Pi Phases in Balanced Fermionic Superfluids on Spin-Dependent Optical Lattices

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π-phases in balanced fermionic superfluids on spin-dependent optical lattices

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We study a balanced two-component system of ultracold fermions in one dimension with attractive interactions and subject to a spin-dependent optical lattice potential of opposite sign for the two components. The ground-state develops a non-trivial superconducting order parameter as the depth of the lattice is increased. The real-space gap parameter changes sign and is analogous to the Fulde-Ferrell-Larkin-Ovchnnikov states discussed in the context of superconductors in magnetic fields. We discuss how to observe these π-phases using the rapid-ramp technique. In addition we discuss laser setups that can produce the required lattices needed for these novel phases to appear.

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One of the most intriguing examples of the interplay of superconductivity and magnetism is the Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) phase, where Zeeman splitting of the Fermi surfaces should lead to spatial oscillations of the pairing amplitude. It is difficult to obtain such phases in superconductors, since the orbital effect of the magnetic field is typically much larger than the spin Zeeman splitting. Several proposals have been made, however they remain controversial [1]. For example, FFLO phase has been discussed in the context of heavy fermion $CeCoIn_5$ superconductors [2, 3], but alternative interpretation in terms of competing magnetic order has also been given [4]. So far the only unambiguous demonstration of FFLO-like physics has been achieved in heterostructures of ferromagnetic and superconducting (F/SC) layers , where proximity coupling through ferromagnetic layers results in superconducting π-junctions (see [1] for a review). We note that π-phases arising from a different mechanism than FFLO have also been discussed for high-T_c cuprates [1, 5].

Recently, in cold atoms, there has been a large body of work, both experimental and theoretical, aimed at achieving FFLO states. The biggest difficulty is that FFLO phases are fragile and extremely susceptible to phase separation and the experimental situation remains controversial [6, 7, 8, 9, 10]. In this paper we propose a novel system of ultracold fermions in an optical lattice [11, 12] which can be used to observe FFLO type states with oscillating pairing amplitude. The system which we discuss is somewhat similar to F/SC heterostructures and should be stable against phase separation. Our proposal relies on the ability to create spin dependent optical lattices and we find that beyond a certain critical strength of such optical potential, the superconducting pairing amplitude becomes a sign changing function (we will refer to such states as as π-phases, see Fig. 2).

The gap profile in the ground-state depends on the wavelength of the lattice, λ, and the strength of the potential, V_0. In Fig. 1 we present the (V_0, λ) phase diagram showing the transitions from constant gap to the

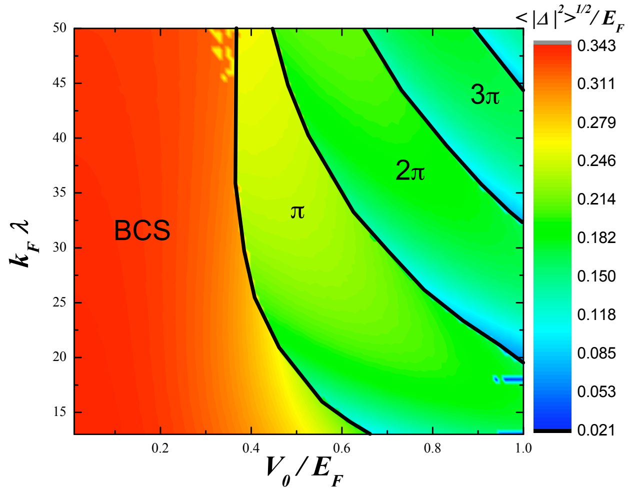

FIG. 1: Phase diagram showing the emergence of π-phases for a spin-dependent lattice potential of wavelength λ and strength V_0 for interaction strength $g_{1D}k_F/E_F = -2.04$ and zero temperature. A gradient of colors gives the average of the absolute value of the gap. The black lines indicate transitions from gap profiles with zero, two, four, six and eight zero-crossings per unit cell. These regions are labelled BCS, π, 2π, and 3π respectively.

π-phases with several zero-crossings in the pairing amplitude. A color gradient gives the root-mean-square value of the gap and the black lines indicate the transitions. We clearly see π-phases occuring in a broad range of λ restricted from below only by the coherence length as we will discuss. The emergence of oscillations in the gap gives clear signatures in the Fourier transform. We will demonstrate how the rapid-ramp techniques can be used to observe these states in time-of-flight measurements. We also suggest ways to make spin dependent large wavelength lattice potentials in the high-field regime as is needed to access π-phases.

The quasi-1D system we study is described by the ef-

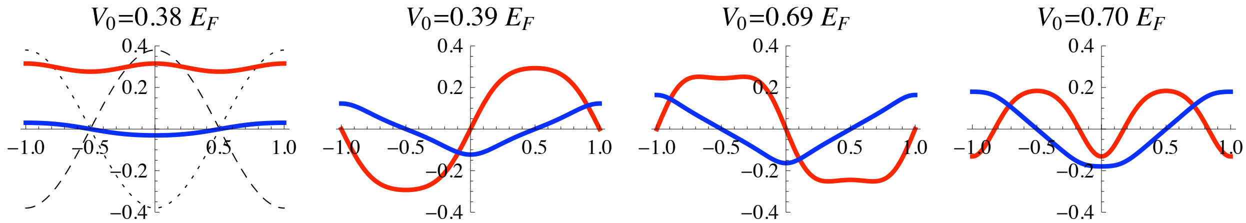

FIG. 2: The polarization $(n_\uparrow(x) - n_\downarrow(x))/k_F$ in blue and pairing $\Delta(x)/E_F$ in red as functions of $2x/\lambda$, for the case of 0-phase (left), π-phase (two figures in the middle), and 2π-phase (right) at zero temperature. The dashed and dotted black lines in the left plot show the spin-dependent lattice potential. Here $k_F\lambda = 30, T = 0, g_{1D}k_F/E_F = -2.04$, and $V_\uparrow(x) = -V_\downarrow(x) = V_0 \cos(2\pi x/\lambda)$.

fective Hamiltonian [13]

$$\begin{aligned} H &- \mu_\downarrow N_\downarrow - \mu_\uparrow N_\uparrow = \\ &\sum_{\sigma=\uparrow\downarrow} \int dx \Psi_\sigma^\dagger(x)[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_\sigma(x) - \mu_\sigma]\Psi_\sigma(x) \\ &+ g_{1D} \int dx \Psi_\uparrow^\dagger(x)\Psi_\downarrow^\dagger(x)\Psi_\downarrow(x)\Psi_\uparrow(x), \end{aligned} \quad (1)$$

where g_{1D} is the effective 1D coupling constant (we use $g_{1D}k_F/E_F = -2.04$ as in [13]). We consider a balanced system but introduce chemical potentials μ_σ since the optical lattice potential is spin-dependent. For the main part of this work we use $V_\uparrow(x) = V_0 \cos(2\pi x/\lambda)$ and $V_\downarrow(x) = -V_\uparrow(x)$ in which case $\mu_\uparrow = \mu_\downarrow$. In the non-interacting system, the spin-dependent lattice spatially displaces the degenerate solution of the two components as V_0 is increased. In a simple-minded picture, pairing of these states will generate spatial variation in the order parameter which is the origin of π-phases.

We solve the Hamiltonian of Eq. (1) in mean-field theory by using the inhomogeneous Bogoliubov-deGennes (BdG) ansatz for the field operator $\Psi_\sigma(x,t) = \sum_k [u_{k\sigma}(x)e^{-i\omega_{k\sigma}t}c_{k\sigma} + \sigma\bar{v}_{k\bar{\sigma}}(x)e^{i\omega_{k\bar{\sigma}}t}c_{k\bar{\sigma}}^\dagger]$, where the c and $c^\dagger$ denote the quasiparticles and the sum runs over $\omega_{k\sigma} > 0$ with k the quasiparticle index composed of a quasimomentum and the band index. The mean-field equations are

$$\begin{aligned} &\begin{bmatrix} H_\sigma & \Delta(x) \\ \bar{\Delta}(x) & -H_{-\sigma} \end{bmatrix} \begin{pmatrix} u_{k\sigma}(x) \\ v_{k\sigma}(x) \end{pmatrix} = \omega_{k\sigma} \begin{pmatrix} u_{k\sigma}(x) \\ v_{k\sigma}(x) \end{pmatrix}, \\ &H_\sigma = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_\sigma(x) - \mu_\sigma + g_{1D}n_{-\sigma}(x), \end{aligned} \quad (2)$$

where $n_\sigma(x) = \langle\Psi_\sigma^\dagger(x)\Psi_\sigma(x)\rangle$ and $\Delta(x) = -g_{1D}\langle\Psi_\downarrow(x)\Psi_\uparrow(x)\rangle$. These equations can be solved self-consistently for densities and gap through

$$\begin{aligned} n_\uparrow(x) &= \sum_{\omega_{k\uparrow}\gtrless 0} f(\omega_{k\uparrow})|u_{k\uparrow}(x)|^2 = \sum_{\tilde{m}=-\infty}^{\infty} n_{\uparrow\tilde{m}} e^{i2\pi\tilde{m}x/\lambda} \\ n_\downarrow(x) &= \sum_{\omega_{k\uparrow}\gtrless 0} f(-\omega_{k\uparrow})|v_{k\uparrow}(x)|^2 = \sum_{\tilde{m}=-\infty}^{\infty} n_{\downarrow\tilde{m}} e^{i2\pi\tilde{m}x/\lambda} \\ \Delta(x) &= g_{1D} \sum_{\omega_{k\uparrow}\gtrless 0} f(\omega_{k\uparrow})u_{k\uparrow}(x)\bar{v}_{k\uparrow}(x) \\ &= \sum_{\tilde{m}=-\infty}^{\infty} \Delta_{\tilde{m}} e^{i2\pi\tilde{m}x/\lambda}, \end{aligned} \quad (3)$$

where $f(\omega_{k\sigma}) = 1/(1 + \exp(\hbar\omega_{k\sigma}/k_BT))$ is the Fermi-Dirac distribution. In Eq. (3) we use the periodicity of the optical lattice to do a Fourier decomposition. Notice that these equations only contain u, v for $\sigma =\uparrow$ [13]. We have explicitly checked the convergence of our numerical solutions by extending the cut-off on the basis size.

The mean-field BdG ansatz does not take into account soft collective modes of the order parameter which, in principle, lead to the power law decay of the superconducting correlations. However, the BdG approach describes the ground state energy in our parameter regime well, which is determined by correlations on the scale of the BCS correlation length [13, 14]. Thus we expect it to also correctly capture the competition between 0- and π-phases.

In Fig. 2 we show $(n_\uparrow(x) - n_\downarrow(x))/k_F$ and $\Delta(x)/E_F$ as functions of $2x/\lambda$ with $k_F\lambda = 30$ for amplitudes $V_0/E_F = 0.38$, 0.39, 0.69 and 0.70. Here we notice a sudden jump in the gap from an even to an odd function (around $x = 0$) at the definite value $V_0/E_F = 0.39$, and again at $V_0/E_F = 0.70$. We will give arguments as to why this occurs in the following sections. The signature of the new phases are even more clear in Fig. 3 which shows the largest components of $|\Delta_{\tilde{m}}|$. Here we see a very clear jump between even and odd Fourier components as V_0 is increased. This transition constitutes the main result of our paper and below we propose a way to observe the π-phases which is clearly distinguishable from other oscillatory behaviors in the gap.

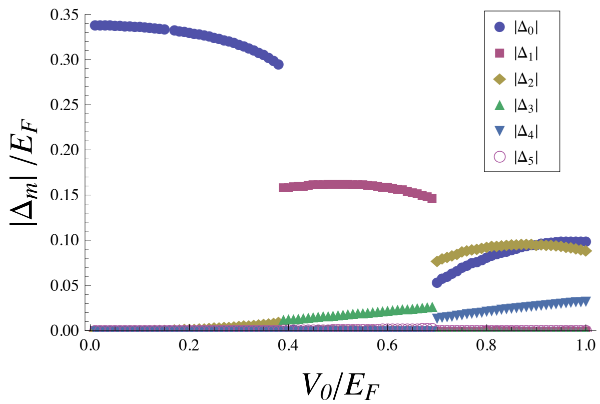

FIG. 3: Plot of the absolute value of the Fourier components of $\Delta(x)$ in Eq. (3) for $\tilde{m} = 0, 1, 2, 3, 4$, and 5 as function of the lattice potential strength V_0 (parameters as in Fig. 2). Only even components are non-zero for $V_0/E_F \leq 0.39$, whereas only odd ones are non-zero for $0.39 \leq V_0/E_F \leq 0.69$, and so forth. This is the tell-tale sign of the transition from the 0- to the π-phase.

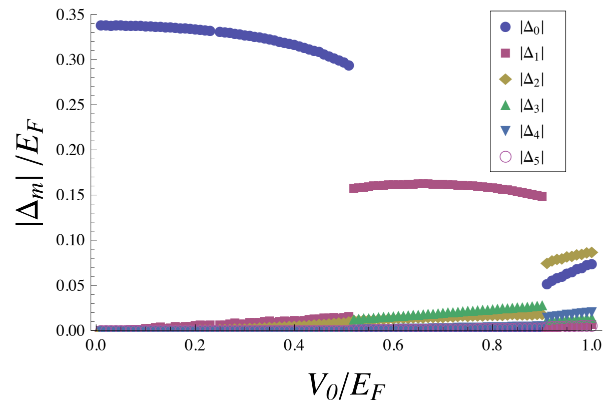

FIG. 4: Same as Fig. 3 but with $V_\uparrow(x) = -3V_\downarrow(x)/2 = V_0 \cos(2\pi x/\lambda)$.

The spin-dependent lattice we use here is invariant under spatial reflection. We can therefore characterize our solutions for $\Delta(x)$ in terms of their parity under reflection. In addition, for the balanced system, we study the symmetry of the lattice potential implies that the densities are even and interchanged every half wavelength ($\lambda/2$). We can use this observation to restrict the functional form of the gap. In the absence of any currents, the gap obeys $\Delta(x + \lambda/2) = \pm\Delta(x)$. Combined with the full periodicity $\Delta(x + \lambda) = \Delta(x)$, we see that either only even or only odd Fourier components survive which facilitates its unmistakable detection.

Our spin-dependent lattice potential effectively acts as a spatially varying magnetic field. In agreement with FFLO states in homogenous systems with a uniform magnetic field, we find a critical value of V_0 for the π-phase to develop, and with increasing V_0 more oscillations in the gap appears. Furthermore, there is a suppresion of the gap magnitude as V_0 grows as seen in the homogeneous case [13]. The difference for our system is that the wavelength of variations in the gap does not vary continuously but changes at discrete values of V_0 since the oscillations must be commensurate with the lattice potential. FFLO states are similar to π-phases since modulation of the pairing field generates a lower energy solution. However, the present proposal differs from FFLO since we do not have a global spin imbalance.

The transition between the different π-phases can be explained by an energy balance argument. They are driven by the competition between interaction (pairing and Hartree terms) and potential energy in the lattice. First consider the situation where the gap and densities are almost constant and even functions, $\Delta^e(x) = \Delta_0$ and $n_\uparrow(x) = n_\downarrow(x) = n_0/2$ ($V_0/E_F \leq 0.38$ in Fig 2), and contrast this with the situation where the gap is an odd function. Let us for simplicity assume the same magnitude of the gap and introduce a corresponding oscillation in the densities, thus $\Delta^o(x) = \Delta_0 \sin(2\pi x/\lambda)$ and $n^o_{\uparrow/\downarrow}(x) = n_0/2 \mp (\delta n/2)\cos(2\pi x/\lambda)$. In the long-wavelength limit we can neglect the kinetic energy and the energy densities of the even and odd state can be written

$$v_{e/o} := \int dx \left[\sum_\sigma V_\sigma(x) n^{e/o}_\sigma(x) + g_{1D} n^{e/o}_\downarrow(x) n^{e/o}_\uparrow(x) + |\Delta^{e/o}(x)|^2/g_{1D} \right] /L, \qquad (4)$$

where L is the system size. From our ansatz we get $v_e = g_{1D}n_0^2/4 + \Delta_0^2/g_{1D}$ and $v_o = g_{1D}n_0^2/4 + \Delta_0^2/(2g_{1D}) - V_0\delta n/2 - (g_{1D}/2)(\delta n/2)^2$. If we determine the density variation in the odd state by requiring minimal energy, we find $v_o = g_{1D}n_0^2/4 + \Delta_0^2/(2g_{1D}) + V_0^2/(2g_{1D})$ from which it follows that the constant even solution is lower in energy until $V_0 = \Delta_0$. Taking $\Delta_0/E_F \sim 0.3$ from Fig. 2 gives $V_0/E_F \sim 0.3$. Numerically we find $\Delta_0/E_F \sim 0.39$, about 30% higher. At small λ we expect large deviations from this estimate. This is caused by the neglected kinetic term that grows with decreasing λ and push the jump to larger V_0.

The transitions we find are very sharp as illustrated in Figs. 2 and 3. The 0- and π-phase have different parities and we therefore have a crossing of ground-states as we tune V_0. We test the stability of our predictions by using a potential that has $V_\uparrow(x) = -3V_\downarrow(x)/2$. As Fig. 4 shows, a sharp transition occurs also in this case. Even though this potential breaks $\lambda/2$ symmetry, there is still conservation of parity, thus $\Delta_{-m} = \pm\Delta_m$, and a sharp transition still occurs. However, we no longer have either even or odd Fourier components.

The results discussed thus far use $k_F\lambda = 30$. For smaller $\lambda \sim \xi$, the lattice drives the system into the normal state before showing any noticeable oscillations of the gap. For the interaction strength $g_{1D}k_F/E_F = -2.04$, we estimate that $k_F\lambda \gtrsim 12$ is necessary to support observable π-phases. In the phase diagram in Fig. 1 the suppression of the gap at the transitions at small λ is clearly seen. Fig. 1 also demonstrates that more zeros of the gap per unit cell could be accessible in experiments. The presented results are for the zero temperature case. We find that our results are robust up to $T \sim 0.1T_F$ for the full phase diagram. Above this temperature the effects are washed out as the gap vanishes rapidly.

In order to detect the π-phase, the rapid ramp technique can be used to transfer opposite spin pairs into molecules on the BEC side of the Feshbach resonance as shown in [15, 16, 17]. If the gap is uniform over the tube we expect to see only one prominent peak at zero momentum in a time-of-flight image. However, if there is an oscillating component, $\Delta \propto \sin(2\pi x/\lambda)$, it translates into molecules going in opposite horizontal directions with velocity $v = h/(m\lambda)$. Assuming a 35 ms vertical drop under gravity and $\lambda = 7.6$ μm, the molecules are displaced horizontally by $x \sim 91$ μm. Leftover unpaired atoms can be separated by a Stern-Gerlach field. In particular for $V_\sigma(x) = -V_{-\sigma}(x)$, π-phases are linked to either only odd or only even Fourier components of the order parameter and after the rapid ramp process, two spots on the screen clearly shows that the system has gone through a π-phase, and rules out other types of oscillation in Δ. On the other hand, phase-separated densities located in the minima of their respective potentials would require many Fourier components.

For experimental realization we focus here on ^{40}K [18]. ^{6}Li is another possible candidate, although we note that a spin-dependent lattice is harder to implement [19]. We assume a 1D geometry of tubes that are optically trapped with a superposed magnetic field to control the interaction via the Feshbach resonance at $B_0 = 202.1$G. Using $N \sim 100$ per tube of length $L \sim 40\mu$m, we have $n \sim 2.5$ μm^{-1} and $k_F = \pi n/2 \sim 3.93$ μm^{-1}. For simplicity we neglect the external confinement.

In the unpolarized system, the appearance of π-phases in the order parameter requires a spin-dependent lattice potential with a wavelength longer than the coherence length. In general we need $\lambda \gtrsim \xi$. To fulfil both requirements multiple lasers should be used [20, 21, 22]. To get spin-dependence there are several proposals and we focus on the one of [22]. The splitting is controlled by the difference in laser intensity and phase of left-circular and right-circular polarized light. Furthermore the transition is between the $^2\text{S}_{1/2}$ and $^2\text{P}_{1/2}$,$^2\text{P}_{3/2}$ lines with optical wavelength $\lambda_{opt} \sim 770$ nm. To change λ in the lattice one changes the angle between laser and tubes. The magnetic field is aligned parallel to the lasers. If θ is the angle between the tubes and these lasers, the lattice wave-length is $\lambda = \lambda_{opt}/2\cos(\theta)$. $k_F\lambda = 30$ translates to 7.6 μm and $\theta \sim 87^o$. Since this is almost perpendicular to the 1D tube, the heating will also be reduced as most of the recoil is absorbed in the confining potential.

In an actual experiment it has been suggested 1D tubes in an intermediate strength 2D optical lattice will give the best conditions for observing the FFLO state in 1D [23], and we expect this to hold for our π-phases as well. Our proposal differs from other studies on FFLO states in cold atom since we use an unpolarized gas. The Fermi surfaces are therefore identical for the two spins and the non-trivial pairing properties of the system are entirely due to the spin-dependent lattice potential.

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