# Packet Loss in a Bufferless Optical WDM Switch Employing Shared Tunable Wavelength Converters 

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#### Abstract

In this paper, we propose an architecture for a bufferless packet optical switch employing the wavelength dimension for contention resolution. The optical packet switch is equipped with tunable wavelength converters shared among the input lines. An analytical model is proposed in order to determine the number of converters needed to satisfy prefixed packet loss probability constraints. This analytical model very accurately fits with simulations results. A sensitivity analysis of the required number of converters as a function of the main system parameters (number of input and output lines, number of wavelengths, . . .) and traffic parameters has been carried out. Making use of the introduced dimensioning procedure we have observed that the proposed architecture allows a saving in terms of employed number of converters with respect to the other architectures proposed in literature. Such a saving can reach about $95 \%$ of the number of converters.


Index Terms-Dimensioning, optical packet switch, performance evaluation, wavelength conversion, wavelength division multiplexing (WDM) networks.

## I. Introduction

WAVELENGTH division multiplexing (WDM) has been rapidly gaining acceptance as the technology able to handle the forecast dramatic increase of bandwidth demand in future networks [1]. Besides the huge amounts of bandwidth, all-optical WDM networks also allow high-speed data transmission without electronic converters at intermediate nodes and transparency with respect to data format to be achieved.

Wavelength-routed WDM network in which lightpaths are set-up on specific wavelengths between pairs of node [2], [3] have been extensively studied in recent past; however, the service evolution and the rapid increase in traffic levels fuel the interest on optical packet switching [4]-[9]. In fact, while current applications of WDM focus on the static usage of individual WDM channels, optical packet switching technologies enable the fast allocation of WDM channels in an on-demand fashion with fine granularities (microsecond time scales). The challenge now is to combine the advantages of the relatively coarse-grained WDM techniques with emerging optical switching capabilities to yield a high-throughput optical platform.

A further reason leading to optical packet switching is its intrinsic flexibility to cheaply support incremental increases of the transmission bit rate [7], so that frequent upgrades of the transmission layer capacity can be envisaged to match increasing bandwidth demand with a minor impact on switching

[^0]nodes. As a consequence, a promising guideline for the network evolution could consists in the migration of most of the switching burden into the optical domain in order to exploit the scalability provides by optical technology to support the progressive increments of transmission capacity of WDM lines. This approach could lead to an effective decoupling between transmission/switching and routing/forwarding functionality. The former should be handled in the optical domain, so as to access the huge fiber bandwidth; the latter should be carried out in the electronic domain, where the routing/forwarding functions based on packet header processing would be performed.

One of the key problems in application of packet switching in optical domain is the handling of packet contentions that take place when two or more incoming packets are directed to the same output line. Various techniques have been examined in literature: buffering, deflection routing, wavelength translation, and wavelength dimension.

The application of buffering technique would make the structure of a optical packet switch strictly close to that of a traditional electronic packet switch, therefore it has been extensively studied, e.g., in [10], [11]. Unfortunately, at least with current technology, optical buffer can be only implemented through a bundle of Fiber Delay Lines (FDLs), with lengths equal to a multiple of a packet duration. This significantly reduces the buffer capacity of an optical packet switch. Thus the number of FDLs is a critical system design parameter because it has an impact on the optical hardware volume, on the switch size and on the noise level due to the transit of optical signal in FDLs.

The deflection routing [12], [13] is simply a multiple path routing technique that allows the contention problems to be solved and the buffer depth and the number of optical gates to be reduced with a sensitive saving in hardware volume and cost. The effectiveness of this technique critically depends on the network topology; as a matter of example, meshed topologies with a high number of interconnections benefit of the largest gain from deflection routing whereas minor advantages arises from more simple topologies.

In order to reduce the buffer size, the wavelength translation technique has been proposed [14], [15], [16]. Packet contentions are handled in the wavelength domain: packets addressed toward the same output are converted to different wavelengths by means of Tunable Optical Wavelength Converters (TOWCs).

The wavelength dimension technique uses the wavelength dimension as a logical buffer in the WDM optical network layer. In [17], a network solution is proposed that eliminates the need for optical buffers by splitting the traffic load on the wavelength channels by using TOWCs. The proposed scheme solves the problem regarding optical buffering; however, it implies a high number of TOWCs, in fact, one TOWC is needed for each input wavelength channel. For example, in [17], it is shown that,
if the traffic intensity per line is 0.8 and the required packet loss probability is equal to $10^{-10}$, an optical switch with 16 input and output fibers and 11 wavelengths per fiber requires only 2816 gates compared to the "traditional" switch, with optical buffers employing only one wavelength, that would require 12288 gates; however, the optical switch needs a high number of converters equal to the number of input fiber multiplied the number of wavelengths supported on each fiber, that is, 176 TOWCs.

As aforementioned, in [17], the number of TOWCs is assumed to be proportional to both the number of input fibers and the number of wavelengths. However, it is to be noted that only a small part of TOWCs is simultaneously utilized; this is due to two main reasons:

- an input channel, identified by the couple "input line, wavelength," at a given instant could not contain packets, whether we suppose to have on the channel a load less than $100 \%$;
- not all of packets contending for the same output line have to be shifted in wavelength because they are already carried by different wavelengths.
In this paper, we propose and analyze a switch architecture in which the TOWCs are shared among the input channels and their number is minimized so that only those TOWCs strictly needed to achieve given performance requirements are employed. An analogous problem has been investigated in [18], where an upper bound of the number of wavelength conversions is derived. In our analysis we generalize the analytical model introduced in [19] and utilized to evaluate the needed number of TOWCs to be employed in the optical switch; in particular the following aspects are addressed:
- the dimensioning of the number of TOWCs needed to satisfy given performance requirements, in terms of packet loss probability;
- a sensitivity analysis with respect to both the optical packet switch parameters and the traffic load;
- the evaluation of the saving of the TOWCs number determined by the proposed architecture with respect to the number of TOWCs needed by the switch architecture proposed in literature;
- the evaluation, by means of an analytical model, of the packet loss probability when a given number of TOWCs is utilized.
This paper is organized as follows. The proposed architecture and the control algorithm of the optical packet switch are discussed, respectively, in Sections II and III; procedures to dimension the number of TOWCs are described in Section IV. In Section V, numerical examples of dimensioning and performance results of the proposed optical switch are given. Finally, in Section VI, we discuss the achieved results and give comments about further research items.


## II. WDM Optical Packet Switch

The considered optical switch architecture is shown in Fig. 1. It has $N$ input and output fibers, each fiber supports a WDM signal with $M$ wavelengths, so an input (or output) channels is characterized by the couple $\left(i, \lambda_{j}\right)$ wherein $i(i \in\{1, \cdots, N\})$ identifies the input/output fiber and $\lambda_{j},(j \in\{1, \cdots, M\})$ identifies the wavelength.

We assume that packets have a fixed size and their arrivals on each wavelength are synchronized on a time-slot basis [20], [21] and a time slot is the time needed to transmit a single packet. Hereafter, the duration $T$ of a time slot is assumed as the time unit.

The switch architecture is equipped with a number $r$ of TOWCs which are shared among the input channels. At each input line, a small portion of the optical power is tapped to the electronic controller not shown in the figure. The switch control unit detects and reads packet headers and drives the space switch matrix and the TOWCs. Incoming packets on each input line are wavelength demultiplexed (DMUXs blocks in Fig. 1). An electronic control logic, on the basis of the routing information contained in each packet header, handles packet contentions and decides which packets have to be wavelength shifted. Packets not requiring wavelength conversion are directly routed toward the output lines (e.g., the packet arriving at the input line 1 and wavelength $\lambda_{1}$ and directed to the output 1, as shown in Fig. 1); on the contrary, packets requiring wavelength conversions will be directed to the pool of $r$ TOWCs and, after a proper wavelength conversion, they will reach the output line (e.g., the packet arriving at the input line $N$ and wavelength $\lambda_{1}$ and directed to the output 1 , as shown in Fig. 1). A detailed description of the control algorithm is mentioned in Section III.

A possible implementation of the switch is illustrated in Fig. 2. It is realized by means of splitters, passive couplers and optical gates. The complexity of the switch expressed in terms of number of optical gates is $C=(N+r) \times M \times N+N \times r$.

As it will be shown later, the converter sharing allows a remarkable reduction of the number of TOWCs with respect to that needed by other switch architectures [16] in which there are as many TOWCs as the input channels are, i.e., $N \cdot M$. Anyway, this assertion is also intuitive because wavelength conversions are not always needed, so a statistical advantage can be achieved by adopting TOWC's sharing.

As a remark, it is to be noted that, the use of shared TOWCs involves two main drawbacks to be dealt with: 1) the enlargement of the switching matrix of a factor equal to the number of used converters $r$ (see Fig. 1) and 2) the introduction of an additional attenuation of the optical signal caused by the twice crossing of the switching matrix. The impact of latter issues strictly depends on the adopted technology and its evaluation is out of the scope of this paper. While, the tradeoff between the positive effect on the switch cost of the sensible reduction of the number of TOWCs and the negative impact of the enlargement of the switch matrix will be here addressed.

## III. Control Algorithm of the Packet Optical Switch

The switch control unit adopts a simple and fair technique in assigning in each time slot the TOWCs to the various output channels. If in a time slot there are conversions to be accomplished, the control unit randomly selects an output fiber, among those having packets to be shifted, and decides the needed conversions by allocating one or more TOWCs and the output channel(s). Afterwards, this operation is repeated by the control unit as long as there are both available TOWCs


Fig. 1. Structure of the bufferless WDM optical switch architecture equipped with wavelength converters.
and packet conflicts to be solved. If all the $r$ TOWCs are allocated but one or more packet contentions are not solved, the relevant packets are lost. So the number of TOWCs must be dimensioned in order to satisfy predefined constraints on the packet loss. The proposed control algorithm has been already described in [18] and we propose it again with a few changes.

Some notations are introduced; let $I_{i, \lambda_{j}}$ be the set of the packets arriving on the wavelength $\lambda_{j},(j \in\{1, \cdots, M\})$ and directed to the output line $i(i \in\{1, \cdots, N\})$. The aim of the algorithm is to determine the packet $o_{i, \lambda_{j}}$, i.e., the packet that has to be switched on the output channel $\left(i, \lambda_{j}\right)$; obviously if the wavelength of the packet is different from that of the assigned output line, wavelength conversion is needed. Finally, $\Lambda$ and $\Gamma$ denote the sets of the wavelengths and the output lines, respectively, that is $\Lambda \equiv\left\{\lambda_{1}, \lambda_{2}, \cdots \lambda_{M}\right\}$ and $\Gamma \equiv\{1,2, \cdots N\}$.

The control algorithm is illustrated in Fig. 3 and it is composed by four steps. Step 1 sets the initial values of the variables and in particular it determines the set $I_{i, \lambda_{j}}$ for all $\lambda_{j}$, $(j \in\{1, \cdots, M\})$ and $i(i \in\{1, \cdots, N\})$. In Step 2 , the control unit determines for each output channel $\left(i, \lambda_{j}\right)$ the packet, if there is, to be transmitted without wavelength conversion; if more than one packet is addressed to a given channel, the control unit randomly selects one of them; moreover, for each output line $i$, the set $\Lambda_{i}^{f}(i \in\{1, \cdots, N\})$ of free wavelengths, i.e., the set of wavelengths on which no packets arrive, is also evaluated in the Step 2. Steps 3 and 4 consider the packets to be switched by employing wavelength conversion; in order to achieve a fair assignment of the converters among the output lines, the con-
trol unit randomly selects the output lines having packets to be wavelength shifted. In Fig. 3, $\Gamma^{\prime}$ denotes the set of output lines still having packets to be wavelength converted. Step 3 checks, for the selected output line $i$, if there are free wavelength channels to which addressing the packets belonging to the set $I_{i}$, that is the set of the packets not selected in Step 2 due to contention events. Denoting by $r^{\prime}$ the number of converters still available during the scheduling process of the packets, the algorithm selects the packets that can be transmitted after a wavelength conversion on the output line $i$ taking into account both the number of free wavelengths and the number $r^{\prime}$ of currently available TOWCs; the number of packets to be transmitted is given by $\min \left(\left|I_{i}\right|,\left|\Lambda_{i}^{f}\right|, r^{\prime}\right)$ while the number of lost packets is given by $\left|I_{i}\right|-\min \left(\left|I_{i}\right|,\left|\Lambda_{i}^{f}\right|, r^{\prime}\right)$ where the notation $|A|$ denotes the cardinality of the set $A$.

## IV. Dimensioning of the Number of TOWCs

We assume that the packet arrivals on the $M N$ channels at each time slot are independent and they occur with probability $a_{o}$. No assumption is done on the arrival dependence of the packets at different time slot, in fact, as we will explain later, the introduced dimensioning methods of the wavelengths converters depends only on the packet arrival probability $a_{o}$ at each time slot; this follows from the fact that the packet arrivals are synchronized on a time slot basis and hence the number of converters needed at a given time slot depends only on the number of packets arriving at such a slot. Analogously owing to the


Fig. 2. Implementation of the optical switch by means of splitters, passive couplers, and optical gates.


Fig. 3. Flow-chart of the switch control algorithm.
bufferless structure of the optical switch, the performance of the switch, expressed in terms of packet loss probability depends only on the traffic intensity $a_{o}$ [22], [23]. Hence, both the converters dimensioning procedures and the switch performances hold for any type of input traffic statistic.

In our analysis, we also assume a symmetric traffic, i.e., each packet has the same probability $1 / N$ to be directed to any given output line. This scenario of symmetric traffic is assumed because it is the one requiring the most severe dimensioning of number of converters; this is due to the fact that in the case of

(b)

Fig. 4. Wavelengths used by the arriving packets (a) before and (b) after the conversion process is accomplished ( $M=4, \beta_{k}=2, W_{k}=2, \sum_{p=q_{1}, q_{2}} A_{k, p}=$ 8).
symmetric traffic we have the lowest packet loss probability and hence, more packets can require wavelength conversions.

In fact, notice that a packet can be lost if either there are not available converters or all of the wavelengths are engaged at the output line where the packet is directed. As the symmetric traffic assumption leads to the smallest number of lost packets due to lack of available wavelengths, we will have more packets requiring wavelength conversions and hence a more severe dimensioning of TOWCs.

In order to carry out the analysis we introduce some notations; we define the random variable $A_{k, j}$ as the number of packet arrivals on the wavelength $\lambda_{j}$, addressed to the output line $k$ at a time-slot at equilibrium. Because, at each time slot, a packet arrives on a given channel with probability $a_{o}$ and it is addressed to each output line with probability $1 / N$, the probability $p_{A_{k, j}}(x)$ that $x$ packets arriving on the wavelength $\lambda_{j}$ are directed to the output line $k$, has the binomial probabilities

$$
\begin{align*}
p_{A_{k, j}}(x)= & \operatorname{Pr}\left\{A_{k, j}=x\right\} \\
= & \binom{N}{x}\left(\frac{a_{o}}{N}\right)^{x}\left(1-\frac{a_{o}}{N}\right)^{N-x} \\
& x=0,1, \cdots N . \tag{4.1}
\end{align*}
$$

In the following, in order to cope with the main dimensioning aspects, we will evaluate the packet loss probability $P_{\text {loss }}$ of the optical switch versus the number of employed TOWCs. This performance index is evaluated in Section IV-B, while in Section IV-A, we will calculate the probability function $p_{W_{k}}(h)$ of the number $W_{k}$ of conversions required by a single output line
$k$; the evaluation of $p_{W_{k}}(h)$ is an intermediate step needed in order to have an expression for $P_{\text {loss }}$.

## A. Evaluation of $p_{W_{k}}(h)$

Let $\beta_{k}$ be the number of wavelengths on which no packets arrive at the output line $k$ in a given time-slot (for example, in Fig. 4 we have $\beta_{k}$ equal to 2 because there are two wavelengths, $\lambda_{3}$ and $\lambda_{4}$, on which no packets arrive). $\beta_{k}$ can be expressed as follows:

$$
\begin{equation*}
\beta_{k}=\sum_{i=1}^{M} \delta\left(A_{k, i}\right) \tag{4.2}
\end{equation*}
$$

where $\delta(x)$ is the discrete impulse

$$
\delta(x)= \begin{cases}1 & \text { if } x=0 \\ 0 & \text { if } x \neq 0\end{cases}
$$

The probability function of $\beta_{k}$ is easily evaluated by means of the following expression:

$$
\begin{equation*}
\operatorname{Pr}\left\{\beta_{k}=j\right\}=\binom{M}{j}\left(1-b_{k, i}\right)^{M-j}\left(b_{k, i}\right)^{j} \tag{4.3}
\end{equation*}
$$

with

$$
\begin{equation*}
b_{k, i}=\operatorname{Pr}\left\{\delta\left(A_{k, i}\right)=0\right\}=\left(1-\frac{a_{o}}{N}\right)^{N} \tag{4.4}
\end{equation*}
$$


(b)

Fig. 5. Conversion process when the arriving number of packets is less than the number of wavelengths $\left(M=4, \beta_{k}=2, W_{k}=1, \sum_{p=q_{1}, q_{2}} A_{k, p}=3\right)$.

Let $p_{W_{k}}(h)$ be the probability function of the number $W_{k}$ of TOWCs; by applying the total probability theorem we can write that

$$
\begin{align*}
p_{W_{k}}(h) & =\operatorname{Pr}\left\{W_{k}=h\right\} \\
& =\sum_{j=0}^{M} \operatorname{Pr}\left\{W_{k}=h / \beta_{k}=j\right\} \operatorname{Pr}\left\{\beta_{k}=j\right\} \tag{4.5}
\end{align*}
$$

wherein we have (4.6) shown at the bottom of the page. To understand (4.6) notice that:

- when $\beta_{k}$ is equal to $j$, there are at most $j$ wavelengths on which the packets can be shifted and hence the number $W_{k}$ of conversions cannot be greater than $j$;
— in order to have a number $W_{k}$ of conversions equal to $j$, the number of packets arriving on the $(M-j)$ remaining wavelengths $\lambda_{q_{1}}, \lambda_{q 2}, \cdots \lambda_{q_{M-j}}$ must be greater than $M$ (this case is illustrated in Fig. 4 where $M=4, \beta_{k}=2$, $W_{k}=2, \sum_{p=q_{1}, q_{2}} A_{k, p}=8$ );
- in order to have a number $W_{k}$ of conversions less than $j(j \neq M)$ it is needed that $M-j+h$ packets arrive on the wavelengths $\lambda_{q_{1}}, \lambda_{q 2}, \cdots, \lambda_{q_{M-j}}: M-j$ of these will be forwarded without conversions on the same arrival wavelengths, the remaining $h$ will be shifted on $h$ of the $j$ available (this case is illustrated in Fig. 5 where $M=4$, $\beta_{k}=2, W_{k}=1, \sum_{p=q_{1}, q_{2}} A_{k, p}=3$ );
- when $j=M$, there is no packet arrival, then the needed number of conversions is zero.
From (4.3), (4.4), and (4.6) we can express (4.5) as shown in (4.7) at the bottom of the next page. We evaluate the survivor

$$
\operatorname{Pr}\left\{W_{k}=h / \beta_{k}=j\right\}= \begin{cases}0 & j<h  \tag{4.6}\\ \operatorname{Pr}\left\{\sum_{p=q_{1}, \cdots q_{M-j}} A_{k, p} \geq M / A_{k, p} \geq 1 p=q_{1}, \cdots q_{M-j}\right\} & (j=h) \text { and }(j \neq M) \\ \operatorname{Pr}\left\{\sum_{p=q_{1}, \cdots q_{M-j}} A_{k, p}=M-j+h / A_{k, p} \geq 1 p=q_{1}, \cdots q_{M-j}\right\} \\ \delta(h) & h+1 \leq j \leq M-1 \\ & j=M .\end{cases}
$$

function of the total number of conversions $W=\sum_{k=1}^{N} W_{k}$ by assuming the variables $W_{k}(k=1, \cdots N)$ statistically independent; notice that in reality the variables $W_{k}$ are not independent because they depend on the variables $A_{k, j}$ which are negatively correlated; in fact fixed a wavelength $\lambda_{j}$, the sum of the variables $A_{k, j}(k=1, \cdots N)$, that is the total number of packets arriving on the wavelength $\lambda_{j}$, is not larger than $N$ and every additional packet destined to the output line $i$ reduces the likelihood of packets directed to the output port $h$, for $h \neq i$; in the extreme case, if $N$ packets arrive during a time slot for a single output port $h$, then no packets can arrive during such time slot for any of the other output lines. Although there exists this dependence, it is shown in [24]-[27] that it is very slight and can be neglected when the traffic offered to the switch is low, that is just the scenario at which we are interested. In fact we have proposed a bufferless switch architecture in which the contention problems are resolved in the wavelength domain; as we will show in Section V, this give rise to a low admissible load per wavelength so we can assume the independence among the variables $A_{k, j}$ ( $k=1, \cdots N ; j=1,2, \cdots M$ ) and consequently the independence among the random variables $W_{k}(k=1,2 \cdots, N)$. Hence, we are able to write the survivor function $f^{W}(h)$ of the total number $W$ of conversions by means of (4.7)

$$
\begin{align*}
f^{W}(h) & =\operatorname{Pr}\{W>h\} \\
& =\operatorname{Pr}\left\{\sum_{k=1}^{N} W_{k}>h\right\} \\
& =\left.\sum_{i=h+1}^{\infty} \stackrel{N}{\otimes}{ }_{k=1}^{N} p_{W_{k}}(x)\right|_{x=i} \tag{4.8}
\end{align*}
$$

where $\stackrel{N}{\otimes}$ denotes the convolution operator applied $N$ times.
The number of TOWCs can be determined by fixing the value of the survivor function expressed by (4.8); e.g., we can fix a threshold value for the packet loss probability due to the unavailability of TOWCs and find by means of (4.8) the minimum value $h$ of $W$ satisfying such a constraint.

An alternative approach that provides a very good approximation of the total number of conversions is given by application of the large deviation theory [28], [29, ch. 3, 4]. Here, we present a plausability argument based upon conjecture. The application of this theory allows us to furnish converters dimensioning formulas valuable in a simple computationally way. In particular in
our analysis we can apply the Gärtner-Ellis theorem [28], [29, ch. 3, 4] that is quite useful when the random variables of interest are not independent and identically distributed but are close to it in some sense. According to this theorem and denoting the variables previously introduced with the apex $(N)$ in order to indicate that they have been evaluated for a optical switch having $N$ input and output lines [for example $W^{(N)}, W_{k}^{(N)}$ denote the total number of conversions and the number of conversions for the output line $k$ respectively when the number of output lines of the optical switch is $N$ ], we can write:

$$
\begin{align*}
f^{W}(N h) & =\operatorname{Pr}\left\{\frac{W^{(N)}}{N}>h\right\} \\
& =\operatorname{Pr}\left\{\frac{\sum_{k=1}^{N} W_{k}^{(N)}}{N}>h\right\}  \tag{4.9}\\
& =e^{-N \operatorname{Inf}_{h^{\prime} \geq h} I\left(h^{\prime}\right)+o(N)}
\end{align*}
$$

where $o(N) \rightarrow 0$ for $N \rightarrow \infty$ and $I\left(h^{\prime}\right)$ is expressed by the following expression:

$$
\begin{equation*}
I\left(h^{\prime}\right)=\sup _{\vartheta}\left(h^{\prime} \vartheta-\varphi(\theta)\right) \tag{4.10}
\end{equation*}
$$

with

$$
\begin{equation*}
\varphi(\theta)=\lim _{N \rightarrow \infty} \varphi_{N}(\theta) \tag{4.11}
\end{equation*}
$$

and

$$
\begin{align*}
\varphi_{N}(\theta) & =\frac{1}{N} \log E\left[\exp \left(\vartheta W^{(N)}\right)\right] \\
& =\frac{1}{N} \log E\left[\exp \left(\vartheta \sum_{k=1}^{N} W_{k}^{(N)}\right)\right] . \tag{4.12}
\end{align*}
$$

In order to evaluate the function $\varphi(\theta)$ we make the following remark: for $N \rightarrow \infty$ the variables $W_{k}^{(N)}(k=1,2 \cdots N)$ become independent; this is owing to a result well known in literature [22], [23], [30, ch. 3] according to which for a fixed

$$
\begin{align*}
p_{W_{k}}(h)= & \binom{M}{h} b_{k, i}^{h}\left(1-b_{k, i}\right)^{M-h} \\
& \cdot \operatorname{Pr}\left\{\sum_{p=q_{1}, \cdots q_{M-h}} A_{k, p} \geq M / A_{k, p} \geq 1 p=q_{1}, \cdots q_{M-h}\right\}+\sum_{j=h+1}^{M-1} b_{k, i}^{j}\left(1-b_{k, i}\right)^{M-j} \\
& \cdot \operatorname{Pr}\left\{\sum_{p=q_{1}, \cdots q_{M-j}} A_{k, p}=M-j+h / A_{k, p} \geq 1 p=q_{1}, \cdots q_{M-j}\right\}+\delta(h)\left(1-b_{k, i}\right)^{M} . \tag{4.7}
\end{align*}
$$

wavelength $\lambda_{j}$ the variables $A_{k, j}^{(N)}$ for $N \rightarrow \infty$ tend to be independent and distributed according to a Poissonian distribution. Hence, $\varphi(\theta)$ can be expressed as follows:

$$
\begin{equation*}
\varphi(\theta)=\log E\left[\exp \left(\vartheta W_{k}^{(\infty)}\right)\right] \tag{4.13}
\end{equation*}
$$

From (4.13), $\varphi(\theta)$ can be interpreted as the moment generation function of the random variable $W_{k}^{(\infty)}$ that is the number of conversions for a generic output line $k$ when the number of input and output lines of the optical switch $N$ tends to infinity; obviously according to the previous remark the moment generation function of the random variable $W_{k}^{(\infty)}$ is evaluated by assuming the variables $A_{k, j}^{(\infty)}$ distributed according to a Poissonian distribution of parameter $a_{o}$.

Finally, after some algebra we can express (4.9) for $h>$ $E\left[W_{k}^{(\infty)}\right][28]$ as follows:

$$
\begin{equation*}
f^{W}(N h)=e^{-N\left(h \vartheta^{*}-\varphi\left(\vartheta^{*}\right)\right)+o(N)} \tag{4.14}
\end{equation*}
$$

where $\theta^{*}=\log y^{*}$ and $y^{*}$ is a root of the equation

$$
\begin{equation*}
p_{W_{k}^{(\infty)}}(0) y+\sum_{i=1}^{M-1} p_{W_{k}^{(\infty)}}(i) y^{i}=0 \tag{4.15}
\end{equation*}
$$

with $p_{W_{k}^{(\infty)}}(h)(h=0,1, \cdots M-1)$ being the probability function of the variable $W_{k}^{(\infty)}$ that according to (4.7) is given by (4.16) shown at the bottom of the page where from (4.4) we obtain $b_{k, i, N \rightarrow \infty}=\lim _{N \rightarrow \infty} b_{k, i}=e^{-a_{o}}$ while as previously mentioned the variables $A_{k, p}^{(\infty)}$ have a Poissonian distribution

$$
\begin{equation*}
p_{A_{k, p}^{(\infty)}}(x)=\operatorname{Pr}\left\{A_{k, p}^{(\infty)}=x\right\}=\frac{a_{o}^{x}}{x!} e^{-a_{o}} \quad x=0,1, \cdots \tag{4.17}
\end{equation*}
$$

The large deviation approach allows a more simple numerical evaluation of the survivor function of the number of converters, but as we will show in the numerical results section, provides less accurate results with respect to the ones obtained by assuming the independence assumption. However the introduced errors are negligible in most cases, so this approach can be utilized to analyze complex switch configurations. On the contrary
the independence assumption is mandatory for the evaluation of the packet loss, so this assumption has to be introduced at least in this issue.

## B. Evaluation of $P_{\text {loss }}$

The purpose of this section is the evaluation of the packet loss probability introduced by the optical packet switch equipped by $0<r<N * M$ TOWCs. This result will be compared with performance arising from packet switch configuration in which $r=0$ (i.e., no conversion) and $r=N * M$ (i.e., full conversion). This will allow the number of TOWCs to be determined and a sensitivity analysis of the dimensioning as a function of the main system parameters (number $N$ of input and output lines, number of wavelengths $M$, traffic intensity $a_{o}, \cdots$ ) to be carried out.

1) Evaluation of $P_{\text {loss }}$ in a Full Conversion Configuration $(r=N * M)$ : We can express the packet loss probability $P_{\text {loss }}^{(1)}$ as follows:

$$
\begin{equation*}
P_{l o s s}^{(1)}=\frac{E\left[N_{l}^{k}\right]}{E\left[N_{o}^{k}\right]} \tag{4.18}
\end{equation*}
$$

where $E[x]$ denotes the expected value of the random variable $x$ and

- $N_{o}^{k}$ is the number of packets offered to the output port $k$; it is easy to show that:

$$
\begin{equation*}
E\left[N_{o}^{k}\right]=M a_{o} \tag{4.19}
\end{equation*}
$$

- $N_{l}^{k}$ is the lost number of packets offered to the output $k$; we can write by applying the total probability theorem:

$$
\begin{equation*}
E\left[N_{l}^{k}\right]=\sum_{i=0}^{N} E\left[N_{l}^{k} / A_{k}=i\right] \operatorname{Pr}\left\{A_{k}=i\right\} \tag{4.20}
\end{equation*}
$$

where $A_{k}=\sum_{j=1}^{M} A_{k, j}$ is the total number of packets arriving at output line $k ; A_{k}$ has got the following probability function:

$$
\begin{equation*}
\operatorname{Pr}\left\{A_{k}=i\right\}=\operatorname{Pr}\left\{\sum_{j=1}^{M} A_{k, j}=i\right\}=\stackrel{\left.\otimes_{j=1}^{\otimes} p_{A_{k}, j}(x)\right|_{x=i}}{ } \tag{4.21}
\end{equation*}
$$

where $p_{A_{k, j}}(x)$ is given by (4.1).

$$
\begin{align*}
p_{W_{k}^{(\infty)}}(h)= & \binom{M}{i} b_{k, i, N \rightarrow \infty}^{h}\left(1-b_{k, i, N \rightarrow \infty}\right)^{M-h} \\
& \cdot \operatorname{Pr}\left\{\sum_{p=q_{1}, \cdots q_{M-h}} A_{k, p}^{(\infty)} \geq M / A_{k, p}^{(\infty)} \geq 1 p=q_{1}, \cdots q_{M-h}\right\}+\sum_{j=h+1}^{M-1} b_{k, i, N \rightarrow \infty}^{j}\left(1-b_{k, i, N \rightarrow \infty}\right)^{M-j} \\
& \cdot \operatorname{Pr}\left\{\sum_{p=q_{1}, \cdots q_{M-j}} A_{k, p}^{(\infty)}=M-j+h / A_{k, p}^{(\infty)} \geq 1 p=q_{1}, \cdots q_{M-j}\right\}+\delta(h)\left(1-b_{k, i, N \rightarrow \infty}\right)^{M} \tag{4.16}
\end{align*}
$$

The term $E\left[N_{l}^{k} / A_{k}=i\right]$, appearing in (4.20), is given by

$$
E\left[N_{l}^{k} / A_{k}=i\right]= \begin{cases}0 & \text { if } 0 \leq i \leq M  \tag{4.22}\\ i-M & \text { if } M<i \leq N M\end{cases}
$$

Expression (4.22) is explained taking into account that according to the conversion process, the output line $k$ is able to send arriving packets $M$ per time-slot; hence a lost phenomenon happens just when $A_{k}>M$ and the lost number of packets is equal to $A_{k}-M$
From (4.18)-(4.22), we obtain the following expression for $P_{\text {loss }}^{(1)}$ :

$$
\begin{align*}
P_{l o s s}^{(1)}=\frac{1}{M a_{o}} & {\left[\sum_{i=M+1}^{N M}(i-M) \stackrel{M}{\otimes}_{j=1}^{M}\binom{N}{x}\right.} \\
& \left.\left.\cdot\left(\frac{a_{o}}{N}\right)^{x}\left(1-\frac{a_{o}}{N}\right)^{N-x}\right|_{x=i}\right] \tag{4.23}
\end{align*}
$$

2) Evaluation of $P_{\text {loss }}$ in No Conversion Configuration ( $r=$ 0 ): As no conversions are accomplished and because we have assumed that the traffic is uniform on the various wavelengths, in order to evaluate $P_{\text {loss }}$, is sufficient to calculate the loss probability of the packets addressed to a generic output channel ( $k, \lambda_{j}$ ). According to the following remark, we can write:

$$
\begin{equation*}
P_{l o s s}^{(2)}=\frac{E\left[N_{l}^{k, j}\right]}{E\left[N_{o}^{k, j}\right]} \tag{4.24}
\end{equation*}
$$

where

- $N_{o}^{k, j}$ is the number of packets offered to the output port $k$ and wavelength $j$, we have that:

$$
\begin{equation*}
E\left[N_{o}^{k, j}\right]=a_{o} \tag{4.25}
\end{equation*}
$$

- $N_{l}^{k, j}$ is the lost number of packets offered to the output $k$ and wavelength $j$; we can express its expected value by applying the total probability theorem:

$$
\begin{equation*}
E\left[N_{l}^{k, j}\right]=\sum_{i=0}^{N} E\left[N_{l}^{k, j} / A_{k, j}=i\right] \operatorname{Pr}\left\{A_{k, j}=i\right\} \tag{4.26}
\end{equation*}
$$

At each time-slot, only one packet can be sent to the output line $k$ and wavelength $j$ and hence:

$$
E\left[N_{l}^{k, j} / A_{k, j}=i\right]=\left\{\begin{array}{ll}
0 & \text { if } 0 \leq i \leq 1  \tag{4.27}\\
i-1 & \text { if } i>1
\end{array} .\right.
$$

From (4.24)-(4.27), and (4.1), we obtain the following expression for $P_{\text {loss }}^{(2)}$ :

$$
\begin{align*}
P_{l o s s}^{(2)} & =\frac{1}{a_{o}}\left[\sum_{i=+1}^{N}(i-1) p_{A_{k, j}}(i)\right] \\
& =\frac{1}{a_{o}}\left[\sum_{i=1}^{N}(i-1)\binom{N}{i}\left(\frac{a_{o}}{N}\right)^{i}\left(1-\frac{a_{o}}{N}\right)^{N-i}\right] \tag{4.28}
\end{align*}
$$

3) Evaluation of $P_{\text {loss }}$ when $r$ TOWCs are Utilized $(0<r<$ $N * M)$ : We can write

$$
\begin{equation*}
P_{l o s s}^{(3)}=\frac{E\left[N_{l}^{k}\right]}{E\left[N_{o}^{k}\right]} \tag{4.29}
\end{equation*}
$$

$E\left[N_{o}^{k}\right]$ is expressed by (4.19) while for $E\left[N_{l}^{k}\right]$ we can write by applying the total probability theorem

$$
\begin{equation*}
E\left[N_{l}^{k}\right]=\sum_{i=0}^{N} E\left[N_{l}^{k} / C_{k}=i\right] \operatorname{Pr}\left\{C_{k}=i\right\} \tag{4.30}
\end{equation*}
$$

where $C_{k}$ is the available number of TOWCs to shift the packets addressed to the output line $k$ and its probability function is given by

$$
\begin{equation*}
\operatorname{Pr}\left\{C_{k}=i\right\}=\sum_{j=0}^{N} \operatorname{Pr}\left\{C_{k}=i / D_{k}=j\right\} \operatorname{Pr}\left\{D_{k}=j\right\} \tag{4.31}
\end{equation*}
$$

where $D_{k}$ is the random variable indicating the selection order of the output line $k$ by the manager of the pool of shared TOWCs. If we form the hypothesis that the selection is accomplished according to a uniform distribution, we can write

$$
\begin{equation*}
\operatorname{Pr}\left\{D_{k}=j\right\}=\frac{1}{N} \quad j=1, \cdots, N \tag{4.32}
\end{equation*}
$$

The conditioned probability appearing in (4.31) is given by

$$
\begin{align*}
& \operatorname{Pr}\left\{C_{k}=i / D_{k}=j\right\} \\
& \quad=\sum_{p=1}^{r} \operatorname{Pr}\left\{C_{k}=i / D_{k}=j, S=p\right\} \operatorname{Pr}\left\{S=p / C_{k}=j\right\} \tag{4.33}
\end{align*}
$$

where the random variables conditioned to the event $D_{k}=j$ are the number of TOWCs utilized by the first $j-1$ selected output lines whose the probability function is easily evaluated by means of the probability functions $p_{W_{k}}(h)$ of the number of conversions $\left\{W_{k} k=z_{1}, \cdots z_{j-1}\right\}$ (see Section IV-A) where $z_{1}, \cdots z_{j-1}$ represent the indexes of the output lines which have been selected by the controller of the pool of shared TOWCs, before the output line $k$ considered. By assuming independence of the random variables $\left\{W_{k} k=z_{1}, \cdots z_{j-1}\right\}$ we can write (4.34) shown at the bottom of the next page.

Obviously, when $S$ is equal to $p$, the number of TOWCs, which can be employed by the output line $k$, is equal to the difference between $r$ and $p$ when $r \geq p$, on the contrary if $r<p$ no TOWCs are available for conversions of the packets arriving to the considered output line $k$; hence

$$
\begin{equation*}
\operatorname{Pr}\left\{C_{k}=i / D_{k}=j, S=p\right\}=\delta(i-\max (0, r-p)) \tag{4.35}
\end{equation*}
$$

In order to evaluate the conditioned expected value of $N_{l}^{k}$ appearing in (4.30) we express it as follows:

$$
\begin{align*}
& E\left[N_{l}^{k} / C_{k}=i\right] \\
& \quad=\sum_{j=0}^{M} E\left[N_{l}^{k} / C_{k}=i, \beta_{k}=j\right] \operatorname{Pr}\left\{\beta_{k}=j / C_{k}=i\right\} \\
& \quad=\sum_{j=0}^{M} E\left[N_{l}^{k} / C_{k}=i, \beta_{k}=j\right] \operatorname{Pr}\left\{\beta_{k}=j\right\} \tag{4.36}
\end{align*}
$$

The last equation follows from the fact that we can assume independent the variables $C_{k}$ and $\beta_{k}$. The term $E\left[N_{l}^{k} / C_{k}=\right.$ $i, \beta_{k}=j$ ] can be evaluated according to the remark done in Section IV-A as shown in (4.37) at the bottom of the page. Finally, from (4.19), (4.29)-(4.37) we can write (4.38) shown at the bottom of the page.

## V. Numerical Results

In this section, numerical results arising from analysis of a bufferless optical packet switch are presented. Such results are compared with data obtained by means of simulation. We assume that the total load offered to each input line is equal to $q$ and hence, if $M$ wavelength channels are used, as the traffic is uniformly distributed on each wavelength channel, we have a offered load per wavelength channel $a_{o}=q / M$. This position analogous to that assumed in [17] is consistent with the goal of using the wavelength dimension as a mean to exclusively solve packet contention and not to increase the system throughput and the transfer capacity.

In the analysis here presented, we consider two traffic types:

1) random traffic where the packets arrivals on the same channel at different time slots are assumed to be with independent;
2) burst traffic where correlated arrivals are considered.

In particular, in the latter case, we model the emission process on each channel at ingress of the switch as a Markovian on-off process in which packets generated in each on period are addressed to the same output line. Both the traffic types are characterized by the traffic intensity $a_{o}$ per channel and in addition the burst traffic is characterized by a value of burstiness $b$, defined as the ratio between the average burst length and the average burst length for a random traffic with the same offered traffic intensity.

In Fig. 6, the simulation and analytical values of the survivor function $f^{W}(h)$ [see (4.8)] of the number of conversions $W$ have been plotted for $q=0.8, M=4$, and $N=16,24$. The analysis is performed for both random and burst traffic; the burst traffic is characterized by a burstiness $b$ equal to 16 . The results show that the analytical model is in very good agreement with simulation results and it provides overestimates of the values of $f^{W}(h)$ : this is due to the negative correlation existing among the variables $A_{h, j}$, in fact, for each wavelength, at most $N$ packets can arrive to the optical switch. Furthermore, as foreseen in Section IV, the dimensioning of the converters does not depend on the considered traffic type but only on its intensity, in fact we obtain the same dimensioning curve for both random and burst traffic. Analogue results are obtained in Figs. 7 and 8 where we have reported, for $N=16, q=0.8$,

$$
\operatorname{Pr}\left\{S=p / D_{k}=j\right\}= \begin{cases}\left.\otimes_{h=z_{1}, \cdots z_{j-1}}^{\otimes} p_{W_{h}}(x)\right|_{x=p} & \text { if } \quad p<r  \tag{4.34}\\ \left.\sum_{p=r}^{(j-1) \cdot(M-1)}{ }_{h=z_{1}, \cdots z_{j-1}} p_{W_{h}}(x)\right|_{x=p} & \text { if } \quad p=r\end{cases}
$$

$$
\begin{align*}
& E\left[N_{l}^{k} / C_{k}=i, \beta_{k}=j\right]=\sum_{p=0}^{(M-j) N} \operatorname{Pr}\left\{\sum_{q=h_{1}, \cdots h_{M-j}} A_{k, q}=p / A_{k, q} \geq 1 \quad q=h_{1}, \ldots h_{M-j}\right\} \\
& \quad \cdot \max (p-(M-j)-\min (j, i), 0) \\
& \quad=\sum_{p=(M-j)+\min (j, i)=1}^{(M-j) N}(p-(M-j)-\min (j, i)) \\
& \cdot \operatorname{Pr}\left\{\sum_{q=h_{1}, \cdots h_{M-j}} A_{k, q}=p / A_{k, q} \geq 1 \quad q=h_{1}, \ldots h_{M-j}\right\} \tag{4.37}
\end{align*}
$$

$$
\begin{align*}
& P_{l o s s}^{(3)}=\frac{1}{N M a_{o}} \sum_{i=0}^{N} \sum_{u=0}^{M} \operatorname{Pr}\left\{\beta_{k}=u\right\} \sum_{\nu=(M-u)+\min (u, i)+1}^{(M-u) N}(\nu-(M-u)-\min (j, i)) \\
& \cdot \operatorname{Pr}\left\{\sum_{q=h_{1}, \cdots h_{M-u}} A_{k, q}=u / A_{k, q} \geq 1 \quad q=h_{1}, \ldots h_{M-u}\right\} \\
& \cdot \sum_{j=1}^{N}\left(\left.\sum_{t=1}^{r-1} \delta(i-\max (0, r-t)) \otimes_{h=z_{1}, \ldots, z_{j-1}} p_{W_{h}}(x)\right|_{x=t}+\left.\delta(i) \cdot \sum_{t=r}^{(j-1) \cdot(M-1)} \otimes_{h=z_{1}, \ldots, z_{j-1}}^{\otimes} p_{W_{h}}(x)\right|_{x=t}\right) \text {. } \tag{4.38}
\end{align*}
$$



Fig. 6. Comparison between the values of simulation and the results of the analytical model for $q=0.8, M=4$ and $N=16,24$.


Fig. 7. Comparison between the values of simulation and the results of the analytical model for $q=0.8, N=16$ and $M=4,6$.
$b=16$, and $M=4,6, f^{W}(h)$ and the packet loss probability $P_{\text {loss }}^{(3)}$, respectively, versus the employed number of TOWCs. As we have used direct simulation techniques which allows us to estimate probabilities greater than around $10^{-6}$, we intend in future works to consider more sophisticated simulation techniques for estimating rare event probabilities [31] and hence to provide further verification of the introduced analytical model.

In the following, we provide results justifying the assumption of symmetric traffic and in particular we want to verify that a more severe dimensioning of converters results from a symmetric traffic scenario with respect to any type of asymmetric traffic. In order to furnish comparison of the two types of traffic, we introduce a more general traffic model; we still assume that the packet arrival probabilities on each channel is $a_{o}$ but the probabilities $p_{i}(i=1,2, \cdots N)$ of a packet to be directed to the


Fig. 8. Values of simulation and results of the analytical model for the packet loss probability of the switch when $N=16, q=0.8, M=4,6$.
various output lines can be different. We also assume the ratio $f=p_{i} / p_{i-1}(i=2, \cdots N)$ to be constant; $f$ is a factor characterizing the unbalancing of traffic. Furthermore we suppose that $f$ can assume values in the range $[1, \infty)$ that is we consider scenarios in which the traffic is unbalanced on the output lines of higher index. It is easy to show that the probabilities $p_{i}$ can be expressed as follows:

$$
p_{i}=\frac{1-f}{1-f^{N}} f^{i-1} \quad i=1,2, \cdots N
$$

From the last expression, we can observe that for $f=1$ we have the case of symmetric traffic while for $f$ growing, the load on the output lines of higher index is greater, up to the condition $f=\infty$ where the whole offered traffic is directed to the output line $N$. Once introduced the traffic model that allows us to characterize an asymmetric traffic scenario, we are able to verify that the in scenario of symmetric traffic a more severe dimensioning of converters is needed. This is illustrated in Fig. 9 where we sketch the packet loss probability as a function of the employed number of converters for $N=16, M=6, q=0.8$ and for values of $f$ equal to $1,1.5,3$. The results reported for the symmetric traffic case $(f=1)$ have been obtained with both the analytic and simulation models while the simulation results have been reported for the unbalanced traffic case ( $f=1.5,3$ ). As you can see and as foreseen, we need a greater number of converters for the symmetric traffic case, in fact in order to reach the minimum packet loss probability we need seven converters for the symmetric traffic case while only five and three converters have to be used for the asymmetric traffic cases when $f=1.5,3$, respectively.

The goodness of the approximation of the survivor function of the number of conversions with the results obtained by applying the Large Deviation theory is illustrated in Figs. 10 and 11. In Fig. 10, we have sketched the survivor function of the number of conversions while in Fig. 11 we illustrate the survivor function of the number of conversions normalized to the input and output lines $N$. The following system and traffic parameters have been


Fig. 9. Packet loss probability versus the used converters for $N=16, M=6$, $q=0.8$ in symmetric $(f=1)$ and asymmetric $(f=1.5,3)$ traffic scenarios.


Fig. 10. Inspection of the goodness of the approximation obtained by applying the Large Deviation theory. The survivor function of the number of conversions is reported as a function of $M=12, q=0.8$, and $N$ equal to $16,32,64$.
adopted: $M=12, q=0.8$ and $N=16,32,64$. These results show that the dimensioning obtained by applying the Large Deviation theory is good with respect to those provided by the analytical model assuming the statistical independence hypothesis; from the figures we can notice that, for a fixed value of the survivor function, the difference of the number of conversions obtained by applying the Large Deviation Theory and the independence assumption, respectively, is not higher than two conversions. For these reasons and owing to the low computational complexity involved in dimensioning of the number of conversions, the next results will be provided by utilizing the Large Deviation theory.

Now we discuss the results achieved in dimensioning of the TOWC's number. The sensitivity of the dimensioning to the


Fig. 11. Inspection of the goodness of the approximation obtained by applying the Large Deviation theory. The survivor function of the number of conversions normalize to $N$ is reported as a function of $M=12, q=0.8$, and $N$ equal to 16, 32, 64.


Fig. 12. Survivor function of the number of required conversions for $N=$ $16, q=0.8$, and varying $M$ from 4 to 16 . The values have been obtained by applying the Large Deviation theory.
variations of the number of wavelengths $M$ from 4 to 16 has been evaluated assuming $q=0.8$ and $N=16$, that corresponds to carrying out an analysis in which the load $a_{o}$ offered per wavelength varies from 0.2 to 0.05 . The results of the dimensioning are illustrated in Figs. 12 and 13 where $f^{W}(h)$ and the survivor function $f^{G}(h)$ of the percentage saving $G$ are plotted, respectively. The percentage saving is defined as $G=100^{*}\left(1-W /\left(N^{*} M\right)\right)$ and it is allows us to evaluate the saving of the number of TOWCs with respect to the maximum number $w_{\max }=N^{*} M$ employed in the switches proposed in


Fig. 13. Survivor function of the number of required conversions versus the gain percentage $G$ for $N=16, q=0.8$, and $M=4,8,12,16$. The values have been obtained by applying the Large Deviation theory.
literature, e.g., [15]. It is easy to show that the survivor function $f^{G}(h)$ of the percentage saving can be expressed as

$$
f^{G}(h)=\operatorname{Pr}\{G>h\}=\operatorname{Pr}\left\{\frac{W}{N}<M^{*}\left(1-\frac{h}{100}\right)\right\}
$$

From Figs. 12 and 13 notice as the sharing of the TOWCs allows to obtain an remarkable saving, for example when $M=$ 16 and fixing a value of $10^{-10}$ for $f^{W}(h)$, it is needed to equip the switch with only ten TOWCs instead of $w_{\max }=16 \times 16=$ 256 , that is the number of TOWCs used in the main architectures proposed in literature in which one converter is dedicated to each input wavelength channel; with this system parameter and required performance we have a consistent percentage saving $g \cong 96 \%$.

Fig. 14 illustrates, for $M=12$ and $q=0.8, f^{W}(h)$ for $N$ varying from $N=16$ to 80 , whereas in Fig. $15 f^{G}(h)$ is plotted. The main comment about these figures is that as expected, when $N$ grows and by fixing $f^{W}(h)$, the number of required conversions increases; in particular for $f^{W}(h) \cong 10^{-10}, h$ is equal to $11,16,19$ for $N=16,48,80$, respectively; on the contrary from Fig. 15, we note that $G$ increases when $N$ increases. This is due to two reasons: 1) the switch is equipped with TOWCs shared among all of the output lines and 2) the distribution of the required number of conversions per each output line does not vary significantly when $N$ is large enough.

Figs. 16 and 17 plot $f^{W}(h)$ and $f^{G}(h)$, respectively, maintaining constant $M=12$ and $N=16$ and varying the load offered per input line $q$ from 0.5 to 0.9 ; as the figures illustrate, the required number of TOWCs needed to obtain a given performance is not particularly sensitive to the load.

The packet loss probability $P_{\text {loss }}^{(3)}$ is shown in Fig. 18 versus the used number $M$ of wavelengths, for $N=16, q=0.8$ and varying the employed number $r$ of TOWCs and in particular we have chosen $r=1,5,9,13, w_{\max }=N \times M$; from Fig. 18 we can observe that:


Fig. 14. Survivor function of the number of required conversions for $M=12$, $q=0.8$, and $N$ varying from 16 to 80 . The values have been obtained by applying the Large Deviation theory.


Fig. 15. Survivor function of the number of required conversions versus the gain percentage $G$; the analysis is carried out assuming $M=12, q=0.8$, and $N$ varies from 16 to 256 . The values have been obtained by applying the Large Deviation theory.

1) the results previously obtained are confirmed; a number of TOWCs much lower than $w_{\max }=N \times M$ is needed to obtain a packet loss probability equal to $P_{\text {loss }}^{(1)}$; as a matter of example, notice that for $r=13 P_{\text {loss }}^{(3)}$ is equal to $P_{\text {loss }}^{(1)}$ in the considered range of wavelengths $M=2 \div 14$.
2) fixing a value of $r$ and increasing $M$, initially $P_{\text {loss }}^{(3)}$ follows $P_{\text {loss }}^{(1)}$ up to a value of wavelength after which $P_{\text {loss }}^{(3)}$ maintains above $P_{\text {loss }}^{(1)}$ decreasing slowly when $M$ is again increased. The slow increase is obviously due to the fact


Fig. 16. Survivor function of the number of required conversions for $M=12$, $N=16$, and $q$ varying from 0.5 to 0.9 . The values have been obtained by applying the Large Deviation theory.


Fig. 17. Survivor function of the number of required conversions versus the gain percentage $G$. The analysis is carried out assuming $M=12, N=16$, and $q$ varying from 0.5 to 0.9 .
to have an insufficient number of TOWCs for that range of wavelengths in which the phenomenon happens.
The advantages of sharing the TOWCs are further illustrated in Fig. 19 that gives the packet loss probability versus the TOWC's number for a $16 \times 16$ switch and $M$ varying from 2 to 14 . For every value of $M$ the trend of $P_{\text {loss }}^{(3)}$ is the same: it decreases as $r$ increases up to reach a value equal to $P_{\text {loss }}^{(1)}$ that depends on the employed number of wavelengths. According to [17] $P_{\text {loss }}^{(1)}$ decreases when $M$ increases being constant (equal to $q=0.8$ ) the traffic per input line. From Fig. 19, notice as, decreasing $M$, less TOWCs are needed to reach the minimum packet loss probability $P_{\text {loss }}^{(1)}$ : this result is not in disagreement


Fig. 18. Packet loss probability of the optical switch for $q=0.8, N=16$ and employed number of converters $r=1,5,9,13, w_{\max }=N \times M ; M$ varies from 2 to 14 .


Fig. 19. Packet loss probability as a function of the employed number of converters for $q=0.8$; the system parameters are $N=16$ and $M$ varying from 2 to 14 .
with that one relative to the Figs. 12 and 13 which show that a more severe dimensioning occurs for smaller values of $M$, in fact the Fig. 19 allows us to evaluate the needed number of TOWCs in order to obtain the packet loss probability $P_{\text {loss }}^{(1)}$ which however is different for different values of $M$.

A sensitivity analysis with respect the switch size $N$ is carried out in Fig. 20, where the packet loss probability $P_{\text {loss }}^{(3)}$ is shown as a function of the number of TOWCs and assuming $N$ as a parameter; additionally we consider $M=12$ and $q=0.8$. We can make the following remarks around the results reported in Fig. 20:


Fig. 20. Packet loss probability as a function of the employed number of converters for $q=0.8$; the system parameters are $M=12$ and $N$ varying from 8 to 80 .

1) for $r=0$, that is when no converter is utilized, $P_{l o s s}^{(3)}$ is equal to $P_{\text {loss }}^{(2)}$ evaluated in Section IV; the values of $P_{\text {loss }}^{(2)}$, obtained varying $N$, are near each other because both the traffic per input channel is the same and the conversion process is not accomplished. For $N \rightarrow \infty, P_{\text {loss }}^{(2)}$ tends to the value obtained assuming the number of arrivals on each output wavelength distributed according to a Poisson random variable [22].
2) Increasing the employed number $r$ of TOWCs, $P_{\text {loss }}^{(3)}$ decreases up to reach the value $P_{\text {loss }}^{(1)}$ at $r=r^{*}(N)$ where $r^{*}(N)$ depends on size $N$ of the switch. For example, from Fig. 20 it is possible to see that for $N=8 r^{*}(N)=9$ while for $N=80 r^{*}(N)=17$. As expected in the range of values of $r \in\left[0, r^{*}(N)\right], P_{\text {loss }}^{(3)}$ becomes greater as $N$ increases. This can be explained by considering that, as shown in Figs. 14 and 15, for $N$ growing the TOWC's dimensioning is more severe. Further notice that, the values of $P_{\text {loss }}^{(1)}$, obtained varying $N$, are near each other because traffic per output line is the same and for $N \rightarrow \infty, P_{\text {loss }}^{(1)}$ tends to a limit value as illustrated in [23].
As a final remark, it is to be noted that, the use of shared TOWCs involves the enlargement of the switching matrix of a factor equal to the number of used TOWCs $r$ (see Figs. 1 and 2). In particular for the implementation of Fig. 2, the complexity $C$ of the switch, in terms of optical gates, is increased to $(N+r) \times M \times N+N \times r$, while the architectures not employing the sharing of the wavelength TOWCs have got a complexity of $C=N \times M \times N$; as a matter of example, for $N=16, q=0.8$ and fixing a performance requirement $P_{l o s s}^{(3)}=10^{-9}$, it needed to use $M=10$ wavelengths and from the dimensioning procedure we evaluate a needed number of TOWCs $r=9$; this increases the complexity $C$ of the switch from 2560 to 4144 optical gates. An accurate evaluation of the impact of the enlarge-
ment of the switching matrix strictly depends on the adopted technology and this is out of the scope of this paper. However, the tradeoff between the positive effect on the switch cost of the sensitive reduction of the number of TOWCs and the negative impact of the enlargement of the switch matrix will be carefully evaluated in further studies.

## VI. Conclusion

We have proposed an architecture for a bufferless optical packet switch making use of the wavelength dimension for contention resolution of packets directed to the same output line. The optical switch architecture is provided with wavelength converters shared among the output lines. An analytical model, allowing the dimensioning of the number of wavelengths converters according to a required performance, has been presented. The results of the analytical model fits very accurately with simulation ones. The model has been applied to carry out a sensitivity analysis of the required number of converters as a function of the main system parameters (number of input and output lines, number of wavelengths,...) and traffic values. We have observed that the proposed architecture allows to save in terms of employed number of converters with respect to the other architectures proposed in literature; as a matter of example, when $M=16, N=16, q=0.8$ it is needed, to equip the switch with only ten converters instead of $w_{\max }=16 \times 16=256$ that is the number of converters used in the main architecture proposed in literature in which one converter is dedicated to each input wavelength channel.

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