Foreword

The Analytic Hierarchy Process (AHP) is a methodology that has been applied successfully to many discrete alternative multiple criteria decision problems in practice. Recently, the rank reversal problem of the original additive AHP has inspired a number of discussions about the mathematical soundness of the way in which it aggregates the local preference statements, and several suggestions have been made to remedy this apparent problem. The current paper provides an interesting method for aggregating preference information within the modeling framework of the Analytic Hierarchy Process, that is robust with respect to rank reversal and can provide additional insights into the decision maker's preference structure. For instance, the proposed method precludes rank reversal if all pairwise comparison statements are consistent, and also if the preference information is inconsistent, as long as it satisfies mild conditions that apply to many if not most preference structures in practice. Hence, the method introduced in this paper serves as a useful alternative to the way the preference information is aggregated in the original AHP. The AHP can take into account both qualitative and quantitative decision criteria. Thus, the topic of this research nicely complements the long-standing research stream in the MDA Project in the area of multicriteria optimization, which deals primarily with quantifiable criteria.

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PAHAP: A Pairwise Aggregated Hierarchical Analysis of Ratio-Scale Preferences

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ABSTRACT

In this paper, we present a Pairwise Aggregated Hierarchical Analysis of Ratio-Scale Preferences (PAHAP), a new method for solving discrete alternative multicriteria decision problems. Following the Analytic Hierarchy Process (AHP), PAHAP uses pairwise preference judgments to assess the relative attractiveness of the alternatives. By first aggregating the pairwise judgment ratios of the alternatives across all criteria, and then synthesizing based on these aggregate measures, PAHAP determines overall ratio scale priorities and rankings of the alternatives which are not subject to rank reversal, provided that certain weak consistency requirements are satisfied. Hence, PAHAP can serve as a useful alternative to the original AHP if rank reversal is undesirable, for instance when the system is open and criterion scarcity does not affect the relative attractiveness of the alternatives. Moreover, the single matrix of pairwise aggregated ratings constructed in PAHAP provides useful insights into the decision maker's preference structure. PAHAP requires the same preference information as the original AHP (or, alternatively, the same information as the Referenced AHP, if the criteria are compared based on average (total) value of the alternatives). As it is easier to implement and interpret than previously proposed variants of the conventional AHP which prevent rank reversal, PAHAP also appears attractive from a practitioner's viewpoint.

Subject Areas: Decision Analysis, Priority Models, and Scaling Methods.

INTRODUCTION

Over the past two decades, a number of methods have been developed which use pairwise comparisons of the alternatives and criteria for solving discrete alternative multicriteria decision problems. One of the most prominent and successful methods is the Analytic Hierarchy Process (AHP), developed by Saaty [15] [16]. However, several researchers have noted a potentially undesirable characteristic of the original AHP, in that the rank order of the existing alternatives may change when the set of alternatives is modified [3] [6] [7] [23] [24] [26]. We will refer to this phenomenon as the "rank reversal" problem.

In certain situations, for instance when the decision maker (DM) is highly inconsistent, or if the addition or removal of alternatives provides additional information which is relevant in evaluating the criterion levels, rank reversal may be legitimate and even desirable. However, rank reversal in the original AHP can occur even if the pairwise comparisons are strictly consistent, and regardless of whether modifying the set of alternatives yields additional information relevant for the overall preferences.

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The following example illustrates that the the original AHP methodology is not appropriate if the principle of independence of irrelevant alternatives applies. Suppose that two candidates applying for a particular job are equally qualified in terms of a specific decision criterion, say "computer skills." The original AHP will divide the relative priority weight for the "computer skills" criterion evenly between the two applicants. However, the similarity of the applicants' computer skills should not render both candidates half as attractive with respect to this criterion.

The potential for rank reversal in such decision problems, even in the absence of inconsistency between the judgments, casts doubt on the validity of the original AHP methodology as a tool of analysis for this hiring decision problem. In general, the rank reversal is critical, because the DM cannot be confident in the methodology of a decision aid that suggests rankings which are subject to unreasonable rank reversals during the course of the problem analysis. If the manager cannot depend on the methodology, the original AHP is of limited value and may introduce unnecessary confusion into the decision-making process. Therefore, the rank reversal phenomenon, if undesirable, should be precluded if the AHP preference modeling framework is to be used as a general guideline for solving discrete alternative multicriteria decision problems.

Of course, there are many decision problems for which rank reversal is quite a reasonable phenomenon, even in the absence of inconsistent judgments. For instance, rank reversal can reflect, in a very specific manner, the relative attractiveness of the alternatives if criterion scarcity is an issue, that is, if the system is closed and resources are limited, and if the introduction of new alternatives can provide relevant additional preference information. Through the way it synthesizes and appropriates the criterion weights, the original AHP implicitly adjusts (pro-rates) the relative preferences of the alternatives for criterion scarcity. In such cases, the original AHP may yield reasonable results, although the fact that rank reversal is possible if all judgments are consistent is still troublesome from a theoretical (mathematical) viewpoint. A well-known example of a decision situation where scarcity clearly plays a role is that a stylish high-fashion hat may be considered very attractive if it is unique, but much less desirable if several identical or near-identical copies exist [4].

Nevertheless, the original AHP in relative measurement mode synthesizes each problem in the same way, regardless of the nature of the decision problem at hand (e.g., whether or not the principle of independence of irrelevant alternatives applies), and the rank reversal phenomenon is intrinsic to the way in which the AHP calculates the priority weights. Thus, the original AHP can generate questionable ratings and rankings, so that there does appear to be a potential problem, and the validity of the original AHP as a generally applicable methodology has been an issue of concern [2] [3] [6] [7] [23] [27] to decision analysts. Schoner, Wedley, and Choo conclude that the original AHP is inconsistent with the principle of independence of irrelevant alternatives [25]. Dyer goes a step further, and remarks that "when the principle of hierarchical decomposition is assumed, the results produced by the AHP are arbitrary" [6, p. 254]. However, Saaty disagrees with Dyer's conclusion, and argues that the AHP should not be considered "as being arbitrary simply because it does not adhere to the axioms and outcomes of utility theory" [18, p. 268].

Several remedies have been proposed to deal with the rank reversal problem in the original AHP, but none of them appears to resolve the problem fully. For instance, it is possible to avoid rank reversal (as long as the criteria themselves do not need to be reevaluated) by selecting the "ratings" approach, as implemented in the Expert Choice software package [9], where alternatives are rated individually with respect to an absolute measurement scale consisting of pre-specified categorical criterion levels. However, in this approach the alternatives are not compared pairwise. Dyer remarks that absolute measurements carry the assumption "that weights on the criteria are independent of the ratings used to measure performance on them. In general, this is not true. Therefore, the rankings produced in this approach will be arbitrary, even though these rankings will not change when new alternatives are added or deleted" [6, p. 256]. An alternative course of action which has been suggested is to pre-screen the set of alternatives, eliminating duplicate or near-duplicate alternatives from the problem prior to the AHP analysis [17], but [6] has shown that one can get rank reversal even in the absence of identical or near-identical alternatives. Troutt [26] gives an example of rank reversal even when the worst alternative is deleted.

Dyer proposes to re-scale the AHP judgments by ensuring "that the criterion weights and the scores of the alternatives on the criteria are normalized with respect to the same range of alternative values" [6, p. 256], in effect transforming the ratio scale measurements to an interval scale similar to that in multiattribute decision theory. While Dyer's proposed transformation precludes rank reversal, has attractive properties, and will lead to the same rankings as those obtained using an additive value function, provided that the decision maker is consistent, [12] and [18] argue that essential information about the fundamental characteristics of the decision maker's preference structure will be lost in the process. Belton and Gear [3] advocate a modified weight normalization where the maximum entry of the weight vector is unity, rather than all entries summing to unity, as in the AHP. Others propose a normalization to the minimum entry [23] [25]. Yet another suggested remedy, discussed by [6] and [11], is to use the super matrix feedback technique [16]. However, it has recently been shown that this approach does not prevent rank reversal under all conditions [21] [22].

It has been argued that the original AHP assumption that the evaluation of criteria is independent of the alternatives is violated in most cases, and instead the criterion weights should be proportional to the average (or total) value of the alternatives on the respective criteria [2] [23] [28]. Moreover, [28] argues that the pairwise judgments in the original AHP may be meaningless, and that different alternatives should be compared pairwise across criteria. The link-pinning methods by [2] and [25] and the Referenced AHP methods proposed by [23] and [25] are examples of methods which implement these ideas. These methods preclude the occurrence of rank reversal.

While link-pinning and Referenced AHP methods may be successful in terms of resolving the issue of rank reversal, and may be theoretically preferred in terms of their fundamental interpretation, their appeal in practice may be somewhat limited, because they are conceptually more abstract and complex, more difficult to implement, and subject to the danger of complicating the pairwise evaluations to the point where the decision maker has difficulty comprehending the scope and consequences of the questions posed. Consequently, it may be problematic to derive the decision maker's true preferences using these methods. A disadvantage of Referenced AHP is that it is necessary to reassess the criterion weights whenever an alternative is added or deleted, since a change in the set of alternatives will generally lead to a change in average (total) value of the alternatives with respect to the criteria [23] [24].

Partly in order to address the above issue, the most recent version of the Expert Choice software implementation of the AHP includes the option of either a distributive or an ideal mode analysis. The distributive mode corresponds with the original AHP model, and is based on the assumption that the system is closed and has scarce resources, while the ideal mode assumes an open system where resource availability does not play a role in assessing the relative attractiveness of the alternatives. Simply stated, in the ideal mode "total preference" is allocated to a fictitious "ideal" alternative that, if it existed, would be preferred under every criterion [9, pp. 86-87]. While in distributive mode the ranks of the alternatives may change as a result of adding a new alternative, in ideal mode the rankings will remain the same. Our methodology, which will be introduced below, does not require an assumption about whether or not the system under consideration is open or closed, and we will focus our comparison on the well-established distributive mode AHP methodology. We will refer to this original or conventional AHP methodology as CAHP.

The purpose of this paper is not to add to the ongoing debate of whether the rank reversal phenomenon is legitimate or not, nor to assess the balance of advantages and disadvantages associated with previously proposed methods which avoid the rank reversal problem. Rather, we note that the rank reversal property of the CAHP may be controversial, and present a new method, Pairwise Aggregated Hierarchical Analysis of Ratio-Scale Preferences (PAHAP), which, while using exactly the same preference information as the CAHP, under rather general conditions overcomes the rank reversal problem in a very simple manner. Alternatively, PAHAP requires the same information as the Referenced AHP, if the criteria are evaluated on the basis of average (total) value.

PAHAP aggregates the problem into a single matrix of pairwise synthesized values, which is then synthesized to determine the final alternative ratings. It is up to the decision maker or analyst to decide whether the CAHP, PAHAP, a different modified AHP technique, or perhaps a multiattribute utility method is most appropriate for the particular application at hand. Therefore, we view PAHAP as complementary to the CAHP, with the attractive property that it precludes rank reversal as long as the pairwise preferences satisfy certain general, relatively weak consistency requirements.

THE CAHP AND PAHAP

In this section, we first review the CAHP, after which we introduce and derive some fundamental properties of the PAHAP method, for the case of relative measurements. We explicitly introduce the problem structure of the CAHP, because it is identical to that of PAHAP and the two methods require exactly the same information from the decision maker. Of course, if the criteria are compared on the basis of average (total) values, the information required is the same as for the Referenced AHP or ideal mode AHP. Hence, while in our illustrations we restrict ourselves to the comparison of PAHAP with the CAHP, the discussion could easily be extended to the Referenced and ideal mode AHP methods. We will also focus our comparison on the distributive mode CAHP. Briefly reviewing the steps of the CAHP is also convenient for the presentation of the basic premise of PAHAP, and provides the basis for comparing and contrasting the two methods.

Conventional AHP (CAHP)

For simplicity of exposition, we will discuss the case of one criterion level only. The extension to multiple levels is straightforward, but notationally cumbersome. We assume that the reader is already familiar with the computational techniques used in the CAHP to derive priority weights from a matrix of pairwise comparisons, such as Saaty's principal eigenvector approach [15] [16]. We will restrict ourselves to reviewing the mathematical constructs of the CAHP to the extent that these are relevant within the context of our paper.

Consider a discrete alternative multicriteria decision problem with alternative set $A=\{A_1, \ldots, A_m\}$, and criterion set $C=\{C_1, \ldots, C_n\}$. Denote the set of indices on the alternatives by $M=\{1, \ldots, m\}$, and the set of indices on the criteria by $L=\{1, \ldots, n\}$. Define the matrix of pairwise comparisons of A_i and A_j, $i, j\in M$, with respect to criterion C_k, by $\mathbf{A}(C_k)=\{a_{ijk}\}$, $i, j\in M$, $k\in L$, as in (1):

$$\mathbf{A}(C_1) = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} a_{111} & a_{121} & \cdots & a_{1m1} \\ a_{211} & a_{221} & \cdots & a_{2m1} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ a_{m11} & a_{m21} & \cdots & a_{mm1} \end{bmatrix} \end{array}, \ldots, \mathbf{A}(C_n) = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} a_{11n} & a_{12n} & \cdots & a_{1mn} \\ a_{21n} & a_{22n} & \cdots & a_{2mn} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ a_{m1n} & a_{m2n} & \cdots & a_{mmn} \end{bmatrix} \end{array}, \tag{1}$$

where a_{ijk} represents the relative importance of A_i to A_j, with respect to criterion k, such that $a_{ijk}\in[1/9, 9]$, for all $i, j\in M$, $k\in L$. In the CAHP, each $\mathbf{A}(Ck)$ is assumed to be reciprocal, that is, $a_{ijk}a_{jik}=1$, for all $i, j\in M$, $k\in L$, so that the complete assessment of each matrix $\mathbf{A}(C_k)$ requires $m(m-1)/2$ pairwise comparisons. The matrix $\mathbf{C}=\{c_{ij}\}$ of pairwise comparisons of the criteria is defined in (2):

$$\mathbf{C} = \begin{array}{c} \\ C_1 \\ C_2 \\ \cdot \\ \cdot \\ \cdot \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} \end{array}. \tag{2}$$

In the CAHP, the usual principal eigenvector approach is used to convert (synthesize) the pairwise judgments in $\mathbf{C}$ to relative importance weights (ratings) w_k for C_k, $k\in L$ [15] [16], where $w_k\geq 0$ for all $k\in L$, and $\Sigma_k w_k=1$. Let $\mathbf{w}=(w_1, \ldots, w_n)$. Similarly, synthesizing $\mathbf{A}(C_k)$ yields ratings w_{ik} for A_i with respect to criterion k. Define $\mathbf{w}_i=(w_{i1}, \ldots, w_{ik})$. The overall CAHP rating r_i of A_i is calculated as the weighted sum $\Sigma_k w_{ik}w_k = \mathbf{w}_i\mathbf{w}^T=r_i$.

Let the binary relation $>_B$ represent "more preferred than," $\sim_B$ "indifferent to," and $<_B$ "less preferred than," with respect to a criterion set B. Hence, denote "A_i is preferred to A_j, with respect to B," by $A_i >_B A_j$. Likewise, if A_i and A_j are equally preferred, based on B, we write $A_i \sim_B A_j$, and $A_i <_B A_j$ represents the case where A_i is less preferred than A_j. Assuming what Saaty and Takizawa describe as a primitive notion of a fundamental scale [20], every pair of alternatives A_i, A_j can be assigned a positive real number $P_B(A_i, A_j)$ which represents their relative preference, such that $A_i >_B A_j$ if and only if (iff) $P_B(A_i, A_j)>1$, $A_i \sim_B A_j$ iff $P_B(A_i, A_j)=1$, and $A_i <_B A_j$ iff $P_B(A_i, A_j)<1$, with respect to B. In the case where the pairwise evaluation is made with respect to a single criterion C_k, $k \in L$, that is, $B=\{C_k\}$, we have $P_B(A_i, A_j)=a_{ijk}$. The pairwise judgments in the CAHP are not required to be consistent, and the preference statements elicited from the decision maker need not be transitive, that is, it is possible to have $a_{ijk}>a_{jqk}>a_{qik}$, for some $i, j, q \in M$, $k \in L$. Of course, such a situation will rarely occur in practice, and should always be examined in further detail if it occurs.

The final CAHP ratings are often used to rank the alternatives, such that $A_i >_C A_j$ iff $r_i > r_j$, and $A_i \sim_C A_j$ iff $r_i = r_j$, where $B=C$. Even though the primitive notion of a fundamental scale has been criticized [6], because its link to classical preference theory has not been well-established, it appears to correspond closely to the way people tend to evaluate alternatives (see, for instance, [13], as communicated by [19]), and is widely used. Therefore, we will use the ratings as the basis of comparison in our paper, in particular in terms of rank reversals.

An important assumption of the CAHP procedure is the principle of hierarchical decomposition evaluation, implying that the evaluation of any pair of alternatives with respect to criterion C_k is independent of all other criteria C_h, $h, k \in L$, $h \neq k$. Interdependence of the criteria can be built into the model by constructing a super matrix [11] [16], but the discussion of this issue is beyond the scope of our paper.

PAHAP: Relative Measurement

In PAHAP, the pairwise comparisons of the criteria can be conducted either without reference to the criterion levels, as in the CAHP, or based on the average (total) value of the alternatives with respect to the criteria, as in the Referenced AHP. If the Referenced AHP approach is used, the criterion weights should be reassessed whenever the set of alternatives is modified [24]. In this paper we will assume that all the comparisons are made according to the framework used in the CAHP. The preference elicitation and representation in the PAHAP is identical to the CAHP, and PAHAP requires no additional effort from the decision maker beyond the CAHP; hence, PAHAP uses the pairwise comparison matrices $\mathbf{A}(C_k)$, $k \in L$, and $\mathbf{C}$ as defined in (1) and (2), and $\mathbf{C}$ is synthesized to determine the criterion weight vector $\mathbf{w}$.

However, while the CAHP assumes independence across all criteria when synthesizing the pairwise preference information, PAHAP assumes that the individual judgments pertaining to a given pair of alternatives are independent of the individual criterion levels of the remaining alternatives. This assumption implies that the individual scores for each pair of alternatives can be aggregated separately across all criteria. If the a_{ijk} truly represent preference intensities and the judgments are made according to the principle of hierarchical decomposition, then this pairwise aggregation of the weighted preference intensities appears justified.

Once the pairwise preferences have been combined according to the formulas below, PAHAP synthesizes the aggregate matrix to arrive at the final alternative ratings. Of course, the aggregation of pairwise preference intensities across criteria implies that the conflicts between the pairwise judgments are not synthesized based on individual criteria, but on the basis of aggregate preference intensities. As we will formally show in Proposition 3 below, a consequence of the order in which PAHAP synthesizes the information is that rank reversal is precluded, as long as certain mild consistency conditions are satisfied.

Given **C** and $\mathbf{A}(C_k)$, $k \in L$, in PAHAP we first determine t_{ij}, a modified measure of the pairwise ratings a_{ijk}, aggregated and weighted across all criteria, as defined in (3):

$$t_{ij} = \sum_{k=1}^{n} \left(\left(\frac{a_{iik}}{a_{iik} + a_{jik}} \right) w_k \right) = \sum_{k=1}^{n} \left(\left(\frac{a_{ijk}}{a_{iik} + a_{ijk}} \right) w_k \right). \tag{3}$$

In fact, t_{ij} simplifies to $\Sigma_k w_k/(1+a_{jik})$, since the diagonal elements of the pairwise comparison matrices are all equal to 1. In Proposition 1, we establish some properties which facilitate a useful interpretation of t_{ij}.

Proposition 1: (a) $t_{ij}+t_{ji}=1$, for all $i, j \in M$; and (b) $0<t_{ij}<1$, for all $i, j \in M$.

Proof: Using the definition of t_{ij} in (3), it follows that $t_{ij}+t_{ji}=\Sigma_k w_k/(1+a_{jik})+\Sigma_k w_k/(1+a_{ijk})=\Sigma_k w_k[1/(1+1/a_{ijk})+1/(1+a_{ijk})]=\Sigma_k w_k[a_{ijk}/(1+a_{ijk})+1/(1+a_{ijk})]=\Sigma_k w_k=1$, for all $i, j \in M$, which proves part (a). The range of a_{ijk} is restricted to $1/9 \le a_{ijk} \le 9$, so that $a_{ijk}>0$, for all i, j, k. Moreover, $w_k \ge 0$ for each $k \in L$, and there exists at least one $h \in L$ such that $w_h>0$, since $\Sigma_h w_h=1$. Therefore, it follows from (3) that $t_{ij}>0$, for all $i, j \in M$, which establishes the required lower bound. As $t_{ji}>0$, it follows from part (a) of this proposition that $t_{ij}=(1-t_{ji})<1$, for all $i, j \in M$, which establishes the upper bound and completes the proof.

From (3) we see that each component $a_{iik}/(a_{iik}+a_{jik})$ of t_{ij} is weighted by the corresponding criterion weight w_k. As the a_{ijk} represent relative preference intensities, it is indeed meaningful to combine the weighted preferences into a single measure t_{ij}, as in (3). The value of t_{ij} may be viewed as a measure of the overall accumulation of evidence, across all criteria in C, considering the comparisons involving both A_i and A_j only, in support of the conjecture that $A_i >_C A_j$. Similarly, t_{ji} is a measure of the cumulative evidence that $A_j >_C A_i$. Assuming the validity of the fundamental ratio scale in the CAHP, a value $a_{ijk}>1$, indicating that $A >_{\{C_k\}} A_j$, contributes more to t_{ij} than to t_{ji}. For instance, if $a_{ijk}=2$, then the contribution of this term to t_{ij} is $.667w_k$, while the contribution to t_{ji} is $.333w_k$. A value $t_{ij}=.5$ would indicate that there is an equal degree of overall evidence in favor of either hypothesis, based on the pairwise comparisons of A_i and A_j.

The ratio p_{ij} in (4) explicitly combines both measures of relative preference, t_{ij} and t_{ji}, in one composite measurement:

$$p_{ij} = \frac{t_{ij}}{t_{ji}}, \quad i, j \in M. \tag{4}$$

Since $0<t_{ij}+t_{ji}=1$, the value of p_{ij} may be viewed as the balance of evidence, contained in **C** and those pairwise judgments in $\mathbf{A}(C_k)$, $k \in L$, which pertain to A_i and A_j only, in support of the hypothesis that $A_i >_C A_j$, taking into account both the degree of relative pairwise preference of A_i over A_j, t_{ij}, and that of A_j over A_i, t_{ji}. A p_{ij} value of 1 would indicate that $A_i \sim_C A_j$

Clearly, p_{ij} is well-defined for any t_{ij}, as $0<t_{ij}$, $t_{ji}<1$, for all $i, j \in M$. The p_{ij} are represented succinctly in the matrix $\mathbf{P}=\{p_{ij}\}$:

$$\mathbf{P} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} \begin{bmatrix} A_1 & A_2 & \cdots & A_m \\ p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}. \tag{5}$$

Proposition 2 shows that, like $\mathbf{A}(C_k)$, **P** is reciprocal. Proposition 3 establishes that p_{ij} depends only on the relative preferences of alternatives A_i and A_j, and not in any way on the preference information related to other pairs of alternatives.

Proposition 2: The matrix **P** is reciprocal.

Proof: From (4) it is immediately clear that $p_{ji}=t_{ji}/t_{ij}=1/p_{ij}$, for all $i, j \in M$, so that **P** is indeed reciprocal.

Proposition 3: The elements p_{ij} and p_{pq} of **P** are independent if and only if $i \neq q$ and $j \neq r$.

Proof: The importance weights of the criteria, w_k, are based solely on **C** and do not involve information about the individual alternatives. Moreover, from (3) and (4) we see that t_{ij} and p_{ij} are determined as a function of a_{iik}, a_{ijk} and w_k, $i, j \in M$; $k \in L$. Thus, p_{ij} is obtained by combining the pairwise judgments of alternatives A_i and A_j only, and is independent of the pairwise comparisons a_{qrk}, $i \neq q, j \neq r$, with or among the remaining alternatives. Conversely, p_{ij} and p_{qr} have several components in common if $i=q$ and/or $j=r$, and hence are not independent.

Proposition 3 formalizes an attractive property of p_{ij} as an overall measure of relative preference, namely that it is not affected by the pairwise judgments on any alternative A_s, $s \neq i, j$. Later in this paper we will see that, under certain mild conditions, the final rankings of the existing alternatives are not affected if a new alternative is added or if an existing one is removed from the problem. Due to the construction of its elements as a ratio of t_{ij} and t_{ji}, **P** provides an interpretation of the aggregate pairwise comparisons, thus contributing potentially valuable information about the decision maker's relative preference structure. Note that the independence property in Proposition 3 does not continue to hold if **C** is evaluated using the Referenced AHP method, since in that case the average (total) criterion values, and therefore **w**, depend on all alternatives.

The final step in PAHAP is to synthesize the aggregate weighted preferences intensities in **P**, to determine the final alternative ratings $\mathbf{s}=(s_1, \ldots, s_m)$, where s_i is the final rating of alternative A_i. As the criterion weights have already been accounted for in the t_{ij}, **s** is determined directly by synthesizing **P**. A synthesis based on the arithmetic mean approach that is also used in the CAHP involves adding all normalized values in row i, and dividing this expression by the number of alternatives:

$$s_i = \frac{\sum_{h=1}^{m} \left(\frac{p_{ih}}{\sum_{j=1}^{m} p_{jh}} \right)}{m}, \, i \in M. \tag{6}$$

It can easily be verified that the s_i in (6) sum to one, as $\Sigma_i s_i = [\Sigma_i \{\Sigma_h (p_{ih} / \Sigma_j p_{ijh})\}]/m = [\Sigma_h \{\Sigma_i p_{ih} / \Sigma_j p_{jh}\}]/m = m/m = 1$. It is also possible to synthesize **P** using the geometric mean method. Unless indicated otherwise, we will use the arithmetic mean synthesis method.

The synthesis of **P** takes account of the potential inconsistencies between the p_{ij} values. In other words, it reconciles the discrepancies among the aggregate pairwise preference measures in **P**. Like the CAHP ratings r_i, PAHAP ratings s_i can be used to rank the alternatives, such that $A_i >_C A_j$ iff $s_i > s_j$, and $A_i \sim_C A_j$ iff $s_i = s_j$. Of course it is possible, and perhaps desirable in practice as the pairwise judgments may be imprecise, to relax the interpretation of the preference ratings, by inferring that alternatives are "equally preferred" as long as their ratings do not differ by more than a pre-specified (usually relatively small) amount. The steps of PAHAP procedure can be summarized as follows:

Step 1: Determine **C** and $\mathbf{A}(C_k)$, for each $k \in L$, in the same way as in the CAHP (or, alternatively, as in the Referenced AHP).

Step 2: Synthesize **C**, resulting in the criterion weight vector **w**.

Step 3: Use $\mathbf{A}(C_k)$, $k=1, \ldots, n$, and **w** to calculate t_{ij} according to equation (3), for all $i, j \in M$.

Step 4: Use (4) and (5) to construct **P**.

Step 5: Synthesize **P**, yielding the overall alternative ratings, $s_1, \ldots, s_m$.

It is also worth noting that in the presence of only two alternatives A_1 and A_2, t_{12} coincides with the final priority rating of alternative A_1 in the CAHP, and the CAHP and PAHAP will always yield the same alternative ratings.

PAHAP: Direct Rating (Absolute Measurement)

Like the CAHP, PAHAP can also be applied if the alternatives are directly rated in terms of each criterion on an absolute scale, rather than in terms of relative measurement. Suppose that the absolute preference rating for $A_i \in A = \{A1, \ldots, Am\}$ with respect to $C_k \in C = \{C1, \ldots, Cn\}$ is given by a'_{ik}, let $\mathbf{A}' = \{a'_{ik}\}$ and let w_k be the relative importance weight of criterion C_k. The a'_{ik} are used to derive t'_{ij}:

$$t'_{ij} = \sum_{k=1}^{n} \left(\frac{a'_{ik}}{a'_{ik} + a'_{jk}} \right) w_k. \tag{7}$$

It is easy to see that t'_{ij} in (7) and t_{ij} in (3) are equivalent if $a_{ijk}=a'_{ik}/a'_{jk}$, that is, if the pairwise judgments correspond exactly to the ratio of the direct ratings. The interpretation of t'_{ij} as a measure of the aggregate preference intensity contributing to the conjecture that $A_i >_C A_j$, based on all pairwise judgments involving this pair of alternatives, is analogous to that of t_{ij}. We can establish matrix $\mathbf{P}'=\{p_{ij}\}$ from the t'_{ij}, as in (8):

$$p'_{ij} = \frac{t'_{ij}}{t'_{ji}}. \tag{8}$$

The characteristics and interpretation of $\mathbf{P}'$, as well as the synthesis of $\mathbf{P}'$ to obtain the final alternative ratings $\mathbf{s}'=(s_1, \ldots, s_m)$, are exactly the same as in the case of relative measurement. Thus, the final ratings for absolute measurements are defined by equation (6), where p_{ij} is replaced by p'_{ij} and s_i by s'_i.

PAHAP: EXAMPLE PROBLEM

We use the following two-criterion, three-alternative problem, previously discussed by [14], to illustrate PAHAP. Let $C=\{C_1, C_2\}$ and $A=\{A_1, A_2, A_3\}$. Suppose the pairwise comparisons are given by:

$$\mathbf{A}(C_1) = \begin{array}{c|ccc} & A_1 & A_2 & A_3 \\ \hline A_1 & 1 & 3 & 1/2 \\ A_2 & 1/3 & 1 & 1/6 \\ A_3 & 2 & 6 & 1 \end{array} \qquad \mathbf{A}(C_2) = \begin{array}{c|ccc} & A_1 & A_2 & A_3 \\ \hline A_1 & 1 & 1/2 & 4 \\ A_2 & 2 & 1 & 8 \\ A_3 & 1/4 & 1/8 & 1 \end{array}.$$

The preferences in this problem are transitive and $a_{ijk}=a_{iqk}/a_{jqk}$, for all i, j, k, so that $\mathbf{A}(C_1)$ and $\mathbf{A}(C_2)$ are perfectly consistent. Like [14], we assume that C_1 and C_2 have equal weights, that is, $w_1=w_2=.5$.

Using PAHAP, the elements of $\mathbf{P}$ and the final ratings s_i are calculated as follows:

$$p_{12} = \frac{.5 \cdot 1/(1+1/3) + .5 \cdot 1/(1+2)}{1-(.5 \cdot 1/(1+1/3) + .5 \cdot 1/(1+2))} = 1.182;$$

$$p_{13} = \frac{.5 \cdot 1/(1+2) + .5 \cdot 1/(1+1/4)}{1-(.5 \cdot 1/(1+2) + .5 \cdot 1/(1+1/4))} = 1.308;$$

$$p_{23} = \frac{.5 \cdot 1/(1+6) + .5 \cdot 1/(1+1/8)}{1-(.5 \cdot 1/(1+6) + .5 \cdot 1/(1+1/8))} = 1.066;$$

$$P_{21} = \frac{1}{1.182} = .846; \quad P_{31} = \frac{1}{1.308} = .765; \quad P_{32} = \frac{1}{1.066} = .938;$$

$$s_1 = \frac{1}{3}\left(\frac{1}{2.611} + \frac{1.182}{3.120} + \frac{1.308}{3.374}\right) = .383;$$

$$s_2 = \frac{1}{3}\left(\frac{.846}{2.611} + \frac{1}{3.120} + \frac{1.066}{3.374}\right) = .320;$$

$$s_3 = \frac{1}{3}\left(\frac{.765}{2.611} + \frac{.938}{3.120} + \frac{1}{3.374}\right) = .297;$$

The PAHAP results are summarized as follows:

		PAHAP			
P	A_1	A_2	A_3	Final Rating s_i	Rank
A_1	1.000	1.182	1.308	.383	1
A_2	.846	1.000	1.066	.320	2
A_3	.765	.938	1.000	.297	3
Total	2.611	3.120	3.374	1.000	

Hence, the final PAHAP rank ordering is $s_1 > s_2 > s_3$. Saaty [14] shows that the final priority scores generated using the CAHP are: r_1=.304, r_2=.358, and r_3=.338, so that $r_2 > r_3 > r_1$. Saaty [14] in fact rounded these figures to the second decimal, but to maintain consistency throughout our paper, we carry the third digit as well. The CAHP calculations were performed using the Expert Choice 8.0 software package [9]. As mentioned earlier, the CAHP results that we report reflect the distributive synthesis mode. The ideal mode applies to open systems only and will not allow changes in rank [9]. Thus, the PAHAP ratings s_i differ substantially from their counterparts r_i computed using the CAHP. Moreover, there is a sizeable discrepancy between the rankings of alternatives.

Thus, the question arises which method, then, might be more appropriate and lead to the correct solution. Recall from Proposition 3 that p_{ij} is an overall measure of by how much A_i is preferred to A_j, independent of the remaining alternatives. We infer from **P** that $A_1 >_C A_2$ in the absence of A_3, as p_{12}=1.182>1, whereas p_{13}=1.308>1, implying that $A_1 >_C A_3$ in the absence of A_2, and finally $A_2 >_C A_3$ without the alternative A_1, as p_{23}=1.066>1. These results are consistent with the PAHAP rankings for the three-alternative problem, but not with the CAHP rankings. A separate CAHP analysis of each combination of two alternatives will yield ratings which are identical to those obtained using PAHAP. In other words, the CAHP rankings will revert to those of PAHAP when any one of the three alternatives is deleted. Hence, one may argue that PAHAP rankings are more appropriate. On the other hand, if the introduction of the third alternative provides the decision maker with relevant additional information about the problem, the reversal of the CAHP rankings may (or may not) be justified. In this case, the elements of the original matrix should be modified.

In the next section, we will prove that under certain general conditions the PAHAP rankings are not subject to rank reversal when the set of alternatives is modified.

RANK PRESERVATION AND RANK REVERSAL IN PAHAP

A Rank Preservation Theorem for PAHAP

We next introduce the notion of "pairwise normal consistency," in Definition 1.

Definition 1: A decision maker is said to be pairwise normally consistent, with respect to criterion set C, if the elements of $\mathbf{P}$ calculated based on C satisfy the condition in (9):

$$\text{If } p_{ij} > 1, \text{ then } \frac{p_{iq}}{p_{jq}} > 1, \quad \text{for all } q;\ i, j, q \in M. \tag{9}$$

Definition 1 implies that, if the degree of overall evidence t_{ij} that $A_i >_C A_j$, based on the pairwise judgments involving these two alternatives only, exceeds the corresponding evidence t_{ji} in support of the hypothesis that $A_j >_C A_i$, then the pairwise preference ratio p_{iq} of A_i over A_q should be greater than that of A_j over A_q, p_{jq}, for any $q \in M$.

In the presence of ratio scale measurements, the pairwise normal consistency condition is weaker than the condition of perfect pairwise consistency, which would imply $a_{ijk} a_{jqk} = a_{iqk}$ for all i, j, $q \in M$, $k \in L$. In fact, the pairwise normal consistency condition in (9) is similar to the transitivity property of (interval scale) value function representations of preference structures (see, e.g., [5], [29]), and will be satisfied in most decision problems. In the case of direct ratings, $p_{ij} = p_{iq}/p_{jq}$, always holds, so that for absolute measurements condition (9) is always satisfied, and the decision maker's preference information is always pairwise normally consistent.

Proposition 4 shows that in the presence of pairwise normal consistency, and the arithmetic mean method is used to synthesize $\mathbf{P}$, it is not difficult to prove that PAHAP is not subject to rank reversal.

Proposition 4: If the decision maker is pairwise normally consistent, and the arithmetic synthesis method in (6) is used to synthesize P, the rank order of the alternatives obtained using PAHAP will remain unchanged if a new alternative is added to or if an existing alternative is removed from the set of alternatives.

Proof: Suppose that we initially have a set of m alternatives, $A = \{A_1, \ldots, A_m\}$, and without loss of generality let $s_q > s_r$, for some q, $r \in M$. Using (6), $s_q > s_r$ yields (10):

$$\frac{\sum_{j=1}^{m} \left(\frac{p_{qj}}{\sum_{i=1}^{m} p_{ij}} \right)}{m} > \frac{\sum_{j=1}^{m} \left(\frac{p_{rj}}{\sum_{i=1}^{m} p_{ij}} \right)}{m}, \tag{10}$$

which in turn implies (11):

$$\sum_{j=1}^{m} \left(\frac{p_{qj} - p_{rj}}{\sum_{i=1}^{m} p_{ij}} \right) > 0. \tag{11}$$

Suppose that a new alternative, A_{m+1}, is added to the problem, let $A^* = A \cup A_{m+1}$, and $M^* = M \cup m+1$. Define the revised ratings by $\mathbf{s}^* = (s_1^*, \ldots, s_{m+1}^*)$. In Proposition 3, we have shown that p_{ij} is independent of p_{qr}, for any q, $r \in M$, $q \neq i$ and $r \neq j$, so that the introduction of A_{m+1} clearly does not affect any p_{ij}, $i, j \neq m+1$. After introducing A_{m+1}, we have (12):

$$s_q^* - s_r^* = \sum_{j=1}^{m+1} \left(\frac{p_{qj} - p_{rj}}{\sum_{i=1}^{m+1} p_{ij}} \right) = \sum_{j=1}^{m} \left(\frac{p_{qj} - p_{rj}}{\sum_{i=1}^{m+1} p_{ij}} \right) + \left(\frac{p_{q,m+1} - p_{r,m+1}}{\sum_{i=1}^{m+1} p_{ij}} \right). \tag{12}$$

From (11) it follows that $\Sigma_{j=1}^{m}(p_{qj}-p_{rj})>0$, and the first term of right-hand side of (12) is strictly positive. Similarly, $p_{q,m+1}-p_{r,m+1}>0$, due to the pairwise normal consistency, and the second term of the right-hand side of (12) is also strictly positive, so that the left-hand side of (12) is strictly positive as well, and the revised final ratings s_q^* and s_r^* satisfy $s_q^*>s_r^*$. Since this argument is true for any q, $r \in M$, $q \neq r$, we have shown that the preference ranking of the existing alternatives is unchanged. The proof of rank preservation when an alternative is deleted from the current set of alternatives is similar.

Note that, as a special case, Proposition 4 implies that the rank order of alternatives is preserved when an exact copy of an existing alternative is added to the current problem, provided that the decision maker is pairwise normally consistent. Of course, the alternative ranking is also unaffected by the introduction of irrelevant alternatives.

Rather than using the arithmetic approach, the preference information can also be synthesized using the geometric mean approach [1]. In Proposition 5 we show that the PAHAP method precludes rank reversal if the judgements are normally consistent and the geometric mean approach is used to synthesize **P**.

Proposition 5: If the decision maker is pairwise normally consistent, and the geometric mean approach is used to synthesize **P**, the rank order of the alternatives obtained using PAHAP will remain unchanged if a new alternative is added to or if an existing alternative is removed from the set of alternatives.

Proof: As in Proposition 4, suppose that we initially have a set of m alternatives, $A=\{A_1, \ldots, A_m\}$, and without loss of generality let $s_q>s_r$, for some q, $r \in M$. Using the geometric mean method, $s_q>s_r$ yields (13):

$$\frac{\left(\prod_{j=1}^{m} p_{qj}\right)^{1/m}}{\sum_{i=1}^{m}\left(\prod_{j=1}^{m} p_{ij}\right)^{1/m}} > \frac{\left(\prod_{j=1}^{m} p_{rj}\right)^{1/m}}{\sum_{i=1}^{m}\left(\prod_{j=1}^{m} p_{ij}\right)^{1/m}}, \tag{13}$$

which in turn implies (14):

$$\prod_{j=1}^{m} p_{qj} > \prod_{j=1}^{m} p_{rj}. \tag{14}$$

Suppose that a new alternative, A_{m+1}, is added to the problem, and define A^*, M^* and $\mathbf{s}^*$ as in Proposition 4. From the pairwise normality property it follows that $p_{q,m+1} > p_{r,m+1}$. Thus, $(\Pi_{j=1}^{m} p_{qj}) p_{q,m+1} > \Pi_{j=1}^{m} p_{rj}) p_{r,m+1}$, and $(\Pi_{j=1}^{m+1} p_{qj})^{1/(m+1)} > \Pi_{j=1}^{m+1} p_{rj})^{1/(m+1)}$. Therefore, (15) holds,

$$\frac{\left(\prod_{j=1}^{m+1} p_{qj}\right)^{1/(m+1)}}{\sum_{i=1}^{m+1}\left(\prod_{j=1}^{m+1} p_{ij}\right)^{1/(m+1)}} > \frac{\left(\prod_{j=1}^{m+1} p_{rj}\right)^{1/(m+1)}}{\sum_{i=1}^{m+1}\left(\prod_{j=1}^{m+1} p_{ij}\right)^{1/(m+1)}}, \tag{15}$$

so that $s_q^* > s_r^*$, for any q, $r \in M$, $q \neq r$. Thus, the preference rankings of the existing alternatives is unchanged if an alternative is added. The proof of rank preservation when an alternative is deleted from the current set of alternatives is similar.

We next use an example to illustrate the rank preservation property of PAHAP when the decision maker is pairwise normally consistent. In this example, applying the CAHP leads to rank reversal.

Example of Rank Preservation in PAHAP (Arithmetic Mean Method)

Consider the following four-criterion, four-alternative example problem with absolute measurement (i.e., direct ratings), previously presented by [6] and [8]:

	C_1	C_2	C_3	C_4
A_1	1	9	1	3
A_2	9	1	9	1
A_3	8	1	4	5
A_4	4	1	8	5.

We assume throughout that the criteria are weighted equally, that is, w_i=.25, for all $i \in L\{1, \ldots, 4\}$. Let us first consider alternatives A_1, A_2, and A_3 only. The final ratings r_i' and s_i' and rankings for this problem determined by the CAHP (see [6]) and PAHAP (using the arithmetic mean method) are as follows:

	CAHP				Final Rating	
	C_1	C_2	C_3	C_4	r_i'	Rank
A_1	1/18	9/11	1/14	3/9	.320	3
A_2	9/18	1/11	9/14	1/9	.336	2
A_3	8/18	1/11	4/14	5/9	.344	1

	PAHAP			Final Rating	
	A_1	A_2	A_3	s_i'	Rank
A_1	1.00	.86	.66	.273	3
A_2	1.16	1.00	.89	.334	2
A_3	1.52	1.12	1.00	.393	1

Note that the final ranking, $r_3' > r_2' > r_1'$ and $s_3' > s_2' > s_1'$, is the same for both methods. After adding the fourth alternative, A_4, the revised ratings $r_i^{*\prime}$ and $s_i^{*\prime}$ and associated rankings are as follows:

	CAHP				Final Rating	
	C_1	C_2	C_3	C_4	$r_i^{*\prime}$	Rank
A_1	1/22	9/12	1/22	3/14	.264	1
A_2	9/22	1/12	9/22	1/14	.243	4
A_3	8/22	1/12	4/22	5/14	.246	2
A_4	4/22	1/12	8/22	5/14	.246	2

	PAHAP				Final Rating	
	A_1	A_2	A_3	A_4	$s_i^{*\prime}$	Rank
A_1	1.00	.86	.66	.66	.193	3
A_2	1.16	1.00	.89	.89	.242	2
A_3	1.52	1.12	1.00	1.00	.282	1
A_4	1.52	1.12	1.00	1.00	.282	1

As we have absolute measurements in this example, the decision maker's evaluations are pairwise normally consistent, and the revised PAHAP ranking of $s_3^{*\prime}=s_4^{*\prime}>s_2^{*\prime}>s_1^{*\prime}$ indeed fully preserves the ranking in the original problem. However, the revised CAHP rank order, $r_1^{*\prime}>r_3^{*\prime}=r_4^{*\prime}>r_2^{*\prime}$, is substantially different from the original CAHP ranking. For example, A_1, which in the three-alternative model ranked last, becomes the highest ranking alternative once A_4 has been added to the problem.

Of course, we reiterate that the property of PAHAP that under conditions of pairwise normal consistency previous rankings are always maintained when altering the set of alternatives may not always be desirable, even if the decision maker is indeed pairwise normally consistent. We believe that the choice of appropriate methodology depends on the nature of the particular decision problem at hand, and in particular on whether adding the new alternative offers additional information relevant for solving the problem as a whole.

Rank Reversal in PAHAP

In this section we will see that, while pairwise normal consistency is a sufficient condition for rank preservation in the PAHAP, rank reversal is possible if the decision maker is not pairwise normally consistent. Consider the following one-criterion, three-alternative problem, with matrix **A** of pairwise judgments and the associated final ratings obtained using the CAHP and PAHAP. Due to the extreme inconsistency, the CAHP results for this one-criterion problem calculated using Expert Choice 8.0 [9] differ from the PAHAP results, perhaps due to the fact that Expert Choice uses the power method rather than the normalized matrix method.

A	A_1	A_2	A_3	CAHP Final Rating r_i	PAHAP Final Rating s_i
A_1	1	3	1/9	.221	.279
A_2	1/3	1	9	.460	.389
A_3	9	1/9	1	.319	.332

It is obvious that the entries of **A** are not pairwise normally consistent. From the above table, we see that the CAHP ranking of $r_2>r_3>r_1$, and the PAHAP ranking of $s_2>s_3>s_1$ are the same. Now, suppose we remove A_3 from the set of alternatives. Since $a_{12}=3>1$, the revised CAHP and PAHAP ratings are $r_1^{*'}=s_1^{*'}=.750$ and $r_2^{*'}=s_2^{*'}=.250$, so that the ranking of A_1 and A_2 has reversed in both methods. Of course, rank reversal would also have occurred in both the CAHP and PAHAP if we would have started with alternatives A_1 and A_2, and added A_3.

Saaty recommends that one always check the validity of the model and the pairwise judgments. This is particularly important if there is substantial conflict between the judgments. To this purpose, he proposes the use of the inconsistency ratio [16], a measure of the degree of conflict among judgments within the CAHP model which should—as a rule of thumb—not exceed .1. In the case of the matrix **A** above, the inconsistency ratio of 4.204 far exceeds .1, and the the decision maker is recommended to validate the model structure and inputs. We do note, however, that if even after careful consideration the decision maker remains confident with his/her (highly inconsistent) judgments, so that the occurrence of inconsistency cannot be eliminated, one can argue that rank reversal may in fact be legitimate, in view of the preference structure. In such cases, the use of either the CAHP or PAHAP may be justified. Alternatively, one could argue that, due to the large extent of the inconsistency, the AHP modeling philosophy as used by both CAHP and PAHAP may be inappropriate for solving the decision problem at hand, as in such a situation the addition or deletion of an alternative may change the problem structure completely.

CONCLUSIONS AND IMPLICATIONS FOR DECISION MAKERS

We introduce a novel ratio-scale based method, PAHAP, for solving discrete alternative multicriteria decision problems. PAHAP preserves its ranking under mild consistency conditions (normal consistency), which apply to the majority of decision problems. The solution procedure does not require any further information from the decision maker beyond the CAHP, does not assume an open or closed system structure, and is easier to implement than previously proposed modified AHP methods developed to solve the rank reversal problem.

The methodology of pairwise aggregating weighted preference intensities across criteria, followed by a final synthesis, appears a logical way of representing the overall alternative ratings. Furthermore, the intermediate product, the **P** matrix, may provide the manager with a detailed insight into the decision problem at hand. In fact, this matrix is a summary of the pairwise comparisons, explains how the final ratings are derived, and can be helpful if the decision maker wishes to take a closer look at his/her pairwise preferences.

Therefore, the implication for decision makers is that the PAHAP ratings offer a reasonable alternative to the CAHP for analyzing discrete alternative multicriteria decision problems, yielding plausible and robust rankings of the alternatives. Moreover, the pairwise aggregated preference measurements provide detailed, potentially useful information about the decision maker's overall pairwise preference structure not revealed by the CAHP. [Received: October 18, 1993. Accepted: June 16, 1994.]

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