\bar{I}/P)}{P} + \frac{\Gamma(2, \lambda_{sp}\bar{I}/P)}{\lambda_{sp}\bar{I}}\right], (13)$$

$$E\left[\frac{1}{\beta}\right] = \left(\frac{P_0}{\lambda_{pd}} + N_0\right) \left[\frac{1 - \exp(-\lambda_{rp}\bar{I}/P)}{P} + \frac{\Gamma(2, \lambda_{rp}\bar{I}/P)}{\lambda_{rp}\bar{I}}\right].$$
(14)

¹The outage performance in high SNR regime $(\frac{1}{\lambda_{sr}N_0}, \frac{1}{\lambda_{rd}N_0} \rightarrow \infty)$ is decided by the behavior of the PDF of SNR at zero [15, Prop. 5].

$$F_{\gamma_1}(x) = 1 - \frac{\lambda_{pr}}{e^{\lambda_{sr}xN_0/P}} \left[\frac{1 - e^{-\lambda_{sp}\bar{l}/P}}{\lambda_{pr} + \lambda_{sr}xP_0/P} - \frac{\bar{l}\lambda_{sp}/\lambda_{sr}}{e^{\lambda_{sp}\bar{l}/P}xP_0} e^{(\bar{l}\lambda_{sr}/\lambda_{sr} + xN_0)(\lambda_{sr}/P + \lambda_{pr}/(xP_0))} Ei(-(\bar{l}\lambda_{sp}/\lambda_{sr} + xN_0)(\lambda_{sr}/P + \lambda_{pr}/(xP_0))) \right].$$
(6)

$$F_{\gamma_2}(x) = 1 - \frac{\lambda_{pd}}{e^{\lambda_{rd}xN_0/P}} \left[\frac{1 - e^{-\lambda_{rp}\bar{I}/P}}{\lambda_{pd} + \lambda_{rd}xP_0/P} - \frac{\bar{I}\lambda_{rp}/\lambda_{rd}}{e^{\lambda_{rp}\bar{I}/P}xP_0} e^{(\bar{I}\lambda_{rp}/\lambda_{rd} + xN_0)(\lambda_{rd}/P + \lambda_{pd}/(xP_0))} Ei(-(\bar{I}\lambda_{rp}/\lambda_{rd} + xN_0)(\lambda_{rd}/P + \lambda_{pd}/(xP_0))) \right].$$
(7)

$$P_{out}(\gamma_{th}) \approx \gamma_{th} \left[\lambda_{sr} \left(\frac{P_0}{\lambda_{pr}} + N_0 \right) \left(\frac{1 - \exp(-\lambda_{sp}\bar{I}/P)}{P} + \frac{\Gamma(2,\lambda_{sp}\bar{I}/P)}{\lambda_{sp}\bar{I}} \right) + \lambda_{rd} \left(\frac{P_0}{\lambda_{pd}} + N_0 \right) \left(\frac{1 - \exp(-\lambda_{rp}\bar{I}/P)}{P} + \frac{\Gamma(2,\lambda_{rp}\bar{I}/P)}{\lambda_{rp}\bar{I}} \right) \right] (18)$$

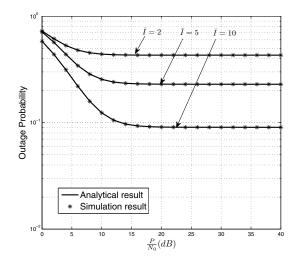


Fig. 2. Outage probability versus maximum transmit power to noise ratio and different interference levels \bar{I} .

Proof: Based on the definition of α , it yields

$$E\left[\frac{1}{\alpha}\right] = E[P_0 \left|h_{pr}\right|^2 + N_0] \times E\left[\frac{1}{\min(\frac{\bar{l}}{\left|h_{sp}\right|^2}, P)}\right],$$
 (15)

where $E[P_0 |h_{pr}|^2 + N_0] = P_0 / \lambda_{pr} + N_0$. After some mathematical operation, we have

$$E\left[\frac{1}{\min(\frac{\bar{I}}{|h_{sp}|^{2}}, P)}\right] = \int_{0}^{\frac{\bar{I}}{P}} \frac{1}{P} f_{|h_{sp}|^{2}}(x) dx + \int_{\frac{\bar{I}}{P}}^{\infty} \frac{x}{\bar{I}} f_{|h_{sp}|^{2}}(x) dx$$
$$= \frac{1}{P} \left(1 - \exp(-\lambda_{sp}\bar{I}/P)\right) + \frac{\lambda_{sp}}{\bar{I}} \int_{\frac{\bar{I}}{P}}^{\infty} x \exp(-\lambda_{sp}x) dx. \quad (16)$$

With the help of [13, Eq.(3.351.2)], it can obtained that

$$\int_{\frac{I}{P}}^{\infty} x \exp(-\lambda_{sp} x) dx = \lambda_{sp}^{-2} \Gamma(2, \lambda_{sp} \overline{I} / P), \qquad (17)$$

where $\Gamma(n, x)$ is the upper incomplete gamma function [13, Eq.(8.352.7)]. Therefore, based on (15)-(17) and after some simplification, we can obtain $E\left[\frac{1}{\alpha}\right]$ as (13). With the similar approach, $E\left[\frac{1}{\beta}\right]$ can also be derived.

With the help of (12)-(14), the asymptotic outage probability can be finally obtained as (18). It can be observed that the

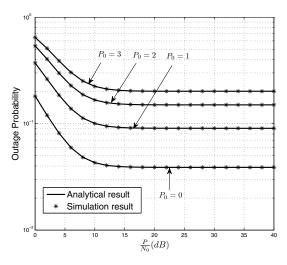


Fig. 3. Outage probability versus the maximum transmit power to noise ratio and different PU_T 's transmit powers P_0 .

primary user's interference introduces penalty items (*i.e.*, $\frac{P_0}{\lambda_{pr}}$ and $\frac{P_0}{\lambda_{pd}}$) in the outage probability of the secondary system, which degrade the outage performance.

V. SIMULATION RESULTS

This section provides simulation results to verify the analytical results. The following analytical results for the cases with PU_T 's interference are calculated from (2), (6) and (7).

Fig. 2 illustrates the outage probability with respect to the maximum transmit power to noise ratio P/N_0 under different interference levels \overline{I} . Without any loss of generality, the following parameter values are used: $\lambda_{pr} = \lambda_{pd} = 0.5$, $\lambda_{sr} = \lambda_{rd} = 1$, $\lambda_{sp} = \lambda_{rp} = 1$, $N_0 = 1$, $P_0 = 1$, and $\gamma_{th} = 0.3$. The analytical results and the simulation results match exactly. A floor in the outage performance curve is observed, which is due to the interference level constraint. Moreover, as the interference threshold increases, the outage probability floor decreases, *i.e.* the outage performance of the system improves.

Fig. 3 illustrates the outage probability under different PU_T 's transmit powers P_0 . The analytical outage performance for the no-primary-user's-interference case, *i.e.* the primary user's interference is not considered (*i.e.* $P_0 = 0$, equivalently), is obtained from the Eq.(8) in [4]. The following parameter

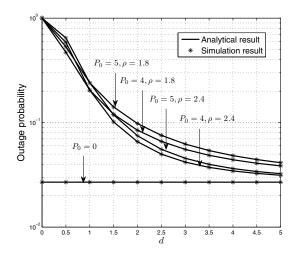


Fig. 4. Outage probability versus the location of PU_T and different PU_T 's transmit powers P_0 with pathloss exponents ρ .

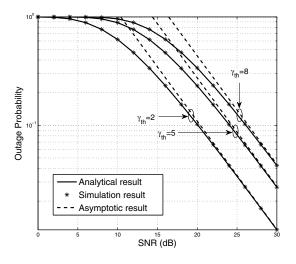


Fig. 5. Outage probability versus the SNR $\frac{1}{\lambda_{sr}N_0}$ for threshold SINR γ_{th} .

values are used: $\lambda_{pr} = \lambda_{pd} = 0.5$, $\lambda_{sr} = \lambda_{rd} = 1$, $\lambda_{sp} = \lambda_{rp} = 1$, $N_0 = 0.5$, $\bar{I} = 5$, and $\gamma_{th} = 0.1$. As expected, the outage probability improves, when the PU_T 's transmit power goes smaller. However, the outage probability of the no-primary-user's-interference case also becomes saturated, which is due to the interference level constraint on the SU_S and SU_R .

Fig. 4 shows the outage probability for different locations of PU_T . It is assumed that $\lambda_{pr}^{-1} = \lambda_{pd}^{-1} = d^{-\rho}$, where ρ is the pathloss exponent and d is distance between PU_T and secondary receiver, and other parameters' configuration is the same as the one in Fig. 3. It can be found that the outage performance is better for the case with smaller PU_T 's transmit power and larger pathloss exponent. As expected, when PU_T locates closer to the secondary receivers, the outage performance of the secondary system deteriorates.

Fig. 5 shows the outage probability in different SNR regimes for the SU_S - SU_R link with various threshold SINR γ_{th} , and the asymptotic results based on (18) are also presented. The following parameter values are used: $\lambda_{pr} = \lambda_{pd} = 1$, $\lambda_{sp} = \lambda_{rp} = 1$, $N_0 = 1$, $P = P_0 = 1$, $\overline{I} = 1$, and $\lambda_{sr} = \lambda_{rd}$. Again, the simulation results match the analytical

results very well, and as the threshold SINR increases, the outage performance degrades. Observe that the asymptotic expressions are highly accurate in the medium and high SNR regimes.

VI. CONCLUSIONS

The impact of the primary user's interference on the secondary relay system for a spectrum-sharing DF CRN is evaluated. Under Rayleigh fading channels, the new, exact expression for outage probability has been derived. Using it, the impact of various key system parameters has been explored in the asymptotic regimes. For more insightful results, in the high SNR regime, the asymptotic outage probability was also derived, resulting in a good approximation for the exact outage performance. Moreover, our analytical and simulation results unsurprisingly reveals that the primary-user interference degrades the reliability of the secondary network. This degradation may be overcome by the use of directional antennas at the primary transmitter or interference cancelation techniques at the secondary receiver, which depend on the specific physical-layer transmission strategies in the CRN. This topic may be investigated in the future.

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