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## OUTPUT, EMPLOYMENT, AND WAGES IN THE SHORT RUN

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### I. INTRODUCTION

This paper began as an attempt to clarify the relation between two alternative theories of distribution.<sup>1</sup> One theory is usually, though not quite accurately, called the marginal productivity theory; it makes the distribution of income depend mainly, but not entirely, on technological conditions. The other theory we shall call the Cambridge theory because it has been argued, in slightly different ways, by Nicholas Kaldor,<sup>2</sup> Joan Robinson, and Luigi Pasinetti; in that theory the distribution of income is made to depend primarily or exclusively on the different propensities to spend and save wage income and profits.

It soon became clear to us that the essence of the relation between the two theories is this: in the marginal productivity theory the main function of the real wage is to clear the labor market, while in the Cambridge theory the main function of the real wage is to clear the commodity market. We were led, by this route, to a slightly novel theory of the determination of aggregate output and employment in the short run.<sup>3</sup> In particular, we pay explicit attention to

1. We are grateful to friends who read an earlier draft of this paper and offered us many interesting, useful and occasionally contradictory suggestions. They include Morris Adelman, Gottfried Haberler, Frank Hahn, Edwin Kuh, Franco Modigliani, E. S. Phelps, Michael Piore, Hugh Rose, Amartya Sen, and John Williamson. We thank the National Science Foundation for financial support under research grant GS 1791.

2. Nicholas Kaldor, "Alternative Theories of Distribution," *Review of Economic Studies*. XXIII (1955-56).

3. Sen formulates the problem in a similar way, but goes in a different direction altogether. A. K. Sen, "Neo-Classical and Neo-Keynesian Theories

the nature of the aggregate supply of output; in this we are returning to the method of the *General Theory*. But we do not assume that the price level adjusts immediately to clear the market for goods, i.e., to equate aggregate supply and aggregate demand, any more than we assume that the money wage adjusts to clear the market for labor. Much does depend, however, on the character and speed of the reaction of prices and wages to disequilibrium in the goods market and to unemployment of labor.

The model we have constructed may help to elucidate two other questions about Keynesian economics. It permits a kind of underemployment equilibrium, although money wages have a certain amount of downward flexibility (thus suggesting that the operational significance of "rigid wages" is merely inability of the money wage to clear the labor market instantaneously). In addition it shows how the real wage may respond in either direction to fluctuations in effective demand, although production exhibits impeccable diminishing returns to labor in the short run.

Our analysis is limited strictly to the short run. Everything that happens is supposed to happen so quickly that the effects of current investment on the size and composition of the capital stock can be neglected. (Current investment is treated as exogenous.) The distributional impact of variations in effective demand can thus be studied in isolation. Moreover, any attempt at a *macroeconomic* application of marginal productivity theory in the long run is open to the usual criticism: except under the most stringent assumptions, there may not be any simply-defined "production function" whose partial derivatives can be interpreted as marginal productivities and related to factor prices. In the short run, with a given inventory of capital goods, this difficulty does not arise. If we think of labor as the only variable factor of production (which amounts to neglecting what Keynes called "marginal user cost"), no more than the usual aggregation problem is involved in treating aggregate output as a function of aggregate employment, whose slope is the short-run marginal product of labor. If the results of this investigation prove acceptable and interesting, then we are faced with the problem of extending them to the long run, i.e., of incorporating a reasonable of Distribution," *Economic Record*, XXXIX (March 1963). Phelps explores yet another route. E. S. Phelps, "Short-Run Employment and Real Wage Rate Under Market-Clearing Prices," *International Economic Review*, forthcoming. Our approach has a lot in common with that of Rose and Williamson. Hugh Rose, "On the Non-Linear Theory of the Employment Cycle," *Review of Economic Studies*, XXXIV (April 1967). John Williamson, "The Price-Price Spiral," *Yorkshire Bulletin of Economic and Social Research*, Vol. 19 (1967), pp. 3-14. An earlier predecessor is Bent Hansen, *A Study in the Theory of Inflation* (London; Allen, 1951).

representation of the way the short-run production function is shifted by current investment.

To simplify the analysis, we have ignored monetary factors. Thus the demand for goods and services is assumed not to depend on the rate of interest or on cash balances. This is odd in a theory in which money wages and prices play an important role; we hope the reader will see that the model could easily be extended to include the standard LM-IS apparatus. In the meanwhile, imagine that the monetary authorities follow the policy of maintaining a constant rate of interest, or a constant ratio of money supply to current-price value of output, or follow some other permissive policy.

## II. BUILDING BLOCKS

Given the stock of capital goods inherited from the past, real output ( $Y$ ) is a function of labor input ( $N$ ) alone. The short-run marginal product of labor is positive. There are likely to be short-run diminishing returns to labor alone, because less efficient capacity must be drawn into use at higher levels of output. But the econometric indications on this point are unclear, perhaps because a component of overhead labor results in a phase of increasing average productivity of labor in the short run, along with decreasing marginal productivity, perhaps because of frictions in the adjustment of employment to changes in output. In any case, we assume that the marginal product of labor is lower at higher outputs. Thus for the short-run production function we have

$$(1) \quad Y = F(N), F'(N) > 0, F''(N) < 0.$$

Under more or less competitive conditions, the aggregate supply of output ( $Y^*$ ) at any price level ( $p$ ) and money wage ( $w$ ) is the profit-maximizing output, or the output at which price equals marginal cost, or, since labor is the only variable input, the output at which the marginal product of labor is equal to the real wage. If we let  $f(\cdot)$  stand for the inverse function of  $F'$ , and let  $v = w/p$  be the real wage, then

$$(2) \quad Y^* = F(f(w/p)) = G(v), G' < 0;$$

aggregate supply at any real wage is the output corresponding to the employment at which the marginal product of labor equals the real wage, and is therefore a decreasing function of the real wage, because of diminishing returns.<sup>4</sup>

4. Under some market structures, naturally, no supply curve exists at all. One can imagine imperfectly-competitive situations in which a curve like (2) makes sense, but with a fixed markup on marginal cost. Then the middle term in (2) would read  $F(f(m \frac{w}{p}))$ , where  $m$  is the markup.

This is a conventional short-run supply curve, in the sense that the stock of capital goods is fixed while employment varies. But there is strong econometric evidence that employment adjusts to changes in the demand for output only with a delay that presumably reflects both uncertainty and frictional costs. At any instant of time, therefore, employment, too, can be regarded as fixed, and the wage bill as a fixed cost. In other words, in the very short run, marginal cost is zero for outputs up to  $F(N)$  and (as a simplification) prohibitively high for larger outputs. The momentary supply curve for output is thus completely inelastic at the level  $F(N)$ :

$$(3) \quad Y^* = F(N).$$

Employment does, however, adjust: toward what? Presumably it adjusts toward the level appropriate to an expected rate of output. A natural choice is the smaller of  $Y^D$  and  $Y^S$ , where  $Y^D$  is aggregate demand for output, to be discussed in a moment, and  $Y^S$  has already been defined. Given a going real wage and rate of employment, firms supply momentarily their current rate of output. They will wish to supply  $Y^S$  when they have had time to adjust employment appropriately. But this intention may be overridden if the limit to output is actually on the demand side. We adopt the simple linear adjustment process that moves employment each instant some part of the way from its current level toward its target level,  $F^{-1}(\min(Y^D, Y^S))$ :

$$(4) \quad N' = \theta(F^{-1}(\min(Y^S, Y^D)) - N);$$

where  $N'$  stands for  $dN/dt$ ,  $F^{-1}(\cdot)$  is the inverse function of  $F$ , and  $\theta$  is a positive constant. For short-run analysis to make sense,  $\theta$  must not be too small. The econometric evidence is that it is not.

On the side of aggregate demand ( $Y^D$ ) we have nothing new to offer. We treat investment ( $I$ ) as exogenous, and we ignore direct monetary influence on demand. To accommodate the Cambridge theory we allow for possibly different marginal propensities to consume ( $1 - s_w$  and  $1 - s_p$ ) wage and profit incomes.

Thus

$$(5) \quad Y^D = I + (1 - s_w)vN + (1 - s_p)(Y^D - vN).$$

Notice that aggregate demand is entirely in real terms. Notice also that we have inserted  $Y^D$  on the right-hand side of (5), and not  $Y$ ; the aggregate demand function gives the sustainable real demand generated by any given rate of investment, wage bill, and propensities to save.<sup>5</sup> Then (3) can be solved to yield

5. It can be shown that inserting actual output on the right-hand side of (5) would not change the qualitative character of our results, but would create a peculiar simultaneity in a model with continuous time.

$$(6) \quad Y^D = \frac{I}{s_p} + \frac{s_p - s_w}{s_p} vN,$$

so that, for given  $I$  and  $N$ , aggregate demand is an increasing function of the real wage, provided that  $s_p > s_w$ . Obviously, if  $s_p = s_w = s$ , then aggregate demand is simply  $I/s$ , investment times the ordinary multiplier. That traditional case is therefore included in the analysis that follows.

At the current moment,  $N$  is historically given. The momentary supply of output is  $Y^*$ ; the demand for output,  $Y^D$ , is an increasing (or perhaps constant) function of  $v$ . There will usually be a positive real wage  $v_0$  at which  $Y^D = Y^*$ ; if the real wage happened to equal  $v_0$ , the goods market would be momentarily cleared. (Even so, the money wage and price level might be changing, as we will discuss soon.) If the current real wage is  $v < v_0$ , there is momentary excess supply of goods,  $Y^* > Y^D$ ; if  $v > v_0$ , there is momentary excess demand for goods,  $Y^* < Y^D$ . We assume that

$$(7) \quad Y = \min(Y^*, Y^D).$$

Remember the distinction between momentary and short-run equilibrium. At any given moment, wages, prices, and employment are given; all the firm has to decide is its output—hence (7). In the short run, both output and employment are variable. In full short-run macroeconomic equilibrium, with a given fixed level of investment, real wages are constant, and firms have no incentive to change either output or employment.

Whenever the real wage is such as to generate excess supply of goods, actual output is limited by effective demand and is, in fact, equal to effective demand.<sup>6</sup> When the real wage is such as to generate excess demand for goods, firms produce whatever their outfit of capital goods and current level of employment permit them to produce and sell.

At the same time, firms reduce or add to current employment, according to (4). If there is momentary excess supply, they lay off workers. If there is momentary excess demand they add workers, unless the going real wage is so high that it would be unprofitable to satisfy the demand or even to produce as much as is now being produced. In that case, they reduce employment (but they raise prices as well, only not all at once).

The picture is described in Figure I, drawn for the current level of  $N$ . If  $v < v_0$ , current output is given by the ordinate of  $Y^D$ ; if  $v > v_0$ ,  $Y = Y^*$ . Meanwhile employment is also changing. The

6. This amounts to assuming away unintended inventory accumulation. To do otherwise would considerably complicate the later dynamic analysis.

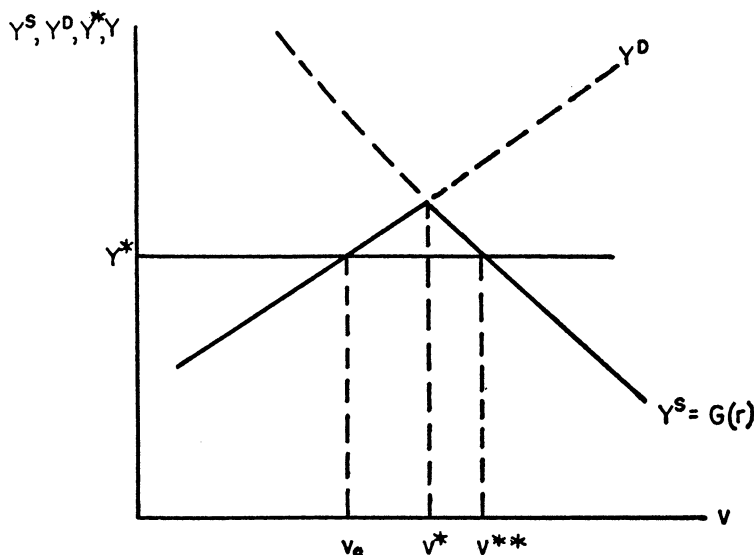


FIGURE I

direction of change can be traced in Figure I, in view of the monotone correspondence between  $Y$  and  $N$  via  $Y = F(N)$ . Employment falls from its current level,  $F^{-1}(Y^*)$  whenever  $Y^*$  is above the inverted-V curve  $\min(Y^S, Y^D)$ , and rises in the opposite case. Thus for  $v < v_0$  and for  $v > v^{**}$  employment falls, for,  $v_0 < v < v^{**}$ , it rises. At  $v = v_0$  and  $v = v^{**}$ , employment is temporarily stationary; in the first case because there is no market for higher output, in the second case because at any higher output marginal cost would exceed price.

The situation of momentary excess demand, arising when  $v > v_0$ , or of short-run excess demand, arising when  $v > v^*$ , offers no special difficulty. We will, of course, assume that excess demand pulls up prices, only not so fast as to clear the market instantaneously.

The excess supply case is more of a problem. At a real wage less than  $v^*$ , or  $v_0$ , the market will take just so much. If producers produce just that much, price exceeds marginal cost. Each producer could increase his profits by selling more at the going price and, as a perfect competitor, he ought to try to do so, and he ought to succeed. But all producers together can sell no more than  $Y^D$  for the going real wage. The situation of excess supply seems to be incompatible with perfect competition. Arrow<sup>7</sup> has suggested

7. Kenneth J. Arrow, "Toward a Theory of Price Adjustment," in M.

that the way out of this dilemma is to recognize that markets necessarily become imperfectly competitive when sales are limited by inadequate effective demand, so that each producer sees himself as selling along a falling demand curve. That will do the trick formally, but the precise mechanism is far from clear. We do not try to settle the issue, if only because we would like the model to be compatible with a variety of market structures including, but not limited to perfect competition. We simply assume, like Patinkin,<sup>8</sup> that, despite the excess of price over marginal cost, producers in the aggregate are restrained from increasing output beyond  $Y = Y^p$  by the *force majeure* of effective demand. Under conditions of aggregate excess supply, however, there may be downward pressure on prices.

It only remains to formalize the dynamics of prices and money wages. As for prices, we make the natural assumption that the relative rate of change of the absolute price level is an increasing function of the proportional short-run excess demand. The price level may be constant if excess demand is zero, but we need not insist on it, especially since monetary policy is assumed to be permissive.

We would like also to allow prices to be partially cost-determined, under some markup formula. The natural hypothesis is that the rate of change of price depends on the rate of change of labor cost per unit of output. There is some recent evidence to suggest that prices do not respond to minor fluctuations in productivity, so that the relevant determinant is unit labor cost at some standard rate of output. In a short-run model, productivity can be appropriately treated as constant (or a trend only) at the standard output, so fluctuations in standard unit labor cost are proportional to fluctuations in the money wage. It is enough,<sup>9</sup> therefore, to add to

Abramovitz *et al.*, *The Allocation of Economic Resources* (Stanford, Calif.: Stanford University Press, 1959), pp. 41-51.

8. Don Patinkin, *Money, Interest, and Prices* (2d ed.: New York: Harper & Row, 1965).

9. We have also studied the behavior of this model under the strong assumption that prices are independent of the current state of demand: there is a target price, determined by a fixed markup on marginal cost at a standard output. Our impression is that this assumption causes no radical change in the behavior of the model, but that the version actually given in the text is both analytically richer and nearer the truth. For example, let  $F(N^*)$  be the standard output, so  $W/F'(N^*)$  and  $WN^*/F(N^*)$  are standard marginal and average costs. Let the target price,  $p^*$ , be a fixed multiple,  $m$ , of either, where  $m$  is presumably related to the subjective elasticity of demand in the usual way. We can replace the right-hand side of (8), or at least the second term, by  $j \left( \frac{p^*}{p} - 1 \right)$ ; so that  $p$  adjusts with a lag toward  $p^*$ .

There remains the definition of  $N^*$ . At one extreme, we can take  $N^* = \text{con-}$

the rate of change of prices a component proportional to the rate of change of the money wage:

$$(8) \quad p'/p = g(Y^p/Y^s) + j w'/w.$$

Here  $g(\cdot)$  is an increasing function which may or may not have the property that  $g(1) = 0$ ; one would expect  $j$  to be between zero and one, and perhaps nearer one. Econometric evidence suggests that the price level rises faster, other things equal, the faster real output is rising; but we have to omit that influence.

There is a possible alternative to (8). We have made the pressure on the price level a function of the ratio of aggregate demand to aggregate supply. Aggregate demand is defined in (5) as the sustainable volume of real expenditures, given the going employment and real wage. That seems unobjectionable whenever actual output  $Y$  is in fact equal to aggregate demand, to the left of  $v_0$  in Figure I. To the right of  $v_0$ , however,  $Y^p$  is a rather notional concept. One might argue that a better index of the pressure on prices is desired expenditure at the actual level of output, namely,  $Y = Y^*$ . Then the numerator of the argument of  $g(\cdot)$  in (8) would be

$$\begin{aligned} I + (1 - s_w)vN + (1 - s_p)(Y - vN) \\ = I + (s_p - s_w)vN + (1 - s_p)Y \\ = X, \text{ say.} \end{aligned}$$

To the left of  $v_0$ , (5) would hold as before; to the right,  $Y$  would be replaced by  $Y^*$ . This alternative formulation of demand pressure in the commodity market makes no qualitative difference to the results. The reason is the following. It is obvious that  $Y^p/Y^s$  is an increasing function of  $v$ . So is  $X/Y^s$  to the left of  $v_0$ , because  $X = Y^p$ . To the right of  $v_0$ ,  $X$  is defined as above with  $Y$  replaced by  $Y^*$ , but obviously  $X/Y^s$  still increases with  $v$ . This is all we require for our analysis.

Money wages can be treated roughly symmetrically. We take the main influence on the rate of change of money wages to be the unemployment rate or, in our language, the ratio of current employment to the supply of labor ( $N^s$ ). The supply of labor may have some elasticity with respect to the real wage even in the short run, but for simplicity we neglect that and think of  $N^s$  as a given constant. We also allow changes in the price level to react back on the rate of change of the money wage:

$$(9) \quad w'/w = h(N/N^s) + kp'/p.$$

stant; at the other, we can take  $N^* =$  the short-run target employment (see (4)). In either case it is straightforward to verify that the resulting system is qualitatively unchanged; i.e., the partial derivatives of (8) with respect to  $N$  and  $v$  have the same sign pattern as in the text.



Undoubtedly  $h(\cdot)$  is an increasing function, but where it crosses zero (or the long-run rate of productivity growth) is an empirical matter. The constant  $k$  is between zero and one; if it is very near one, the wage bargain is very nearly struck in real terms, and the expected rate of change of prices is very nearly accurate. A value of  $k$  closer to zero means more money illusion in the labor market or a stronger tendency to underestimate changes in the price level. Some econometric estimates suggest that  $k$  may be less than one-half.

In (2)–(5) and (7)–(9) we have seven equations in the seven unknown time functions  $Y^*$ ,  $Y^p$ ,  $Y^s$ ,  $Y$ ,  $N$ ,  $p$  and  $w$ , with  $v = w/p$ . We turn now to the short-run flow equilibrium of this system and to its ultra-short-run dynamics.

### III. THE WORKING OF THE MODEL

We study the trajectories of our system in the  $(N, v)$ -plane. The first step is to find, for each real wage, the short-run equilibrium level of employment. From (4), the answer is  $F^{-1}(\min(Y^s, Y^p))$ . Thus the locus we seek is a transform of the solid inverted- $V$  curve in Figure I. Along the falling branch of that curve, the construction is straightforward:  $Y = Y^s = G(v)$  according to (2), and so equilibrium employment is simply  $F^{-1}(G(v))$ , a decreasing function of the real wage. Along the rising branch in Figure I, where  $Y = Y^p$ , the situation is less simple because — see (6) — the position of the aggregate demand curve itself depends on the current level of employment. We must find, for each real wage, a level of employment which will yield an aggregate demand at that real wage whose production will require the same amount of employment: that is to say, we must solve the equation  $Y^p = A + BvN = F(N)$ , where  $A$  and  $B$  are shorthand for the appropriate constants in (6).<sup>1</sup> Along

this branch of the locus,  $\frac{dN}{dv} = \frac{BN}{F'(N) - Bv} > 0$  because  $Y^p < Y^s$

implies that the real wage, which is  $F'$  evaluated at  $F^{-1}(Y^s)$ , must be less than  $F'(N) = F'(F^{-1}(Y^p))$ , and  $B$  is between zero and one. So the transformed curve has positive slope all along this branch, and the picture is given by the inverted- $V$  curve in Figure II. If the supply of labor places an absolute limit on employment and

1. The equation will usually have two roots for a given real wage; it is the smaller root that counts because aggregate supply rather than aggregate demand will be binding at the higher  $N$ . To put it another way, at the higher  $N$ , the wage will be greater than the marginal product, a possibility we have already ruled out.

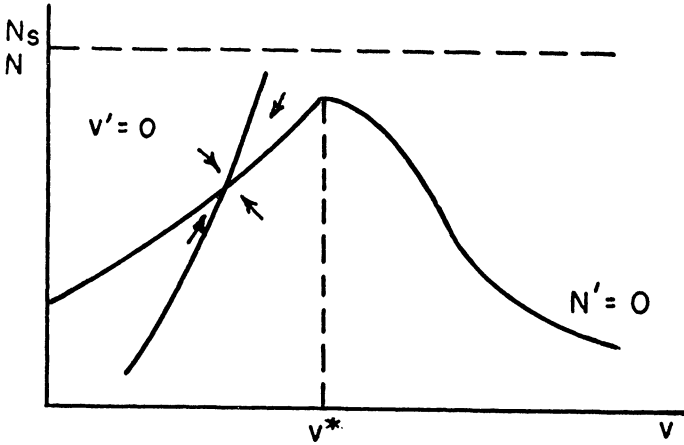


FIGURE II

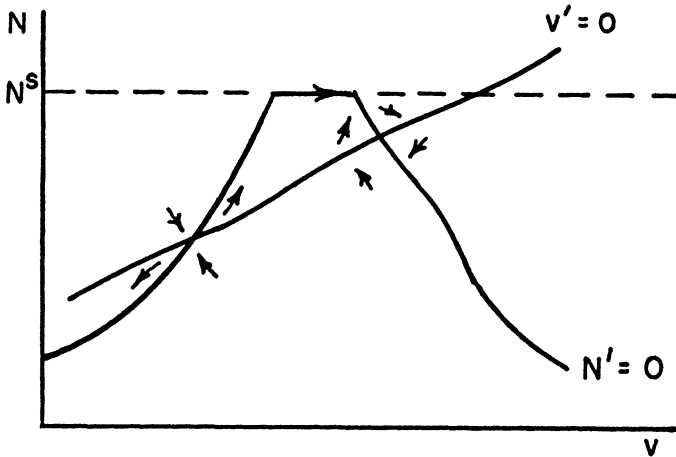


FIGURE III

that limit is effective, then Figure III describes the situation. At any point in either diagram, the volume of unemployment is measured by the vertical distance to the line  $N = N^s$ .

At any point on the locus we have just constructed,  $N$  is constant, at least for the instant. At any point above the locus,  $N$  is decreasing; and at any point below it,  $N$  is increasing.

The next move is to study the ultra-short-run dynamics of the real wage. Equations (8) and (9) can be solved simultaneously for the rates of change of the price level and money wage:

$$(10) \quad p'/p = \frac{g(Y^D/Y^S) + jh(N/N^S)}{1 - jk}$$

$$w'/w = \frac{h(N/N^S) + kg(Y^D/Y^S)}{1 - jk}$$

and therefore

$$(11) \quad v'/v = w'/w - p'/p = (1 - jk)^{-1} [(1 - j)h(N/N^S) - (1 - k)g(Y^D/Y^S)].$$

Since  $Y^D$  is a function of  $v$  and  $N$ , and  $Y^S$  a function of  $v$  alone, (10) permits us to calculate whether  $v$  is rising or falling at any point in the  $(N, v)$  - plane. The locus along which  $v$  is temporarily constant — with  $p$  and  $w$  rising or falling at the same proportional rate — is defined by

$$(12) \quad h(N/N^S) = \frac{1 - k}{1 - j} g\left(\frac{A + BvN}{G(v)}\right).$$

For the moment let us rewrite (11) and (12) to simplify notation:

$$(11') \quad v'/v = L(N) - C(N, v)$$

$$(12') \quad v'/v = 0 \text{ whenever } L(N) = C(N, v).$$

The natural presumption is that the locus defined by (12) or (12'), along which  $v' = 0$ , should be positively sloped in the  $(N, v)$  plane: a higher real wage strengthens the pressure of demand on supply in the commodity market and drives the price level up faster; the money wage can keep pace only at a lower unemployment rate. But from (11') it is seen that  $dN/dv = C_v/(L' - C_N)$ , and all three derivatives involved are positive. The sign of  $dN/dv$  is therefore the sign of  $L' - C_N$ . It can be negative. The economic reason is that an increase in employment at constant real wage adds to aggregate demand for goods. If it adds enough, the "natural presumption" is reversed. A higher real wage by itself always tends to make the price level rise and the real wage fall; but higher employment might not only cause the money wage to rise but also stimulate demand enough to make the price level rise still faster and the real wage fall still faster. In that case it would take a reduction in employment to stabilize the real wage at a higher level. It is easy to show from (12) that the natural presumption is more likely to hold the larger is  $k$  and the smaller is  $B$ , i.e., the less *de facto* money illusion there is in the labor market, and the smaller the difference between  $s_p$  and  $s_w$ . (We shall allow for the possibility that the locus (12) should be downward sloping in the  $(N, v)$  plane, but this should be thought of as an unlikely case.)

It follows also from (11') that  $v$  is always increasing to the left of the locus (11'), and decreasing to the right.

We must now superimpose the curve defined by (12) on Figures II and III. In both diagrams we let the locus (12) slope upward; in Figure II it intersects the constant- $N$  locus only on the branch where  $Y = Y^p$  while in Figure III it intersects each branch once. Figures IV and V (the right-hand intersection) exemplify two other possibilities, when the constant- $v$  locus slopes downward throughout. We mention, though we do not intend to pursue

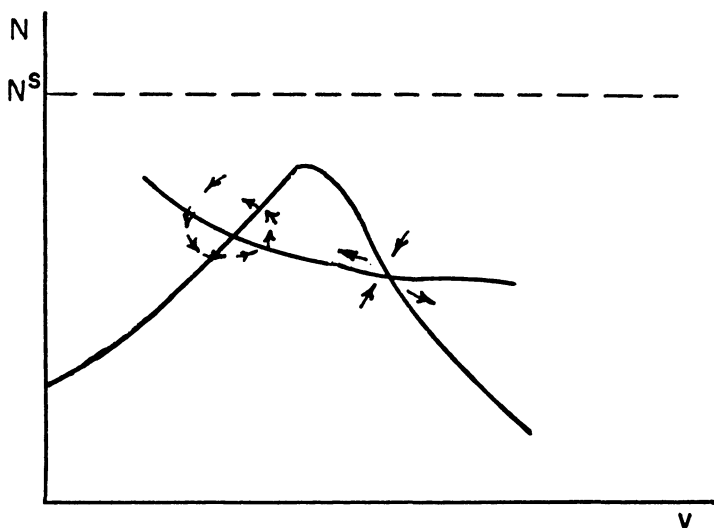


FIGURE IV

all logical possibilities, that the constant- $v$  locus may in principle have several upward and downward sloping segments.<sup>2</sup> Whether it does so or not, it may intertwine with the constant- $N$  locus and intersect any number of times, yielding alternately stable and unstable equilibria (see Figure VI). One could also discuss what happens at the axes, but neither case, zero real wages nor zero employment, seems worth the trouble.

2. It is easily verified that, for any  $N$ ,  $v'$  is a decreasing function of  $v$ , so  $v' = 0$  for only one  $v$ ; but for given  $v$  there may be more than one  $N$  at which  $v' = 0$ . We can, however, provide a condition which is sufficient to rule out there being more than one upward sloping segment and one downward sloping segment in the  $v' = 0$  curve: if

$$\frac{h''}{N_s^2} - \left( \frac{1-k}{1-j} \right) \left\{ \frac{Bv}{G(v)} \right\}^2 g''$$

is one-signed, e.g.,  $h''$  is concave and  $g''$  is convex, or vice versa. Note that (12') may not have solutions in the positive orthant for all values of  $v$ , although it will have solutions for all values of  $N$ .

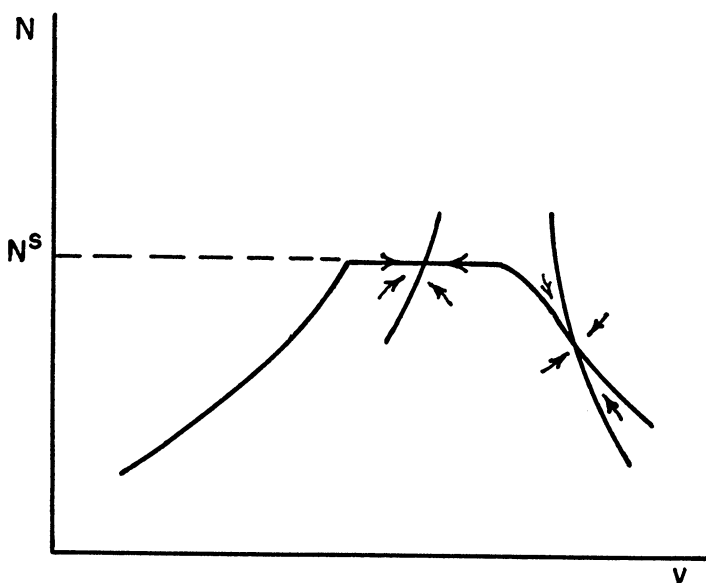


FIGURE V

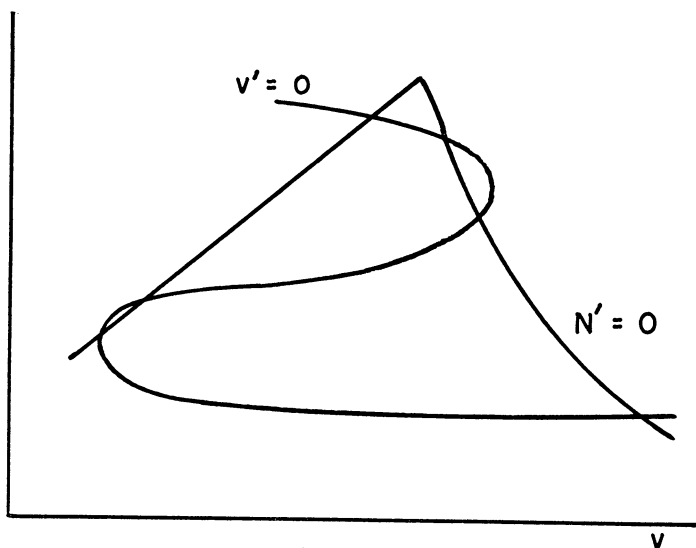


FIGURE VI

Each intersection of the two curves represents a possible short-run equilibrium of the system. We are using the word "equilibrium" in a slightly extended sense.<sup>3</sup> At each intersection in Figures II-V,

3. It is what Hansen, *op. cit.*, calls a "quasi-equilibrium."

the real wage, the level of employment, and the level of real output are all constant,<sup>4</sup> and have no inherent tendency to change. In that sense each intersection is a short-run equilibrium.

On the other hand, the money wage and the price level may both be rising or falling, so long as they are rising or falling at the same percentage rate. Moreover, except in the very special case that the intersection occurs right at the cusp of the constant- $N$  curve, at  $v = v^*$ , the commodity market is not cleared at "equilibrium." In Figure II, or at the left-hand intersection in Figures III and IV, there is excess supply. Price exceeds marginal cost;<sup>5</sup> the marginal product of labor exceeds the real wage. But the special kind of market imperfection associated with inadequate effective demand keeps output from rising; the price level may be falling but, if it is, the money wage is falling at the same rate. Presumably, then, there is unemployment. If there were not, the money wage would not fall, the real wage would be rising and this could not be an intersection point.

In Figure V, or at the right-hand intersection in Figures III and IV, there is excess demand. There is no incentive to expand or contract output and employment because price is equal to marginal cost. The price level is inflating, but the money wage is just keeping pace, so that the real wage does not budge. Neither, therefore, does output or employment. There may be unemployment in a short-run "equilibrium" with excess demand for goods, but only because the money wage is rising as fast as the price level despite the unemployment.

(It is at this point that our model calls out most obviously for an explicit monetary system. But if we were to introduce one, the economy could no longer be analyzed two-dimensionally; absolute prices and wages would enter. So we continue to assume monetary policy to be permissive.)

Not every intersection in Figures II-V represents a stable short-run equilibrium. Some are stable, but some are unstable. To distinguish, we can trace the motion of the system near each possible equilibrium point, using the rules already derived;  $N$  rises (falls) at any point above (below) the constant- $N$  locus;  $v$  rises (falls) at any point to the left (right) of the constant- $v$  locus. Arrows are sketched in each diagram to illustrate the character of the trajectories.

4. It would be easy to introduce a steady trend-increase in productivity, as in Williamson, *op. cit.* Then in a short-run equilibrium employment would be constant, but output and the real wage would rise at the trend rate.

5. By more than the conventional fixed markup, if there is one.

For example, the single equilibrium point in Figure II is a stable node. The rules of the game will carry any initial point into the equilibrium, and with very little in the way of fluctuation. Once a moving point enters the angular regions southwest or northeast of the equilibrium it can never again leave. The equilibrium itself is one in which output is limited by effective demand.

Figure III has two equilibrium points. The left-hand one is unstable. More precisely it is a saddlepoint. Trajectories approach it and then move away. (There are two singular motions which do approach the saddlepoint in infinite time, but they can be neglected — they represent the possibility of starting a pendulum so that it will just come to a dead stop in a vertical position, upside-down!) At the right-hand equilibrium the limit to output is on the supply side. It is a stable equilibrium, either a node or a focus. If it is a focus, trajectories will spiral in on it; employment and the real wage will converge in damped oscillations to their equilibrium values. Thus the configuration in Figure III offers the possibility that the ultimate outcome depends on initial conditions. From some starting points the economy gets trapped into a situation of falling employment and real wage; from others it is attracted to an excess-demand equilibrium. Of course, the equilibrium in question is only short-run.

Figure IV is the mirror image of Figure III: the left-hand demand-limited equilibrium is a stable focus or node. The right-hand supply-limited equilibrium is an unstable saddlepoint. Figure V is the mirror image of Figure II: the equilibria shown are both stable nodes. The left-hand alternative is at maximum employment. Presumably the money wage is rising, but so is the price level, because aggregate demand exceeds what the labor force is capable of producing. That configuration is not very plausible. If  $N^s$  represents the maximum possible employment, one would expect the constant- $v$  locus to become horizontal as it approached  $N = N^s$  from below. The money wage would always outstrip the price level as the unemployment rate fell near zero. The left-hand equilibrium point in Figure V would become more like the right-hand equilibrium in Figure III.

As an illustration we give the detailed local stability analysis for Figure III. The equations of motion near the right-hand (supply-limited) equilibrium are:

$$\begin{aligned} v' &= v(L(N) - C(N, v)) \\ N' &= \theta(F^{-1}(G(v)) - N). \end{aligned}$$

They have a linear approximation

$$v' = -v^*C_v^*(v - v^*) + v^*(L'^* - C_N^*)(N - N^*)$$

$$N' = \theta \frac{G'^*}{F'^*} (v - v^*) - \theta(N - N^*)$$

where the asterisk means evaluation at the equilibrium point. The character of the motion near  $(N^*, v^*)$  is determined by the roots  $Z$  of the characteristic equation

$$\begin{vmatrix} -v^*C_v^* - Z & v^*(L'^* - C_N^*) \\ \theta \frac{G'^*}{F'^*} & -\theta - Z \end{vmatrix}$$

$$= Z^2 + (\theta + v^*C_v^*)Z + \theta(v^*C_v^* - \frac{G'^*}{F'^*}v^*(L'^* - C_N^*)) = 0.$$

Thus

$$2Z = -(\theta + v^*C_v^*) \pm ((\theta + v^*C_v^*)^2 - 4\theta v^*C_v^* + 4\theta v^* \frac{G'^*}{F'^*} (L'^* - C_N^*))^{1/2}.$$

If the discriminant is negative, the equilibrium point is a focus, but a stable focus because the real part of the characteristic roots is definitely negative. Output, employment, and real wages will approach  $F(N^*)$ ,  $N^*$ ,  $v^*$  in damped oscillations. The combination of employment lag, real-wage dynamics, and different propensities to spend generates a kind of cycle. If the discriminant is positive, the equilibrium point is a node, but a stable node. The square root term must be between zero and  $\theta + v^*C_v^*$  because the terms after the first are all negative ( $G' < 0$  and  $L' - C_N > 0$  in Figure III). Output, employment, and real wage approach their equilibrium values with at most one turning point.

It is possible to describe the dependence of the solution on  $\theta$ , the speed of adjustment of employment to output. The discriminant is quadratic in  $\theta$ . For  $\theta = 0$ , and therefore for small  $\theta$ , the discriminant is positive and the solution nonoscillatory. The same is true for large  $\theta$ . In between there may — or may not — be a range of  $\theta$  for which the solution cycles.

At the left-hand equilibrium point in Figure III, the equation for  $v'$  is unchanged, but  $N' = \theta(F^{-1}(A + BvN) - N)$ . The characteristic equation of the linear approximation becomes

$$\begin{vmatrix} -v^*C_v^* - Z & v^*(L'^* - C_N^*) \\ \theta \frac{BN^*}{F'^*} & \theta \left( \frac{Bv^*}{F'^*} - 1 \right) - Z \end{vmatrix}$$



$$= Z^2 + \left( v^* C_v^* + \theta \frac{F'^* - Bv^*}{F'^*} \right) Z + \theta \frac{(F'^* - Bv^*)}{F'^*} v^* C_v^* - \frac{v^* \theta B N^*}{F'^*} (L^* - C_N^*) = 0.$$

Since

$\frac{v\theta}{F'} > 0$ , the product of the roots has the sign of  $(F'^* - Bv^*)C_v^* - BN^*(L^* - C_N^*)$ . This number is negative when the  $v$ -stationary cuts the  $N$ -stationary from above. The roots must therefore be real and of opposite sign, so the equilibrium is a saddle-point and unstable.

The other configurations illustrated in Figures II, IV and V can be analyzed similarly. The only kinds of equilibria that can arise are stable foci and nodes, and (unstable) saddlepoints. There are no unstable nodes or foci, and no centers (closed cycles). The equilibria in Figures II and V can only be nodes. Figure IV is like Figure III, with the right- and left-hand equilibria interchanged.

We make one further qualitative remark about dynamics. Suppose  $s_p = s_w$ , so  $B = 0$ . Then aggregate demand for goods is independent of the distribution of income and the picture is as in Figure VII. It is easily seen graphically that an equilibrium point at which output is limited by effective demand is necessarily a

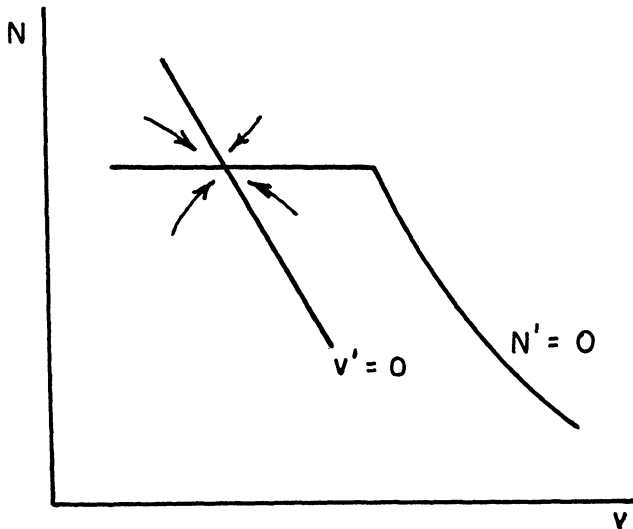


FIGURE VII

stable node. Indeed, with  $B = 0$  the roots of the characteristic equation are simply  $-v^*C_v^*$  and  $-\theta$ . No possibility of oscillations arises. Qualitatively, then, if the marginal propensities to spend wages and profits are nearly the same, the approach to a demand-limited equilibrium is essentially monotone.

#### IV. DISPLACEMENT OF SHORT-RUN EQUILIBRIUM

We turn next to the important question of comparative statics: what happens to the real wage, employment, the level of output — and therefore to the share of wages — when a short-run equilibrium is disturbed by a variation in investment, say? Not surprisingly, the qualitative character of the answer is different for stable and unstable equilibria. Since only stable equilibria are of real interest, we concentrate on that case.

There are two classes of stable short-run equilibrium points, and they have to be analyzed separately. Supply-limited equilibrium is illustrated by the right-hand singular point in Figure III and Figure V. Demand-limited equilibrium is illustrated by the equilibrium point in Figure II and the left-hand one in Figure IV. We take supply-limited equilibrium first, because it is somewhat simpler.

In any supply-limited equilibrium,  $Y = Y^s = G(v)$ . The equilibrium point is, therefore, determined by the two equations

$$(13) \quad G(v) = F(N)$$

$$(13') \quad L(N) = C(N, v; A, B).$$

The second of these equations is simply (12') rewritten to display the parameters  $A$  and  $B$ ; for the exact form of the equation see (12). A shift in investment amounts to a shift in  $A (= I/s_p)$  in the same direction. If  $A$  changes, (13) is unaffected. The aggregate supply curve remains where it was. But (13'), the locus of constant real wages, does shift. One can see directly from (12) that it shifts to the left. Let  $A$  increase and consider any fixed  $N$ : the left-hand side of (12) is unchanged, so  $v$  must change to keep the argument of  $g(\cdot)$  constant; a decrease in  $v$  increases the denominator and reduces the numerator, offsetting the initial change in  $A$ . It follows that an increase in investment will shift any stable supply-limited equilibrium upward and to the left along the aggregate supply curve. The real wage will fall; output and employment will rise; and the rate of inflation will also increase. The effect on relative shares will depend entirely on the "elasticity of substitu-

tion" associated with  $F(N)$ .<sup>6</sup> If it is less than one, the share of wages falls; if it exceeds one, the share of wages rises.

It is important to understand the mechanism that leads to this result. The increase in aggregate demand accelerates the inflation of the price level; the money wage does not initially keep pace, so the real wage falls and permits an increase in employment. Employment rises until, for Phillips-curve reasons, the money wage rises as fast as the price level and the real wage stabilizes. It is evident from (10) that the rate of inflation is faster in the new equilibrium. (Since we are dealing with a stable equilibrium, the new equilibrium will actually be approached.)

For completeness, we point out that a glance at Figure IV will show that if the equilibrium is unstable the results are just the opposite. An increase in  $A$  will increase the real wage and reduce employment, thus reversing the usual Keynesian proposition about the effect of an increase in the level of investment; but this is of no practical interest. The situation of equilibrium at maximum feasible employment, illustrated in Figure V, is very simple; the real wage falls, but employment and output do not increase because they cannot. The rate of inflation increases. But this situation is fundamentally implausible, as we mentioned earlier, because one would expect the money wage to rise fast enough at maximum employment to carry the real wage with it.

A shift in a demand-limited equilibrium is more complicated to analyze — and the results are less clear-cut — because both curves shift when  $A$  changes. The equilibrium point is defined by (13') and

$$(13'') \quad A + BvN = F(N).$$

Just as before, the locus of constant real wages shifts to the left when investment increases. From (13''), the locus of constant employment shifts to the left too: for given  $N$ , a higher value of  $A$  must be offset by a lower  $v$  to preserve the equality. One cannot read the outcome unambiguously from the diagram, and we must make a closer analysis.

Total differentiation of (13') and (13'') with respect to  $A$  yields

$$\begin{bmatrix} -C_v & L' - C_N \\ BN & Bv - F' \end{bmatrix} \begin{bmatrix} dv/dA \\ dN/dA \end{bmatrix} = \begin{bmatrix} C_A \\ -1 \end{bmatrix}$$

whence

6. The quotation marks are to remind the reader that there need not be any constant-returns-to-scale production function in labor and capital underlying  $F(N)$ . Even if there is not, for this calculation one pretends that there is.

$$(14) \quad \begin{aligned} dv/dA &= D^{-1}(L' - C_N - C_A(F' - Bv)) \\ dN/dA &= D^{-1}(C_v - C_A BN) \end{aligned}$$

where  $D = C_v(F' - Bv) - (L' - C_N)BN$  is known from earlier analysis to be negative if the equilibrium is unstable and positive if it is stable.

$C_A$  is easily seen from (12) to be positive. Moreover, by partial differentiation of (11),

$$C_A = C_N/Bv = C_v/(BN - Y^p G'/G).$$

It follows at once that an increase in investment always increases employment and the rate of inflation if the equilibrium is stable. It is also true that  $dN/dA < 0$  at an unstable equilibrium but this is less meaningful. As for the effect of an increase in investment on the real wage, one can show — by eliminating  $C_A$  in favor of  $C_v$  — that  $dv/dA < 0$  at an unstable equilibrium. Unfortunately there is no unambiguous answer at a stable equilibrium. But one can show — by eliminating  $C_A$  in favor of  $C_N$  — that  $dv/dA$  is positive or negative at a stable equilibrium according as  $L'$  is greater or less than  $C_N F'/Bv = \frac{1-k}{1-j} \frac{g'F'}{G(v)}$ . Thus the real wage can go either way.

The other demand-side parameter that can vary is  $B$ .  $B$  rises with  $A$  constant if  $s_w$  falls, and falls if  $s_w$  rises. (A change in  $s_p$  changes both  $A$  and  $B$ .) It is straightforward that

$$\begin{aligned} dv/dB &= D^{-1}(vN(L' - C_N) - C_B(F' - Bv)) \\ dN/dB &= D^{-1}(C_v vN - C_B BN) \end{aligned}$$

and  $C_B = C_N N/B = C_v vN/(BN - Y^p G'/G) > 0$ . A little more calculation shows that  $dv/dB$  and  $dN/dB$  have the same signs as  $dv/dA$  and  $dN/dA$ . So a fall in the propensity to save wages acts like an increase in investment.

We can also consider the displacement effects of changes in  $k$  and  $j$ . Those parameters affect the location of the constant- $v$  locus only. It is clear from (12) that an increase in  $k$ , or a decrease in  $j$ , shifts the constant- $v$  locus to the right. We see then from Figures II and IV that the effect on a stable demand-limited equilibrium is to increase the equilibrium employment and the real wage. (If  $s_p = s_w$ , the real wage rises but employment is unchanged.) Similarly, Figures III and V show that the effect on a stable supply-limited equilibrium is to increase the equilibrium real wage but to decrease equilibrium employment. (The effects at unstable equilibria are just the reverse.) In words, if money wages become more sensitive to changes in commodity prices or prices less sensitive

to changes in standard unit labor costs, the equilibrium real wage will always be higher. In an excess-supply situation, employment will increase as well; if there is already excess demand, employment will fall.

#### V. EFFECTIVE DEMAND AND THE DISTRIBUTION OF INCOME

The theory presented here gives a determinate answer, though not a simple one, to the question: how does the share of wages in total income vary when effective demand varies in the short run? When output is limited by supply, the answer is the conventional one, depending on the "elasticity of substitution" or, more accurately, on the speed with which the short-run marginal product of labor falls as employment rises. When output is limited by effective demand, the answer is more complicated still. We observe that the share of wages is  $vN/(A + BvN) = \frac{1}{A/vN + B}$ . It fol-

lows that if investment (and therefore  $A$ ) rises, the share of wages will rise or fall according as the sum of the elasticities of  $v$  and  $N$  with respect to  $A$  exceed or fall short of one. This condition can be explored further with the aid of (14), but we have not been able to reduce it to any very simple form.

Indeed, we have already shown that the effect of shifts in aggregate demand on the real wage rate is ambiguous, depending on the money wage and price response mechanisms in an important way. It will be remembered that Keynes, in the *General Theory*, held that price was always equal (or proportional) to marginal cost, so that in the short run the real wage would always fall as employment rose and vice versa. The early empirical work of Dunlop<sup>7</sup> and Tarshis<sup>8</sup> seemed to subvert this idea. In reviewing the situation, Keynes<sup>9</sup> took a cautious view, apparently not quite convinced that the facts had been properly got at, but apparently willing to jettison that part of the theory if the data demanded it. Later statistical work, with better data and more appropriate concepts, appears to confirm Dunlop and Tarshis.<sup>1</sup> The real wage does not appear to fall, or fall relative to trend, in cyclical upswings. Nor

7. John T. Dunlop, "The Movement of Real and Money Wages," *Economic Journal*, XLVIII (Sept. 1938).

8. Lorie Tarshis, "Changes in Real and Money Wages," *Economic Journal*, XLIX (Mar. 1939).

9. J. M. Keynes, "Relative Movements of Real Wages and Output," *Economic Journal*, XLIX (Mar. 1939).

1. See Edwin Kuh, "Unemployment, Production Functions, and Effective Demand," *Journal of Political Economy*, LXXXIV (June 1966), 238-49.

does the real wage seem to have any other pronounced pattern in the course of short-run economic fluctuations. We are not unhappy, therefore, with a theory that permits the real wage to rise when effective demand increases and fall when effective demand falls, but does not require it. Since the outcome, in this theory, depends so much on the wage and price adjustment mechanism, and since those mechanisms must be expected to change from time to time, there is no reason to expect consistent behavior over long periods of time.

We come now to the relation between the Cambridge and marginal-productivity theories of distribution. There is a sense in which the marginal-productivity theory can be said to hold at any supply-limited equilibrium, and the Cambridge theory can be said to hold at any demand limited equilibrium. At any supply-limited equilibrium, and only at supply-limited equilibria, the real wage is equal to the short-run marginal product of labor (perhaps modified by monopoly). At any demand-limited equilibrium, and only at demand-limited equilibria, it is true that

$$vN/Y = \frac{s_p}{s_p - s_w} - \frac{1}{s_p - s_w} \frac{I}{Y};$$

the standard equation of the Cambridge theory (although it is important to realize that  $Y$  and  $N$  are here *unknowns*, not given, as in Kaldor). If the economic system runs, or is run, in such a way as to keep it near the intersection of the aggregate supply and aggregate demand curves, then both theories can hold determinately and simultaneously. Under other circumstances, one or the other will be "true" at any time, but neither provides a complete or determinate theory unless supplemented by a market mechanism — the one we have described here, or some other.

The market mechanism described in this paper has an important asymmetry, with an important asymmetrical consequence. At a short-run equilibrium the real wage may equal the marginal product of labor (if the equilibrium is supply-limited) or fall short of the marginal product of labor (if the equilibrium is demand-limited). But the real wage can never exceed the marginal product of labor. This asymmetry arises because price may equal or exceed marginal cost, but cannot be less. Under conditions of inadequate demand, price may exceed marginal cost, but there is no tendency for output and employment to increase, precisely because demand is inadequate. The price level may fall, of course, but whether or not the price level falls relative to marginal cost depends on the behavior of the money wage. But there is no symmetrical situation. Marginal cost can never exceed price in short-run equilibrium because there is no

hindrance to a reduction in output and employment under those circumstances. Output may be constrained by effective demand below the supply curve, but it cannot be dragged above the supply curve because there is no obligation to produce unprofitable items of output.

## VI. NEXT STEPS

We have already mentioned some directions in which this theory needs to be extended. First and foremost, it needs a monetary mechanism. We have refrained from providing one in this exposition to keep the analysis two-dimensional. Under our assumptions, the dynamics and comparative statics could be analyzed in terms of the real wage and the level of employment. As soon as an explicit monetary system is introduced the analysis will have to be three-dimensional, in terms of the money wage, price level, and employment. We do not think that offers difficulties of principle, but there will be a loss of transparency. We intend later to extend the model in this direction.

Second, the model needs to be extended to the long run. The first requirement is to find a representation of the shift in short-run production possibilities brought about by current investment. The easiest course is to suppose that the short-run production function is simply a section of a long-run production function in capital and labor. If this is too great a stretch of the imagination, there are more plausible — but less maneuverable — alternatives; see, for example, Solow, Tobin, Weizsäcker, and Yaari<sup>2</sup> and Attiyeh.<sup>3</sup> Depending on how this task is accomplished, there may or may not arise the further question of the choice of labor-intensity for current investment. When a choice of technique is available, the current and prospective price configuration will have an influence on the labor-intensity selected for any given increase in capacity. Moreover, since the price configuration has a lot to do with the profitability of any given investment, it will have an influence on the amount of capacity installed. In the long-run context, investment cannot be treated as exogenous, even as an approximation.

A third extension has to do with the equation (7), stating that actual output is the smaller of aggregate momentary supply and

2. R. M. Solow, J. Tobin, C. von Weizsäcker, M. Yaari, "Neo-classical Growth with Fixed Factor Proportions," *Review of Economic Studies*, XXXIII (Apr. 1966).

3. Richard S. Attiyeh, "Estimation of a Fixed Coefficients Vintage Model of Production," *Yale Economic Essays*, Vol. 7 (Spring 1967), pp. 5-40.

aggregate demand. This amounts to assuming away unintended inventory changes (intended inventory change is included in section I). It would be more realistic to suppose that when supply exceeds demand, actual output is somewhere between, and inventories are built up; and that, when demand exceeds supply, some of the excess demand is met out of inventories. By itself, that amendment might not be difficult. But as soon as one allows for an inventory policy, and the notion that some part of aggregate demand is intended to build stocks toward a target level, a new dynamic element is added to the model and it becomes much more complex.

Finally, we call attention to the price and wage adjustment equations (8) and (9). As they stand, especially the wage equation, they concede quite a lot to money illusion or systematic underextrapolation of price changes. That is not so bad in a short-run model, especially since the real wage does turn out to be constant in equilibrium. Most empirical studies of wage behavior suggest that  $k$  is considerably less than one. In a long-run context, however, one might prefer assumptions that guarantee that any prolonged rate of inflation will come to be expected, and built into money wage determination. One way to accomplish that would be to drop the last term in (9) and add instead a symbol representing the expected rate of inflation. The expected rate of inflation could then itself be governed by one of the usual differential equations for adaptive or extrapolative expectations. Such a model would behave in the short run much like (9) with  $k < 1$ , and in the long run much like (9) with  $k = 1$ , or at least nearer unity.

If, in fact,  $k = 1$ , (9) by itself determines the rate of change of the real wage. ("The wage bargain is in real terms.") The locus of constant real wage in our diagrams would be a horizontal line at a height corresponding to the employment at which  $h(N/N^s) = 0$ . The rest of the analysis would go pretty much as before; (8) would merely determine the rate of inflation. If, on the other hand,  $j$  were equal to one in (8), the locus of constant real wage would be a vertical line. If  $g(1) = 0$ , the only equilibrium point would be at the intersection of the aggregate demand and supply curves, with the rate of inflation determined by (9). The model will not function with  $j = k = 1$ .