OUTPUT FEEDBACK CONTROL OF LINEAR MULTIPASS PROCESSES

E. Rogers and D.H. Owens

Centre for Industrial Modelling and Control and
Division of Dynamics and Control, University of
Strathclyde, Glasgow G1 1XJ, United Kingdom.

ABSTRACT

An error actuated output feedback controller for a sub-class of linear multipass processes designated as 'differential unit memory' is defined. Further, the design of this controller for closed-loop stability is considered. In particular, a recently developed computationally feasible stability test is used to present some preliminary work on this problem.

1. Introduction

Multipass processes are a class of dynamic systems characterised [1] by a repetitive action with interaction between successive passes. Industrial examples include [1] *longwall coal cutting and metal rolling. Further, these* processes can [1] exhibit undesirable characteristics which require unique control action.

Previous work [2] has developed a rigorous stability theory for multipass processes using an abstract representation which includes the class of so-called [1] differential unit memory linear multipass processes as a special case. The members of this class are described by the state-space model.

$$\begin{aligned} &\dot{X}_{k+1}(t) = AX_{k+1}(t) + BU_{k+1}(t) + B_oY_k(t), X_{k+1}(0) = 0 \\ &Y_{k+1}(t) = CX_{k+1}(t), 0 \le t \le \alpha, k \ge 0 \\ &X_k(t) \epsilon R^n, Y_k(t) \epsilon R^m, U_k(t) \epsilon R^l \end{aligned} \tag{1}$$

Here $Y_k(t)$ is the kth pass profile, $X_k(t)$ is the kth pass state vector, $U_k(t)$ is the kth pass control input and the pass length α is assumed finite.

The definition of, together with conditions for, stability of (1) can be found in [1]. Further, the testing of these conditions has been considered in [3]. This has resulted in a computationally feasible simulation based test. Note, however, that no work has yet been reported on the design of output feedback based control schemes. Consequently this paper defines a so-called current pass error actuated proportional output feedback controller for (1). Additionally, this recently developed simulation based test is used to present some preliminary work on its design for closed-loop stability.

2. Stability

The result of this section is based on the so-called [3] associated conventional linear system of (1) defined as

$$\begin{aligned} &\dot{X}(t) = AX(t) + B_oY(t), X(O) = 0 \\ &W(t) = CX(t) \\ &or \quad W = LY \quad where \end{aligned} \tag{2}$$

$$(LY)(t) = C\int_o^t e^{A(t-\tau)} \quad B_oY(\tau)d\tau \tag{3}$$

Hence, in effect, (2) has been obtained from (1) by setting B = 0, dropping the pass subscript and ignoring the pass length. Further, (2) is assumed to be controllable, observable and stable. In addition, it is assumed that its step response matrix $W^1(t) = C\int_o^t e^{At^1}B_odt^1$ is available from appropriate *simulation studies.*

To proceed, let f be a scalar continuous function defined on any finite interval [0,t]. Then $N_t(f)$ denotes the norm of f on [0,t], i.e.

$$N_t(f) \triangleq |f(0^+)| + \sum_{j=1}^{k} |f(t_j) - f(t_{j-1})| + |f(t) - f(t_k)| \tag{4}$$

where $0 = t_o < t_1 < t_2 <$ - - - - - are the local minima and maxima of f on [0, + ∞] and k is the largest integer satisfying $t_k \le t$. For t = + ∞,

$$N_\infty(t) = \sup_{t \ge 0} N_t(f) = \mathop{limit}_{t \to +\infty} \quad N_t(f) \tag{5}$$

whenever the limit exists. Note also that the computation of (4) and (5) is a simple exercise [4] from graphical display of f.

Suppose, therefore, that (5) is applied to each element in turn of $W^1(t)$ and denote the resulting matrix by $\|W^1{}_\infty\|p$. Then the following result, *for a proof see [3], constitutes* a sufficient condition for stability of (1) where r (.) denotes the spectral radius.

<u>THEOREM 1</u>: (1) is stable if

$$r\,(\|W^1{}_\infty\|p) < 1 \tag{6}$$

The testing of (6) for a given example is straightforward [3], consisting, essentially, of appropriate simulation studies to obtain W^1 and the subsequent computation of $r(\|W^1{}_\infty\|p)$. Hence this test is clearly computationally feasible and well suited to inclusion within a computer aided design package.

3. Output Feedback Control

One approach (for others see [3]) to altering the dynamic characteristics of (1) is to follow standard linear systems theory and employ current pass error actuated output feedback control. Thus a current pass error actuated proportional output feedback controller for (1) takes the form:

$$U_{k+l}(t) = Ke_{k+1}(t), 0 \le t \le \alpha, k \ge 0 \tag{7}$$

where K is an l x m real constant matrix, $e_{k+1}(t) = r_{k+1}(t) - Y_{k+1}(t)$ is the current pass error vector, and $r_{k+1}(t) \epsilon R^m$ represents desired behaviour on pass k+1. Suppose also that (7) is applied to (1). Then it follows immediately that the resulting closed-loop system is stable if theorem 1 holds for the linear operator defined by substituting A-BKC for A in (3).

Consider now the problem of designing (7) to stabilise (1). Then a fundamental question to be answered is when, and under what conditions, does such a stabilising control law exist. This is termed [3] the existence problem for (7) applied to (1) and its solution in the general case could prove a formidable task. For one special case, however, the following result provides a solution.

THEOREM: Suppose that $m = l$ and that the matrices A, B, B_o and C are given (after use of a state transformation if appropriate [3]) by $A = -A_o^{-1} A_1$, $B = A_o^{-1}$, $B_o = I_m$ and $C = I_m$ respectively, where A_o and A_1 are real constant matrices. Suppose also that $A_o^{-1} A_1$ has a diagonal canonical form and set

$$K = \rho A_o - A_1 \tag{8}$$

where ρ is a positive real scalar. Then $\| W_{\infty}^{1} \|_{\rho} = \frac{1}{\rho} \quad I_m$ and hence by theorem 1 the closed-loop system is stable for all choices of $\rho > 1$.

Note: Theorem 2 relates to the important practical case when the so-called derived conventional linear system, [3], of (1) has the structure of a multivariable first order lag.

4. Conclusions

A current pass error actuated proportional output feedback controller for differential unit memory linear multipass processes has been defined. Further, the design of this controller for closed-loop stability has been considered. In particular, a solution to this problem in one special case has been developed.

References

[1] Edwards, J.B. and Owens, D.H., 1982, 'Analysis and Control of Linear Multipass Processes' (Wiley Research Press).

[2] Owens, D.H., 1977, 'Stability of Linear Multipass Processes', Proc. I.E.E., 124, (11), pp1079-1082.

[3] Rogers, E., 1987, 'Feedback and Stability Theory for Linear Multipass Processes', Research Report, The Queen's University of Belfast.

[4] Owens, D.H. and Chotai, A., 1983, 'Robust Controllers for Linear Dynamic Systems Using Approximate Models' Proc. I.E.E., 130 (2), pp45-56.