## Viewpoints

## OUTRANKING PROBABILITIES AND THE SYNTHESIS OF FORECASTS

RECENTLY in a series of papers and a monograph<sup>1-4</sup> Bunn has developed a methodology for the synthesis of forecasting models. My purpose here is to argue that Bunn's conceptual framework is not as sound as he suggests.

Roughly Bunn's argument runs as follows. In the usual Bayesian approach to forecasting <sup>5,6</sup> there is a set of possible forecasting models  $\{M_i | i = 1, 2, ..., n\}$ ; a set of forecasting distributions conditional on the models for the future observation  $y, \{P(y|M_i)|i = 1, 2, ..., n\}$ ; and a set of probabilities  $\{P(M_i)|i = 1, 2, ..., n\}$ , where  $P(M_i)$  is usually interpreted as the probability that the model  $M_i$  is "true". The Bayesian forecast is then the marginal distribution:

$$\sum_{i=1}^{n} P(y|M_i) P(M_i).$$
 (1)

Bunn rightly points out that the interpretation of  $P(M_i)$  is far from clear. What is it to say that a forecasting model is "true"? The truth or falsity of a model is not experimentally verifiable and so its probability is a concept that is fraught with difficulties of interpretation. Bunn's solution to the problem is to interpret  $P(M_i)$  as an outranking probability.

 $P(M_i)$  is to be interpreted as the probability that the model  $M_i$  will out-perform all the other models in the set, i.e. the probability that the prediction of model  $M_i$ , presumeably  $E(y|M_i)$ , is nearer y than any other prediction. It is certainly true that this outranking probability is easier to handle conceptually than the probability that a model is true. Each possible outranking event is observationally verifiable.

Unfortunately for Bunn in giving  $P(M_i)$  a simple interpretation, he makes something else in his forecasting system quite meaningless, namely the marginal distribution of y given by (1). Under the usual Bayesian formulation, this averaging is justified as a meaningful expectation; the  $P(M_i)$  give the probabilities of the conditioning events in the  $P(y|M_i)$ . Under Bunn's formulation they do not. He himself admits this implicitly when he does not update  $P(M_i)$  through the usual likelihood based on  $P(y|M_i)$ , but rather on an outranking likelihood.

Thus, I find Bunn's claim to have produced a conceptually satisfactory framework for forecasting very unconvincing. Indeed, I do not accept that the usual interpretation of  $P(M_i)$  is as impossible to handle as he claims. There is a very illuminating discussion of this problem initiated by Marschak<sup>7</sup> which shows that a rigorous and clear conceptual definition of  $P(M_i)$  is elusive. But is this surprising?

Throughout history, philosophers have played with the concept of truth as a relation between a model and the external world and they have come to no very clear conclusion. Yet we all use the concept. We do not refuse to discuss the truth or realism of a mathematical model just because we have no adequate conceptual definition. We use our intuition and, by and large, it gets us by. The  $P(M_i)$  do have a meaning for most Bayesians and when required we can quantify them. Indeed doing so is precisely as easy (or difficult) as giving a prior distribution for any parameter in a model. Bunn uses such prior distributions.

Whilst I realise that this pragmatic answer is not perfectly satisfactory as a reply to Bunn's criticism, it points, I believe, in a more hopeful direction than his outranking probabilities. Certainly it does not lead to a nonsense interpretation of the marginal distribution (1).

Before closing it might be appropriate to make some comments on the practical results obtained by Bunn. He may reasonably point to these as an argument for adopting his techniques. Bunn's methods of updating the  $P(M_i)$  with outranking likelihoods is clearly highly correlated with the usual Bayesian updating based upon  $P(y|M_i)$  likelihoods. Thus one would expect the two methods to perform similarly, and any practical results in favour of his point to similar favourable results for the conventional Bayesian methods.

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