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Research Article

Output Feedback Adaptive Stabilization of Uncertain Nonholonomic Systems

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This paper investigates the problem of output feedback adaptive stabilization control design for a class of nonholonomic chained systems with uncertainties, involving virtual control coefficients, unknown nonlinear parameters, and unknown time delays. The objective is to design a robust nonlinear output-feedback switching controller, which can guarantee the stabilization of the closed loop systems. An observer and an estimator are employed for states and parameters estimates, respectively. A constructive controller design procedure is proposed by applying input-state scaling transformation, parameter separation technique, and backstepping recursive approach. Simulation results are provided to show the effectiveness of the proposed method.

1. Introduction

The control and feedback stabilization problems of nonholonomic systems have been widely studied by many researchers. It is well known that control of nonholonomic systems is extremely challenging, largely due to the impossibility of asymptotically stabilizing nonholonomic systems via smooth time-invariant state feedback, a well-recognized fact pointed out in [1, 2]. In order to overcome this obstruction, a number of approaches have been proposed for the problem, which mainly include discontinuous feedback, time-varying feedback, and hybrid stabilization. The discontinuous feedback stabilization was first proposed by [3], and then further discussion was made in [4–7]; especially an elegant discontinuous coordinate transformation approach is proposed in [5] for the stabilization problem of nonholonomic systems. Meanwhile, the smooth time-varying feedback control strategies also have drawn much attention [8–11].

As pointed out in [9], many nonlinear mechanical systems with nonholonomic constraints can be transformed, either locally or globally, to the nonholonomic systems in the so-called chained form. So far, there have been a number of controller design approaches [8–25] for such chained nonholonomic systems. Recently, adaptive control strategies have been proposed to stabilize the nonholonomic systems. For instance, the problem of adaptive state-feedback control is studied in [15–19], while output feedback controller design in [20–24]. Considering the actual modeling perspective, time delay should be taken into account. The problem of state feedback stabilization is studied for the delayed nonholonomic systems in [25, 26]. However, the virtual control coefficients and unknown parameter vector are not considered in its system models. Here, an iterative controller design method will be proposed for the output feedback adaptive stabilization of the concerned delayed nonholomic systems.

In this paper, we study a class of chained nonholonomic systems with strong nonlinear drifts, and the problem of adaptive output-feedback stabilization for the concerned nonholonomic systems is investigated. The constructive design method proposed in this note is based on a combined application of the input scaling technique, the backstepping recursive approach, and the novel Lyapunov-Krasovskii functionals. The switching control strategy for the first subsystem is employed to achieve the asymptotic stabilization.

The rest of this paper is organized as follows. In Section 2, the problem formulation and some preliminary knowledge are given. Section 3 presents the controller design procedure and stability analysis. Section 4 gives the switching control strategy. In Section 5, numerical simulations testify to the effectiveness of the proposed method, and Section 6 summarizes the paper.

2. Problem Formulation and Preliminaries

In this paper, we deal with a class of nonholonomic systems described by

$$\begin{aligned}
\dot{x}_0(t) &= d_0 u_0(t) + \phi_0(t, x_0(t)), \\
\dot{x}_1(t) &= d_1 u_0(t) x_2(t) + \varphi_1(u_0(t), y(t), y(t-\tau_1)) \\
&\quad + \phi_1(t, u_0(t), x_0(t), x(t), \theta), \\
&\vdots \\
\dot{x}_{n-1}(t) &= d_{n-1} u_0(t) x_n(t) + \varphi_{n-1}(u_0(t), y(t), y(t-\tau_{n-1})) \\
&\quad + \phi_{n-1}(t, u_0(t), x_0(t), x(t), \theta), \\
\dot{x}_n(t) &= d_n u_1(t) + \varphi_n(u_0(t), y(t), y(t-\tau_n)) \\
&\quad + \phi_n(t, u_0(t), x_0(t), x(t), \theta), \\
y(t) &= [x_0(t), x_1(t)]^T,
\end{aligned} \tag{1}$$

where $[x_0(t), x(t)]^T = [x_0(t), x_1(t), \ldots, x_n(t)]^T \in R^{n+1}$, $u(t) = [u_0(t), u_1(t)]^T \in R^2$, and $y(t) \in R^2$ are system states, control input, and measurable output, respectively; $\theta \in R^m$ is an unknown parameter vector; ϕ_0 (known) and ϕ_i $(1 \le i \le n)$ (unknown) denote the possible modeling error and neglected dynamics; φ_i $(1 \le i \le n)$ are known modeled dynamics, which contain output delays; τ_i $(1 \le i \le n)$ are unknown constants, and d_i $(0 \le i \le n)$ referred to the respective virtual control coefficients.

In this paper, we make the following assumptions on the virtual control directions d_i and nonlinear functions φ_i, ϕ_i in system (1).

Assumption 1. d_0 is a known constant and the sign of $\overline{d}_n$ is known, where $\overline{d}_n = d_1 d_2 \cdots d_n$.

Assumption 2. There exist known smooth nonnegative functions $\overline{\phi}_o$ and $\overline{\phi}_i$ $(1 \le i \le n)$ such that

$$\begin{aligned}
\phi_0(t, x_0(t)) &= x_0 \overline{\phi}_0(x_0(t)), \\
|\phi_i(t, u_0(t), x_0(t), x(t), \theta)| & \\
&\le |x_1| \overline{\phi}_i(u_0(t), x_0(t), x_1(t), \theta).
\end{aligned} \tag{2}$$

for all $(t, u_0(t), x_0(t), x(t), \theta) \in R_+ \times R \times R \times R^n \times R^m$.

Assumption 3. For every $1 \le i \le n$, the nonlinear function φ_i satisfies inequality

$$\begin{aligned}
|\varphi_i(u_0(t), y(t), y(t-\tau_i))| &\le |x_1(t)| \psi_i(u_0(t), y(t)) \\
&\quad + |x_1(t) x_1(t-\tau_i)| \\
&\quad \times \overline{\varphi}_i(u_0(t), y(t), y(t-\tau_i)),
\end{aligned} \tag{3}$$

in which $\overline{\varphi}_i$ and ψ_i are known smooth nonnegative nonlinear functions.

Remark 4. Compared with some existing literatures in recent years, the structure of our concerned system (1) is more general. For instance, in [15], it is assumed that not only the virtual control directions $d_i = 1$ and the dynamics ϕ_i satisfy $\phi_i = \widetilde{\phi}_i^T \theta$, but also the modeled dynamics φ_i do not exist. In [22], the virtual control coefficients and time delays have not been considered, and the expression $\phi_i = \widetilde{\phi}_i^T \theta$ is also required. While $d_i = 1$ and φ_i and unknown parameters θ are not existent, system (1) degenerates to the one studied in [21]. When $\varphi_i = 0$, together with $\phi_i = \widetilde{\phi}_i^T \theta$, system (1) becomes the considered system in [23].

Remark 5. Note that here we only use the sign of $\overline{d}_n = d_1 d_2 \cdots d_n$ without any knowledge of individual virtual control direction d_i $(1 \le i \le n)$. Moreover, Assumptions 2 and 3 are imposed on the nonlinear functions ϕ_i and φ_i, respectively. In fact, if the modeled dynamics φ_i do not involve time delays, inequality (3) is reduced into

$$|\varphi_i(u_0(t), y(t))| \le |x_1(t)| \psi_i(u_0(t), y(t)). \tag{4}$$

It can be seen that the above inequality condition is used in some existing literatures, such as [20, 21], and so on.

Our object of this paper is to design adaptive output feedback control laws under Assumptions 1–3, such that the system states $(x_0(t), x(t))$ converge to zero, while other signals of the closed-loop system are bounded. The designed control laws can be expressed in the following form:

$$\begin{aligned}
u_0 = \mu_0(y(t)), \qquad u_1 = \mu_1(y(t), \nu(t)), \\
\dot{\nu}(t) = \varrho(y(t), \nu(t)).
\end{aligned} \tag{5}$$

Next, we list some lemmas which will be applied in the coming controller design.

Lemma 6 (see [27]). *For any real-valued continuous function $f(x, y)$, where $x \in R^n, y \in R^m$, there are smooth functions $a(x) \ge 0, b(y) \ge 0, c(x) \ge 1, d(y) \ge 1$ such that*

$$|f(x, y)| \le a(x) + b(y), \quad |f(x, y)| \le c(x) d(y). \tag{6}$$

Lemma 7 (see [19]). *For any continuous function $\mu_0(t)$ there exist two strictly positive real numerates $p_{\min}$ and $p_{\max}$ such that the unique solution $P(t)$ of the following matrix differential equation:*

$$\dot{P} = P\left(A - \mu_0(t)L\right)^T + \left(A - \mu_0(t)L\right)P - PC^TCP + I, \quad P(0) = P_0 > 0, \tag{7}$$

satisfies $p_{\min}I \le P(t) \le p_{\max}I,\ t \ge 0$.

By Lemma 6 and Assumption 1, we know that there exist smooth functions $\omega_i \ge 1$, and $\zeta_i \ge 1$ such that

$$\begin{aligned}&\left|\phi_i\left(t, u_0(t), x_0(t), x(t), \theta\right)\right| \\ &\quad \le \left|x_1\right| \omega_i\left(u_0(t), x_0(t), x_1(t)\right)\zeta_i(\theta).\end{aligned} \tag{8}$$

Furthermore, we denote $\vartheta = \sum_{i=1}^{n}\zeta_i(\theta)$; then it yields

$$\begin{aligned}&\left|\phi_i\left(t, u_0(t), x_0(t), x(t), \theta\right)\right| \\ &\quad \le \left|x_1\right| \omega_i\left(u_0(t), x_0(t), x_1(t)\right)\vartheta.\end{aligned} \tag{9}$$

3. Output Feedback Adaptive Stabilization Control Design

In this paper, we design control laws $u_0(t)$ and $u_1(t)$ separately to globally asymptotically stabilize the system (1). According to the structure of system (1), we can see that when $x_0(t)$ converges to zero, $x_i(t)$ $(1 \le i \le n)$ will be uncontrollable. A widely used method to design control law $u_1(t)$ is to introduce a discontinuous input scaling transformation (12). On the other hand, the control directions d_i are unknown; then we should employ another coordinate transformation to overcome the obstacle.

3.1. State Coordinate Transformation. Firstly, we design the coordinate transformation as follows:

$$\overline{x}_i(t) = \overline{d}_{i-1}x_i(t), \quad 1 \le i \le n, \tag{10}$$

where $\overline{d}_0 = 1$ and $\overline{d}_{i-1} = d_1 d_2 \cdots d_{i-1}$ $(1 \le i \le n+1)$. Then, the system (1) can be transformed into

$$\begin{aligned}\dot{x}_0(t) &= d_0 u_0(t) + \phi_0\left(t, x_0(t)\right), \\ \dot{\overline{x}}_1(t) &= u_0(t)\overline{x}_2(t) + \varphi_1\left(u_0(t), y(t), y(t-\tau_1)\right) \\ &\quad + \phi_1\left(t, u_0(t), x_0(t), x(t), \theta\right), \\ &\ \ \vdots \\ \dot{\overline{x}}_n(t) &= \overline{d}_n u_1(t) + \overline{d}_{n-1}\varphi_n\left(u_0(t), y(t), y(t-\tau_n)\right) \\ &\quad + \overline{d}_{n-1}\phi_n\left(t, u_0(t), x_0(t), x(t), \theta\right).\end{aligned} \tag{11}$$

Next, the following input-state scaling discontinuous transformation is introduced:

$$z_i(t) = \frac{\overline{x}_i(t)}{u_0^{n-i}(t)}, \quad 1 \le i \le n. \tag{12}$$

Under the new $z(t)$-coordinates, the $\overline{x}(t)$-subsystem (10) is changed into

$$\begin{aligned}\dot{z}_1(t) &= z_2(t) - (n-1)\frac{\dot{u}_0(t)}{u_0(t)}z_1(t) \\ &\quad + \frac{1}{u_0^{n-1}(t)}\left(\varphi_1 + \phi_1\right), \\ \dot{z}_i(t) &= z_{i+1}(t) - (n-i)\frac{\dot{u}_0(t)}{u_0(t)}z_i(t) \\ &\quad + \frac{1}{u_0^{n-i}(t)}\left(\overline{d}_{i-1}\varphi_i + \overline{d}_{i-1}\phi_i\right), \\ \dot{z}_n(t) &= \overline{d}_n u_1(t) + \overline{d}_{n-1}\varphi_n + \overline{d}_{n-1}\phi_n.\end{aligned} \tag{13}$$

Next, we can design the control laws $u_0(t)$ and $u_1(t)$ to asymptotically stabilize the states $x_0(t)$ and $z(t)$, respectively. Rewrite system (13) in the compact form

$$\dot{z}(t) = \left(A - L\frac{\dot{u}_0(t)}{u_0(t)}\right)z(t) + Bu_1(t) + \Psi + \Phi, \tag{14}$$

where

$$A = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix}, \qquad L = \begin{bmatrix} n-1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \overline{d}_n \end{bmatrix}, \qquad \Psi = \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_n \end{bmatrix}, \qquad \Phi = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_n \end{bmatrix} \tag{15}$$

with

$$\begin{aligned}\Psi_i &= \overline{d}_{i-1}\frac{\varphi_i\left(u_0(t), y(t), y(t-\tau_i)\right)}{u_0^{n-i}(t)}, \\ \Phi_i &= \overline{d}_{i-1}\frac{\phi_i^d\left(t, u_0(t), x_0(t), x(t), \theta\right)}{u_0^{n-i}(t)}.\end{aligned} \tag{16}$$

In order to obtain the estimation for the nonlinear functions Ψ_i and Φ_i, the following lemmas are given.

Lemma 8. *For every $1 \le i \le n$, there exists smooth nonnegative function $\widetilde{\omega}_i(u_0(t), x_0(t), z_1(t))$ such that*

$$\left|\Phi_i\right| \le \left|\overline{d}_{i-1}\right| \cdot \left|z_1(t)\right| \widetilde{\omega}_i\left(u_0(t), x_0(t), z_1(t)\right)\vartheta. \tag{17}$$

Lemma 9. *For every $1 \le i \le n$, there exist smooth nonnegative functions $\widetilde{\psi}_i, \widetilde{\varphi}_i, f_{i1}, f_{i2}$ such that*

$$\begin{aligned}\left|\Psi_i\right| &\le \left|\overline{d}_{i-1}\right| \cdot \left|z_1(t)z_1(t-\tau_i)\right| \\ &\quad \times \widetilde{\varphi}_i\left(u_0(t), u_0(t-\tau_i), y(t), y(t-\tau_i)\right) \\ &\quad + \left|\overline{d}_{i-1}\right| \cdot \left|z_1(t)\right| \widetilde{\psi}_i\left(u_0(t), y(t)\right) \\ &\le \left|\overline{d}_{i-1}\right| \cdot \left|z_1(t)z_1(t-\tau_i)\right| f_{i1}\left(u_0(t), y(t)\right)\end{aligned}$$

$$\times f_{i2}(u_0(t-\tau_i), y(t-\tau_i)) + |\overline{d}_{i-1}| \cdot |z_1(t)| \widetilde{\psi}_i(u_0(t), y(t)). \quad (18)$$

Remark 10. By lemmas and assumptions before, Lemmas 8 and 9 can be derived easily, and then the proof is omitted.

3.2. Observer Design. Define the following filter/estimator:

$$\dot{\xi}_0(t) = \left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right)\xi_0(t) + PC^T(y(t) - C\xi_0(t)), \quad (19)$$

$$\dot{v}(t) = \left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right)v(t) + e_n u_1(t), \quad (20)$$

$$\dot{P} = P\left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right)^T + \left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right)P - PC^TCP + I, \quad (21)$$

where $y(t) = z_1(t), e_n = [0,\ldots,1]^T, \xi_0 = [\xi_{01},\ldots,\xi_{0n}]^T, v = [v_1,\ldots,v_n]^T$, $A_0 = A - KC, C = [1,0,\ldots,0]$, $K = [k_1,\ldots,k_n]^T$, and k_i $(1 \le i \le n)$ are design parameters to be determined later. Let $\widehat{z}(t) = \xi_0(t) + \overline{d}_n v, \sigma(t) = z(t) - \overline{d}_n v(t)$; then, the estimation error $\varepsilon(t) = z(t) - \widehat{z}(t)$ and the newly defined parameter $\sigma(t)$ satisfy the dynamical equations

$$\begin{aligned}\dot{\varepsilon}(t) &= \left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)} - PC^TC\right)\varepsilon(t) \\ &\quad + (K - PC^T)z_1(t) + PC^TC\sigma(t) + \Psi + \Phi, \quad (22)\\ \dot{\sigma}(t) &= \left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right)\sigma(t) + Kz_1(t) + \Psi + \Phi.\end{aligned}$$

3.3. Control Design. In this section, the intergrator backstepping approach will be used to design the control laws $u_0(t)$ and $u_1(t)$ subject to $x_0(t_0) \neq 0$. The case that the initial condition $x_0(t_0) = 0$ will be treated in Section 4.

Step 0. At this step, control law $u_0(t)$ will be designed, which is essential to guarantee the effectiveness of the subsequent steps. For the $x_0(t)$-subsystem, choose the control $u_0(t)$ as follows:

$$u_0(t) = -\lambda_0 x_0(t) - \lambda_0 x_0(t)\overline{\phi}_0(x_0(t)), \quad (23)$$

where λ_0 is a constant satisfying $\lambda_0 d_0 > 1$. Introduce the Lyapunov function candidate $V_0 = (1/2)x_0^2(t)$, and the time derivative of V_0 satisfies

$$\begin{aligned}\dot{V}_0 &= -\lambda_0 d_0 x_0^2(t) - \lambda_0 d_0 x_0^2(t)\overline{\phi}_0(x_0(t)) \\ &\quad + x_0(t)\phi_0(t, x_0(t)) \quad (24)\\ &\le -\lambda_0 d_0 x_0^2(t) \triangleq -c_0 x_0^2(t),\end{aligned}$$

where $c_0 = \lambda_0 d_0 > 1$. This indicates that $x_0(t)$ converges to zero exponentially.

Since $\overline{\phi}_0(x_0(t))$ is a smooth function, then there exist a constant $M_0 > 1$, such that $|\overline{\phi}_0(x_0(t))| \le M_0$ for $|x_0(t)| \le 1$. Therefore, the following inequality is true with $|x_0(t)| \le 1$:

$$\dot{V}_0 \ge -(\lambda_0 d_0 + \lambda_0 d_0 M_0 + M_0)x_0^2(t) \triangleq -\rho x_0^2(t), \quad (25)$$

which implies that when $|x_0(t)| \le 1$, the state $x_0(t)$ converges to zero with a rate less than a certain constant ρ. It is $x_0(t)$ which does not become zero in any time instant. Therefore, the adopted input-state scaling discontinuous transformation in (12) is effective.

According to the design of control law $u_0(t)$ in (23), it can be computed that

$$\begin{aligned}\frac{\dot{u}_0(t)}{u_0(t)} &= -\lambda_0 d_0 - (\lambda_0 d_0 - 1)\overline{\phi}_0(x_0(t)) \\ &\quad - \lambda_0 d_0 x_0(t)\frac{\partial\overline{\phi}_0(x_0(t))}{\partial x_0(t)} \\ &\quad + \frac{x_0(t)\overline{\phi}_0(x_0(t))}{1 + \overline{\phi}_0(x_0(t))}\frac{\partial\overline{\phi}_0(x_0(t))}{\partial x_0(t)} \quad (26)\\ &\triangleq \beta + \widetilde{\phi}_0(x_0(t)),\end{aligned}$$

where $\beta = -\lambda_0 d_0$ and $\widetilde{\phi}_0 = -(\lambda_0 d_0 - 1)\overline{\phi}_0(x_0(t)) - \lambda_0 d_0 x_0(t)(\partial\overline{\phi}_0(x_0(t))/\partial x_0(t)) + (x_0(t)\overline{\phi}_0(x_0(t))/(1 + \overline{\phi}_0(x_0(t))))(\partial\overline{\phi}_0(x_0(t))/\partial x_0(t))$.

Remark 11. From (26), we know that β is a constant and $\widetilde{\phi}_0(x_0(t))$ is a function with respect to $x_0(t)$. Moreover, we can conclude that $\widetilde{\phi}_0(x_0(t))$ is smooth because $\overline{\phi}_0(x_0(t))$ is a nonnegative smooth function.

Denote $A_1 = A_0 - KC - L\beta$; we can choose appropriate design parameters k_i $(1 \le i \le n)$ such that A_1 is Hurwitz. Then there exists a positive definite matrix Q satisfying $QA_1 + A_1^TQ = -\mu I$, and μ is a positive constant.

Step 1. For $z_1(t)$-subsystem in (13),

$$\begin{aligned}\dot{z}_1(t) &= z_2(t) - (n-1)\frac{\dot{u}_0(t)}{u_0(t)}z_1(t) \\ &\quad + \frac{1}{u_0^{n-1}(t)}(\varphi_1 + \phi_1) \quad (27)\\ &= \varepsilon_2(t) + \xi_{02}(t) + \overline{d}_n v_2(t) \\ &\quad - (n-1)\frac{\dot{u}_0(t)}{u_0(t)}z_1(t) + \Psi_1 + \Phi_1,\end{aligned}$$

let $\eta_1(t) = z_1(t)$, and $\eta_2(t) = v_2(t) - \alpha_1$. Introduce the following Lyapunov functional:

$$V_1 = \overline{V}_1 + \widetilde{V}_1, \quad (28)$$

where

$$\overline{V}_1 = \varepsilon^T(t) P^{-1}\varepsilon(t) + \sigma^T(t) Q\sigma(t) + \frac{1}{2}\eta_1^2(t) + \frac{\left|\overline{d}_n\right|}{2}\widetilde{\Theta}_1^T\widetilde{\Theta}_1$$

$$\widetilde{V}_1 = \left(4\ell_1 + \delta_2\|Q\|^2\right)\sum_{j=1}^{n}\int_{t-\tau_j}^{t}\eta_1^4(\sigma) f_{j2}^4(u_0(\sigma), \ y(\sigma))\,d\sigma + \frac{n}{2}\int_{t-\tau_1}^{t}\eta_1^2(\sigma) f_{12}^2(u_0(\sigma), y(\sigma))\,d\sigma, \quad (29)$$

with ℓ_1, δ_2 being positive constants to be designed; $\widetilde{\Theta}_1 = \Theta_1 - \widehat{\Theta}_1$, where Θ_1 is an unknown parameter vector to be specified later, and $\widehat{\Theta}_1$ is an estimate of Θ_1.

Associated with (22) and (27), the time derivatives of $\overline{V}_1$ and $\widetilde{V}_1$ can be calculated, respectively, that

$$\begin{aligned}
\dot{\overline{V}}_1 &= 2\varepsilon^T(t) P^{-1}\left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)} - PC^TC\right)\varepsilon(t) \\
&\quad + 2\varepsilon^T(t) P^{-1}\left(K - PC^T\right) z_1(t) \\
&\quad + 2\varepsilon^T(t) C^TC\sigma(t) + 2\varepsilon^T(t) P^{-1}\Psi \\
&\quad + 2\varepsilon^T(t) P^{-1}\Phi + 2\sigma^T(t) Q\left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right)\sigma(t) \\
&\quad + 2\sigma^T(t) QKz_1(t) + 2\sigma^T(t) Q\Psi \\
&\quad + 2\sigma^T(t) Q\Phi - 2\varepsilon^T(t)\left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right)^T P^{-1}\varepsilon(t) \\
&\quad + 2\varepsilon^T(t) C^TC\varepsilon(t) - \varepsilon^T(t) P^{-2}\varepsilon(t) \\
&\quad + \eta_1(t)\left[\varepsilon_2(t) + \xi_{02}(t) + \overline{d}_n v_2(t) - (n-1)\frac{\dot{u}_0(t)}{u_0(t)}z_1(t)\right. \\
&\qquad \left. + \Psi_1 + \Phi_1\right] - \left|\overline{d}_n\right|\widetilde{\Theta}_1^T\dot{\widehat{\Theta}}_1 \\
&= -\varepsilon^T(t) P^{-2}\varepsilon(t) - \mu\sigma^T(t)\sigma(t) \\
&\quad + 2\varepsilon^T(t) P^{-1}\left(K - PC^T\right) z_1(t) + 2\varepsilon^T(t) P^{-1}\Psi \\
&\quad + 2\varepsilon^T(t) P^{-1}\Phi + 2\varepsilon^T(t) C^TC\sigma(t) \\
&\quad - 2\sigma^T(t) QL\widetilde{\phi}(x_0(t))\sigma(t) + 2\sigma^T(t) QKz_1(t) \\
&\quad + 2\sigma^T(t) Q\Psi + 2\sigma^T(t) Q\Phi + \eta_1(t)\Psi_1 + \eta_1(t)\Phi_1 \\
&\quad - (n-1)\frac{\dot{u}_0(t)}{u_0(t)}\eta_1^2(t) + \eta_1(t)\varepsilon_2(t) - \varepsilon^T(t) C^TC\varepsilon(t) \\
&\quad + \eta_1(t)\left[\xi_{02}(t) + \overline{d}_n v_2(t)\right] - \left|\overline{d}_n\right|\widetilde{\Theta}_1^T\dot{\widehat{\Theta}}_1,
\end{aligned} \quad (30)$$

$$\begin{aligned}
\dot{\widetilde{V}}_1 &= \left(4\ell_1 + \delta_2\|Q\|^2\right)\sum_{j=1}^{n}\eta_1^4(t) f_{j2}^4(u_0(t), y(t)) + \frac{n}{2}\eta_1^2(t) \\
&\quad \times f_{12}^2(u_0(t), y(t)) - \left(4\ell_1 + \delta_2\|Q\|^2\right)\sum_{j=1}^{n}\eta_1^4\left(t-\tau_j\right) \\
&\quad \times f_{j2}^4\left(u_0\left(t-\tau_j\right), y\left(t-\tau_j\right)\right) - \frac{n}{2}\eta_1^2(t-\tau_1) \\
&\quad \times f_{12}^2\left(u_0(t-\tau_1), y(t-\tau_1)\right).
\end{aligned} \quad (31)$$

For some terms on the right-hand side of (30), the following estimations (32)–(34) should be conducted. Firstly, by Lemma 8 and Young's inequality, we can obtain that there exist positive constants ℓ_1, δ_1 to make the following inequalities hold:

$$\begin{aligned}
\eta_1(t)\Phi_1 &\le \eta_1^2(t) + \frac{1}{4}\eta_1^2(t)\,\bar{\omega}_1^2(u_0(t), x_0(t), z_1(t))\,\vartheta^2 \\
&\le \eta_1^2(t) + \frac{1}{4}\eta_1^2(t)\,\bar{\omega}_1^2(u_0(t), x_0(t), z_1(t))\,\vartheta_1, \\
2\varepsilon^T(t) P^{-1}\Phi &\le \frac{1}{4\ell_1}\varepsilon^T(t) P^{-2}\varepsilon(t) \\
&\quad + 4\ell_1\sum_{j=1}^{n}\eta_1^2(t)\,\bar{\omega}_j^2(u_0(t), x_0(t), z_1(t))\,\overline{d}_{j-1}^2\vartheta^2 \\
&\le \frac{1}{4\ell_1}\varepsilon^T(t) P^{-2}\varepsilon(t) + 4\ell_1\sum_{j=1}^{n}\eta_1^2(t) \\
&\quad \times \bar{\omega}_j^2(u_0(t), x_0(t), z_1(t))\,\vartheta_1, \\
2\sigma^T(t) Q\Phi &\le \frac{1}{\delta_1}\sigma^T(t)\sigma(t) \\
&\quad + \delta_1\|Q\|^2\sum_{j=1}^{n}\eta_1^2(t)\,\bar{\omega}_j^2(u_0(t), x_0(t), z_1(t))\,\vartheta_1,
\end{aligned} \quad (32)$$

where $\vartheta_1 = \vartheta^2 + \sum_{j=1}^{n-1}\overline{d}_j^2\vartheta^2$. Next, employ Lemma 9 and Young's inequality, and we have

$$\begin{aligned}
&\eta_1(t)\Psi_1 \\
&\quad \le \eta_1^2(t)\,\bar{\psi}_1(u_0(t), y(t)) + \frac{1}{2}\eta_1^4(t) f_{11}^2(u_0(t), y(t)) \\
&\qquad + \frac{1}{2}\eta_1^2(t-\tau_1) f_{12}^2\left(u_0(t-\tau_1), y(t-\tau_1)\right), \\
&2\varepsilon^T(t) P^{-1}\Psi \\
&\quad \le \frac{1}{4\ell_1}\varepsilon^T(t) P^{-2}\varepsilon(t) + 4\ell_1\sum_{j=1}^{n}\Psi_j^2 \\
&\quad \le \frac{1}{4\ell_1}\varepsilon^T(t) P^{-2}\varepsilon(t)
\end{aligned}$$

$$
\begin{aligned}
&+ 8\ell_1 \sum_{j=1}^{n} \eta_1^2(t)\, \widetilde{\psi}_j^2(u_0(t), y(t))\, \overline{d}_{j-1}^2 \\
&+ 4\ell_1 \sum_{j=1}^{n} \eta_1^4(t-\tau_j)\, f_{j2}^4(u_0(t-\tau_j), y(t-\tau_j)) \\
&+ 4\ell_1 \sum_{j=1}^{n} \eta_1^4(t)\, f_{j1}^4(u_0(t), y(t))\, \overline{d}_{j-1}^4 \\
\le{}& \frac{1}{4\ell_1} \varepsilon^T(t) P^{-2} \varepsilon(t) \\
&+ 8\ell_1 \sum_{j=1}^{n} \eta_1^2(t)\, \widetilde{\psi}_j^2(u_0(t), y(t))\, d \\
&+ 4\ell_1 \sum_{j=1}^{n} \eta_1^4(t-\tau_j)\, f_{j2}^4(u_0(t-\tau_j), y(t-\tau_j)) \\
&+ 4\ell_1 \sum_{j=1}^{n} \eta_1^4(t)\, f_{j1}^4(u_0(t), y(t))\, d, \\
&2\sigma^T(t) Q\Psi \\
\le{}& \frac{1}{\delta_2} \sigma^T(t)\sigma(t) + 2\delta_2 \|Q\|^2 \sum_{j=1}^{n} \eta_1^2(t)\, \widetilde{\psi}_j^2(u_0(t), y(t))\, d \\
&+ \delta_2 \|Q\|^2 \sum_{j=1}^{n} \eta_1^4(t-\tau_j)\, f_{j2}^4(u_0(t-\tau_j), y(t-\tau_j)) \\
&+ \delta_2 \|Q\|^2 \sum_{j=1}^{n} \eta_1^4(t)\, f_{j1}^4(u_0(t), y(t))\, d,
\end{aligned}
\tag{33}
$$

where $d = 1 + \sum_{j=1}^{n-1} \overline{d}_j^2 + \sum_{j=1}^{n-1} \overline{d}_j^4$, and δ_2 is a positive constant.

By completing the square, the following estimations are also true:

$$
\begin{aligned}
\eta_1(t)\,\varepsilon_2(t) &\le \frac{1}{4\ell_1} \varepsilon^T(t) P^{-2} \varepsilon(t) + \ell_1 P_{\max}^2 \eta_1^2(t), \\
2\varepsilon^T(t) P^{-1} K z_1(t) &\le \frac{1}{4\ell_1} \varepsilon^T(t) P^{-2} \varepsilon(t) + 4\ell_1 K^T K \eta_1^2(t), \\
-2\varepsilon^T(t) C^T z_1(t) &\le \frac{1}{2} \varepsilon^T(t) C^T C \varepsilon(t) + 2\eta_1^2(t), \\
2\varepsilon^T(t) C^T C \sigma(t) &\le \frac{1}{2} \varepsilon^T(t) C^T C \varepsilon(t) + 2\sigma^T(t)\sigma(t), \\
2\sigma^T(t) Q K z_1(t) &\le \sigma^T(t)\sigma(t) + K^T Q^T Q K \eta_1^2(t).
\end{aligned}
\tag{34}
$$

Substitute (31)–(34) into $\dot{V}_1$, it yields

$$
\begin{aligned}
\dot{V}_1 ={}& \dot{\overline{V}}_1 + \dot{\widetilde{V}}_1 \\
\le{}& -\left(1 - \frac{1}{\ell_1}\right) \varepsilon^T(t) P^{-2} \varepsilon(t) \\
&- \overline{c}_1 \eta_1^2(t) - (n-1)\, \widetilde{\phi}(x_0(t))\, \eta_1^2(t) \\
&- \overline{\mu} \sigma^T(t)\sigma(t) - \widetilde{\phi}(x_0(t))\, \sigma^T(t) [QL + LQ]\, \sigma(t) \\
&- \left|\overline{d}_n\right| \widetilde{\Theta}_1^T \dot{\widehat{\Theta}}_1 \\
&- \frac{n-1}{2} \eta_1^2(t-\tau_1)\, f_{12}^2(u_0(t-\tau_1), y(t-\tau_1)) \\
&+ \overline{d}_n \eta_1(t) \left[\Theta_1^T \Upsilon_1 + \upsilon_2(t)\right],
\end{aligned}
\tag{35}
$$

where $\overline{\mu} = \mu - 1/\delta_1 - 1/\delta_2 - 3$, $\overline{c}_1 = c_1 - 3 - K^T Q^T Q K - 4\ell_1 K^T K - \ell_1 P_{\max}^2 + (n-1)\beta$, $\Theta_1^T = (1/\overline{d}_n)[1, d, \vartheta_1]$, and $\Upsilon_1 = [\Upsilon_{11}, \Upsilon_{12}, \Upsilon_{13}]^T$ with

$$
\begin{aligned}
\Upsilon_{11} ={}& c_1 \eta_1(t) + \xi_{02}(t) \\
&+ \eta_1(t)\, \widetilde{\psi}_1(u_0(t), y(t)) + \frac{1}{2} \eta_1^3(t)\, f_{11}^2(u_0(t), y(t)) \\
&+ \left(4\ell_1 + \delta_2 \|Q\|^2\right) \sum_{j=1}^{n} \eta_1^3(t)\, f_{j2}^4(u_0(t), y(t)) \\
&+ \frac{n}{2} \eta_1(t)\, f_{12}^2(u_0(t), y(t)), \\
\Upsilon_{12} ={}& 8\ell_1 \sum_{j=1}^{n} \eta_1(t)\, \widetilde{\psi}_j^2(u_0(t), y(t)) \\
&+ \left(4\ell_1 + \delta_2 \|Q\|^2\right) \sum_{j=1}^{n} \eta_1^3(t)\, f_{j1}^4(u_0(t), y(t)), \\
\Upsilon_{13} ={}& \left(4\ell_1 + \delta_1 \|Q\|^2\right) \sum_{j=1}^{n} \eta_1(t)\, \widetilde{\omega}_j^2(u_0(t), x_0(t), z_1(t)) \\
&+ \frac{1}{4} \eta_1(t)\, \widetilde{\omega}_1^2(u_0(t), x_0(t), z_1(t)).
\end{aligned}
\tag{36}
$$

Choose the virtual control function α_1 and the adaptation law of $\widehat{\Theta}_1$ as follows:

$$
\alpha_1 = -\widehat{\Theta}_1^T \Upsilon_1, \tag{37}
$$

$$
\dot{\widehat{\Theta}}_1 = \operatorname{sign}\left(\overline{d}_n\right) \Upsilon_1 \eta_1(t). \tag{38}
$$

Notice that $\bar{d}_n\eta_1(t)\eta_2(t) \le \eta_1^2(t) + (\bar{d}_n^2/4)\eta_2^2(t)$, then it follows from (35)–(38) that

$$\dot{V}_1 \le -\left(1-\frac{1}{\ell_1}\right)\varepsilon^T(t)P^{-2}\varepsilon(t) - \bar{\mu}\sigma^T(t)\sigma(t) - (\bar{c}_1-1)\eta_1^2(t) - (n-1)\tilde{\phi}(x_0(t))\eta_1^2(t) - \tilde{\phi}(x_0(t))\sigma^T(t)[QL+LQ]\sigma(t) - \frac{n-1}{2} \times \eta_1^2(t-\tau_1) f_{12}^2(u_0(t-\tau_1), y(t-\tau_1)) + \frac{\bar{d}_n^2}{4}\eta_2^2(t). \quad (39)$$

Step 2. Introduce the new variable $\eta_3(t) = v_3(t) - \alpha_2$, where α_2 is regarded as the virtual control input, and take the Lyapunov functional as

$$V_2 = V_1 + \frac{1}{2}\eta_2^2(t) + \frac{1}{2}\tilde{\Theta}_2^T\tilde{\Theta}_2, \quad (40)$$

where $\tilde{\Theta}_2 = \Theta_2 - \hat{\Theta}_2$, Θ_2 is an unknown parameter vector to be defined later, and $\hat{\Theta}_2$ is an estimate of Θ_2. Then, combined with (20), (37), and (39), we have

$$\dot{V}_2 = \dot{V}_1 + \eta_2(t)\Big\{ -k_2 v_1(t) - (n-2)\beta v_2(t) - (n-2)\tilde{\phi}_0(x_0(t))v_2(t) + \eta_3(t) + \alpha_2 - \frac{\partial\alpha_1}{\partial\hat{\Theta}_1^T}\dot{\hat{\Theta}}_1 - \frac{\partial\alpha_1}{\partial\xi_{02}}\dot{\xi}_{02} - \frac{\partial\alpha_1}{\partial x_0}\dot{x}_0 - \frac{\partial\alpha_1}{\partial u_0}\dot{u}_0 - \frac{\partial\alpha_1}{\partial z_1}\left[\xi_{02} - (n-1)\frac{\dot{u}_0(t)}{u_0(t)}z_1(t)\right] - \frac{\partial\alpha_1}{\partial z_1}\varepsilon_2(t) - \frac{\partial\alpha_1}{\partial z_1}\Psi_1 - \frac{\partial\alpha_1}{\partial z_1}\Phi_1 - \frac{\partial\alpha_1}{\partial z_1}\bar{d}_n v_2(t)\Big\} - \tilde{\Theta}_2^T\dot{\hat{\Theta}}_2. \quad (41)$$

Using Lemmas 8 and 9 and Young's inequality, the following inequalities hold:

$$-\frac{\partial\alpha_1}{\partial z_1}\eta_2(t)\Psi_1 \le \frac{1}{2}\eta_1^2(t) + \frac{1}{2}\left(\frac{\partial\alpha_1}{\partial z_1}\right)^2\tilde{\psi}_1^2(u_0(t), x_0(t), z_1(t))\eta_2^2(t) + \frac{1}{2}\left(\frac{\partial\alpha_1}{\partial z_1}\right)^2\eta_1^2(t) f_{11}^2(u_0(t), y(t))\eta_2^2(t) + \frac{1}{2}\eta_1^2(t-\tau_1) f_{12}^2(u_0(t-\tau_1), y(t-\tau_1)),$$

$$-\frac{\partial\alpha_1}{\partial z_1}\eta_2(t)\Phi_1 \le \frac{1}{2}\eta_1^2(t) + \frac{1}{2}\left(\frac{\partial\alpha_1}{\partial z_1}\right)^2\tilde{\omega}_1^2(u_0(t), x_0(t), z_1(t))\eta_2^2(t)\vartheta^2,$$

$$-\frac{\partial\alpha_1}{\partial z_1}\eta_2(t)\varepsilon_2(t) \le \frac{1}{\ell_2}\varepsilon^T(t)P^{-2}\varepsilon(t) + \frac{\ell_2}{4}p_{\max}^2\left(\frac{\partial\alpha_1}{\partial z_1}\right)^2\eta_2^2(t). \quad (42)$$

By the above inequalities, we get

$$\dot{V}_2 \le -\left(1-\frac{1}{\ell_1}-\frac{1}{\ell_2}\right)\varepsilon^T(t)P^{-2}\varepsilon(t) - \bar{\mu}\sigma^T(t)\sigma(t) - (\bar{c}_1-2)\eta_1^2(t) - (n-1)\tilde{\phi}(x_0(t))\eta_1^2(t) - \tilde{\phi}(x_0(t))\sigma^T(t)[QL+LQ]\sigma(t) - \frac{n-2}{2}\eta_1^2(t-\tau_1)f_{12}^2(u_0(t-\tau_1), y(t-\tau_1)) + \eta_2(t) \times \Big\{ -k_2 v_1(t) - (n-2)\beta v_2(t) - (n-2)\tilde{\phi}(x_0(t)) \times v_2(t) - \frac{\partial\alpha_1}{\partial\hat{\Theta}_1}\dot{\hat{\Theta}}_1 - \frac{\partial\alpha_1}{\partial\xi_{02}}\dot{\xi}_{02} - \frac{\partial\alpha_1}{\partial x_0}\dot{x}_0 + \eta_3(t) - \frac{\partial\alpha_1}{\partial u_0}\dot{u}_0 - \frac{\partial\alpha_1}{\partial z_1}\left[\xi_{02} - (n-1)\frac{\dot{u}_0(t)}{u_0(t)}z_1(t)\right] + \frac{1}{2}\left(\frac{\partial\alpha_1}{\partial z_1}\right)^2\eta_1^2(t) f_{11}^2(u_0(t), y(t))\eta_2(t) + \alpha_2 + \frac{1}{2}\left(\frac{\partial\alpha_1}{\partial z_1}\right)^2\tilde{\psi}_1^2(u_0(t), x_0(t), z_1(t))\eta_2(t) + \frac{\ell_2}{4}p_{\max}^2\left(\frac{\partial\alpha_1}{\partial z_1}\right)^2\eta_2(t) + \Theta_2^T\Upsilon_2\Big\} - \tilde{\Theta}_2^T\dot{\hat{\Theta}}_2, \quad (43)$$

where $\Theta_2^T = [\vartheta^2, \bar{d}_n^2, \bar{d}_n]$ and $\Upsilon_2 = [(1/2)(\partial\alpha_1/\partial z_1)^2 \tilde{\omega}_1^2\eta_2(t),\ \eta_2(t)/4, -(\partial\alpha_1/\partial z_1)v_2(t)]^T$. By taking the adaptation law $\dot{\hat{\Theta}}_2 = \Upsilon_2\eta_2(t)$ and the virtual control function α_2 as

$$\alpha_2 = -c_2\eta_2(t) + k_2 v_1(t) + (n-2)\beta v_2(t) + (n-2)\tilde{\phi}(x_0(t))v_2(t) + \frac{\partial\alpha_1}{\partial z_1}\left[\xi_{02} - (n-1)\frac{\dot{u}_0(t)}{u_0(t)}z_1(t)\right] + \frac{\partial\alpha_1}{\partial x_0}\dot{x}_0 + \frac{\partial\alpha_1}{\partial u_0}\dot{u}_0 + \frac{\partial\alpha_1}{\partial\hat{\Theta}_1}\dot{\hat{\Theta}}_1 + \frac{\partial\alpha_1}{\partial\xi_{02}}\dot{\xi}_{02}$$

$$
\begin{aligned}
&- \frac{1}{2}\left(\frac{\partial \alpha_1}{\partial z_1}\right)^2 \eta_1^2(t) f_{11}^2(u_0(t), y(t)) \eta_2(t) \\
&- \widehat{\Theta}_2^T \Upsilon_2 - \frac{\ell_2}{4} p_{\max}^2 \left(\frac{\partial \alpha_1}{\partial z_1}\right)^2 \eta_2(t) \\
&- \frac{1}{2}\left(\frac{\partial \alpha_1}{\partial z_1}\right)^2 \widetilde{\psi}_1^2(u_0(t), x_0(t), z_1(t)) \eta_2(t),
\end{aligned} \tag{44}
$$

we can obtain

$$
\begin{aligned}
\dot{V}_2 \le &- \left(1 - \frac{1}{\ell_1} - \frac{1}{\ell_2}\right) \varepsilon^T(t) P^{-2} \varepsilon(t) \\
&- \overline{\mu} \sigma^T(t) \sigma(t) - (\overline{c}_1 - 2) \eta_1^2(t) - c_2 \eta_2^2(t) \\
&- (n-1) \widetilde{\phi}(x_0(t)) \eta_1^2(t) \\
&- \widetilde{\phi}(x_0(t)) \sigma^T(t) [QL + LQ] \sigma(t) + \eta_2(t) \eta_3(t) \\
&- \frac{n-2}{2} \eta_1^2(t-\tau_1) f_{12}^2(u_0(t-\tau_1), y(t-\tau_1)).
\end{aligned} \tag{45}
$$

Step 3. Define that $\eta_4(t) = v_4(t) - \alpha_3$, where α_3 is the virtual control input, and consider the following Lyapunov functional:

$$
V_3 = V_2 + \frac{1}{2}\eta_3^2(t) + \frac{1}{2}\widetilde{\Theta}_3^T \widetilde{\Theta}_3. \tag{46}
$$

The time derivative of V_3 along the estimator system (20) satisfies

$$
\begin{aligned}
\dot{V}_3 = \dot{V}_2 + \eta_3(t) \times \Bigg\{ &- k_3 v_1(t) - (n-3) \beta v_3(t) \\
&- (n-3) \widetilde{\phi}(x_0(t)) v_3(t) + \eta_4(t) \\
&+ \alpha_3 - \frac{\partial \alpha_2}{\partial \widehat{\Theta}_1} \dot{\widehat{\Theta}}_1 - \frac{\partial \alpha_2}{\partial \widehat{\Theta}_2} \dot{\widehat{\Theta}}_2 - \frac{\partial \alpha_2}{\partial \xi_{02}} \dot{\xi}_{02} - \frac{\partial \alpha_2}{\partial x_0} \dot{x}_0 \\
&- \frac{\partial \alpha_2}{\partial u_0} \dot{u}_0 - \frac{\partial \alpha_1}{\partial z_1} \left[\xi_{02} - (n-1) \frac{\dot{u}_0(t)}{u_0(t)} z_1(t)\right] \\
&- \frac{\partial \alpha_2}{\partial v_1} \dot{v}_1 - \frac{\partial \alpha_2}{\partial v_2} \dot{v}_2 - \frac{\partial \alpha_1}{\partial z_1} \varepsilon_2(t) - \frac{\partial \alpha_2}{\partial z_1} \Psi_1 - \frac{\partial \alpha_2}{\partial z_1} \Phi_1 \\
&- \frac{\partial \alpha_2}{\partial z_1} \overline{d}_n v_2(t) \Bigg\} - \widetilde{\Theta}_3^T \dot{\widehat{\Theta}}_3.
\end{aligned} \tag{47}
$$

By similar conduction method in (42), we have

$$
\begin{aligned}
&- \frac{\partial \alpha_2}{\partial z_1} \eta_3(t) \Psi_1 \\
&\quad \le \frac{1}{2}\eta_1^2(t) + \frac{1}{2}\left(\frac{\partial \alpha_2}{\partial z_1}\right)^2 \widetilde{\psi}_1^2(u_0(t), x_0(t), z_1(t)) \eta_3^2(t) \\
&\qquad + \frac{1}{2}\left(\frac{\partial \alpha_2}{\partial z_1}\right)^2 \eta_1^2(t) f_{11}^2(u_0(t), y(t)) \eta_3^2(t) \\
&\qquad + \frac{1}{2}\eta_1^2(t-\tau_1) f_{12}^2(u_0(t-\tau_1), y(t-\tau_1)), \\
&- \frac{\partial \alpha_2}{\partial z_1} \eta_3(t) \Phi_1 \\
&\quad \le \frac{1}{2}\eta_1^2(t) + \frac{1}{2}\left(\frac{\partial \alpha_2}{\partial z_1}\right)^2 \widetilde{\omega}_1^2(u_0(t), x_0(t), z_1(t)) \eta_3^2(t) \vartheta^2, \\
&- \frac{\partial \alpha_2}{\partial z_1} \eta_3(t) \varepsilon_2(t) \\
&\quad \le \frac{1}{\ell_3} \varepsilon^T(t) P^{-2} \varepsilon(t) + \frac{\ell_3}{4} p_{\max}^2 \left(\frac{\partial \alpha_2}{\partial z_1}\right)^2 \eta_3^2(t),
\end{aligned} \tag{48}
$$

where $\ell_3 > 0$ is a scalar. Based on (48), it yields

$$
\begin{aligned}
\dot{V}_3 \le &- \left(1 - \frac{1}{\ell_1} - \frac{1}{\ell_2} - \frac{1}{\ell_3}\right) \varepsilon^T(t) P^{-2} \varepsilon(t) - \overline{\mu} \sigma^T(t) \sigma(t) \\
&- (\overline{c}_1 - 3) \eta_1^2(t) - (n-1) \widetilde{\phi}(x_0(t)) \eta_1^2(t) \\
&- \widetilde{\phi}(x_0(t)) \sigma^T(t) [QL + LQ] \sigma(t) \\
&- \frac{n-3}{2} \eta_1^2(t-\tau_1) f_{12}^2(y(t-\tau_1)) \\
&+ \eta_3(t) \Bigg\{ \eta_2(t) - k_3 v_1(t) - (n-3) \beta v_3(t) - (n-3) \\
&\qquad \times \widetilde{\phi}(x_0(t)) v_3(t) + \eta_4(t) + \alpha_3 - \frac{\partial \alpha_2}{\partial \widehat{\Theta}_1} \dot{\widehat{\Theta}}_1 \\
&\qquad - \frac{\partial \alpha_2}{\partial \widehat{\Theta}_2} \dot{\widehat{\Theta}}_2 - \frac{\partial \alpha_1}{\partial \xi_{02}} \dot{\xi}_{02} - \frac{\partial \alpha_1}{\partial x_0} \dot{x}_0 - \frac{\partial \alpha_1}{\partial u_0} \dot{u}_0 \\
&\qquad - \frac{\partial \alpha_1}{\partial z_1} \left[\xi_{02} - (n-1) \frac{\dot{u}_0(t)}{u_0(t)} z_1(t)\right] \\
&\qquad - \frac{\partial \alpha_2}{\partial v_1} \dot{v}_1 - \frac{\partial \alpha_2}{\partial v_2} \dot{v}_2 + \frac{\ell_3}{4} p_{\max}^2 \left(\frac{\partial \alpha_2}{\partial z_1}\right)^2 \eta_3(t) \\
&\qquad + \frac{1}{2}\left(\frac{\partial \alpha_2}{\partial z_1}\right)^2 \eta_1^2(t) f_{11}^2(u_0(t), y(t)) \eta_3(t)
\end{aligned}
$$

$$+\frac{1}{2}\left(\frac{\partial\alpha_2}{\partial z_1}\right)^2\widetilde{\psi}_1^2(u_0(t),x_0(t),z_1(t))$$
$$\times\eta_3(t)+\Theta_3^T\Upsilon_3\Big\}-\widetilde{\Theta}_3^T\dot{\widehat{\Theta}}_3, \tag{49}$$

where $\Theta_3^T=[\vartheta^2,\overline{d}_n]$ and $\Upsilon_3=[(1/2)(\partial\alpha_2/\partial z_1)^2\widetilde{\omega}_1^2\eta_3(t), -(\partial\alpha_2/\partial z_1)\upsilon_2(t)]^T$. Choose the tuning function $\pi_3\Upsilon_3\eta_3(t)$, and the virtual control function α_3 as follows:

$$\alpha_3=-c_3\eta_3(t)-\eta_2(t)+k_3\upsilon_1(t)$$
$$+(n-3)\beta\upsilon_3(t)+(n-3)\widetilde{\phi}(x_0(t))\upsilon_3(t)+\frac{\partial\alpha_2}{\partial\widehat{\Theta}_1}\dot{\widehat{\Theta}}_1$$
$$+\frac{\partial\alpha_2}{\partial\widehat{\Theta}_2}\dot{\widehat{\Theta}}_2+\frac{\partial\alpha_2}{\partial\xi_{02}}\dot{\xi}_{02}+\frac{\partial\alpha_2}{\partial x_0}\dot{x}_0+\frac{\partial\alpha_2}{\partial u_0}\dot{u}_0$$
$$+\frac{\partial\alpha_2}{\partial z_1}\left[\xi_{02}-(n-1)\frac{\dot{u}_0(t)}{u_0(t)}z_1(t)\right]+\frac{\partial\alpha_2}{\partial\upsilon_1}\dot{\upsilon}_1$$
$$+\frac{\partial\alpha_2}{\partial\upsilon_2}\dot{\upsilon}_2-\frac{\ell_3}{4}p_{\max}^2\left(\frac{\partial\alpha_2}{\partial z_1}\right)^2\eta_3(t)$$
$$-\frac{1}{2}\left(\frac{\partial\alpha_2}{\partial z_1}\right)^2\eta_1^2(t)f_{11}^2(u_0(t),y(t))\eta_3(t)-\widehat{\Theta}_3^T\Upsilon_3$$
$$-\frac{1}{2}\left(\frac{\partial\alpha_2}{\partial z_1}\right)^2\widetilde{\psi}_1^2(u_0(t),x_0(t),z_1(t))\eta_3(t). \tag{50}$$

Under the virtual control function α_3 and the tuning function π_3 defined above, the derivative of V_3 becomes that

$$\dot{V}_3\leq-\left(1-\frac{1}{\ell_1}-\frac{1}{\ell_2}-\frac{1}{\ell_3}\right)\varepsilon^T(t)P^{-2}\varepsilon(t)$$
$$-(\overline{c}_1-3)\eta_1^2(t)-c_2\eta_2^2(t)-c_3\eta_3^2(t)$$
$$-\overline{\mu}\sigma^T(t)\sigma(t)-\widetilde{\phi}(x_0(t))\sigma^T(t)[QL+LQ]\sigma(t)$$
$$-(n-1)\widetilde{\phi}(x_0(t))\eta_1^2(t)+\eta_3(t)\eta_4(t)-\widetilde{\Theta}_3^T\left(\dot{\widehat{\Theta}}_3-\pi_3\right)$$
$$-\frac{n-3}{2}\eta_1^2(t-\tau_1)f_{12}^2(u_0(t-\tau_1),y(t-\tau_1)). \tag{51}$$

Step i ($4\leq i\leq n$). Assume that, at Step i–1, a virtual control function α_{i-1}, a tuning function π_{i-1}, and a Lyapunov functional V_{i-1} have been designed in such a way that

$$\dot{V}_{i-1}\leq-\left(1-\sum_{j=1}^{i-1}\frac{1}{\ell_j}\right)\varepsilon^T(t)P^{-2}\varepsilon(t)$$
$$-(\overline{c}_1-i+1)\eta_1^2(t)-\sum_{j=2}^{i-1}c_j\eta_j^2(t)$$
$$-\overline{\mu}\sigma^T(t)\sigma(t)+\eta_{i-1}(t)\eta_i(t)-\widetilde{\phi}(x_0(t))\sigma^T(t)$$
$$\times[QL+LQ]\sigma(t)-\frac{n-i+1}{2}$$
$$\times\eta_1^2(t-\tau_1)f_{12}^2(u_0(t-\tau_1),y(t-\tau_1))$$
$$-\widetilde{\Theta}_3^T\left(\dot{\widehat{\Theta}}_3-\pi_{i-1}\right)-(n-1)\widetilde{\phi}(x_0(t))\eta_1^2(t)$$
$$-\sum_{j=3}^{i-2}\frac{\partial\alpha_j}{\partial\widehat{\Theta}_3}\left(\dot{\widehat{\Theta}}_3-\pi_{i-1}\right)\eta_{j+1}(t). \tag{52}$$

Let $\eta_{i+1}(t)=\upsilon_{i+1}(t)-\alpha_i$, where α_i is regarded as the virtual control input, and choose Lyapunov functional as

$$V_i=V_{i-1}+\frac{1}{2}\eta_i^2(t). \tag{53}$$

Based on (52), the time derivative of V_i satisfies

$$\dot{V}_i=\dot{V}_{i-1}+\eta_i(t)\Big\{-k_i\upsilon_1(t)-(n-i)\beta\upsilon_i(t)$$
$$-(n-i)\widetilde{\phi}(x_0(t))\upsilon_i(t)+\eta_{i+1}(t)$$
$$-\frac{\partial\alpha_{i-1}}{\partial\widehat{\Theta}_1}\dot{\widehat{\Theta}}_1-\frac{\partial\alpha_{i-1}}{\partial\widehat{\Theta}_2}\dot{\widehat{\Theta}}_2-\frac{\partial\alpha_{i-1}}{\partial\widehat{\Theta}_3}\dot{\widehat{\Theta}}_3$$
$$-\frac{\partial\alpha_{i-1}}{\partial z_1}\left[\xi_{02}-(n-1)\frac{\dot{u}_0(t)}{u_0(t)}z_1(t)\right]$$
$$+\alpha_i-\frac{\partial\alpha_{i-1}}{\partial u_0}\dot{u}_0-\frac{\partial\alpha_{i-1}}{\partial\xi_{02}}\dot{\xi}_{02}$$
$$-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\upsilon_j}\dot{\upsilon}_j-\frac{\partial\alpha_{i-1}}{\partial z_1}\varepsilon_2(t)-\frac{\partial\alpha_{i-1}}{\partial z_1}\Psi_1$$
$$-\frac{\partial\alpha_{i-1}}{\partial x_0}\dot{x}_0-\frac{\partial\alpha_{i-1}}{\partial z_1}\Phi_1-\frac{\partial\alpha_{i-1}}{\partial z_1}\overline{d}_n\upsilon_2(t)\Big\}. \tag{54}$$

Next, we estimate the following terms in the right-hand side of (53) by Lemmas 8 and 9 and Young's inequality as follows:

$$-\frac{\partial\alpha_{i-1}}{\partial z_1}\eta_i(t)\Psi_1$$
$$\leq\frac{1}{2}\eta_1^2(t)+\frac{1}{2}\left(\frac{\partial\alpha_{i-1}}{\partial z_1}\right)^2\widetilde{\psi}_1^2(u_0(t),x_0(t),z_1(t))\eta_i^2(t)$$
$$+\frac{1}{2}\left(\frac{\partial\alpha_{i-1}}{\partial z_1}\right)^2\eta_1^2(t)f_{11}^2(u_0(t),y(t))\eta_i^2(t)$$
$$+\frac{1}{2}\eta_1^2(t-\tau_1)f_{12}^2(u_0(t-\tau_1),y(t-\tau_1)),$$
$$-\frac{\partial\alpha_{i-1}}{\partial z_1}\eta_i(t)\Phi_1$$

$$\leq \frac{1}{2}\eta_1^2(t) + \frac{1}{2}\left(\frac{\partial\alpha_{i-1}}{\partial z_1}\right)^2 \tilde{\omega}_1^2\eta_i^2(t)\vartheta^2,$$

$$-\frac{\partial\alpha_{i-1}}{\partial z_1}\eta_i(t)\varepsilon_2(t)$$

$$\leq \frac{1}{\ell_i}\varepsilon^T(t)P^{-2}\varepsilon(t) + \frac{\ell_i}{4}p_{\max}^2\left(\frac{\partial\alpha_{i-1}}{\partial z_1}\right)^2\eta_i^2(t). \tag{55}$$

Choosing the virtual control function α_i as

$$\begin{aligned}
\alpha_i = & -c_i\eta_i(t) - \eta_{i-1}(t) + k_i v_1(t) \\
& + (n-i)\beta v_i(t) + (n-i)\tilde{\phi}(x_0(t))v_i(t) \\
& + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial v_j}\dot{v}_j + \frac{\partial\alpha_{i-1}}{\partial\widehat{\Theta}_1}\dot{\widehat{\Theta}}_1 + \frac{\partial\alpha_{i-1}}{\partial\widehat{\Theta}_2}\dot{\widehat{\Theta}}_2 \\
& + \frac{\partial\alpha_{i-1}}{\partial x_0}\dot{x}_0 + \sum_{j=3}^{i-2}\frac{\partial\alpha_j}{\partial\widehat{\Theta}_3}\Upsilon_i\eta_{j+1}(t) + \frac{\partial\alpha_{i-1}}{\partial u_0}\dot{u}_0 \\
& + \frac{\partial\alpha_{i-1}}{\partial z_1}\left[\xi_{02} - (n-1)\frac{\dot{u}_0(t)}{u_0(t)}z_1(t)\right] \\
& - \frac{\ell_i}{4}p_{\max}^2\left(\frac{\partial\alpha_{i-1}}{\partial z_1}\right)^2\eta_i(t) + \frac{\partial\alpha_{i-1}}{\partial\widehat{\Theta}_3}\pi_i \\
& - \frac{1}{2}\left(\frac{\partial\alpha_{i-1}}{\partial z_1}\right)^2\eta_1^2(t)f_{11}^2(y(t))\eta_i(t) + \frac{\partial\alpha_{i-1}}{\partial\xi_{02}}\dot{\xi}_{02} - \widehat{\Theta}_3^T\Upsilon_i,
\end{aligned} \tag{56}$$

and the tuning function $\pi_i = \pi_{i-1} + \Upsilon_i\eta_i(t)$ with $\Upsilon_i = [(1/2)(\partial\alpha_{i-1}/\partial z_1)^2\tilde{\omega}_1^2\eta_i(t),\ -(\partial\alpha_{i-1}/\partial z_1)v_2(t)]^T$. Then, we can show that

$$\begin{aligned}
\dot{V}_i \leq & -\left(1 - \sum_{j=1}^{i}\frac{1}{\ell_j}\right)\varepsilon^T(t)P^{-2}\varepsilon(t) \\
& - (\bar{c}_1 - i)\eta_1^2(t) - \sum_{j=2}^{i}c_j\eta_j^2(t) \\
& - \bar{\mu}\sigma^T(t)\sigma(t) - \tilde{\phi}(x_0(t))\sigma^T(t)[QL + LQ]\sigma(t) \\
& - (n-1)\times\tilde{\phi}(x_0(t))\eta_1^2(t) \\
& - \widetilde{\Theta}_3^T(\dot{\Theta}_3 - \pi_i) - \sum_{j=3}^{i-1}\frac{\partial\alpha_j}{\partial\widehat{\Theta}_3}\eta_{j+1}(t)\left(\dot{\widehat{\Theta}}_3 - \pi_i\right) \\
& - \frac{n-i}{2}\eta_1^2(t-\tau_1)f_{12}^2(u_0(t-\tau_1), y(t-\tau_1)) \\
& + \eta_i(t)\eta_{i+1}(t).
\end{aligned} \tag{57}$$

At the last step ($i = n$), the true input $u_1(t)$ will be designed on the basis of the virtual control $\alpha_i's$ and the Lyapunov function V_{n-1} introduced before.

The actual control input $u_1(t)$ can be designed as

$$\begin{aligned}
u_1(t) = & -c_n\eta_n(t) - \eta_{n-1}(t) + k_n v_1(t) \\
& + \frac{\partial\alpha_{n-1}}{\partial\widehat{\Theta}_1}\dot{\widehat{\Theta}}_1 + \frac{\partial\alpha_{n-1}}{\partial\widehat{\Theta}_2}\dot{\widehat{\Theta}}_2 + \frac{\partial\alpha_{n-1}}{\partial\xi_{02}}\dot{\xi}_{02} \\
& + \frac{\partial\alpha_{n-1}}{\partial x_0}\dot{x}_0 + \frac{\partial\alpha_{n-1}}{\partial u_0}\dot{u}_0 \\
& + \frac{\partial\alpha_{n-1}}{\partial z_1}\left[\xi_{02} - (n-1)\frac{\dot{u}_0(t)}{u_0(t)}z_1(t)\right] - \widehat{\Theta}_3^T\Upsilon_n \\
& + \frac{\partial\alpha_{n-1}}{\partial\widehat{\Theta}_3}\pi_n - \frac{1}{2}\left(\frac{\partial\alpha_{n-1}}{\partial z_1}\right)^2\eta_1^2(t)f_{11}^2(y(t))\eta_n(t) \\
& - \frac{\ell_n}{4}p_{\max}^2\left(\frac{\partial\alpha_{n-1}}{\partial z_1}\right)^2\eta_n(t) \\
& + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial v_j}\dot{v}_j + \sum_{j=3}^{n-2}\frac{\partial\alpha_j}{\partial\widehat{\Theta}_3}\Upsilon_n\eta_{j+1}(t),
\end{aligned} \tag{58}$$

and the update law $\dot{\widehat{\Theta}}_3 = \pi_n$ with $\pi_n = \pi_{n-1} + \Upsilon_n\eta_n(t)$ and $\Upsilon_n = [(1/2)(\partial\alpha_{n-1}/\partial z_1)^2\tilde{\omega}_1^2\eta_n(t), -(\partial\alpha_{n-1}/\partial z_1)v_2(t)]^T$. Eventually, it can be achieved that

$$\begin{aligned}
\dot{V}_n \leq & -\left(1 - \sum_{j=1}^{n}\frac{1}{\ell_j}\right)\varepsilon^T(t)P^{-2}\varepsilon(t) \\
& - \bar{\mu}\sigma^T(t)\sigma(t) - (\bar{c}_1 - n)\eta_1^2(t) - \sum_{j=2}^{n}c_j\eta_j^2(t) - (n-1) \\
& \times\tilde{\phi}(x_0(t))\eta_1^2(t) - \tilde{\phi}(x_0(t))\sigma^T(t)[QL + LQ]\sigma(t).
\end{aligned} \tag{59}$$

3.4. Stability Analysis. Notice that $\tilde{\phi}(x_0(t))$ tends to zero as $x_0(t)$ converges to origin, and $\delta_1, \delta_2, \ell_i, c_i$ $(1 \leq i \leq n)$ in (59) are positive design parameters. Therefore, by an appropriate parameter choice, there exist positive constants $\lambda_i > 0$ $(1 \leq i \leq n+2)$ such that

$$\begin{aligned}
\dot{V}_n \leq & -\sum_{j=1}^{n}\lambda_j\eta_j^2(t) - \lambda_{n+1}\varepsilon^T(t)P^{-2}\varepsilon(t) \\
& - \lambda_{n+2}\sigma^T(t)\sigma(t).
\end{aligned} \tag{60}$$

It can be seen that $\eta_i(t), \varepsilon(t), \sigma(t), \widetilde{\Theta}_1, \widetilde{\Theta}_2, \widetilde{\Theta}_3$ are bounded. Since θ and d_i are unknown bounded parameters, $\widehat{\Theta}_1, \widehat{\Theta}_2, \widehat{\Theta}_1$ are bounded. According to estimator equations (19)–(21), it can be deduced that the boundedness of $z_1(t) = \eta_1(t)$ guarantees the boundedness of $\xi_0(t)$, and then $v_1(t) = (1/\bar{d}_n)(z_1(t) - \sigma_1(t))$ and α_1 are also bounded. By similar analysis, we can conclude that all signals of the closed loop system are bounded.

By LaSalle invariant Theorem, it further achieves that $\eta_i(t), \varepsilon(t), \sigma(t), \widetilde{\Theta}_1, \widetilde{\Theta}_2, \widetilde{\Theta}_3 \to 0$ as $t \to \infty$. By the controller design procedure, we get that $\xi_0(t), v(t), \alpha_i, u_1(t)$ asymptotically tend to zero. Then, the definitions $\widehat{z}(t) = \xi_0(t) + \overline{d}_n v(t)$ and $z(t) = \varepsilon(t) + \widehat{z}(t)$ show the asymptotical convergence of $\widehat{z}(t)$ and $z(t)$. Finally, from the transformations (10) and (12), we know $x_i(t) = (1/\overline{d}_n)u_0^{n-i}(t)z_i(t)$, which indicates that the states $x_i(t)$ asymptotically converge to zero with the initial condition $x_0(t_0) \neq 0$.

For purposes of analysis, we can rewrite the system (14) as follows:

$$\dot{z}(t) = \left(A_1 - L\overline{\phi}_0(x_0(t))\right)z(t) + Kz_1(t) + Bu_1(t) + \Psi + \Phi. \quad (61)$$

To solve the above differential equation, we have

$$\begin{aligned} z(t) &= e^{(A_1 - L\overline{\phi}_0(x_0(t)))t} z(t_0) \\ &\quad + \int_{t_0}^{t} e^{(A_1 - L\overline{\phi}_0(x_0(t)))(t-s)} \left[Kz_1(s) + Bu_1(s) + \Psi + \Phi\right] ds. \end{aligned} \quad (62)$$

Notice that $A_1 = A - KC - L\beta$ is *Hurwitz*, and $\overline{\phi}_0(x_0(t))$ tends to zero as $x_0(t) \to 0$, then by Lemmas 8 and 9, there exist constants $\varrho_1 > 0, \varrho_2 > 0$ such that

$$\begin{aligned} |z(t)| &\le \varrho_1 e^{-\varrho_2 t} |z(t_0)| \\ &\quad + \int_{t_0}^{t} \varrho_1 e^{-\varrho_2 (t-s)} \left[\|K\| \cdot |z_1(s)| + \|B\| \right. \\ &\quad \left. \cdot |u_1(s)| + \|\Psi\| + \|\Phi\|\right] ds \\ &\le \varrho_1 e^{-\varrho_2 t} |z(t_0)| \\ &\quad + \varrho_1 e^{-\varrho_2 t} \int_{t_0}^{t} e^{\varrho_2 s} \left[\|K\| \cdot |z_1(s)| + \|B\| \cdot |u_1(s)| \right. \\ &\quad \left. + |z_1(s)| \widetilde{G}_1 + |z_1(s)| \widetilde{G}_2\right] ds, \end{aligned} \quad (63)$$

where $\widetilde{G}_1$ is a nonnegative smooth function of $d_i, u_0(s), u_0(s-\tau_i), y(s), y(s-\tau_i)$, and $\widetilde{G}_2$ is a nonnegative smooth function of $d_i, u_0(s), x_0(s), z_1(s), \vartheta$.

Since $x_0(t), x_1(t), u_0(t)$ and the system parameters are all bounded, then $\widetilde{G}_1, \widetilde{G}_2$ in (63) are also bounded. Employing the convergence of $x_0(t), z_1(t), u_1(t)$, we can get that $z(t)$-system is globally asymptotically convergent. From the introduced transformations before, it can be deduced that system (1) is also asymptotically convergent. Now, we can express the following theorem.

Theorem 12. *For system* (1)*, under Assumptions 1–3, if the control strategies* (23) *and* (58) *are applied with an appropriate choice of the design parameters, the global asymptotic stabilization of the closed loop system is achieved for* $x_0(t_0) \neq 0$.

In the next section, we will deal with the stability analysis of the closed loop as long as the initial condition $x_0(t_0)$ is zero.

4. Switching Controller

Several switching controllers have been proposed in some existing literatures. As well known, the choice of a constant feedback for $u_0(t)$ may lead to a finite escape. In this note, the following switching category can be designed for the stabilization of system (1) with the initial sate $x_0(t_0) = 0$. Choosing controller $u_0(t)$ as

$$u_0(t) = \operatorname{sign}(d_0) u_0^*, \quad \text{when } |x_0(t)| \le \varrho_3 < x_0^*, \quad (64)$$

where $u_0^* > 0$ and $\varrho_3 > 0$ are constants.

Since $x_0(t_0) = 0$, then $\dot{x}_0(t_0)$ with $u_0(t)$ can be deduced

$$\dot{x}_0(t_0) = |d_0| u_0^* + \phi(t, x_0(t_0)) = |d_0| u_0^* > 0, \quad (65)$$

then during the initial small time period, $x_0(t)$ is increasing and satisfies $|x_0(t)| + |x_0(t)|\overline{\phi}_0(x_0(t)) < |d_0|u_0^*$.

Choose x_0^* that satisfy

$$|x_0^*| + |x_0^*| \overline{\phi}_0(x_0^*) = |d_0| u_0^*. \quad (66)$$

Obviously, $x_0(t)$ is increasing when $x_0(t) \le x_0^*$. When $|x_0(t)| \le \varrho < x_0^*$, choose the controller $u_0(t) = \operatorname{sign}(d_0)u_0^*$, and the controller $u_1(t)$ can be designed according to the simple nonlinear backstepping iterative approach. Since $|x_0(t)| > \varrho_3$, at t_s, we switch the control laws $u_0(t)$ and $u_1(t)$ into (23) and (58), respectively.

Theorem 13. *For system* (1)*, under Assumptions 1–3, if above switching control strategy is applied with an appropriate choice of the design parameters, then the closed-loop system is globally asymptotic regulated at the origin for* $x_0(t_0) = 0$.

5. Simulation Example

In this section, a numerical example will be given to illustrate that the proposed systematic control law design method is effective. Consider the following system:

$$\begin{aligned} \dot{x}_0(t) &= d_0 u_0(t) + x_0(t)^3, \\ \dot{x}_1(t) &= d_1 u_0(t) x_2(t) + \frac{1}{2} \ln\left(1 + x_1^2(t)\right) e^{x_0(t)} \\ &\quad \times x_1^2(t - 0.3) + x_1(t) \theta_1^{x_1(t)}, \\ \dot{x}_2(t) &= d_2 u_1(t) + x_1(t) e^{x_0(t-0.2)} \\ &\quad \times x_1^3(t - 0.2) + \ln\left(1 + (\theta_2 x_2(t))^2\right), \\ y(t) &= \left[x_0(t), x_1(t)\right]^T, \end{aligned} \quad (67)$$

where d_0, d_1, d_2 are virtual control directions with d_1, d_2 unknown and d_0 known, and the sign of $\overline{d}_2 = d_1 d_2$ is also known. θ_1, θ_2 are unknown bounded parameters. Next, we consider to design the controller $u_0(t)$ and $u_1(t)$ to asymptotically stabilize system (67) by the measurable

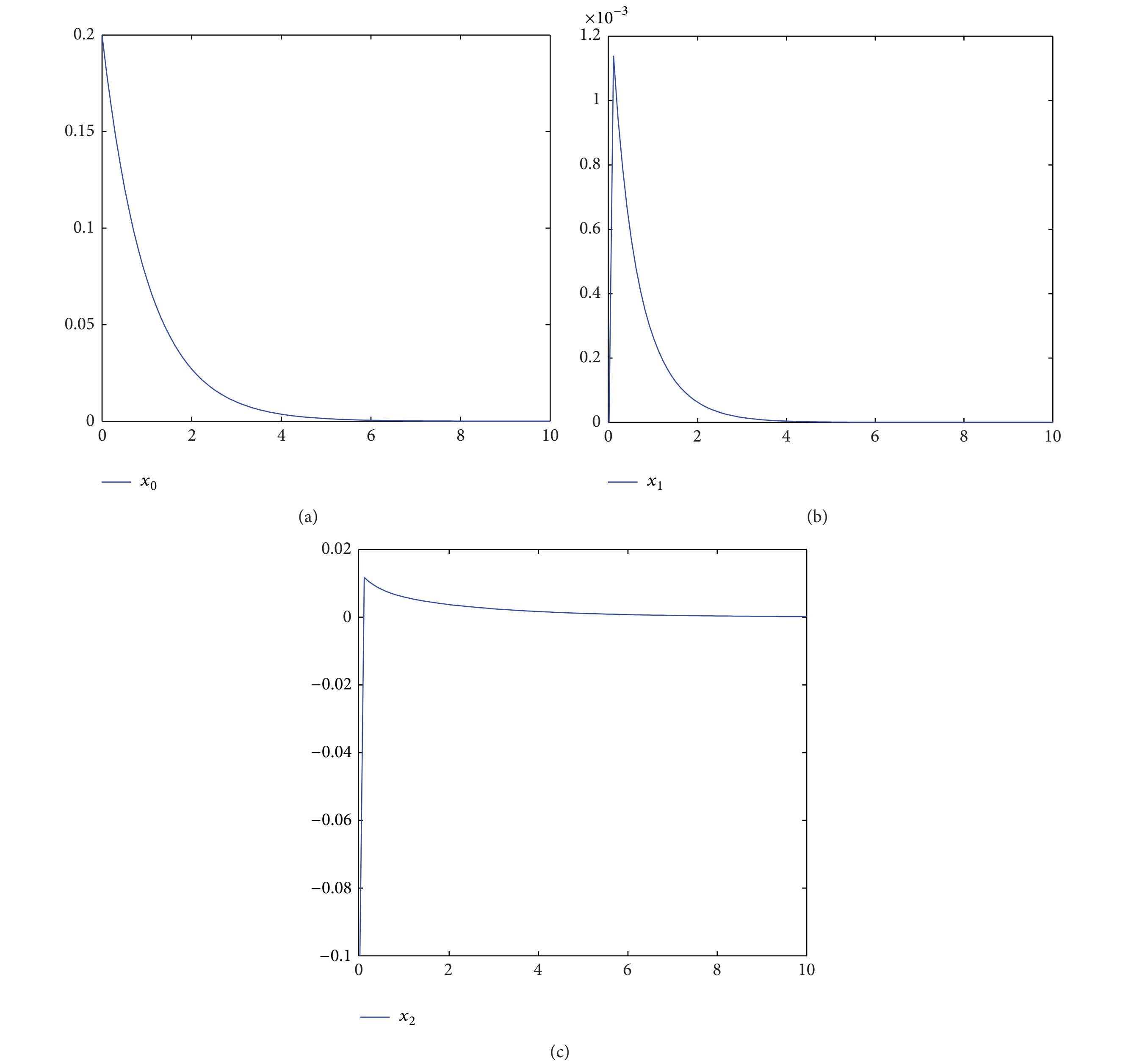

Figure 1: States $x_0(t), x_1(t), x_2(t)$.

output. We assume that $x_0(t_0) \neq 0$ and make the following estimation for some nonlinear terms in system (67):

$$\begin{aligned} x_1(t)\,\theta_1^{x_1(t)} &\leq |x_1(t)|\, e^{(1/2)x_1^2(t)}\vartheta, \\ \ln\left(1 + (\theta_2 x_2(t))^2\right) &\leq |x_1(t)|\,\vartheta, \end{aligned} \tag{68}$$

where $\vartheta = e^{(1/2)\ln^2\theta_1} + |\theta_2|$.

Firstly, we introduce the following transformation:

$$\bar{x}_1(t) = x_1(t), \qquad \bar{x}_2(t) = d_1 x_2(t), \tag{69}$$

and then the system (67) can be rewritten as

$$\begin{aligned} \dot{x}_0(t) &= d_0 u_0(t) + x_0(t)^3, \\ \dot{\bar{x}}_1(t) &= u_0(t)\,\bar{x}_2(t) + \frac{1}{2}\ln\left(1 + x_1^2(t)\right) e^{x_0(t)} \\ &\quad \times x_1^2(t - 0.3) + x_1(t)\,\theta_1^{x_1(t)}, \\ \dot{\bar{x}}_2(t) &= \bar{d}_2 u_1(t) + d_1 x_1(t)\, e^{x_0(t-0.2)} \\ &\quad \times x_1^3(t - 0.2) + d_1 \ln\left(1 + (\theta_2 x_2(t))^2\right), \end{aligned} \tag{70}$$

where $\bar{d}_2 = d_1 d_2$, and assume that the sign of $\bar{d}_2$ is known.

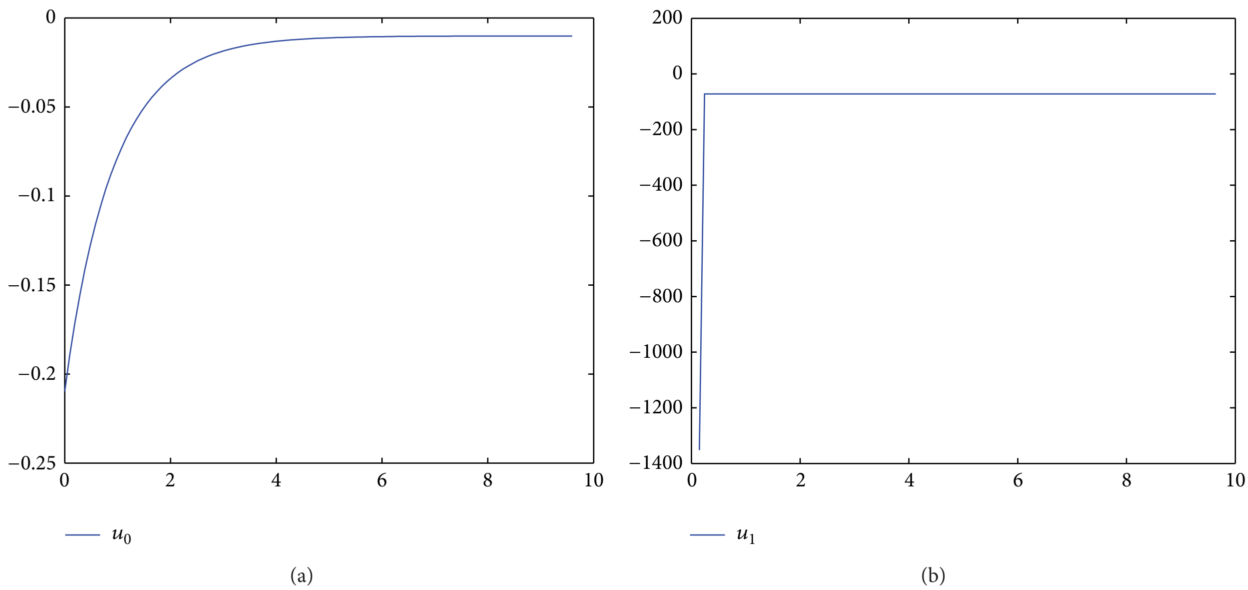

FIGURE 2: Controllers $u_0(t)$ and $u_1(t)$.

Next, make the following input scaling transformation for $\overline{x}(t)$-system:

$$z_1(t) = \frac{\overline{x}_1(t)}{u_0(t)}, \qquad z_2(t) = \overline{x}_2(t), \tag{71}$$

and then the transformed system is

$$\dot{z}(t) = \left(A - L\frac{\dot{u}_0(t)}{u_0(t)}\right) z(t) + Bu_1(t) + \Psi + \Phi, \tag{72}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad L = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \overline{d}_2 \end{bmatrix}, \qquad \Psi = \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix}, \qquad \Phi = \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix},$$

$$\Psi_1 = \frac{\ln\left(1 + x_1^2(t)\right) e^{x_0(t)} x_1^2(t - 0.3)}{2u_0(t)},$$
$$\Psi_i = d_1 x_1(t) e^{x_0(t-0.2)} x_1^3(t - 0.2), \tag{73}$$

$$\Phi_1 = \frac{x_1(t)\,\theta_1^{x_1(t)}}{u_0(t)},$$

$$\Phi_2 = d_1 \ln\left(1 + (\theta_2 x_2(t))^2\right) . \frac{\dot{u}_0(t)}{u_0(t)}.$$

Design the following controller $u_0(t)$:

$$u_0(t) = -c_0 x_0(t) - c_0 x_0(t)^3, \tag{74}$$

and then $\dot{u}_0(t)/u_0(t)$ can be calculated as follows:

$$\frac{\dot{u}_0(t)}{u_0(t)} = -c_0 d_0 - 3c_0 d_0 x_0(t) + \frac{x_0^2(t) + 3x_0^4(t)}{1 + x_0^2(t)}. \tag{75}$$

For system (72), constructing the following estimator:

$$\dot{\xi}_0(t) = \left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right) \xi_0(t) + PC^T\left(y(t) - C\xi_0(t)\right),$$

$$\dot{v}(t) = \left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right) v(t) + e_n u_1(t),$$

$$\dot{P} = P\left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right)^T + \left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right) P - PC^T CP + I, \tag{76}$$

where $y(t) = z_1(t), e_n = [0, 1]^T, \xi_0 = [\xi_{01}, \xi_{02}]^T, v = [v_1, v_2]^T$, $A_0 = A - KC$, $C = [1, 0]$, and $K = [k_1, k_2]^T$. The design of k_1, k_2 can guarantee that $A_1 = A_0 - KC - L\beta$ is *Hurwitz*. It is further achieved that there exists plosive definite matrix Q satisfying $QA_1 + A_1^T Q = -\mu I$, in which $\mu > 0$ is a constant. Denote $\widehat{z}(t) = \xi_0(t) + \overline{d}_n v$, $\sigma(t) = z(t) - \overline{d}_n v(t)$ and $\varepsilon(t) = z(t) - \widehat{z}(t)$, and then the observation error $\varepsilon(t)$ and parameter invariable $\sigma(t)$ satisfy

$$\dot{\varepsilon}(t) = \left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)} - PC^T C\right) \varepsilon(t)$$
$$+ \left(K - PC^T\right) z_1(t) + PC^T C\sigma(t) + \Psi + \Phi, \tag{77}$$
$$\dot{\sigma}(t) = \left(A_0 - L\frac{\dot{u}_0(t)}{u_0(t)}\right) \sigma(t) + Kz_1(t) + \Psi + \Phi.$$

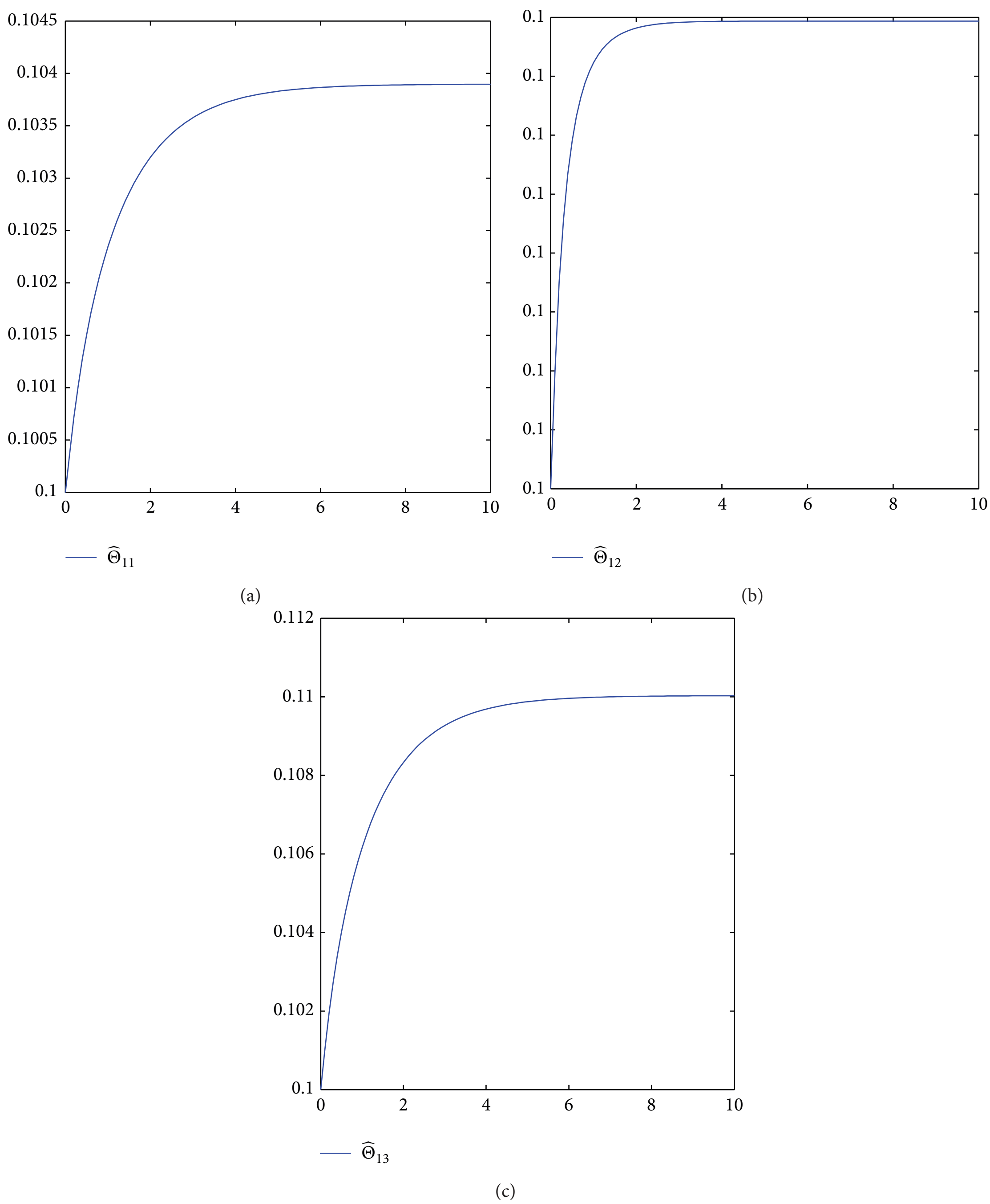

Figure 3: Parameters $\widehat{\Theta}_{11}, \widehat{\Theta}_{12}, \widehat{\Theta}_{13}$.

Define the invariable that $\eta_1(t) = z_1(t), \eta_2(t) = v_2(t) - \alpha_1$. According to the iterative procedure in Section 3, we can design the virtual control function and controller $u_1(t)$ as

$$\alpha_1 = -\widehat{\Theta}^T \Upsilon_1 = -\left[\widehat{\Theta}_{11}, \widehat{\Theta}_{12}, \widehat{\Theta}_{13}\right]\left[\Upsilon_{11}, \Upsilon_{12}, \Upsilon_{13}\right]^T,$$

$$\begin{aligned} u_1(t) = {} & -c_2\eta_2(t) + k_2 v_1(t) + \frac{\partial \alpha_1}{\partial \widehat{\Theta}_1^T}\dot{\widehat{\Theta}}_1 + \frac{\partial \alpha_1}{\partial \xi_{02}}\dot{\xi}_{02} \\ & + \frac{\partial \alpha_1}{\partial u_0(t)}\dot{u}_0(t) + \frac{\partial \alpha_1}{\partial z_1(t)}\left[\xi_{02} - \frac{\dot{u}_0(t)}{u_0(t)} z_1(t)\right] \\ & - \frac{\ell_2}{4}\left(P_{12}^2 + P_{22}^2\right)\left(\frac{\partial \alpha_1}{\partial z_1(t)}\right)^2 \eta_2(t) \\ & - \frac{1}{2}\left(\frac{\partial \alpha_1}{\partial z_1(t)}\right)^2 e^{2x_0(t)} \eta_1^2(t)\eta_2(t) - \widehat{\Theta}_2^T \Upsilon_2, \end{aligned} \tag{78}$$

where

$$\begin{aligned} \Upsilon_{11} = {} & c_1\eta_1(t) + \xi_{02}(t) + \frac{1}{2}\eta_1^3(t) e^{2x_0(t)} \\ & + \left[\frac{\ell_1}{8} + \frac{\delta_1}{32}\|Q\|^2\right]\eta_1^7(t) u_0^8(t) \end{aligned}$$

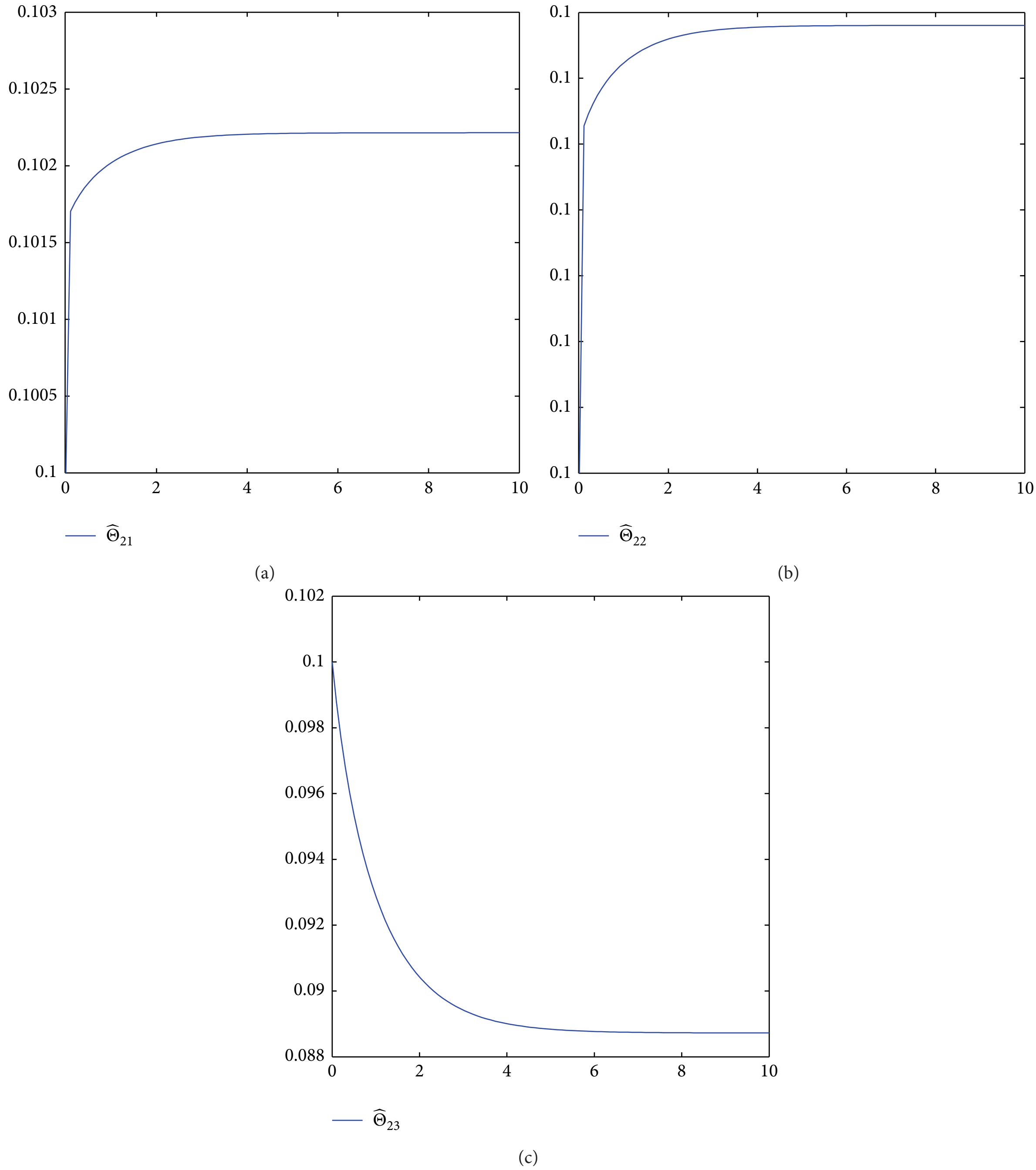

FIGURE 4: Parameters $\widehat{\Theta}_{21}, \widehat{\Theta}_{22}, \widehat{\Theta}_{23}$.

$$+\left[2\ell_1 + \frac{\delta_1}{2}\|Q\|^2\right]\eta_1^{11}(t)\, e^{4x_0(t)} u_0^{12}(t) + \frac{1}{4}\eta_1^3(t)\, u_0^4(t),$$

$$\Upsilon_{12} = 2\ell_1\eta_1^3(t)\, e^{4x_0(t)} + 2\ell_1\eta_1^3(t)\, u_0^4(t) + \frac{\delta_1}{2}\|Q\|^2\eta_1^3(t)\, e^{4x_0(t)} + \frac{\delta_1}{2}\|Q\|^2\eta_1^3(t)\, u_0^4(t),$$

$$\Upsilon_{13} = \left[\frac{1}{4} + 4\ell_1 + \delta_2\|Q\|^2\right]\eta_1(t)\, e^{z_1^2(t)u_0^2(t)} + \left[4\ell_1 + \delta_2\|Q\|^2\right]\eta_1(t)\, u_0^2(t),$$

$$\widehat{\Theta}_2^T = \left[\widehat{\Theta}_{21}, \widehat{\Theta}_{22}, \widehat{\Theta}_{23}\right],$$

$$\Upsilon_2 = \left[\frac{1}{4}\left(\frac{\partial\alpha_1}{\partial z_1(t)}\right)^2 e^{z_1^2(t)u_0^2(t)}\eta_2(t), \frac{1}{4}\eta_2(t), -\frac{\partial\alpha_1}{\partial z_1(t)}\upsilon_2(t)\right]^T. \tag{79}$$

The adaption laws of the parameter invariable in controller $u_1(t)$ are chosen as

$$\dot{\widehat{\Theta}}_1 = \operatorname{sign}\left(\overline{d}_2\right)\Upsilon_1\eta_1(t), \qquad \dot{\widehat{\Theta}}_2 = \Upsilon_2\eta_2(t). \tag{80}$$

For simulation use, we pick the unknown parameters $d_1 = 1.5, d_2 = 2.5, \theta_1 = \theta_2 = 0.5$. In addition, we take the other controller design parameters as $c_0 = 1$, $c_1 = 130$, $c_2 = 2$, $k_1 = 4$, $k_2 = 1$, $\ell_1 = 2$, $\ell_2 = 3, \delta_1 = \delta_2 = 4$. Moreover, The initial state condition is $[0.2, 0, \ -0.1]^T$. Simulation results are shown in Figures 1, 2, 3, and 4. It is obvious that the states $x_0(t), x_1(t), x_2(t)$ and control input $u_0(t), u_1(t)$ converge to zero, and the parameters estimation invariable tend to constants.

6. Conclusion

The output-feedback adaptive stabilization was investigated for a class of nonholonomic systems with unknown virtual control coefficients, nonlinear uncertainties, and unknown time delays. In order to overcome the difficulties, we introduce suitable transformation and novel Lyapunov-Krasovskii functionals, and then a recursive technique is given to design the adaptive controller. To make the input-state scaling transformation effective, the switching control strategy is employed to achieve the asymptotic stabilization.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publishing of this paper.

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