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Output Regulation for Voltage Control in DC Networks With Time-Varying Loads

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Abstract—**In this letter, we propose a novel control scheme for regulating the voltage in Direct Current (DC) networks. More precisely, the proposed control scheme is based on the output regulation methodology and, differently from the results in the literature, where the loads are assumed to be constant, we consider *time-varying* loads whose dynamics are described by a class of nonlinear differential equations. We prove that the proposed control scheme achieves voltage regulation ensuring the stability of the overall network.**

Index Terms—**DC networks, voltage control, nonlinear output regulation.**

I. INTRODUCTION

THE RECENT wide spread of renewable energy sources, electronic appliances and batteries motives the design and operation of Direct Current (DC) networks, which are generally more efficient and reliable than AC networks [1].

In order to guarantee a proper and safe functioning of the overall network and the appliances connected to it, the main control objective in DC networks is voltage stabilization (see for instance [2]–[9]). In [2], new passivity properties using a Krasovskii's type Lyapunov function as storage function are presented for control of Brayton-Moser systems. A robust decentralized control scheme is presented in [3], where the loads are assumed to be measurable. A nonlinear adaptive

control scheme is designed in [4] to increase the stability margin of DC networks with unknown constant power loads. An input-to-state stability (ISS)-like Lyapunov function is obtained and used for control design in [5], ensuring voltage stability with known constant loads. Robust and decentralized passivity-based control schemes are proposed in [6], [7] to achieve voltage regulation with unknown constant ZIP loads. In [8], the authors study the conditions under which PI controllers (locally) passivate the generation units and the constant ZIP loads. In [9], a systematic and constructive design based on the port-Hamiltonian framework is proposed. However, all these works provide stability guarantees only in presence of constant load components, while loads are in practice *time-varying*.

Nowadays, the ever increasing electrification of transportation and buildings may indeed increase the demand fluctuations, putting a strain on the system stability [10], [11]. For this reason, the resilience and reliability of the power grid may benefit from the design and analysis of control strategies that theoretically guarantee the system stability in presence of time-varying loads [12], [13]. Indeed, [10] and [11] show that loads can be described for instance by stochastic processes (e.g., Ito calculus) or dynamical models, respectively, attracting much research attention (see for instance [14], [15] for AC networks and [12], [13] for DC networks). However, [12], [13] do not provide any stability or convergence guarantee. Differently, in this letter we adopt a load model similar to but more general than the one in [15], and propose a control scheme based on the output regulation methodology [16], guaranteeing voltage regulation in presence of *time-varying* loads.

The main contributions of this letter can be summarized as follows: (i) the voltage control problem in DC networks including *time-varying* loads is formulated as a standard output regulation problem; (ii) we consider *time-varying* impedance and current load components; (iii) we describe each load component as the output of a large class of nonlinear dynamical exosystem, as it is customary in output regulation theory [16]; (iv) the proposed control scheme achieves voltage regulation ensuring the stability of the overall network.

Notation: The set of positive (nonnegative) real numbers is denoted by $\mathbb{R}_{>0}$ ($\mathbb{R}_{\geq 0}$). Let **0** be the vector of all zeros or the null matrix of suitable dimension(s) and let $\mathbb{1}_n \in \mathbb{R}^n$
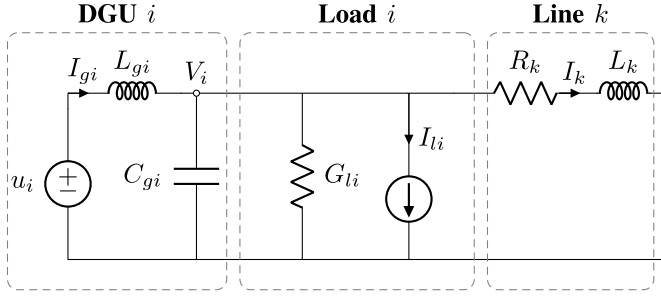
Fig. 1. Electrical scheme of DGU i, load i and transmission line k, with $i \in \mathcal{V}$ and $k \in \mathcal{E}$.

be the vector containing all ones. The i-th element of vector x is denoted by x_i. A steady-state solution to the system $\dot{x} = \zeta(x)$, is denoted by $\bar{x}$, i.e., $\mathbf{0} = \zeta(\bar{x})$. Given a vector $x \in \mathbb{R}^n$, $[x] \in \mathbb{R}^{n \times n}$ indicates the diagonal matrix whose diagonal entries are the components of x. Let $A \in \mathbb{R}^{n \times n}$ be a matrix. In case A is a positive definite (positive semi-definite) matrix, we write $A > \mathbf{0}$ ($A \geq \mathbf{0}$). The $n \times n$ identity matrix is denoted by $\mathbb{I}_n$. Let $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$ be vectors, then we define $\mathrm{col}(x, y) := (x^\top \ y^\top)^\top \in \mathbb{R}^{n+m}$. Consider the vector $x \in \mathbb{R}^n$ and functions $g : \mathbb{R}^n \to \mathbb{R}^{n \times m}$, $h : \mathbb{R}^n \to \mathbb{R}^n$, then the Lie derivative of $h(x)$ along $g(x)$ is defined as $L_g h(x) := \frac{\partial h(x)}{\partial x} g(x)$, with $\frac{\partial h(x)}{\partial x} = \mathrm{col}(\frac{\partial h_1(x)}{\partial x}, \ldots, \frac{\partial h_n(x)}{\partial x})$ and $\frac{\partial h_i(x)}{\partial x} = (\frac{\partial h_i(x)}{\partial x_1} \ \cdots \ \frac{\partial h_i(x)}{\partial x_n})$, for $i = 1, \ldots, n$. The bold symbols denote the solutions to Partial Differential Equations (PDEs).

II. PROBLEM FORMULATION

In this section, we introduce the DC network model together with the dynamics of the load components, which are considered as the outputs of nonlinear dynamical exosystems. Then, the main control objective concerning the voltage regulation is introduced.

A. DC Network Model

The model of the considered DC network includes Distributed Generation Units (DGUs), loads and transmission lines (see for instance [17]–[19] and the references therein). Fig. 1 illustrates the architecture of the considered DC network and the meaning of the used symbols. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a connected and undirected graph describing the DC network topology. The nodes and the edges are denoted by $\mathcal{V} = \{1, \ldots, n\}$ and $\mathcal{E} = \{1, \ldots, m\}$, respectively. Then, let $\mathcal{A} \in \mathbb{R}^{n \times m}$ denote the corresponding incidence matrix, whose entries are given by $\mathcal{A}_{ik} = +1$ if i is the positive end of k, $\mathcal{A}_{ik} = -1$ if i is the negative end of k, and $\mathcal{A}_{ik} = 0$ otherwise. Then, the dynamics of the overall network can be written compactly as

$$
\begin{aligned}
L_g \dot{I}_g &= -V + u \\
C_g \dot{V} &= I_g + \mathcal{A}I - [G_l]V - I_l \\
L \dot{I} &= -\mathcal{A}^\top V - RI,
\end{aligned} \tag{1}
$$

where $I_g, V, u : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, $I : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$, $L_g, C_g \in \mathbb{R}^{n \times n}_{>0}$ and $R, L \in \mathbb{R}^{m \times m}_{>0}$ are diagonal matrices. Also, $G_l, I_l : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ are *time-varying* signals. More precisely, we assume that G_l, I_l are the output vectors of nonlinear dynamical exosystems, whose

dynamics are introduced in the next subsection. To refer to the load types above, the letters Z and I, respectively, are often used in the literature (see for instance [6]).

B. Exosystems Model

In this letter, we consider the dynamics of the components of the ZI load, i.e., G_l, I_l, as the outputs of (known) nonlinear dynamical exosystems, as it is customary in output regulation theory [16]. Let y denote G or I in case of Z or I loads, respectively. Then, the exosystem dynamics can be expressed as follows:

$$
\begin{aligned}
\dot{d}^a_{yi} &= 0 \\
\dot{d}^b_{yi} &= s_{yi}(d^b_{yi}) \\
y_{li} &= \Gamma_{yi} \, \mathrm{col}\!\left(d^a_{yi}, d^b_{yi} \right),
\end{aligned} \tag{2}
$$

where $d^a_{yi} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, $d^b_{yi} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_d}$ are the states of the exosystem describing the constant and *time-varying* components of y_{li}, respectively, $s_{yi} : \mathbb{R}^{n_d} \to \mathbb{R}^{n_d}$, and $\Gamma_{yi} \in \mathbb{R}^{1 \times (n_d+1)}$, $n_d \in \mathbb{R}_{>0}$ being the dimension of the time-varying component. Then, (2) can be written compactly as

$$
\begin{aligned}
\dot{d}_y &= S_y(d_y) \\
y_l &= \Gamma_y d_y,
\end{aligned} \tag{3}
$$

where $d_y : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n(n_d+1)}$ is defined as $d_y := \mathrm{col}(d^a_{y1}, d^b_{y1}, \ldots, d^a_{yn}, d^b_{yn})$, $y_l : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, $S_y : \mathbb{R}^{n(n_d+1)} \to \mathbb{R}^{n(n_d+1)}$ is defined as $S_y := \mathrm{col}(0, s_{y1}, \ldots, 0, s_{yn})$, and $\Gamma_y := \mathrm{blockdiag}(\Gamma_{y1}, \ldots, \Gamma_{yn}) \in \mathbb{R}^{n \times n(n_d+1)}$.

C. Control Objective

Before introduce the main control objective of this letter, we notice that for a constant input $\bar{u}$, the steady-state solution $(\bar{I}_g, \bar{V}, \bar{I}, \bar{d}_I, \bar{d}_G)$ to (1) and (3) satisfies

$$
\begin{aligned}
\bar{V} &= \bar{u} & \text{(4a)} \\
\Gamma_I \bar{d}_I + [\Gamma_G \bar{d}_G]\bar{V} - \bar{I}_g &= \mathcal{A}\bar{I} & \text{(4b)} \\
\bar{I} &= -R^{-1}\mathcal{A}^\top \bar{V} & \text{(4c)} \\
\mathbf{0} &= S_I(\bar{d}_I) & \text{(4d)} \\
\mathbf{0} &= S_G(\bar{d}_G). & \text{(4e)}
\end{aligned}
$$

Then, the control objective concerning the steady-state value of the voltages is defined as follows:

Objective 1 (Voltage Regulation):

$$
\lim_{t \to \infty} V(t) = V^*, \tag{5}
$$

$V^*_i \in \mathbb{R}_{>0}$ being the voltage reference at node $i \in \mathcal{V}$.

III. OUTPUT REGULATION BASED CONTROLLER DESIGN

In this section, we formulate the voltage control problem as a standard output regulation problem [16] in order to design a control scheme achieving Objective 1.

Let the network state $x : \mathbb{R}_{\geq 0} \to \mathbb{R}^{m+2n}$ and the exosystems state $d : \mathbb{R}_{\geq 0} \to \mathbb{R}^{2n(n_d+1)}$ be defined as $x := \mathrm{col}(I_g, V, I)$ and $d := \mathrm{col}(d_I, d_G)$, respectively, and $u : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ be the

control input. Then, we can rewrite (1) and (3) as the following composite system:

$$\dot{x} = f(x, d) + g(x, d)u \tag{6a}$$
$$\dot{d} = S(d) \tag{6b}$$
$$h(x, d) = V - V^*, \tag{6c}$$

where $h(x, d)$ is the output mapping, $S(d) := \text{col}(S_I(d), S_G(d))$, $g(x, d) := \text{col}(L_g^{-1}, \mathbf{0}_{n \times n}, \mathbf{0}_{m \times n})$ and

$$f(x, d) := \begin{pmatrix} -L_g^{-1}V \\ C_g^{-1}(I_g + \mathcal{A}I - [\Gamma_G d_G]V - \Gamma_I d_I) \\ L^{-1}(-\mathcal{A}^\top V - RI) \end{pmatrix}.$$

Now, we compute the relative degree of system (6), which will be used in the following subsections for analyzing the zero dynamics of system (6). Let

$$f_a(x, d) := \text{col}(f(x, d), S(d))$$
$$g_a(x, d) := \text{col}(g(x, d), \mathbf{0}_{2n(n_d+1) \times n}), \tag{7}$$

then, based on the definition [16, Definition 2.47], the relative degree of the system (6) is computed in the following lemma.

Lemma 1 (Relative Degree of System (6)): For each $i = 1, \ldots, n$, the i-th output h_i of system (6) has relative degree $r_i = 2$ for all the trajectories (x, d).

Proof: System (6) satisfies

$$L_{g_a}h(x, d) = \mathbf{0}_{n \times n}$$
$$L_{g_a}L_{f_a}h(x, d) = C_g^{-1}L_g^{-1}, \tag{8}$$

which concludes the proof. ∎

Before introducing the output regulation methodology, we show in the following lemma that there exists a state-feedback controller that asymptotically stabilizes system (6a) when the load components are constant. More precisely, similarly to [2]–[9], we provisionally assume that G_l and I_l in (1) are constant vectors. This result is indeed needed for the solvability of the output regulation problem we introduce in the next subsection (see [16, Assumption 3.2]).

Lemma 2 (Stabilizability of System (6a) With Constant Loads): Consider system (6a) with $d = d^*$, $d^* = \text{col}(d_I^*, d_G^*) \in \mathbb{R}^{2n(n_d+1)}$ being any constant vector. Let $K_x := (-K \ \mathbf{0}_{n \times n} \ \mathbf{0}_{n \times m})$, where $K \in \mathbb{R}^{n \times n}$ is a positive definite diagonal matrix. Then, system (6a) in closed-loop with the state-feedback controller

$$u = K_x x \tag{9}$$

asymptotically converges to the equilibrium point $(\bar{I}_g, \bar{V}, \bar{I})$, satisfying (4a)–(4c).

Proof: Consider the following Lyapunov function

$$S(x) = (I_g - \bar{I}_g)^\top L_g(I_g - \bar{I}_g) + (V - \bar{V})^\top C_g(V - \bar{V}) + (I - \bar{I})^\top L(I - \bar{I}). \tag{10}$$

Then, the derivative of the Lyapunov function (10) along the solutions to (6a) satisfies

$$\dot{S}(x) = -(I_g - \bar{I}_g)^\top K(I_g - \bar{I}_g) - (V - \bar{V})^\top [\Gamma_G d_G^*](V - \bar{V}) - (I - \bar{I})^\top R(I - \bar{I})$$
$$\leq 0, \tag{11}$$

where the inequality follows from $K, R > \mathbf{0}$ and $[\Gamma_G d_G^*] \geq \mathbf{0}$. Then, as a preliminary result we can conclude that the solutions to the closed-loop system (6a), (9) are bounded. Moreover, according to LaSalle's invariance principle, these solutions converge to the largest invariant set contained in $\Omega := \{I_g, I, V : I_g = \bar{I}_g, I = \bar{I}\}$. Hence, the behavior of the closed-loop system (6a), (9) on the set Ω can be described by

$$\mathbf{0} = -V - K\bar{I}_g \tag{12a}$$
$$C\dot{V} = \bar{I}_g + \mathcal{A}\bar{I} - [\Gamma_G d_G^*]V - \Gamma_I d_I^* \tag{12b}$$
$$\mathbf{0} = -\mathcal{A}^\top V - R\bar{I}. \tag{12c}$$

Then, it follows from (12a) that V is also constant on the largest invariant set Ω, concluding the proof. ∎

In the following, we briefly recall for the readers' convenience some concepts of the output regulation methodology. Then, we propose a control scheme for the problem of voltage regulation in DC networks including *time-varying* loads.

A. Output Regulation Methodology

We now define the nonlinear output regulation problem for system (6) as follows.

Problem 1 (Nonlinear Output Regulation): Let the initial condition $(x(0), d(0))$ of system (6) be sufficiently close to the equilibrium point $(\bar{x}, \bar{d})$ satisfying (4). Then, design a static state feedback controller

$$u(t) = k(x(t), d(t)), \tag{13}$$

such that the closed-loop system (6), (13) has the following two properties:

Property 1: The trajectories $\text{col}(x(t), d(t))$ of the closed-loop system exist and are bounded for all $t \geq 0$.

Property 1: The trajectories $\text{col}(x(t), d(t))$ of the closed-loop system satisfy $\lim_{t \to \infty} h(x, d) = \mathbf{0}_n$, achieving Objective 1.[1]

If there exists a controller such that the closed-loop system satisfies Properties 1 and 2, we say that the (local) nonlinear output regulation problem (Problem 1) is *solvable*. Now, in analogy with [16, Assumption 3.1′], we introduce the following assumption.

Assumption 1 (Stability of Exosystem): The equilibrium $\bar{d}$ of the exosystem (6b) is Lyapunov stable and there exists an open neighborhood $\mathcal{D}$ of $d = \bar{d}$ in which every point is Poisson stable [16, Remark 3.2].

We need the above assumption for establishing the necessary condition for the solvability of Problem 1. Then the solvability of Problem 1 is established in the following theorem.

Theorem 1 (Solvability and Regulator Equation): Let Assumption 1 hold. Problem 1 is solvable if and only if there exist smooth functions $\boldsymbol{x}(d)$ and $\boldsymbol{u}(d)$ defined for $d \in \mathcal{D}$ such that

$$\frac{\partial \boldsymbol{x}(d)}{\partial d} S(d) = f(\boldsymbol{x}(d), d) + g(\boldsymbol{x}(d), d)\boldsymbol{u}(d) \tag{14a}$$
$$\mathbf{0}_n = h(\boldsymbol{x}(d), d). \tag{14b}$$

[1]Note that Property 2 implies $\bar{x} = \text{col}(\bar{I}_g, V^*, \bar{I})$.

Proof: See[2] [16, Th. 3.8]. ∎

The Partial Differential Equation (PDE) (14a) together with (14b) is called *regulator equation*. It can be inferred from Theorem 1 that the solvability of the regulator equation (14) is equivalent to the solvability of Problem 1.

B. Controller Design

In this subsection, a novel control scheme is designed for solving Problem 1 and, consequently, achieving Objective 1 in presence of *time-varying* loads. More precisely, we first analyze the zero dynamics of system (6) in order to make the regulator equation (14) simpler. Then we present the proposed control scheme.

Let $x(d)$ in (14) be partitioned as $x(d) = \mathrm{col}(x^a(d), x^b(d))$, with $x^a(d) = \mathrm{col}(I_g(d), V(d))$ and $x^b(d) = I(d)$. Then, consider the following PDE:

$$\frac{\partial x^b(d)}{\partial d} S(d) = \varrho\left(x^b(d), d\right), \tag{15}$$

where

$$\varrho\left(x^b(d), d\right) = -L^{-1}\left(\mathcal{A}^\top V^* + RI(d)\right). \tag{16}$$

Moreover, let $I_g(d)$, $V(d)$ be given by

$$V(d) = V^*$$
$$I_g(d) = -\mathcal{A}I(d) + [\Gamma_G d_G]V^* + \Gamma_I d_I. \tag{17}$$

Recalling that for each $i = 1, \ldots, n$, the i-th output h_i of system (6) has relative degree equal to 2 (see Lemma 1), equation (17) follows from considering the output and its first-time derivative being identically zero. In the following theorem, we propose a controller solving Problem 1.

Theorem 2 (Controller Design): Let Assumption 1 hold. Consider system (6) in closed-loop with

$$u = u_e^*(x(d), d) + K_x(x - x(d)), \tag{18}$$

where

$$\begin{aligned} u_e^*(x(d), d) &= V^* + L_g \mathcal{A}L^{-1}\mathcal{A}^\top V^* + L_g \mathcal{A}L^{-1}RI(d) \\ &\quad + L_g \Gamma_I S_I(d_I) + L_g[V^*]\Gamma_G S_G(d_G), \end{aligned} \tag{19}$$

and $I(d)$ is the solution to (15). Then, the trajectories of the closed-loop system (6), (18) starting sufficiently close to $(\bar{I}_g, V^*, \bar{I}, \bar{d}_I, \bar{d}_G)$ are bounded and converge to the set where the voltage is equal to the corresponding desired reference value V^*, achieving Objective 1.

Proof: In analogy with [16, Th. 3.26], we first compute the following matrix:

$$H_e(x, d) = \begin{pmatrix} h(x, d) \\ L_{f_a} h(x, d) \end{pmatrix}. \tag{20}$$

Then we notice that the solution to $H_e(x, d) = \mathbf{0}_{2n}$ for system (6) can be expressed as follows:

$$V = V^*$$
$$I_g = -\mathcal{A}I + [\Gamma_G d_G]V^* + \Gamma_I d_I. \tag{21}$$

Therefore, there exist the partition $x^a := \mathrm{col}(I_g, V)$, $x^b := I$ and sufficiently smooth function $\delta(x^b, d) := \mathrm{col}(-\mathcal{A}I + [\Gamma_G d_G]V^* + \Gamma_I d_I, V^*)$ such that $H_e(x, d)|_{x^a = \delta(x^b, d)} = \mathbf{0}_{2n}$. Recalling that for each $i = 1, \ldots, n$, the i-th output h_i of system (6) has relative degree equal to 2 (see Lemma 1), we compute the equivalent control input $u_e(x, d)$ by posing the second-time derivate of the output mapping (6c) equal to zero, i.e.,

$$L_{f_a}^2 h(x, d) + L_{g_a} L_{f_a} h(x, d) u_e(x, d) = \mathbf{0}_n, \tag{22}$$

that is,

$$\begin{aligned} u_e(x, d) &= V + L_g C_g^{-1}[\Gamma_G d_G]\left(I_g + \mathcal{A}I - [\Gamma_G d_G]V \right. \\ &\quad \left. - \Gamma_I d_I\right) + L_g \mathcal{A}L^{-1}\left(\mathcal{A}^\top V + RI\right) \\ &\quad + L_g \Gamma_I S_I(d_I) + L_g[V]\Gamma_G S_G(d_G). \end{aligned} \tag{23}$$

Now, let $u_e^*(x, d) := u_e(x, d)|_{x^a = \delta(x^b, d)}$. By replacing V and I_g in (23) with the right-hand side of (21), we obtain

$$\begin{aligned} u_e^*(x, d) &= V^* + L_g \mathcal{A}L^{-1}\mathcal{A}^\top V^* + L_g \mathcal{A}L^{-1}RI \\ &\quad + L_g \Gamma_I S_I(d_I) + L_g[V^*]\Gamma_G S_G(d_G). \end{aligned} \tag{24}$$

According to Lemma 1, the zero dynamics of (6) can be expressed as

$$L\dot{I} = -\mathcal{A}^\top V^* - RI$$
$$\dot{d} = S(d), \tag{25}$$

which can be rewritten as

$$\dot{x}^b = \varrho(x^b, d), \tag{26a}$$
$$\dot{d} = S(d), \tag{26b}$$

Now, we replace x^b in (26a) with the solution $x^b(d)$ to (15); therefore, $\varrho(x^b(d), d)$ can be given by (16). Moreover, by observing that the matrix $\frac{\partial \varrho(x^b, d)}{\partial x^b}|_{(x,d)=(\bar{x},\bar{d})} = -R$ is negative definite, then, according to [16, Corollary 3.27], we know that the solution to (14) exists and can be given by

$$x(d) = \begin{pmatrix} -\mathcal{A}I(d) + [\Gamma_G d_G]V^* + \Gamma_I d_I \\ V^* \\ I(d) \end{pmatrix}$$
$$u(d) = u_e^*(x(d), d), \tag{27}$$

where $I(d)$ is the solution to (15) and $u_e^*(x(d), d)$ is given by (19). Consequently, according to Theorem 1, Problem 1 is solvable. Properties 1 and 2 in Problem 1 imply that the trajectories of the closed-loop system (6), (18) starting sufficiently close to $(\bar{I}_g, V^*, \bar{I}, \bar{d}_I, \bar{d}_G)$ are bounded and converge to the set where the voltage is equal to V^*, achieving Objective 1. ∎

Remark 1 (Controller Properties): Note that the structure of the control scheme we propose in this letter is more complex than other control schemes proposed in [2]–[9]. More precisely, the proposed control scheme is distributed, requires some information about the network parameters and the exosystems, which can be determined in practice from data analysis and engineering understanding. Also, a current sensor is required at each node to measure the generated current I_g. Note that, this higher complexity is associated with the more
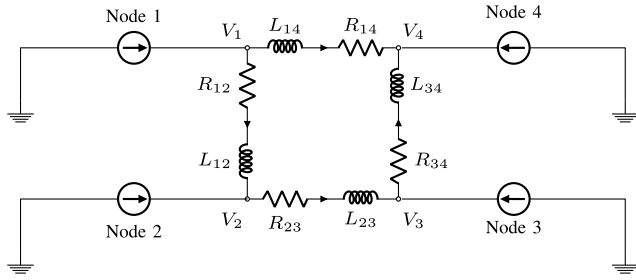
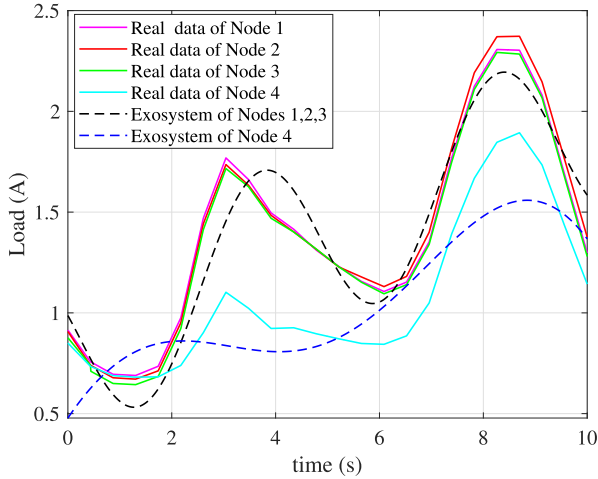Fig. 2. Scheme of the considered microgrid with 4 nodes [18], [19].

Fig. 3. Comparison between the load current profile obtained from the database [25] and the load current profile produced by the considered exosystems (similar results are obtained for the load conductance profile).

Fig. 4. Proposed controller: time evolution of the voltages at each node together with the corresponding desired values (dashed lines).

challenging control objective we achieve. Indeed, differently from [2]–[9], the proposed control scheme achieves voltage regulation in DC networks affected by *time-varying* rather than constant loads. Moreover, we notice that $I(d)$ can be approximated via the approximation methods proposed for instance in [16, Ch. 4], [20], [21].

IV. SIMULATION RESULTS

In this section, the performance of the proposed method is verified in simulation. We consider a DC network composed of 4 nodes as illustrated in Fig. 2, whose electric parameters are equal to those reported in [18, Tables 2, 3] and are identical or very similar to those used in [2], [7]–[9], [17], [19], [22] for simulations and in [6], [23], [24] for experimental validation in DC microgrids test facilities. In the following, we assume that there is a mismatch between the actual load profile and the one generated by the corresponding exosystem, showing that the controlled system is input-to-state stable (ISS) with respect to such a mismatch and the voltages are kept very close to the desired references.

Let the system initially be at the steady-state with $I_l(0) = \mathrm{col}(30, 15, 30, 26)$ A and $G_l(0) = \mathrm{col}(0.07, 0.05, 0.06, 0.08)$ Ω^{-1}. Then, we suppose that at the time instant $t = 1$ s the exosystems produce the following load variations: $\Delta I_l = 1.43 \sin(0.08t - 0.12) + 0.45 \sin(1.37t - 3.5) + 1$ A for Nodes 1, 2 and 3, $\Delta I_l = 12.41 \sin(0.477t - 1.1) + 11.98 \sin(0.495t +$
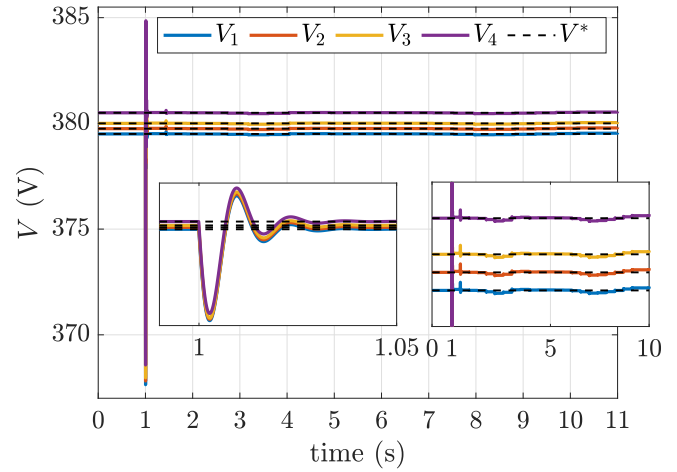
$1.97) + 0.5$ A for Node 4, and $\Delta G_l = 0.005 \Delta I_l \ \Omega^{-1}$, i.e., the exosystem (6) can be expressed as

$$\dot{d}_{yi}^{a} = 0$$

$$\dot{d}_{yi}^{b} = \begin{pmatrix} 0 & -\omega_{yi}^{\alpha} & 0 & 0 \\ \omega_{yi}^{\alpha} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\omega_{yi}^{\beta} \\ 0 & 0 & \omega_{yi}^{\beta} & 0 \end{pmatrix} d_{yi}^{b}$$

$$y_{li} = \Gamma_{yi} \mathrm{col}\left(d_{yi}^{a}, d_{yi}^{b}\right), \tag{28}$$

where $d_{yi}^{a} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, $d_{yi}^{b} : \mathbb{R}_{\geq 0} \to \mathbb{R}^4$ are the states of the exosystem, ω_{yi}^{α}, ω_{yi}^{β} are equal to 0.08 and 1.37 rad/s for Nodes 1, 2 and 3, and 0.477 and 0.495 rad/s for Node 4, respectively. Moreover, the elements of the matrix Γ_{yi} can be obtained by the amplitude and phase of the sinusoidal terms in ΔI_l and ΔG_l, where y denotes G or I in case of impedance or current loads, respectively. Then, at the time instant $t = 1$ s, we let the load vary according to the real values obtained from the dataset[3] [25], while the controller uses the information of the exosystems, which generate load trajectories that are different from the real ones (see Fig. 3). We can observe from Fig. 4 that, despite the mismatch between the actual load profile and the one generated by the corresponding exosystems, the voltage at each node is kept very close to the corresponding reference, showing that the controlled system is ISS with respect to such a mismatch, achieving in practice Objective 1 (we have also tested the case without mismatch, obtaining exact convergence to the voltage references).

Moreover, we compare our controller with the one proposed in [17], which is designed to deal with constant loads only. We can clearly observe from Fig. 5 that the controller in [17] is not capable to achieve voltage regulation.

Finally, although the estimate of the region of attraction of the equilibria is out of the scope of this letter, we have verified

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[3]Note that the dataset [25] provides hourly load profile data. However, given the fast dynamics of our system, it does not make sense to show simulations of 24 hours. Since the real load profile looks like a sinusoidal signal, we have then reproduced the same signal (in terms of amplitude) with a higher frequency.
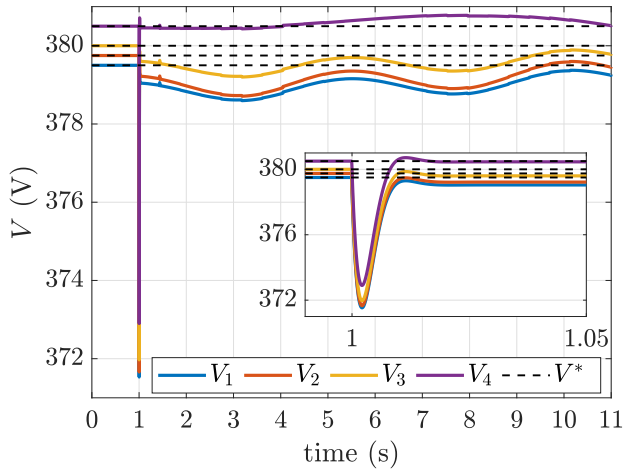
Fig. 5. Controller [17]: time evolution of the voltages at each node together with the corresponding desired values (dashed lines).

in simulation that such a region is very large, especially when compared with linear control techniques.

V. CONCLUSION AND FUTURE WORK

In this letter, we have considered *time-varying* dynamics for the load components of a DC power network. Then, we have proposed a control scheme based on the output regulation methodology to achieve voltage regulation and guarantee the stability of the overall network. Future research directions include the use of robust output regulation theory to tackle the problem of voltage regulation and current sharing in uncertain DC networks.

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