

# OVERVIEW OF MAGNET MEASUREMENT METHODS

*K.N. Henrichsen*

CERN, Geneva, Switzerland

## **Abstract**

Electromagnets used as beam guiding elements in particle accelerators and storage rings require very tight tolerances on their magnetic fields along the particle path. Also large volumes of magnetic field in modern spectrometer magnets must be mapped in order to allow for the precise tracking of secondary particles. The article describes the methods and equipment used for these types of magnetic measurements. Short descriptions are given of the magnetic resonance techniques, the fluxmeter method, the Hall generator and the fluxgate magnetometer. A few of the more exotic methods are also briefly mentioned. References of historical nature as well as citations of recent work are indicated. It is mentioned when sensors and associated equipment are commercially available.

## **1. INTRODUCTION**

Before computers became common tools, electromagnets were designed using analytical calculations or by measuring representative voltage maps in electrolytical tanks and resistive sheets. Magnetic measurements on the final magnets and even on intermediate magnet models were imperative at that time.

Nowadays it has become possible to calculate strength and quality of magnetic fields with an impressive accuracy. However, the best and most direct way to verify that the expected field quality has been reached is to perform magnetic measurements on the finished magnet. It is also the most efficient way of verifying the quality of series produced electromagnets in order to monitor wear of tooling during production.

It is curious to note that while most measurement methods have remained virtually unchanged for a very long period, the equipment has been subject to continual development. In the following only the more commonly used methods will be discussed. It is noticeable that these methods are complementary and that a wide variety of the equipment is readily available from industry. For the many other existing measurement methods, a more complete discussion can be found in the two classical bibliographical reviews [1, 2]. An interesting description of early measurement methods can be found in [3].

## **2. MEASUREMENT METHODS**

### **2.1 Choice of measurement method**

The choice of measurement method depends on several factors. The field strength, homogeneity and variation in time, as well as the required accuracy all need to be considered. Also the number of magnets to be measured can determine the method and equipment to be deployed. As a guide, Fig. 1 shows the accuracy which can be obtained in an absolute measurement as a function of the field level, using commercially available equipment. An order of magnitude may be gained by improving the methods in the laboratory.

### **2.2 Magnetic resonance techniques**

The nuclear magnetic resonance technique is considered as the primary standard for calibration. It is frequently used, not only for calibration purposes, but also for high precision

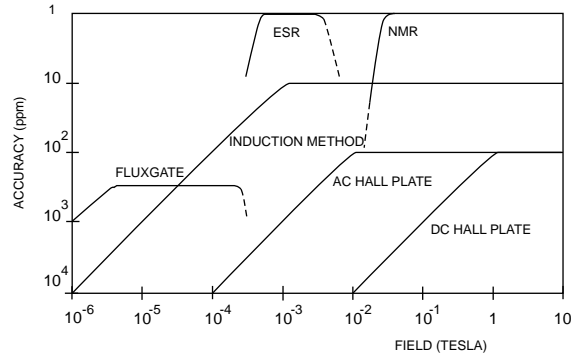


Fig. 1 Measurement methods: accuracies and ranges

solids by two independent research teams [6–8]. Since then, the method has become the most important way of measuring magnetic fields with very high precision. Based on an easy and precise frequency measurement it is independent of temperature variations. Commercially available instruments measure fields in the range from .011 T up to 13 T with an accuracy better than 10 ppm.

In practice, a sample of water is placed inside an excitation coil, powered from a radio-frequency oscillator. The precession frequency of the nuclei in the sample is measured either as nuclear induction (coupling into a detecting coil) or as resonance absorption [9]. The measured frequency is directly proportional to the strength of the magnetic field with coefficients of 42.57640 MHz/T for protons and 6.53569 MHz/T for deuterons. The magnetic field is modulated with a low-frequency signal in order to determine the resonance frequency [10].

The advantages of the method are its very high accuracy, its linearity and the static operation of the system. The main disadvantage is the need for a rather homogeneous field in order to obtain a sufficiently coherent signal. A small compensation coil, formed on a flexible printed circuit board and providing a field gradient, may be placed around the probe when used in a slightly inhomogeneous field. A correction of the order of 0.2 T/m may be obtained [10]. The limited sensitivity and dynamic range also set limits to the suitability of this method. It is, however possible to use several probes with multiplexing equipment, if a measurement range of more than half a decade is needed.

Pulsed NMR measurements have been practiced for various purposes [11, 12], even at cryogenic temperatures [13]. But equipment for this type of measurement is not yet commercially available.

Finally, it should be mentioned that a rather exotic method of NMR measurements using water flowing in a small tube has given remarkably good results at low fields [14–16]. It fills the gap in the measurement range up to 11 mT, for which NMR equipment is not yet commercially available. In addition, it provides a method of measurement in environments of strong ionizing radiation such as in particle accelerators. It was tested for measurements in the bending magnets installed in LEP. A resolution of 0.0001 mT was reached in the range from the remanent field of 0.5 mT, up to the maximum field of 112.5 mT and a corresponding reproducibility was observed [17]. The remarkable sensitivity and resolution of this measurement method makes it very suitable for absolute measurements at low fields. In fact, it was even possible to detect the Earth magnetic field outside the magnet, corresponding to an excitation frequency of about 2 kHz. However, the operation of this type of equipment is rather complicated due to the relatively long time delays in the measurement process.

Electron spin resonance (ESR) [18–21] is a related and very precise method for measuring weak fields. It is now commercially available in the range from 0.55 mT to 3.2 mT with a reproducibility of 1 ppm and is a promising tool in geology applications.

Magnetic resonance imaging (MRI) has been proposed for accelerator magnet measurements [22]. It is a very promising technique which has proven its quality in other applications. However, the related signal processing requires powerful computing facilities,

### 2.3 The fluxmeter method

This method is based on the induction law. The change of flux in a measurement coil will induce a voltage across the coil terminals. It is the oldest of the currently used methods for magnetic measurements, but it can be very precise [23]. It was used by Wilhelm Weber in the middle of the last century [24] when he studied the variations in strength and direction of the earth's magnetic field. Nowadays it has become the most important measurement method for particle accelerator magnets. It is also the most precise method for determining the direction of the magnetic flux lines; this being of particular importance in accelerator magnets. The coil geometry is often chosen to suit a particular measurement. One striking example is the Flux ball [25] whose complex construction made it possible to perform point measurements in inhomogeneous fields.

Measurements are performed either by using fixed coils in a dynamic magnetic field, or by moving the coils in a static field. The coil movement may be a rotation through a given angle, a continuous rotation or simply a movement from one position to another. Very high resolution may be reached in field mapping using this method [26].

Very high resolution may also be reached in differential fluxmeter measurements using a pair of search coils connected in opposition, with one coil moving and the other fixed, thus compensating fluctuations in the magnet excitation current and providing a much higher sensitivity when examining field quality. The same principle is applied in harmonic coil measurements, but with both coils moving. A wide variety of coil configurations are used, ranging from the simple flip-coil to the complex harmonic coil systems used in fields of cylindrical symmetry.

#### 2.3.1 Induction coils

The coil method is particularly suited for measurements with long coils in particle accelerator magnets [27, 28], where the precise measurement of the field integral along the particle trajectory is the main concern. Long rectangular coils were usually employed and are still used in magnets with a wide horizontal aperture and limited gap height. In this case, the geometry of the coil is chosen so as to link with selected field components [29]. The search coil is usually wound on a core made from a mechanically stable material, in order to ensure a constant coil area, and the wire is carefully glued to the core. Special glass or ceramics with low thermal dilatation are often used as core materials. During coil winding the wire must be stretched so that its residual elasticity assures a well-defined geometry and mechanical stability of the coil.

Continuously rotating coils with commutating polarity were already employed in 1880 [3]. The harmonic coil method has now become very popular for use in magnets with circular cylindrical geometry, in particular superconducting beam transport magnets. The coil support is usually a rotating cylinder. This method has been developed since 1954 [30, 31]. The induced signal from the rotating coil was often transmitted through slip rings to a frequency selective amplifier (frequency analyzer), thus providing analog harmonic analysis.

The principle of a very simple harmonic coil measurement is illustrated in Fig. 2. The radial coil extends through the length of the magnet and is rotated around the axis of the magnet. As the coil rotates, it will cut the radial flux lines. A number of flux measurements are done between predefined angles and will permit the precise and simultaneous determination of the strength, quality and geometry of the magnetic field. A Fourier analysis of the measured flux distribution will result in a precise description of the field parameters in terms of the harmonic coefficients:

$$B_r(r, \varphi) = B_0 \sum_{n=1}^{\infty} \left( \frac{r}{r_0} \right)^{n-1} (b_n \cos n\varphi + a_n \sin n\varphi)$$

where  $B_0$  is the amplitude of the main harmonic and  $r_0$  is a reference radius.  $b_n$  and  $a_n$  are the harmonic coefficients. In this notation  $b_1$  will describe the normal dipole coefficient,  $b_2$  the

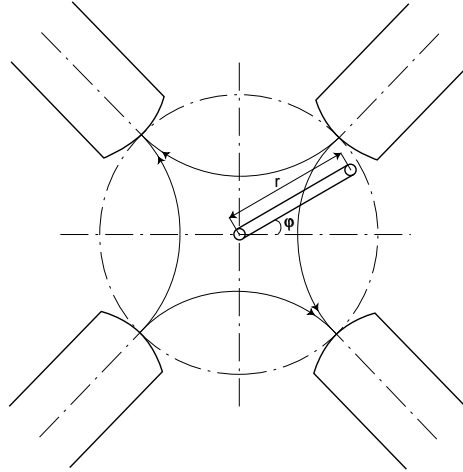


Fig. 2 Harmonic coil measurement

With the advent of modern digital integrators and angular encoders, harmonic coil measurements have improved considerably and are now considered as the best choice for most types of particle accelerator magnets, in particular those designed with cylindrical symmetry [32]. In practice, the coil is rotated one full turn in each angular direction while the electronic integrator is triggered at the defined angles by an angular encoder connected to the axis of the coil. In order to speed up the calculation of the Fourier series, it is an advantage to choose a binary number (e.g. 512) of measurement points. A compensating coil, connected in series and rotated with the main coil may be used to suppress the main field component and thus increase the sensitivity of the system for measurements of field quality. Dynamic fields are measured with a static coil linking to selected harmonics [33]. The harmonic coil measurement principle and its related equipment was described in detail in [34]. A thorough description of the general theory including detailed error analysis can be found in [35]. The practical use of the harmonic coil method for large scale measurements in superconducting magnets was described in [36, 37] and more recent developments in [38–42]

Another induction measurement consists of moving a stretched wire in the magnetic field, thus integrating the flux cut by the wire [43]. It is also possible to measure the flux change while varying the field and keeping the wire in a fixed position. Tungsten is often selected as wire material, if the wire cannot be placed in a vertical position. The accuracy is determined by the mechanical positioning of the wire. Sensitivity is limited, but can be improved by using a multi-wire array. This method is well suited to geometry measurements, to the absolute calibration of quadrupole fields and in particular to measurements in strong magnets with very small aperture.

The choice of geometry and method depends on the useful aperture of the magnet. The sensitivity of the fluxmeter method depends on the coil surface and the quality of the integrator. The coil-integrator assembly can be calibrated to an accuracy of a few tens of ppm in a homogeneous magnetic field by reference to a nuclear magnetic resonance probe, but care must be taken not to introduce thermal voltages in the related cables and connectors. It must also be avoided to induce erratic signals from wire loops exposed to magnetic flux changes. Not only the equivalent surface of the search coil must be measured, but also its median plane which often differs from its geometric plane due to winding imperfections. In the case of long measurement coils, it is important to ensure very tight tolerances on the width of the coil. If the field varies strongly over the length of the coil, it may be necessary to examine the variation of the effective width.

The main advantage of search coil techniques is the possibility of a very flexible design of the coil. The high stability of the effective coil surface is another asset. The linearity and the wide dynamic range also plays an important role. The technique can be easily adapted to measurements at cryogenic temperatures. After calibration of the coils at liquid nitrogen

other hand, the need for relatively large induction coils and their related mechanical apparatus, which is often complex, may be a disadvantage. Finally, measurements with moving coils are relatively slow.

### 2.3.2 The flux measurement

Induction coils were originally used with ballistic galvanometers and later with more elaborate fluxmeters [44]. The coil method was improved considerably with the development of photoelectric fluxmeters [45] which were used for a long period of time. The measurement accuracy was further improved with the introduction of the classic electronic integrator, the Miller integrator. It remained necessary, however, to employ difference techniques for measurements of high precision [46]. Later, the advent of digital voltmeters made fast absolute measurements possible and the Miller integrator has become the most popular fluxmeter. With the development of solid state d.c. amplifiers, this integrator has become inexpensive and is often used in multi-coil systems.

Figure 3 shows an example of such an integrator. It is based on a d.c. amplifier with a very low input voltage offset and a very high open-loop gain. The thermal variation of the integrating capacitor is the most critical problem. The integrating components are therefore mounted in a temperature-controlled oven. Another problem is the decay of the output signal through the capacitor and the resetting relay. So, careful protection and shielding of these components is essential in order to reduce the voltages across the critical surface resistances.

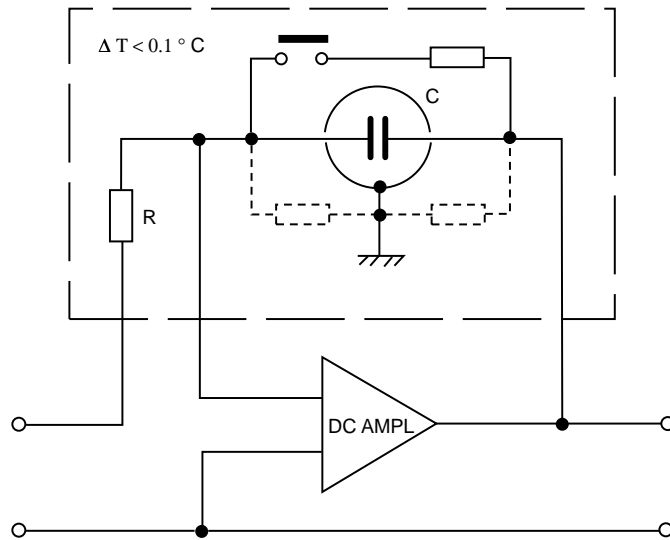


Fig. 3 Analog integrator

The dielectric absorption of the integrating capacitor sets a limit to the integrator precision. A suitable integrating resistor is much easier to find. Most metal-film resistors have stabilities and temperature characteristics matching those of the capacitor. The sensitivity of the integrator is limited by the d.c. offset and low frequency input noise of the amplifier. A typical value is  $0.5\ \mu\text{V}$  which must be multiplied by the measurement time in order to express the sensitivity in terms of flux. Thermally induced voltages may cause a problem, so care must be taken in the choice of cables and connectors. In tests at CERN the overall stability of the integrator time constant proved to be better than 50 ppm over a period of three months. A few electronic fluxmeters have been developed by industry and are commercially available.

In more recent years, a new type of digital fluxmeter has been developed, which is based on a high quality d.c. amplifier connected to a voltage-to-frequency converter (VFC) and a counter. The version shown in Fig. 4 was developed at CERN and is now commercially

of the VFC. Two counters are used in order to measure with continuously moving coils and to provide instant readings of the integrator. One of the counters can then be read and reset while the other is active. In this way no cumulative errors will build up. This fluxmeter has a linearity of 50 ppm. Its sensitivity is limited by the input amplifier, as in the case of the analog amplifier.

This system is well adapted to digital control but imposes limits on the rate of change of the flux since the input signal must never exceed the voltage level of the VFC. The minimum integration period over the full measurement range must be of the order of one second to obtain a reasonable resolution.

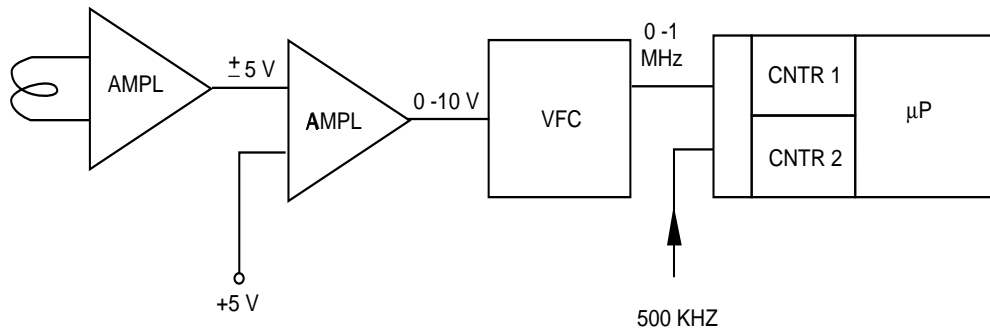


Fig. 4 Digital integrator

## 2.4 The Hall generator method

E.H. Hall discovered in 1879 that a very thin metal strip immersed in a transverse magnetic field and carrying a current developed a voltage mutually at right angles to the current and field that opposed the Lorentz force on the electrons [47]. In 1910 the first magnetic measurements were performed using this effect [48]. It was, however, only around 1950 that suitable semiconductor materials were developed [49–51] and since then the method has been used extensively. It is a simple and fast measurement method, providing relatively good accuracy, and therefore the most commonly used in large-scale field mapping [52–54]. The accuracy can be improved at the expense of measurement speed.

### 2.4.1 Hall-probe measurements

The Hall generator provides an instant measurement, uses very simple electronic measurement equipment and offers a compact probe, suitable for point measurements. A large selection of this type of gaussmeter is now commercially available. The probes can be mounted on relatively light positioning gear [54]. Considerable measurement time may be gained by mounting Hall generators in modular multi-probe arrays and applying multiplexed voltage measurement. Also simultaneous measurements in two or three dimensions may be carried out with suitable probe arrays [55, 56]. The wide dynamic range and the possibility of static operation are other attractive features.

However, several factors set limits on the obtainable accuracy. The most serious is the temperature coefficient of the Hall voltage. Temperature stabilization is usually employed in order to overcome this problem [57], but increases the size of the probe assembly. The temperature coefficient may also be taken into account in the probe calibration by monitoring the temperature during measurements [58]. It depends, however, also on the level of the magnetic field [58], so relatively complex calibration tables are needed. Another complication can be that of the planar Hall effect [59], which makes the measurement of a weak field component normal to the plane of the Hall generator problematic if a strong field component is present parallel to this plane. This effect limits the use in fields of unknown geometry and in particular its use for determination of field geometry.

Last but not least is the problem of the non linearity of the calibration curve. since the Hall

better linearity and has a smaller active surface than the classical rectangular generator. Its magnetic center is, therefore, better defined, so it is particularly well suited for measurements in strongly inhomogeneous fields. Special types, which have a smaller temperature dependence, are available on the market, but these show a lower sensitivity.

The measurement of the Hall voltage sets a limit of about 20  $\mu\text{T}$  on the sensitivity and resolution of the measurement, if conventional d.c. excitation is applied to the probe. This is mainly caused by thermally induced voltages in cables and connectors. The sensitivity can be improved considerably by application of a.c. excitation [61, 62]. A good accuracy at low fields can then be achieved by employing synchronous detection techniques for the measurement of the Hall voltage [63].

Special Hall generators for use at cryogenic temperatures are also commercially available. Although they show a very low temperature coefficient, they unfortunately reveal an additional problem at low temperatures. The so-called "Shubnikov-de Haas effect" [64, 65] shows up as a field dependent oscillatory effect of the Hall coefficient which may amount to about one per cent at high fields, depending on the type of semiconductor used for the Hall generator. This adds a serious complication to the calibration. The problem may be solved by locating the Hall generator in a heated anticryostat [66]. The complications related to the planar Hall effect are less important at cryogenic temperatures and are discussed in detail in [67]. Altogether, the Hall generator has proved very useful for measurements at low temperature [68].

#### 2.4.2 Calibration

Hall generators are usually calibrated in a magnet in which the field is measured simultaneously using the nuclear magnetic resonance technique. The calibration curve is most commonly represented in the form of a polynomial of relatively high order (7 or 9) fitted to a sufficiently large number of calibration points. This representation has the advantage of a simple computation of the magnetic induction from a relatively small table of coefficients.

A physically better representation is the use of a piecewise cubic interpolation through a sufficient number of calibration points which were measured with high precision. This can be done in the form of a simple Lagrange interpolation or even better with a cubic spline function. The advantage of the spline function comes from its minimum curvature and its "best approximation" properties [69]. The function adjusts itself easily to nonanalytic functions and is very well suited to interpolation from tables of experimental data. The function is defined as a piecewise polynomial of third degree passing through the calibration points such that the derivative of the function is continuous at these points. Very efficient algorithms can be found in the literature [70]. The calculation of the polynomial coefficients may be somewhat time-consuming but need only be done once at calibration time. The coefficients (typically about 60 for the bipolar calibration of a cruciform Hall generator) can be easily stored in a microprocessor device [57, 63] and the subsequent field calculations are very fast. The quality of the calibration function can be verified from field values measured between the calibration points. A well designed Hall-probe assembly can be calibrated to a long term accuracy of 100 ppm. The stability may be considerably improved by powering the Hall generator permanently and by keeping its temperature constant.

### 2.5 Fluxgate magnetometer

The fluxgate magnetometer [71] is based on a thin linear ferromagnetic core on which detection and excitation coils are wound. The measurement principle is illustrated in Fig. 5. In its basic version, it consists of three coils wound around a ferromagnetic core: an a.c. excitation winding A, a detection winding B that indicates the zero field condition and a d.c. bias coil C that creates and maintains the zero field. In practice the coils are wound coaxially in subsequent layers. The core is made up from a fine wire of Mumetal or a similar material that has an almost rectangular hysteresis curve. The method was introduced in the 1930's and was also named "peaking strip". It is restricted to use with low fields, but has the advantage of offering a linear measurement and is well suited for static operation. As a directional device with very high

Earth magnetic field. Much more complex coil configurations are applied for precision measurements and in cases where the measured field should not be distorted by the probe. The most interesting application is now in space research and important developments of this technique have taken place over the last decades [72–74]. The use of modern materials for magnetic cores has improved the sensitivity to about 20 pT and can assure a wide dynamic range. The upper limit of the measurement range is usually of the order of a few tens of mT, but can be extended by applying water cooling to the bias coil. Fluxgate magnetometers with a typical range of 1 mT and a resolution of 1 nT are commercially available from several sources. They have many other practical applications, for example in navigation equipment.

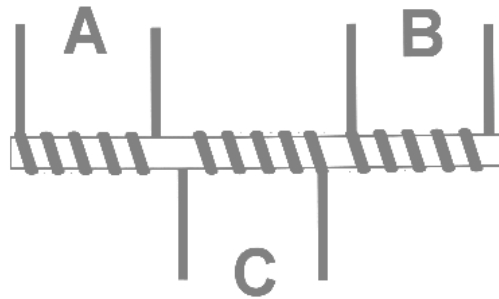


Fig. 5 Fluxgate magnetometer

## 2.6 Magneto-resistivity effect

Magneto-resistivity was discovered by W. Thomson in 1856 [75]. It was exploited quite early and a commercial instrument already existed at the end of last century. Technical problems were, however, important [76]. Dependence on temperature and mechanical stress, combined with difficulties of manufacture and problems with electrical connections, caused a general lack of reliability in this measurement method. As with the Hall generator, it was only when semiconductor materials became available that the method turned into a success. Then inexpensive magneto-resistors came on the market and were used also for magnetic measurements [77]. A more recent application for field monitoring was implemented in one of the large LEP spectrometers [78].

## 2.7 Visual field mapping

The best known visual field mapper is made by spreading iron powder on a horizontal surface placed near a magnetic source, thus providing a simple picture of the distribution of flux lines. Another very classical way of observing flux-line patterns is to place a free-moving compass needle at different points in the volume to be examined and note the direction of the needle. This compass method was applied, long before the discovery of electromagnetism, for studies of the variations in the direction of the earth's magnetic field. Another visual effect may be obtained by observing the light transmission through a colloidal suspension of diamagnetic particles, subject to the field [79, 80].

### 2.7.1 Faraday effect

The magneto-optical rotation of the plane of polarization of polarized light (Faraday effect) is a classical method for the visualization of magnetic fields. A transparent container filled with a polarizing liquid and placed inside the magnet gap may visualize for example the field pattern in a quadrupole by observation through polarization filters placed at each end of the magnet. The rotation of the plane is proportional to the field strength and the length of the polarizing medium and may give a certain indication of the field geometry. This measurement principle has proved useful for measurements of transient magnetic fields [81, 82]. It is less convincing when applied to the precise determination of magnet geometry, even though modern image



### 2.7.2 Floating-wire method

Floating-wire measurements were quite popular in the past [83]. If a current-carrying conductor is stretched in a magnetic field, it will curve subject to the electromagnetic force and describe the path of a charged particle with a momentum corresponding to the current and the mechanical tension in the wire. A flexible annealed aluminium wire was used in order to reduce effects of stiffness and gravity. This method has now been entirely replaced by precise field mapping and simulation of particle trajectories by computer programs.

## 2.8 Measurements based on particle beam observation

A method for the precise measurement of the beam position with respect to the magnetic center of quadrupole magnets installed in particle accelerators has been developed over the last decade [84, 85]. The procedure consists of modulating the field strength in individual lattice quadrupoles while observing the resulting beam orbit oscillations. Local d.c. orbit distortions are applied in the search for the magnetic center. This so-called K-modulation provides a perfect knowledge of the location of the particle beam with respect to the center of a quadrupole. In addition, it may provide other very useful observations for operation and adjustment of the accelerator [86]. This is obviously of particular importance for superconducting accelerators [87]. It is very difficult to provide a superconducting quadrupole magnet with a direct optical reference to its magnetic center, so errors caused by changes of temperature profiles and other phenomena may build up as time passes.

The method may be further improved by synchronous detection of the oscillation, so that its phase can be identified. The sensitivity of the detection is impressive. Experience from LEP [88] showed that an absolute accuracy of 0.05 mm in both the vertical and horizontal plane could be obtained. Furthermore it was observed that a modulation of the quadrupole field of about 300 ppm could be clearly detected, which means that measurements may be carried out on colliding beams while particle physics experiments are taking place. This measurement method also played an important role for adjustments of the so-called Final Focus Beams [89, 90].

## 3. CONCLUDING REMARKS

Proven measurement methods and powerful equipment is readily available for most of the measurement tasks related to beam-guiding magnets as well as for spectrometer magnets. It is therefore prudent to examine existing possibilities carefully before launching the development of a more exotic measurement method. Many unnecessary costs and unpleasant surprises can be avoided by choosing instruments which are commercially available. The measurement methods described above are complementary and the use of a combination of two or more of these will certainly meet most requirements. Already at an early stage of the system design, particular attention must be drawn to definitions of geometry and the future alignment considerations.

In the field of new technologies, there are two methods which merit consideration. Magnet resonance imaging is a promising technique which could find a lasting application. Also the use of superconducting quantum interference devices (SQUIDS) might in the long run become an interesting alternative as an absolute standard and for measurements of weak fields [91, 92]. The complexity of these methods is still at a level which prevents current laboratory use.

## REFERENCES

- [1] J.L. Symonds, "Methods of measuring strong magnetic fields", *Rep. Progr. Phys.*, 18 (1955) 83-126.
- [2] C. Germain, "Bibliographical review of the methods of measuring magnetic fields", *Nucl. Instr. and Meth.*, 21 (1963) 17-46.
- [3] L.W. McKeehan, "The measurement of magnetic quantities", *J. Opt. Soc. Amer.*, 19 (1929) 213-242.
- [4] J.J. Rabi, J.R. Zacharias, S. Millman, P. Kusch, "A new method of measuring nuclear magnetic moment", *Phys. Rev.*, 53 (1938) 318.
- [5] J.J. Rabi, S. Millman, P. Kusch, J.R. Zacharias, "The molecular beam resonance method for measuring nuclear magnetic moments", *Phys. Rev.*, 55 (1939) 526-535.
- [6] E.M. Purcell, H.C. Torrey, R.V. Pound, "Resonance absorption by nuclear magnetic moments in a solid", *Phys. Rev.*, 69 (1946) 37-38.
- [7] F. Bloch, W.W. Hansen, M. Packard, "Nuclear induction", *Phys. Rev.*, 69 (1946) 127.
- [8] F. Bloch, W.W. Hansen, M. Packard, "The nuclear induction experiment", *Phys. Rev.*, 70 (1946) 474-485.
- [9] N. Bloembergen, E.M. Purcell, R.V. Pound, "Relaxation effects in nuclear magnetic resonance absorption", *Phys. Rev.*, 73 (1948) 679-712.
- [10] K. Borer, G. Fremont, "The nuclear magnetic resonance magnetometer type 9298", *CERN* 77-19 (1977).
- [11] W.G. Clark, T. Hijmans, W.H. Wong, "Multiple coil pulsed NMR method for measuring the multipole moments of particle accelerator bending magnets", *J. Appl. Phys.* 63 (1988) 4185-4186.
- [12] R. Prigl, U. Haeberlen, K. Jungmann, G. zu Putlitz, P. von Walter, "A high precision magnetometer based on pulsed NMR", *Nucl. Instr. and Meth.*, A 374 (1996) 118-126.
- [13] W.G. Clark, J.M. Moore, W.H. Wong, "Multiple coil pulsed magnetic resonance method for measuring cold SSC dipole magnet field quality", *Supercollider 2: Proc. of the 2nd International Industrial Symposium on the Supercollider*, Miami Beach, USA (1990), 405-414.
- [14] G. Suryan, "Nuclear resonance in flowing liquids", *Proc. Indian Acad. Sci.*, A33 (1951) 107-111.
- [15] C. Sherman, "High-Precision Measurement of the Average Value of a Magnetic Field over an Extended Path in Space", *Rev. Sci. Instr.*, 30 (1959) 568-575.
- [16] J.M. Pendlebury, K. Smith, P. Unsworth, G.L. Greene, W. Mampe, "Precision field averaging NMR magnetometer for low and high fields, using flowing water", *Rev. Sci. Instr.*, 50 (1979) 535-540.
- [17] P. Galbraith, K.N. Henrichsen, L. Jansak, J. Kvitkovic, "Flowing Water NMR Measurements in the LEP Dipole", *CERN AT/95-50 and CERN LEP2 Note 95-38*

- [18] D. Bourdel, J. Pescia, P. Lopez, "Un magnétomètre a résonance électronique, optimisation des performances", *Rev. Phys. Appl.*, 5 (1970) 187-190.
- [19] F. Hartmann, "Resonance magnetometers", *IEEE Trans. on Magn.*, MAG-8 (1) (1972) 66-75.
- [20] D. Duret, M. Beranger, M. Moussavi, P. Turek, J.J. Andre, "A new ultra-low-field ESR spectrometer", *Rev. Sci. Instr.*, 62 (3) (1991) 685-694.
- [21] N. Kernevez, D. Duret, M. Moussavi, J.-M. Leger, "Weak field NMR and ESR spectrometers and magnetometers", *IEEE Trans. on Magn.*, 28 (5) (1992) 3054-3059.
- [22] D.A. Gross, "Magnetic field measurement with NMR imaging", *Proc. of the ICFA Workshop on Superconducting Magnets and Cryogenics*, Brookhaven National Lab., Upton, USA (May 1986) 309-311.
- [23] J.H. Coupland, T.C. Randle, M.J. Watson, "A magnetic spectrometer with gradient field", *IEEE Trans. on Magn.*, MAG-17 (1981) 1851-1854.
- [24] W. Weber, "Ueber die Anwendung der magnetischen Induction auf Messung der Inclination mit dem Magnetometer", *Ann. der Physik*, 2 (1853) 209-247.
- [25] W.F. Brown, J.H. Sweer, "The Fluxball", *Rev. Sci. Instr.*, 16 (1945) 276-279.
- [26] A. Daël, H. Desportes, F. Kircher, C. Lesmond, R. Duthil, Y. Pabot, J. Thinel, "Construction and test of a superconducting high accuracy solenoid for the SMC polarized target" *Int. J. Mod. Phys. A*, 2B (HEACC'92) (1993) 650-652.
- [27] E.A. Finlay, J.F. Fowler, J.F. Smee, "Field measurements on model betatron and synchrotron magnets", *J. Sci. Instr.*, 27 (1950) 264-270.
- [28] B.C. Brown, "Fundamentals of magnetic measurements with illustrations from Fermilab experience", *Proc. of the ICFA Workshop on Superconducting Magnets and Cryogenics*, Brookhaven National Lab., Upton, USA (May 1986) 297-301.
- [29] B. de Raad, "Dynamic and static measurements of strongly inhomogeneous magnetic fields", Thesis, Delft (1958) 55-67.
- [30] W.C. Elmore, M.W. Garrett, "Measurement of two-dimensional fields, Part I: Theory", *Rev. Sci. Instr.*, 25, (1954) 480-485.
- [31] I.E. Dayton, F.C. Shoemaker, R.F. Mozley, "Measurement of two-dimensional fields, Part II: Study of a quadrupole magnet", *Rev. Sci. Instr.*, 25, (1954) 485-489.
- [32] C. Wyss, "A measuring system for magnets with cylindrical symmetry", *Proc. 5th Int. Conf. on Magnet Technology (MT-5)*, Frascati, Italy (1975) 231-236.
- [33] G.H. Morgan, "Stationary coil for measuring the harmonics in pulsed transport magnets", *Proc. 4th Int. Conf. on Magnet Technology*, Brookhaven National Lab., Upton, USA (1972) 787-790.
- [34] L. Walckiers, "The harmonic-coil method", *CERN Accelerator School*, Montreux, Switzerland, CERN 92-05 (1992) 138-166.
- [35] W.G. Davies, "The theory of the measurement of magnetic multipole fields with rotating

- [36] H. Brück, R. Meinke, P. Schmüser, "Methods for magnetic measurements of the superconducting HERA magnets", *Kerntechnik*, 56 (1991) 248-256.
- [37] P. Schmüser, "Magnetic measurements of superconducting magnets and analysis of systematic errors", CERN Accelerator School, Montreux, Switzerland, CERN 92-05 (1992) 240-273.
- [38] M.I. Green, R. Sponsel, C. Sylvester, "Industrial harmonic analysis system for magnetic measurements of SSC collider arc and high energy booster corrector magnets", *Supercollider 5: Proc. of the 5th International Industrial Symposium on the Supercollider*, San Francisco, USA (1993) 711-714.
- [39] R. Thomas, G. Ganetis, J. Herrera, R. Hogue, A. Jain, W. Louie, A. Marone, P. Wanderer, "Performance of field measuring probes for SSC magnets", *Supercollider 5: Proc. of the 5th International Industrial Symposium on the Supercollider*, San Francisco, USA (1993) 715-718.
- [40] J. Billan, J. Buckley, R. Saban, P. Sievers, L. Walckiers, "Design and Test of the Benches for the Magnetic Measurement of the LHC Dipoles", *IEEE Trans. on Magnetics*, vol. 30, no. 4 (MT-13) (1994) 2658-2661.
- [41] J. Buckley, D. Richter, L. Walckiers, R. Wolf, "Dynamic magnetic measurements of superconducting magnets for the LHC", *IEEE Trans. Appl. Supercond.*, vol. 5, no. 2 (ASC'94) (1995) 1024-1027.
- [42] J. Billan, S. De Panfilis, D. Giloteaux, O. Pagano, "Ambient temperature field measuring system for LHC superconducting dipoles", *IEEE Trans. on Magnetics*, vol. 32, no. 4 (MT-14) (1996) 3073-3076.
- [43] D. Zangrando, R.P. Walker, "A stretched wire system for accurate integrated magnetic field measurements in insertion devices", *Nucl. Instr. and Meth., A* 376 (1996) 275-282.
- [44] M.E. Grassot, "Fluxmètre", *J. de Phys.*, 4 (1904) 696-700.
- [45] R.F. Edgar, "A new photoelectric hysteresigraph", *Trans. Amer. Inst. Elect. Eng.*, 56 (1937) 805-809.
- [46] G.K. Green, R.R. Kassner, W.H. Moore, L.W. Smith, "Magnetic measurements", *Rev. Sci. Instr.*, 24 (1953) 743-754.
- [47] E.H. Hall, "On a new action of the magnet on electric currents", *Amer. J. Math.*, 2 (1879) 287-292.
- [48] W. Peukert, "Neues Verfahren zur Messung magnetischer Felder", *Elektrotechn. Zeitschr.*, 25 (1910) 636-637.
- [49] G.L. Pearson, "A magnetic field strength meter employing the Hall effect in germanium", *Rev. Sci. Instr.*, 19 (1948) 263-265.
- [50] H. Welker, "Ueber neue halbleitende Verbindungen", *Z. Naturforschung*, 7a (1952) 744-749.
- [51] H. Welker, "Neue Werkstoffe mit grossem Hall-Effekt und grosser Widerstandsänderung im Magnetfeld", *Elektrotechn. Zeitschr.*, 76 (1955) 513-517.

- [52] E. Acerbi, J. Faure, B. Laune, J.P. Penicaud, M. Tkatchenko, "Design and magnetic results on a 3 tesla, 10 weber spectrometer magnet at Saclay", IEEE Trans. on Magn., MAG-17 (MT-7) (1981) 1610-1613.
- [53] C. Bazin, S. Costa, Y. Dabin, G. Le Meur, M. Renard, "The DM2 solenoidal detector on DCI at Orsay", IEEE Trans. on Magn., MAG-17 (1981) 1840-1842.
- [54] D. Swoboda, "The polar co-ordinate magnetic measurement system for the axial field spectrometer magnet at the ISR-CERN", IEEE Trans. on Magn., MAG-17 (1981) 2125-2128.
- [55] S. Kawahito, S.O. Choi, M. Ishida, T. Nakamura, "Micromachined Hall elements for two-dimensional magnetic-field sensing", Sensors and Actuators A, 40 (1994) 141-146.
- [56] J. Kvitkovic, M. Majoros, "Three axis cryogenic Hall sensor", 6th European Magnetic Materials and Applications Conference, Vienna, Austria, (1995).
- [57] K. Brand, G. Brun, "A digital teslameter", CERN 79-02 (1979).
- [58] M.W. Poole, R.P. Walker, "Hall effect probes and their use in a fully automated magnetic measuring system", IEEE Trans. on Magn., MAG-17 (1981) 2129-2132.
- [59] C. Goldberg, R.E. Davis, "New galvanometric effect", Phys. Rev., 94, (1954) 1121-1125.
- [60] J. Hauesler, H.J. Lippmann, "Hallgeneratoren mit kleinem Linearisierungsfehler", Solid State Electron., 11 (1968) 173-182.
- [61] J.J. Donoghue, W.P. Eatherly, "A new method for precision measurement of the Hall and magneto-resistive coefficients", Rev. Sci. Instr., 22 (1951) 513-516.
- [62] C.D. Cox, "An a.c. Hall effect gaussmeter", J. Sci. Instr., 41 (1964) 695-691.
- [63] K.R. Dickson, P. Galbraith, "A digital micro-teslameter", CERN 85-13 (1985).
- [64] J. Babiskin, "Oscillatory galvanomagnetic properties of bismuth single crystals in longitudinal magnetic fields", Phys. Rev., 107 (1957) 981-992.
- [65] H.P.R. Frederikse, W.R. Hosler, "Oscillatory galvanomagnetic effects in n-type indium arsenide", Phys. Rev., 110 (1958) 880-883.
- [66] M. Polak, "Low temperature InSb Hall plate with suppressed de Haas-Shubnikov effect", Rev. Sci. Instr. 44 (1973) 1794-1795.
- [67] M. Polak, I. Hlasnik, "Planar Hall effect in heavy doped n-InSb and its influence on the measurement of magnetic field components with Hall generators at 4.2 K", Solid State Electron. 13 (1970) 219-227.
- [68] J. Kvitkovic, M. Polak, "Cryogenic Microsize Hall Sensor", European Conference on Applied Superconductivity (EUCAS), Gottingen, Germany (1993) 1629-1632.
- [69] J.L. Walsh, J.H. Ahlberg, E.N. Nilson, "Best approximation properties of the spline fit", J. Math. Mech., 11 (1962) 225-234.
- [70] A. Ralston, H. Wilf, (ed.), "Mathematical methods for digital computers", Vol. 2, New York, Wiley, 1967.

- [71] J.M. Kelly, "Magnetic field measurements with peaking strips", *Rev. Sci. Instr.*, 22 (1951) 256-258.
- [72] D.I. Gordon, R.E. Brown, "Recent advances in fluxgate magnetometry", *IEEE Trans. on Magn.*, MAG-8, (1972) 76-82.
- [73] F. Primdahl, "The fluxgate magnetometer", *J. Phys. E: Sci. Instr.*, 12 (1979) 241-253.
- [74] O.V. Nielsen, T. Johansson, J.M. Knudsen, F. Primdahl, "Possible magnetic experiments on the surface of Mars", *J. Geophys. Res.*, 97 (1992) 1037-1044.
- [75] W. Thomson, "On the magnetization of the electric conductivity of metals", *Philosoph. Trans.*, 146 (1856) 736-751.
- [76] P. Kapitza, "The study of the specific resistance of bismuth crystals and its change in strong magnetic fields and some allied problems", *Proc. Roy. Soc. A*, 119 (1928) 358.
- [77] E. Welch, P.R. Mace, "Temperature stabilized magneto-resistor for 0.1 % magnetic field measurement", *Proc. 3rd Int. Conf. on Magnet Technology, Hamburg, Germany (1970)* 1377-1391.
- [78] G. Brouwer, F.J.G.H. Crijns, A.C. König, J.M. Lubbers, C.L.A. Pols, D.J. Schotanus, K. Freudenreich, J. Ovnlee, D. Luckey, F. Wittgenstein, "Large scale application of magnetoresistors in the magnetic field measuring system of the L3 detector", *Nucl. Instr. and Meth.*, A 313, (1992) 50-62.
- [79] J.K. Cobb, J.J. Muray, "Magnetic center location in multipole fields", *Nucl. Instr. and Meth.*, 46 (1967) 99-105.
- [80] D. Trbojevic, P. Cameron, G.L. Ganetis, M.A. Goldman, R. Gupta, M. Harrison, M.F. Hemmer, F.X. Karl, A. Jain, W. Louie, S. Mulhall, S. Peggs, S. Tepikian, R. Thomas, P. Wanderer, "Alignment and survey of the elements in RHIC", 1995 Particle Accelerator Conference, Dallas, Texas, USA (May 1995) 2099-2021.
- [81] J. Malecki, M. Surma, J. Gibalewicz, "Measurements of the intensity of transient magnetic fields by the Faraday effect", *Acta Phys. Polon.*, 16 (1957) 151-156.
- [82] J.L. Robertson, D.T. Burns, D. Hardie, "Measurement of a magnetic field integral using the Faraday effect", *Nucl. Instr. and Meth.*, 203 /1982) 87-92.
- [83] L.G. Ratner, R.J. Lari, "A precision system for measuring wire trajectories in magnetic fields", *Proc. International Symposium on Magnet Technology, Stanford, USA (1965)* 497-504.
- [84] D. Rice, G. Aharonian, K. Adams, M. Billing, G. Decker, C. Dunnam, M. Giannella, G. Jackson, R. Littauer, B. McDaniel, D. Morse, S. Peck, L. Sakazaki, J. Seeman, R. Siemann, R. Talman, "Beam diagnostic instrumentation at CESR", *IEEE Trans. on Nucl. Sci.*, NS-30 (1983) 2190-2192.
- [85] P. Rojsel, "A beam position measurement system using quadrupole magnets magnetic centra as the position reference", *Nucl. Instr. and Meth.*, A 343, (1994) 371-382.
- [86] R. Brinkmann, M. Boge, "Beam-based alignment and polarization optimization in the HERA electron ring", 4th European Particle Accelerator Conference, London, (1994) 938-940.

- [87] J. Deregel, P. Genevey, J.-M. Rifflet, P. Galbraith, K.N. Henrichsen, "Proposal of a K-modulation system for the LHC quadrupoles", CERN LHC Project Report 4, (March 1996).
- [88] I. Barnett, A. Beuret, B. Dehning, P. Galbraith, K. Henrichsen, M. Jonker, M. Placidi, R. Schmidt, L. Vos, J. Wenninger, I. Reichel, F. Tecker, "Dynamic beam based alignment", presented at the 6th Beam Instrumentation Workshop, Vancouver, Canada (1994).
- [89] F. Bulos, D. Burke, R. Helm, J. Irwin, A. Odian, G. Roy, R. Ruth, N. Yamamoto, "Beam-based alignment and tuning procedures for e+e- collider final focus systems", 1991 Particle Accelerator Conference, San Francisco, USA (May 1991) 3216-3218.
- [90] P. Tenenbaum, D. Burke, R. Helm, J. Irwin, P. Raimondi, K. Oide, K. Flottmann, "Beam based alignment of the Final Focus Test Beam", 1995 Particle Accelerator Conference, Dallas, Texas, USA (May 1995) 2096-2098.
- [91] G.L. Romani, "SQUID instrumentation in the measurement of biomagnetic fields", Proc. 9th Int. Conf. on Magnet Technology (MT-9), Zurich, Switzerland (1985) 236-242.
- [92] D. Drung, "Recent LTS SQUID Developments", European Conference on Applied Superconductivity (EUCAS), Gottingen, Germany (1993) 1287-1294.