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Pairwise Soft Connected in Soft Bitopological Spaces

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ABSTRACT
In this paper, we introduce the notion of p -separated soft sets based on the soft space (X,η_{12},E) which generate by soft bitopological space (X,η_1,η_2,E) and study some of its properties. Based on this notion we introduce the notions of p -soft connected(disconnected) spaces and study some of their characterizations and properties. Also, we study the connected of p -soft sets by using the soft space (X,η_{12},E). Some examples have given to support these concepts.

Keywords
Soft set; Soft topology; Soft bitopological spaces; p-separated sets; p -separated soft sets; p -soft connected spaces; p -soft disconnected spaces; p -connected soft set and pairwise soft continuous.

1. INTRODUCTION
Molodtsov [19] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory to several directions such as smoothness of functions, game theory, Riemann Integration, and theory of measurement. In recent years, development in the fields of soft set theory and its application has been taking place in a rapid pace. This is because of the general nature of parameterized expressed by a soft set. Shabir and Naz [26] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. I. Zorlutuna et al. [28] studied some new properties of soft continuous mappings and gave some new characterizations of sot continuous, soft open, soft closed mappings and also soft homeomorphisms. In 2012, E. Peyghan, B. Samadi and A. Tayebi [?] introduced and explore the notions of soft connectedness in soft topological spaces. Also, in 2014, M. Al-Khafaj and M. Mahmood [1] introduced and studied some properties of soft connected spaces and soft locally connected spaces. Ittanagi [10] introduced the concept of soft bitopological space and studied some types of soft separation axiom for soft bitopological spaces from his point of view. Kandil et al. [[13],[16]]introduced some structures of soft bitopological spaces. Recently, Kandil et al. [15] introduced some types of pairwise soft open (continuous) mappings and some related results.

The purpose of this article is to introduce and study the concept of soft connectedness in soft bitopological spaces. We study the concepts of pairwise separated soft sets, pairwise soft connected (disconnected) spaces and pairwise connected soft sets. The rest of this paper is organized as follows. In section (2) we introduced briefly the notions of soft set, soft topology, soft bitopological spaces, soft mapping and some related topics. In Section (3), we introduce the notion of pairwise separated soft sets and give some characterizations of these soft sets. In Section (3), we introduce the notions of pairwise soft connected (disconnected) spaces and investigate some of their properties. In section (5), we give the concept of p -connected soft sets and some related properties are studied. The last section summarizes the conclusions.

2. 2. PRELIMINARIES
In this section, we briefly review some concepts and some related results of soft set, soft topological space and soft bitopological space which are needed to used in current paper. For more details about these concepts you can see [[10], [5], [6], [7], [8], [9], [11], [13], [14], [19], [20], [21], [25], [26], [3], [27]].

Let X be an initial universe, E be a set of parameters and $P(X)$ be the power set of X.

Definition 2.1 *[21] A pair (F,E) is called a soft set over X, where F is a mapping given by $F:E\to P(X)$. A soft set can also be defined by the set of ordered pairs*

$$(F,E)=\{(e,F(e)):e\in E,\ F:E\to P(X)\}.$$

From now on, $SS(X)_E$ denotes the family of all soft sets over X with a fixed set of parameters E.

For two soft sets $(F,E),(G,E)\in SS(X)_E$, (F,E) is called a soft subset of (G,E), denoted by $(F,E)\tilde{\subseteq}(G,E)$, if $F(e)\subseteq G(e)$, $\forall e\in E$. In this case, (G,E) is called a soft superset of (F,E). In addition, the union of soft sets (F,E) and (G,E), denoted by $(F,E)\tilde{\cup}(G,E)$, is the soft set (H,E) which defined as $H(e)=F(e)\cup G(e)$, $\forall e\in E$. Moreover, the intersection of soft sets (F,E) and (G,E)

, denoted by $(F,E)\tilde{\cap}(G,E)$, is the soft set (M,E) which defined as $\forall e \in E$. The complement of a soft set (F,E), denoted by $(F,E)^c$, is defined as $(F,E)^c = (F^c,E)$, where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X \setminus F(e)$, $\forall e \in E$. The difference of soft sets (F,E) and (G,E), denoted by $(F,E)\setminus(G,E)$, is the soft set (H,E), which defined as $H(e) = F(e)\setminus G(e)$, $\forall e \in E$. Clearly, $(F,E)\setminus(G,E) = (F,E)\tilde{\cap}(G,E)^c$. A soft set (F,E) is called a null soft set, denoted by $(\tilde{\phi},E)$, if $F(e)=\phi$, $\forall e \in E$. Moreover, a soft set (F,E) is called an absolute soft set, denoted by $(\tilde{X},E)$, if $F(e) = X$, $\forall e \in E$. Clearly, we have $(\tilde{\phi},E)^c = (\tilde{X},E)$ and $(\tilde{X},E)^c = (\tilde{\phi},E)$. Moreover, a soft set (G,E) is said to be a finite soft set if $G(e)$ is a finite set for all $e \in E$. Otherwise, it is called an infinite soft set. A soft set (G,E) over X is called a proper soft subset of $(\tilde{X},E)$ if $(G, E) \subsetneq (\tilde{X}, E)$.

Definition 2.2 („,) *A soft set $(F,E) \in SS(X)_E$ is said to be a soft point in $(\tilde{X},E)$ if there exist $x \in X$ and $e \in E$ such that $F(e)=\{x\}$ and $F(e')=\phi$ for each $e' \in E\setminus\{e\}$. This soft point is denoted by (x_e,E) or x_e, i.e.,*

$x_e : E \to P(X)$ is a mapping defined as

$$x_e(a) = \begin{cases} \{x\} & if \quad e = a, \\ \phi & if \quad e \neq a \end{cases} \qquad \text{for all } a \in E.$$

A soft point (x_e,E) is said to be belonging to the soft set (G,E), denoted by $x_e \tilde{\in} (G,E)$, if $x_e(e) \subseteq G(e)$, i.e., $\{x\} \subseteq G(e)$. Clearly, $x_e \tilde{\in} (G,E)$ if and only if $(x_e,E) \tilde{\subseteq} (G,E)$. In addition, two soft points x_{e_1}, y_{e_2} over X are said to be equal if $x = y$ and $e_1 = e_2$. Thus, $x_{e_1} \neq y_{e_2}$ iff $x \neq y$ or $e_1 \neq e_2$.

The family of all soft points in $(\tilde{X},E)$ is denoted by $\xi(X)_E$.

Proposition 2.1 *[3] The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as a union of all soft points belonging to it, i.e.,* $(G,E) = \bigcup\{(x_e,E) : x_e \tilde{\in} (G,E)\}$.

Proposition 2.2 *[3] Let (G,E), (H,E) be two soft sets over X. Then,*

(1) $x_e \tilde{\in} (G,E) \Leftrightarrow x_e \tilde{\notin} (G,E)^c$.

(2) $x_e \tilde{\in} (G,E)\tilde{\cup}(H,E) \Leftrightarrow x_e \tilde{\in} (G,E)$ or $x_e \tilde{\in} (H,E)$.

(3) $x_e \tilde{\in} (G,E)\tilde{\cap}(H,E) \Leftrightarrow x_e \tilde{\in} (G,E)$ and $x_e \tilde{\in} (H,E)$.

(4) $(G,E) \tilde{\subseteq} (H,E) \Leftrightarrow [x_e \tilde{\in} (G,E) \Rightarrow x_e \tilde{\in} (H,E)]$.

For more details for soft point you can see [[20],[3],[22]].

Definition 2.3 *[26], [23] Let η be a collection of soft sets over a universe X with a fixed set of parameters E, i.e., $\eta \subseteq SS(X)_E$. The collection η is called a soft topology on X if it satisfies the following axioms:*

(1) $(\tilde{X},E),(\tilde{\phi},E) \in \eta$,

(2) The union of any number of soft sets in η belongs to η,

(3) The intersection of any two soft sets in η belongs to η.

The triple (X,η,E) is called a soft topological space. Any member of η is said to be an open soft set in (X,η,E). A soft set (F,E) over X is said to be a closed soft set in (X,η,E), if its complement $(F,E)^c$ is an open soft set in (X,η,E). We denote the family of all closed soft sets by η^c.

Definition 2.4 *[26] Let (X,η,E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft closure of (F,E), denoted by $scl_\eta(F,E)$, is defined by* $scl_\eta(F,E) = \bigcap\{(H,E) \in \eta^c : (F,E) \tilde{\subseteq} (H,E)\})$.

Definition 2.5 *[1] The two soft sets $(A,E),(B,E)$ in a soft topological space (X,η,E) are called separated soft sets if* $scl_\eta(A,E)\tilde{\cap}(B,E) = scl_\eta(B,E)\tilde{\cap}(A,E) = (\tilde{\phi},E)$

.**Definition 2.6** *[1] A soft topological space (X,η,E) is called soft connected space if X cannot be expressed as a union of two separated soft sets.*

$scl_\eta(A,E)\widetilde{\cap}(B,E)=scl_\eta(B,E)\widetilde{\cap}(A,E)=(\widetilde{\phi},E)$

.**Definition 2.7** *[1] A soft set* (G,E) *in a soft topological space* (X,η,E) *is called a disconnected soft set if there exist two separated soft sets* $(A,E),(B,E)$ *such that* $(G,E)=(A,E)\widetilde{\cup}(B,E)$. *Otherwise,* (G,E) *is called a connected soft set.*

Definition 2.8 *[17],[27] Let* $SS(X)_E$ *and* $SS(Y)_K$ *be two families of soft sets. Let* $u:X\to Y$ *and* $p:E\to K$ *be mappings. We define a soft mapping* $f_{pu}:SS(X)_E\to SS(Y)_K$ *as:*

(1) If $(G,E)\in SS(X)_E$. Then, the image of (G,E) under f_{pu}, written as $f_{pu}(G,E)=(f_{pu}(G),p(E))$, is a soft set in $SS(Y)_K$ such that

$$f_{pu}(G,E)(k)=\begin{cases}\bigcup_{e\in p^{-1}(k)} u[G(e)] & if \quad p^{-1}(k)\neq\phi,\\ \phi & if \quad p^{-1}(k)=\phi\end{cases}$$

for all $k\in K$.

(2) If $(H,K)\in SS(Y)_K$. Then, the inverse image of (H,K) under f_{pu}, written as $f_{pu}^{-1}(H,K)=(f_{pu}^{-1}(H),E)$, is a soft set in $SS(X)_E$ such that

$$f_{pu}^{-1}(H,K)(e)=u^{-1}[H(p(e))]\ \ \forall\, e\in E.$$

Theorem 2.1 *[17] Let* $SS(X)_E$ *and* $SS(Y)_K$ *be two families of soft sets. For a soft mapping* $f_{pu}:SS(X)_E\to SS(Y)_K$ *we have the following:*

(1) $f_{pu}(\widetilde{\phi},E)=(\widetilde{\phi},K)$ and $f_{pu}(\widetilde{X},E)\widetilde{\subseteq}(\widetilde{Y},K)$.

(2) $f_{pu}[\bigcup_{i\in I}(G_i,E)]=\bigcup_{i\in I}f_{pu}(G_i,E)$ and $f_{pu}[\bigcap_{i\in I}(G_i,E)]\widetilde{\subseteq}\bigcap_{i\in I}f_{pu}(G_i,E)$.

(3) $(G,E)\widetilde{\subseteq}(M,E)\Rightarrow f_{pu}(G,E)\widetilde{\subseteq}f_{pu}(M,E)$

. **Theorem 2.2** *[17],[18] Let* $SS(X)_E$ *and* $SS(Y)_K$ *be two families of soft sets. For a soft mapping* $f_{pu}:SS(X)_E\to SS(Y)_K$ *we have the following:*

(1) $f_{pu}^{-1}(\widetilde{\phi},K)=(\widetilde{\phi},E)$ and $f_{pu}^{-1}(\widetilde{Y},K)=(\widetilde{X},E)$.

(2) $f_{pu}^{-1}[\bigcup_{i\in I}(H_i,K)]=\bigcup_{i\in I}f_{pu}^{-1}(H_i,K)$ and $f_{pu}^{-1}[\bigcap_{i\in I}(H_i,K)]=\bigcap_{i\in I}f_{pu}^{-1}(H_i,K)$.

(3) $(H,K)\widetilde{\subseteq}(N,K)\Rightarrow f_{pu}^{-1}(H,K)\widetilde{\subseteq}f_{pu}^{-1}(N,K)$

. **Definition 2.9** *[27] Let* $SS(X)_E$ *and* $SS(Y)_K$ *be two families of soft sets. A soft mapping* $f_{pu}:SS(X)_E\to SS(Y)_K$ *is called soft surjective(soft injective) mapping if* u, p *are surjective (injective) mappings, respectively. A soft mapping which is a soft surjective and soft injective mapping is called a soft bijection mapping.*

Theorem 2.3 *[27] Let* $SS(X)_E$ *and* $SS(Y)_K$ *be two families of soft sets. For a soft mapping* $f_{pu}:SS(X)_E\to SS(Y)_K$ *we have the following:*

(1) $f_{pu}^{-1}[(\widetilde{Y},K)\setminus(H,K)]=(\widetilde{X},E)\setminus[f_{pu}^{-1}(H,K)]$ for any $(H,K)\in SS(Y)_K$.

(2) $(G,E)\widetilde{\subseteq}f_{pu}^{-1}[f_{pu}(G,E)]$ for any $(G,E)\in SS(X)_E$. If f_{pu} is a soft injective, the equality holds.

(3) $f_{pu}[f_{pu}^{-1}(H,K)]\widetilde{\subseteq}(H,K)$ for any $(H,K)\in SS(Y)_K$. If f_{pu} is a soft surjective, the equality holds.

Definition 2.10 *[28] Let* (X,η,E) *and* (Y,σ,K) *be two soft topological spaces. A soft mapping* $f_{pu}:(X,\eta,E)\to(Y,\sigma,K)$ *is said to be a soft continuous if the inverse image of any open soft set in* (Y,σ,K) *is an open soft set in* (X,η,E), *i.e.,* $f_{pu}^{-1}(H,K)\in\eta$ *for any* $(H,K)\in\sigma$.

Definition 2.11 *[4] A triple* (X,τ_1,τ_2) *is called a bitopological space [briefly, bts], where* τ_1, τ_2 *are arbitrary topologies on* X. *The collection* τ_{12} *is a supra topology on* X, *where*

$$\tau_{12}=\{G\subseteq X:G=G_1\cup G_2;G_i\in\tau_i,i=1,2\}$$

Definition 2.12 *[4] Let* (X,τ_1,τ_2) *be a bts. The sub* A,B *of* X *are said to be* p *-separated sets in* (X,τ_1,τ_2) *if where* $cl_{\tau_{12}}(A)=\{F\in\tau_{12}^c:A\subseteq F\}$.

Definition 2.13 *[4] A bts* (X,τ_1,τ_2) *is called* p *-disconnected if* X *can be expressed as a union of two* *nonempty p -separated sets. Otherwise, (X,τ_1,τ_2) is called p -connected.*

Definition 2.14 *[10] A quadrable system (X,η_1,η_2,E) is called a soft bitopological space [briefly, sbts], where η_1, η_2 are arbitrary soft topologies on X with a fixed set of parameters E.*

Definition 2.15 *[13] Let (X,η_1,η_2,E) be a sbts. A soft set (G,E) over X is said to be a pairwise open soft set in (X,η_1,η_2,E) [briefly, p -open soft set] if there exist an open soft set (G_1,E) in η_1 and an open soft set (G_2,E) in η_2 such that $(G,E)=(G_1,E)\tilde{\cup}(G_2,E)$. A soft set (G,E) over X is said to be a pairwise closed soft set in (X,η_1,η_2,E) [briefly, p -closed soft set] if its complement is a p -open soft set in (X,η_1,η_2,E). Clearly, a soft set (F,E) over X is a p -closed soft set in (X,η_1,η_2,E) if there exist an η_1-closed soft set (F_1,E) and an η_2-closed soft set (F_2,E) such that $(F,E)=(F_1,E)\tilde{\cap}(F_2,E)$.*

The family of all p -open (p -closed) soft sets in a sbts (X,η_1,η_2,E) is denoted by η_{12} (η_{12}^c), respectively.

Theorem 2.4 *[13] Let (X,η_1,η_2,E) be a sbts. The family of all p -open soft sets η_{12} is a supra soft topology on X, where*

$\eta_{12}=\{(G_1,E)\tilde{\cup}(G_2,E):(G_i,E)\in\eta_i, i=1,2\}$.

The triple (X,η_{12},E) is the supra soft topological space associated to the sbts (X,η_1,η_2,E).

Lemma 2.1 *[11] For any sbts (X,η_1,η_2,E), we have (X,η_1^e,η_2^e) is an ordinary bitopological space, for all $e\in E$, where $\eta_i^e=\{G(e):(G,E)\in\eta_i\}$, $i=1,2$, and $\eta_{12}^e=\eta_{12}(e)=\{H(e):(H,E)\in\eta_{12}\}$.*

Definition 2.16 *[13] Let (X,η_1,η_2,E) be a sbts and let $(G,E)\in SS(X)_E$. The pairwise soft closure of (G,E), denoted by $scl_{\eta_{12}}(G,E)$, is defined by*

$$scl_{\eta_{12}}(G,E)=\tilde{\bigcap}\{(F,E)\in\eta_{12}^c:(G,E)\tilde{\subseteq}(F,E)\}$$

.Clearly, $scl_{\eta_{12}}(G,E)$ is the smallest p -closed soft set contains (G,E).

Theorem 2.5 *[13] Let (X,η_1,η_2,E) be a sbts and $(G,E)\in SS(X)_E$. Then,*

$$x_e\tilde{\in} scl_{\eta_{12}}(G,E)\Leftrightarrow(O_{x_e},E)\tilde{\cap}(G,E)\neq(\tilde{\phi},E),$$

$\forall\,(O_{x_e},E)\in\eta_{12}(x_e)$ where (O_{x_e},E) is any p-open soft set contains x_e and $\eta_{12}(x_e)$ is the family of all p-open soft sets contains x_e.

For more details about the properties of pairwise soft closure operator see [13].

Definition 2.17 *[13] Let (X,η_1,η_2,E) be a sbts and let $(G,E)\in SS(X)_E$. The pairwise soft kernel of (G,E) [briefly, $sker_{\eta_{12}}(G,E)$], is the intersection of all p -open soft supersets of (G,E), i.e.,*

$$sker_{\eta_{12}}(G,E)=\tilde{\bigcap}\{(H,E)\in\eta_{12}:(G,E)\tilde{\subseteq}(H,E)\}$$

Definition 2.18 *[13] A soft set (G,E) is said to be a pairwise Λ - soft set in a sbts (X,η_1,η_2,E) [briefly, $p\Lambda$ -soft set] if $sker_{\eta_{12}}(G,E)=(G,E)$.*

Theorem 2.6 *[13] Every p -open soft set is a $p\Lambda$ -soft set.*

Theorem 2.7 *[13] Let (X,η_1,η_2,E) be a sbts. Then, the class of all $p\Lambda$ -soft sets is an Alexandroff soft topology on X. This soft topology we denoted by $\eta_{p\Lambda}$. The triple $(X,\eta_{p\Lambda},E)$ is the soft topological space associated to the sbts (X,η_1,η_2,E), induced by the family of all $p\Lambda$ -soft sets.*

Theorem 2.8 *[13] Let (X,η_1,η_2,E) be a sbts. Then,*

$$\eta_1\cup\eta_2\subseteq\eta_{12}\subseteq\eta_{p\Lambda}\subseteq SS(X)_E.$$

Definition 2.19 *[14] A sbts (X,η_1,η_2,E) is said to be a pairwise soft T_1^* [briefly PST_1^*] if for each $x_\alpha, y_\beta\in\xi(X)_E$ with $x_\alpha\neq y_\beta$, there exist $(G,E),(H,E)\in\eta_{12}$ such that $x_\alpha\tilde{\in}(G,E)$, $y_\beta\tilde{\notin}(G,E)$ and $y_\beta\tilde{\in}(H,E)$, $x_\alpha\tilde{\notin}(H,E)$.*

Theorem 2.9 *[11] A sbts (X,η_1,η_2,E) is a PST_1^* iff every soft point either p -open soft set or p -closed soft set.*

Definition 2.20 *[16] Let (X,η_1,η_2,E) and (Y,σ_1,σ_2,K) be two soft bitopological spaces. A soft mapping $f_{pu}:(X,\eta_1,\eta_2,E)\to(Y,\sigma_1,\sigma_2,K)$ is said to be a pairwise soft continuous [briefly, p -soft* *continuous] if the inverse image of any* p *-open soft set in* (Y,σ_1,σ_2,K) *is a* p *-open soft set in* (X,η_1,η_2,E)*, i.e.,* $f_{pu}^{-1}(H,K)\in\eta_{12}$ *for any* $(H,K)\in\sigma_{12}$.

Theorem 2.10 *[13] Let* (X,η_1,η_2,E) *be a sbts and let* $Y\subseteq X$*. Then,* $(Y,\eta_{1Y},\eta_{2Y},E)$ *is also soft bitopological space where*
$\eta_{iY}=\{(\tilde{Y},E)\tilde{\cap}(G_i,E):(G_i,E)\in\eta_i\}$, $i=1,2$
and $\tilde{Y}(e)=Y,\ \forall\, e\in E$. This soft bitopological space we will called soft bitopological subspace of (X,η_1,η_2,E).

Theorem 2.11 *[16] Let* (X,η_1,η_2,E) *and* (Y,σ_1,σ_2,K) *be two soft bitopological spaces and let* $f_{pu}:(X,\eta_1,\eta_2,E)\to(Y,\sigma_1,\sigma_2,K)$ *be a soft mapping. Then, the following statements are equivalent:*

(1) f_{pu} is a p -soft continuous.

(2) $f_{pu}^{-1}(F,K)\in\eta_{12}^c$ for all $(F,K)\in\sigma_{12}^c$.

(3) $f_{pu}[scl_{\eta_{12}}(G,E)]\tilde{\subseteq} scl_{\sigma_{12}}[f_{pu}(G,E)]$ for any $(G,E)\in SS(X)_E$.

(4) $scl_{\eta_{12}}[f_{pu}^{-1}(H,K)]\tilde{\subseteq} f_{pu}^{-1}[scl_{\sigma_{12}}(H,K)]$ for any $(H,K)\in SS(Y)_K$.

(5) $f_{pu}^{-1}[sint_{\sigma_{12}}(H,K)]\tilde{\subseteq} sint_{\eta_{12}}[f_{pu}^{-1}(H,K)]$ for any $(H,K)\in SS(Y)_K$.

3. PAIRWISE SEPARATED SOFT SETS

Definition 3.1 ***Let*** (X,η_1,η_2,E) ***be a sbts and let*** $(G,E),(H,E)$ ***are non-null soft sets over*** X ***. The soft sets*** (G,E) ***and*** (H,E) ***are said to be pairwise separated soft sets [briefly, separated soft sets] if***

$$scl_{\eta_{12}}(G,E)\tilde{\cap}(H,E)=(\tilde{\phi},E),$$
$$(G,E)\tilde{\cap}\, scl_{\eta_{12}}(H,E)=(\tilde{\phi},E).$$

Proposition 3.1 *Let* (X,η_1,η_2,E) *be a sbts. Then, every* p *-separated soft sets are disjoint soft sets.*

Remark 3.1 *The converse of Proposition 3.1 may not be true as shown by the following example.*

Example 3.1 *Let* $X=\{x,y,z,w\}$, $E=\{e_1,e_2\}$ *and let*

$$\eta_1=\{(\tilde{\phi},E),(\tilde{X},E),(G,E),(H,E)\}$$
$$\eta_2=\{(\tilde{\phi},E),(\tilde{X},E),(M,E),(N,E)\}$$
$$(G,E)=\{(e_1,\{x,w\}),(e_2,\{z,w\})\},$$
$$(H,E)=\{(e_1,\{y,z\}),(e_2,\{x,y\})\},$$
$$(M,E)=\{(e_1,\{w\}),(e_2,\{y,w\})\},$$
$$(N,E)=\{(e_1,\{x,w\}),(e_2,X)\}.$$

It is easy to verify that

$\eta_{12}=\{(\tilde{\phi},E),(\tilde{X},E),(G,E),(H,E),(M,E),(N,E),(P_1,E),(P_2,E)\}$ where

$$(P_1,E)=\{(e_1,\{x,w\}),(e_2,\{y,z,w\})\},$$
$$(P_2,E)=\{(e_1,\{y,z,w\}),(e_2,\{x,y,w\})\}.$$

Therefore, the family of all p -closed soft sets is $\eta_{12}^c=\{(\tilde{\phi},E),(\tilde{X},E),(G,E)^c,(H,E)^c,(M,E)^c,(N,E)^c,(P_1,E)^c,(P_2,E)^c\}$

where

$$(G,E)^c=\{(e_1,\{y,z\}),(e_2,\{x,y\})\},$$
$$(H,E)^c=\{(e_1,\{x,w\}),(e_2,\{z,w\})\},$$
$$(M,E)^c=\{(e_1,\{x,y,z\}),(e_2,\{x,z\})\}$$
$$(N,E)^c=\{(e_1,\{y,z\}),(e_2,\phi)\},$$
$$(P_1,E)^c=\{(e_1,\{y,z\}),(e_2,\{x\})\},$$

$(P_2,E)^c=\{(e_1,\{x\}),(e_2,\{z\})\}$. Now, let

$$(F_1,E)=\{(e_1,\{x\}),(e_2,\{x\})\},$$

$(F_2,E)=\{(e_1,\{y\}),(e_2,\{y\})\}$ and

$$(F_3,E)=\{(e_1,\{x\}),(e_2,\{w\})\}.$$

It is clear that (F_2,E) and (F_3,E) are p -separated soft sets. Although the soft sets (F_1,E) and (F_2,E) are disjoint, we find that their are not p -separated soft sets because

$$scl_{\eta_{12}}(F_2,E)\tilde{\cap}(F_1,E)=\{(e_1,\phi),(e_2,\{x\})\}\neq(\tilde{\phi},E)$$

Proposition 3.2 *Let* (X,η_1,η_2,E) *be a sbts and let* $(G,E),(H,E)$ *are non-null soft sets over* X*. If*

$scl_{\eta_{12}}(G,E)\tilde{\cap} scl_{\eta_{12}}(H,E)=(\tilde{\phi},E)$, *then* (G,E) *and* (H,E) *are* p *-separated soft sets.*

Proof. Straightforward.

Note: From Propositions 3.1 and 3.2 we deduce that the concept of p-separated soft sets is a weaker than of the condition of disjoint pairwise soft closure of soft sets, but it is a stronger than of the concept of disjoint soft sets.

Remark 3.2 *The converse of Proposition 3.2 may not be true as shown by the following example.* **Example 3.2** *Let* $X=\{x,y,z\}$, $E=\{e_1,e_2\}$ *and let*

$$\eta_1=\{(\tilde{\phi},E),(\tilde{X},E),(G_1,E),(G_2,E),(G_3,E)\},$$

$$\eta_2=\{(\tilde{\phi},E),(\tilde{X},E),(H_1,E),(H_2,E),(H_3,E)\}$$

such that

$(G_1,E)=\{(e_1,\{x,z\}),(e_2,\{x,y\})\}$,
$(G_2,E)=\{(e_1,\{y,z\}),(e_2,\{y,z\})\}$
$(G_3,E)=\{(e_1,\{z\}),(e_2,\{y\})\}$, and

$(H_1,E)=\{(e_1,\{x,y\}),(e_2,\{x,z\})\}$,
$(H_2,E)=\{(e_1,\{z\}),(e_2,\{x,y\})\}$,
$(H_3,E)=\{(e_1,\phi),(e_2,\{x\})\}$.

It is clear that (X,η_1,η_2,E) is a sbts. Moreover,

$$\eta_{12}=\{(\tilde{\phi},E),(\tilde{X},E),(G_1,E),(G_2,E),(G_3,E),(H_1,E),(H_2,E),(H_3,E),(P,E)\}$$

where

$$(P,E)=\{(e_1,\{y,z\}),(e_2,X)\}.$$

Consequently,

$$\eta_{12}^c=\{(\tilde{\phi},E),(\tilde{X},E),(G_1^c,E),(G_2^c,E),(G_3^c,E),(H_1^c,E),(H_2^c,E),(H_3^c,E),(P^c,E)\}$$

where

$(G_1^c,E)=\{(e_1,\{y\}),(e_2,\{z\})\}$,
$(G_2^c,E)=\{(e_1,\{x\}),(e_2,\{x\})\}$,
$(G_3^c,E)=\{(e_1,\{x,y\}),(e_2,\{x,z\})\}$,
$(H_1^c,E)=\{(e_1,\{z\}),(e_2,\{y\})\}$,
$(H_2^c,E)=\{(e_1,\{x,y\}),(e_2,\{z\})\}$,
$(H_3^c,E)=\{(e_1,X),(e_2,\{y,z\})\}$,
$(P^c,E)=\{(e_1,\{x\}),(e_2,\phi)\}$.

Now, let

$(M,E)=\{(e_1,\{y\}),(e_2,\{y\})\}$ and
$(N,E)=\{(e_1,\phi),(e_2,\{x\})\}$. Then,

$scl_{12}(M,E)=(H_3^c,E)$ and $scl_{12}(N,E)=(G_2^c,E)$. It is clear that (M,E), (N,E) are p-separated soft sets but $scl_{12}(M,E)\tilde{\cap} scl_{12}(N,E)=\{(e_1,\{x\}),(e_2,\phi)\}\neq(\tilde{\phi},E)$.

Proposition 3.3 *Let* (X,η_1,η_2,E) *be a sbts. If* (G,E), (H,E) *are* p *-separated soft sets,* $(M,E)\tilde{\subseteq}(G,E)$ *and* $(N,E)\tilde{\subseteq}(H,E)$*, then* $(M,E),(N,E)$ *are also* p *-separated soft sets.*

Proof. From Theorem 3.11 in [13], we have $scl_{\eta_{12}}(M,E)\tilde{\subseteq} scl_{\eta_{12}}(G,E)$. Since $(N,E)\tilde{\subseteq}(H,E)$, then

$$scl_{\eta_{12}}(M,E)\tilde{\cap}(N,E)\tilde{\subseteq} scl_{\eta_{12}}(G,E)\tilde{\cap}(H,E)=(\tilde{\phi},E)$$

Therefore, $scl_{\eta_{12}}(M,E)\tilde{\cap}(N,E)=(\tilde{\phi},E)$. By similar we can prove that $scl_{\eta_{12}}(N,E)\tilde{\cap}(M,E)=(\tilde{\phi},E)$. Hence, $(M,E),(N,E)$ are p-separated soft sets.

Proposition 3.4 *Let* (X,η_1,η_2,E) *be a sbts and let* (G,E) *and* (H,E) *are* p *-open soft sets. Then,* (G,E) *and* (H,E) *are* p *-separated soft sets if and only if* (G,E) *and* (H,E) *are disjoint.* **Proof.** The first direction is obvious from Proposition 3.1. Now, suppose that (G,E) and (H,E) are disjoint p-open soft sets, then $(G,E)\tilde{\subseteq}(H,E)^c\in\eta_{12}^c$. It follows that $scl_{\eta_{12}}(G,E)\tilde{\subseteq}(H,E)^c$ implies $scl_{\eta_{12}}(G,E)\tilde{\cap}(H,E)=(\tilde{\phi},E)$. By similar way we can show that $scl_{\eta_{12}}(H,E)\tilde{\cap}(G,E)=(\tilde{\phi},E)$. Hence, $(G,E),(H,E)$ are p-separated soft sets.

Proposition 3.5 *Let* (X,η_1,η_2,E) *be a sbts and let* (F,E) *and* (M,E) *are* p *-closed soft sets. Then,* (F,E) *and* (M,E) *are* p *-separated soft sets if and only if* (F,E) *and* (M,E) *are disjoint.*

Proof. Straightforward.

Theorem 3.1 *Let* (X,η_1,η_2,E) *be a* PST_1^* *and let* $(G,E),(H,E)$ *be two finite and disjoint soft sets. Then* (G,E) *and* (H,E) *are* p *-separated soft sets.*

Proof. Since (X,η_1,η_2,E) is a PST_1^*, then every soft point is a p -closed soft set. Therefore, every finite soft set is a p -closed soft set. Since (G,E) and (H,E) are finite soft sets, then (G,E) and (H,E) are p -closed soft sets. It follows by Proposition 3.5 that (G,E) and (H,E) are p -separated soft sets.

Theorem 3.2 *Let* (X,η_1,η_2,E) *be a sbts,* $Y\subseteq X$ *and let* $(G,E),(H,E)\tilde{\subseteq}(\tilde{Y},E)\tilde{\subseteq}(\tilde{X},E)$. *Then,*

if $(G,E),(H,E)$ are p -separated soft sets in (X,η_1,η_2,E), then their are p -separated soft sets in $(Y,\eta_{1Y},\eta_{2Y},E)$ where $\eta_{iY}=\{(\tilde{Y},E)\tilde{\cap}(G,E):(G,E)\in\eta_i\}$, $i=1,2$.

Proof. $scl_{\eta_{12Y}}(G,E)=$

$\bigcap\{(F,E):(F,E)\in\eta_{12Y}^c:(G,E)\tilde{\subseteq}(F,E)\}$
$=\bigcap\{(\tilde{Y},E)\tilde{\cap}(M,E):(M,E)\in\eta_{12}^c,(G,E)\tilde{\subseteq}(\tilde{Y},E)\tilde{\cap}(M,E)\}$
$\tilde{\subseteq}\bigcap\{(\tilde{Y},E)\tilde{\cap}(M,E):(M,E)\in\eta_{12}^c:(G,E)\tilde{\subseteq}(M,E)\}$
$=(\tilde{Y},E)\tilde{\cap}[\bigcap\{(M,E):(M,E)\in\eta_{12}^c:(G,E)\tilde{\subseteq}(M,E)\}]$ $=(\tilde{Y},E)\tilde{\cap}scl_{\eta_{12}}(G,E)$. Hence, $scl_{\eta_{12Y}}(G,E)\tilde{\subseteq}(\tilde{Y},E)\tilde{\cap}scl_{\eta_{12}}(G,E)$. It follows that $scl_{\eta_{12Y}}(G,E)\tilde{\cap}(H,E)\tilde{\subseteq}(\tilde{Y},E)\tilde{\cap}(H,E)\tilde{\cap}scl_{\eta_{12}}(G,E)$. Therefore, $scl_{\eta_{12Y}}(G,E)\tilde{\cap}(H,E)\tilde{\subseteq}(H,E)\tilde{\cap}scl_{\eta_{12}}(G,E)=(\tilde{\phi},E)$. Consequently, $scl_{\eta_{12Y}}(G,E)\tilde{\cap}(H,E)=(\tilde{\phi},E)$. By similar way we can prove that $scl_{\eta_{12Y}}(H,E)\tilde{\cap}(H,E)=(\tilde{\phi},E)$. Hence, $(G,E),(H,E)$ are p -separated soft sets in $(Y,\eta_{1Y},\eta_{2Y},E)$.

Theorem 3.3 *Let* $f_{pu}:(X,\eta_1,\eta_2,E)\to(Y,\sigma_1,\sigma_2,K)$ *be a* p *-soft continuous and soft surjective mapping. If* $(M,K),(N,K)$ *are* p *-separated soft sets in* (Y,σ_1,σ_2,K), *then* $f_{pu}^{-1}(M,K),f_{pu}^{-1}(N,K)$ *are* p *-separated soft sets in* (X,η_1,η_2,E).

Proof. Since $(M,K),(N,K)$ are p -separated soft sets in (Y,σ_1,σ_2,K), then $scl_{\sigma_{12}}(M,K)\tilde{\cap}(N,K)=(\tilde{\phi},K)$ and $(M,K)\tilde{\cap}scl_{\sigma_{12}}(N,K)=(\tilde{\phi},K)$. Since f_{pu} is a p -soft continuous mapping, then we deduce by Theorem 2.11 that $scl_{\eta_{12}}[f_{pu}^{-1}(M,K)]\tilde{\subseteq}f_{pu}^{-1}[scl_{\sigma_{12}}(M,K)]$ which implies that

$f_{pu}^{-1}(N,K)\tilde{\cap}scl_{\eta_{12}}[f_{pu}^{-1}(M,K)]\tilde{\subseteq}$
$f_{pu}^{-1}(N,K)\tilde{\cap}f_{pu}^{-1}[scl_{\sigma_{12}}(M,K)]$

$=f_{pu}^{-1}[(N,K)\tilde{\cap}scl_{\sigma_{12}}(M,K)]$ [by Theorem 2.2 (2)]

$=f_{pu}^{-1}[(\tilde{\phi},K)]$

$=(\tilde{\phi},E)$ [by Theorem 2.2 (1)].

Hence, $f_{pu}^{-1}(N,K)\tilde{\cap}scl_{\eta_{12}}[f_{pu}^{-1}(M,K)]=(\tilde{\phi},E)$ .Similarly, we can prove that $f_{pu}^{-1}(M,K)\tilde{\cap}scl_{\eta_{12}}[f_{pu}^{-1}(N,K)]=(\tilde{\phi},E)$. Since f_{pu} is a soft surjective mapping, then $f_{pu}^{-1}(M,K)\neq(\tilde{\phi},E)$ and $f_{pu}^{-1}(N,K)\neq(\tilde{\phi},E)$. Consequently, $f_{pu}^{-1}(M,K)$, $f_{pu}^{-1}(N,K)$ are p -separated soft sets in (X,η_1,η_2,E).

Theorem 3.4 *Let* (X,η_1,η_2,E) *be a sbts. Then,*

if $(G,E),(H,E)$ are p -separated soft sets in (X,η_1,η_2,E), then $G(e),H(e)$ are p -separated sets in (X,η_1^e,η_2^e), for all $e\in E$.

Proof. For any sbts (X,η_1,η_2,E), $e\in E$ we have $\eta_{12}^e=\eta_{12}(e)=\{G(e):(G,E)\in\eta_{12}\}$ [by Lemma 2.1]. Since $scl_{\eta_{12}}(G,E)=\bigcap\{(F,E)\in\eta_{12}^c:(G,E)\tilde{\subseteq}(F,E)\}$, then $scl_{\eta_{12}}(G,E)(e)=\bigcap\{F(e)\in\eta_{12}^c(e):G(e)\subseteq F(e)\}$

. Therefore, $scl_{\eta_{12}}(G,E)(e)=cl_{\eta_{12}^{e}}G(e)$. Now, since $(G,E),(H,E)$ are p-separated soft sets in (X,η_1,η_2,E), then $scl_{\eta_{12}}(G,E)\tilde{\cap}(H,E)=(\tilde{\phi},E)$ and $scl_{\eta_{12}}(H,E)\tilde{\cap}(G,E)=(\tilde{\phi},E)$ implies $[scl_{\eta_{12}}(G,E)\tilde{\cap}(H,E)](e)=\phi$ and $[scl_{\eta_{12}}(H,E)\tilde{\cap}(G,E)](e)=\phi$. Therefore, $scl_{\eta_{12}}G(e)\cap H(e)=\phi$ and $scl_{\eta_{12}}H(e)\cap G(e)=\phi$. It follows by Definition 2.13 that, $G(e),H(e)$ are p-separated sets in (X,η_1^e,η_2^e).

Theorem 3.5 *Let* (X,η_1,η_2,E) *be a sbts. Then,*

If $(G,E),(H,E)$ are p-separated soft sets in (X,η_1,η_2,E), then their are soft separated sets in $(X,\eta_{p\wedge},E)$.

Proof. It follows from the fact that $scl_{\eta_{p\wedge}}(G,E)\subseteq scl_{\eta_{12}}(G,E),\ \forall\,(G,E)\in SS(X)_E$

4. PAIRWISE SOFT DISCONNECTED (CONNECTED) SPACES

Definition 4.1 *Let* (X,η_1,η_2,E) *be a sbts. A* p*-separated soft sets* $(G,E),(H,E)$ *in* (X,η_1,η_2,E) *are said to be pairwise soft separation of* $(\tilde{X},E)$ *[briefly,* p*-soft separation] if* $(\tilde{X},E)=(G,E)\tilde{\cup}(H,E)$. *In this case we say that* $(\tilde{X},E)$ *has a* p*-soft separation.*

Definition 4.2 *A sbts* (X,η_1,η_2,E) *is said to be a pairwise soft disconnected space [briefly,* p*-soft disconnected] if* $(\tilde{X},E)$ *has a* p*-soft separation. Otherwise,* (X,η_1,η_2,E) *is called pairwise soft connected space [briefly,* p*-soft connected], i.e., a sbts* (X,η_1,η_2,E) *is a* p*-soft connected if* $(\tilde{X},E)$ *cannot be represented as the union of two* p*-separated soft sets.*

Example 4.1 *From Example 3.1,* (X,η_1,η_2,E) *is a* p*-soft disconnected because* (G,E) *and* (H,E) *are form a* p*-soft separation of* (X,E)

where

$(G,E)=\{(e_1,\{x,w\}),(e_2,\{z,w\})\}$,
$(H,E)=\{(e_1,\{y,z\}),(e_2,\{x,y\})\}$.

Also, From Example 3.2, we deduce that (X,η_1,η_2,E) is a p-soft disconnected because (G_3,E) and (H_1,E) are form a p-soft separation of (X,E).

Proposition 4.1 *Let* (X,η_1,η_2,E) *be a sbts. Then,*

(1) If $\eta_1=\eta_2=\{(\tilde{\phi},E),(\tilde{X},E)\}$, then (X,η_1,η_2,E) is a p-soft connected.

(2) If $\eta_{12}=SS(X)_E, |X|>1$, then (X,η_1,η_2,E) is a p-soft disconnected.

(3) If $\eta_{12}=\{(\tilde{\phi},E),(\tilde{X},E),(G,E)\}$, then (X,η_1,η_2,E) is a p-soft connected.

Proof. (1): By given we deduce that $\eta_{12}=\{(\tilde{\phi},E),(\tilde{X},E)\}$. So, for any two soft sets $(G,E),(H,E)$ over X we have $scl_{\eta_{12}}(G,E)=scl_{\eta_{12}}(H,E)=(\tilde{X},E)$. Consequently, we cannot represented $(\tilde{X},E)$ as a union of two p-separated soft sets in (X,η_1,η_2,E). Hence, (X,η_1,η_2,E) is a p-soft connected.

(2): Since $\eta_{12}=SS(X)_E, |X|>1$, then every soft set is a p-closed soft set. It follows that for every soft point (x_e,E) in $(\tilde{X},E)$ we have $scl_{\eta_{12}}(x_e,E)=(x_e,E)$ and $scl_{\eta_{12}}(x_e,E)^c=(x_e,E)^c$. So, $(x_e,E),(x_e,E)^c$ are p-separated soft sets and $(\tilde{X},E)=(x_e,E)\tilde{\cup}(x_e,E)^c$. Hence, (X,η_1,η_2,E) is a p-soft disconnected.

(3): Assume that (X,η_1,η_2,E) is a p-soft disconnected. Then there exist two non-null soft sets $(M,E),(N,E)$ such that $scl_{12}(M,E)\tilde{\cap}(N,E)=scl_{12}(N,E)\tilde{\cap}(M,E)$ $=(\tilde{\phi},E)$ and $(\tilde{X},E)=(M,E)\tilde{\cup}(N,E)$. Therefore, $scl_{12}(M,E)=scl_{12}(N,E)=(G,E)^c$. It follows that $(M,E)\tilde{\subseteq}(G,E)$ and $(N,E)\tilde{\subseteq}(G,E)$ implies $(M,E)\tilde{\cup}(N,E)\tilde{\subseteq}(G,E)$. Thus, $(\tilde{X},E)=(G,E)$, a contradicts with that $(\tilde{X},E)\neq(G,E)$. Hence, (X,η_1,η_2,E) is a p-soft connected space.

Theorem 4.1 *Let* (X,η_1,η_2,E) *be a sbts. Then the following are equivalent:*

(1) (X,η_1,η_2,E) is p-soft connected.

(2) $(\tilde{X},E)$ cannot represented as a union of two non-null disjoint p-open soft sets.

(3) $(\tilde{X},E)$ cannot represented as a union of two non null disjoint p-closed soft sets.

(4) $(\tilde{X},E)$ has no proper soft subset which is both p-open and p-closed soft set other than $(\tilde{\phi},E)$.

Proof. $(1)\Rightarrow(2)$: Assume that there exist two non-null p-open soft sets $(G,E),(H,E)$ such that $(G,E)\tilde{\cap}(H,E)=(\tilde{\phi},E)$ and $(\tilde{X},E)=(G,E)\tilde{\cup}(H,E)$. Since $(G,E)\tilde{\cap}(H,E)=(\tilde{\phi},E)$, then $(G,E)\tilde{\subseteq}(H,E)^c$, $(H,E)\tilde{\subseteq}(G,E)^c$. Therefore, $scl_{\eta_{12}}(G,E)\tilde{\cap}(H,E)=(\tilde{\phi},E)$ and $scl_{\eta_{12}}(H,E)\tilde{\cap}(G,E)=(\tilde{\phi},E)$. It follows that $(\tilde{X},E)$ has a p-soft separation, i.e., (X,η_1,η_2,E) is p-soft connected which contradicts with (1).

$(2)\Rightarrow(3)$: Assume that there exist two non-null p-closed soft sets $(F,E),(M,E)$ such that $(F,E)\tilde{\cap}(M,E)=(\tilde{\phi},E)$ and $(\tilde{X},E)=(F,E)\tilde{\cup}(M,E)$. Then, $(F,E)^c,(M,E)^c$ are non-null p-open soft sets and $(F,E)^c\tilde{\cup}(M,E)^c=(\tilde{X},E)$, which contradicts with (2).

$(3)\Rightarrow(4)$: Assume that there exists $(G,E)\subsetneq(\tilde{X},E)$, $(N,E)\neq(\tilde{\phi},E)$ such that (N,E) is both p-open and p-closed soft set. Then (N,E), $(N,E)^c$ are non-null disjoint p-closed soft sets and $(\tilde{X},E)=(N,E)\tilde{\cup}(N,E)^c$, this contradicts with (3). $(4)\Rightarrow(1)$: Assume that (X,η_1,η_2,E) is a p-soft disconnected. Then there exists p-separated soft sets $(G,E),(H,E)$ such that $(\tilde{X},E)=(G,E)\tilde{\cup}(H,E)$. Therefore, $(G,E)^c\tilde{\cap}(H,E)^c=(\tilde{\phi},E)$ implies $(H,E)^c\tilde{\subseteq}(G,E)$ and $(G,E)^c\tilde{\subseteq}(H,E)$. Since $scl_{\eta_{12}}(G,E)\tilde{\cap}(H,E)=(\tilde{\phi},E)$, then $scl_{\eta_{12}}(G,E)\tilde{\subseteq}(H,E)^c\tilde{\subseteq}(G,E)$. Hence, (G,E) is p-closed soft set. Similarly, (H,E) is p-closed soft set. On the other hand, by Proposition 3.1 we deduce that $(G,E)\tilde{\subseteq}(H,E)^c$. Therefore, $(G,E)=(H,E)^c$. It follows that $(H,E)^c$ is p-closed soft set. Hence, (H,E) is non-null p-closed and p-open soft set, which contradicts with (4).

Corollary 4.1 *A sbts* (X,η_1,η_2,E) *is a* p*-soft connected space if and only if the only soft sets over* X *which are* p*-open and* p*-closed soft sets are* $(\tilde{X},E)$ *and* $(\tilde{\phi},E)$.

Example 4.2 *Let* $X=\{a,b,c,d\}$, $E=\{e_1,e_2,e_3\}$ *and let*

$$\eta_1=\{(\tilde{\phi},E),(\tilde{X},E),(G_1,E),(G_2,E),(G_3,E),(G_4,E)\}$$

$$\eta_2=\{(\tilde{\phi},E),(\tilde{X},E),(H_1,E),(H_2,E)\}$$

where,

$(G_1,E)=\{(e_1,\{a,c\}),(e_2,\{a,b,c\}),(e_3,\{c,d\})\}$,
$(G_2,E)=\{(e_1,\phi),(e_2,\{a,c\}),(e_3,\{d\})\}$,

$(G_3,E)=\{(e_1,\{c\}),(e_2,\{b\}),(e_3,\phi)\}$,
$(G_4,E)=\{(e_1,\{c\}),(e_2,\{a,b,c\}),(e_3,\{d\})\}$,

$(H_1,E)=\{(e_1,\{a,b\}),(e_2,\{a,c\}),(e_3,\{a,d\})\}$,
$(H_2,E)=\{(e_1,\{b\}),(e_2,\{c\}),(e_3,\{a,d\})\}$.

Then, (X,η_1,η_2,E) is a sbts and therefore

$$\eta_{12}=\{(\tilde{\phi},E),(\tilde{X},E),(G_1,E),(G_2,E),(G_3,E),(G_4,E),(H_1,E),(H_2,E),(P_1,E),(P_2,E),(P_3,E),(P_4,E),(P_5,E)\}$$

where

$(P_1,E)=\{(e_1,\{a,b,c\}),(e_2,\{a,b,c\}),(e_3,\{a,c,d\})\}$, $(P_2,E)=\{(e_1,\{b\}),(e_2,\{a,c\}),(e_3,\{a,d\})\}$,

$(P_3, E) = \{(e_1, \{a,b,c\}), (e_2, \{a,b,c\}), (e_3, \{a,d\})\}$, $(P_4, E) = \{(e_1, \{b,c\}), (e_2, \{b,c\}), (e_3, \{a,d\})\}$,

$(P_5, E) = \{(e_1, \{b,c\}), (e_2, \{a,b,c\}), (e_3, \{a,d\})\}$.

It is easily to seen that the only soft sets over X which are p-open and p-closed soft sets are $(\tilde{X}, E)$ and $(\tilde{\phi}, E)$. Consequently, by Corollary 4.1 we deduce that (X, η_1, η_2, E) is a p-soft connected space.

Theorem 4.2 *Let* (X, η_1, η_2, E) *be a sbts. Then the following are equivalent:*

(1) (X, η_1, η_2, E) is p-soft disconnected.

(2) $(\tilde{X}, E)$ can represented as a union of two non-null disjoint p-open soft sets.

(3) $(\tilde{X}, E)$ can represented as a union of two non null disjoint p-closed soft sets.

(4) $(\tilde{X}, E)$ has a proper soft subset which is both p-open and p-closed soft set other than $(\tilde{\phi}, E)$.

Proof. Similar to the prove of Theorem 4.1.

Remark 4.1 *Let* (X, η_1, η_2, E) *be a* p*-soft connected space and let* $e \in E$. *Then* (X, η_1^e, η_2^e) *may not be* p*-connected space as shown in the following example.*

Example 4.3 *Let* $X = \{a,b,c\}$, $E = \{e_1, e_2\}$ *and let*

$$\eta_1 = \{(\tilde{\phi}, E), (\tilde{X}, E), (G, E)\},$$

$$\eta_2 = \{(\tilde{\phi}, E), (\tilde{X}, E), (H, E)\},$$

Where,

$(G, E) = \{(e_1, \{a\}), (e_2, \{b,c\})\}$ and $(H, E) = \{(e_1, \{b,c\}), (e_2, \{a,c\})\}$.

Then, (X, η_1, η_2, E) is a sbts. It is Clear that $\eta_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (G, E), (H, E)\}$. Therefore, (X, η_1, η_2, E) is a p-soft connected because we cannot represented $(\tilde{X}, E)$ as a union of two non-null disjoint p-open soft sets. On the other hand, $\eta_{12}(e_1) = \{(\phi, X, \{a\}, \{b,c\}\}$. Hence, $(X, \eta_1^{e_1}, \eta_2^{e_1})$ is a p-disconnected space because $\{a\}$ is both p-open and p-closed set.

Remark 4.2 *Let* (X, η_1, η_2, E) *be a* p*-soft disconnected space and let* $e \in E$. *Then* (X, η_1^e, η_2^e) *may not be* p*-disconnected space as shown in the following example.*

Example 4.4 *Let* $X = \{a,b,c\}$, $E = \{e_1, e_2\}$ *and let*

$$\eta_1 = \{(\tilde{\phi}, E), (\tilde{X}, E), (M, E), (N, E)\}$$
$$\eta_2 = \{(\tilde{\phi}, E), (\tilde{X}, E), (K, E)\},$$

Where,

$(M, E) = \{(e_1, \{b\}), (e_2, \{b\})\}$,
$(N, E) = \{(e_1, X), (e_2, \{b\})\}$, and
$(K, E) = \{(e_1, \phi), (e_2, \{a,c\})\}$.

Then, (X, η_1, η_2, E) is a sbts. It is Clear that $\eta_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (M, E), (N, E), (K, E), (P, E)\}$ where, $(P, E) = \{(e_1, \{b\}), (e_2, X)\}$. Since $\{(e_1, X), (e_2, \{b\})\}$ is both p-open and p-closed soft set, then by Theorem 4.2 (4) we have (X, η_1, η_2, E) is a p-soft disconnected.

Now, $\eta_1^{e_1} = \{\phi, \{b\}, X\}$ and $\eta_2^{e_1} = \{\phi, X\}$.

So, $\eta_{12}^{e_1} = \{\phi, X, \{b\}\}$. Obvious that $(X, \eta_1^{e_1}, \eta_2^{e_1})$ is a p-connected space. We can show that $(X, \eta_1^{e_2}, \eta_2^{e_2})$ is a p-disconnected space.

Theorem 4.3 *Let* (X, η_1, η_2, E) *be sbts and let* $Y \subseteq X$. *Then,* $(Y, \eta_{1Y}, \eta_{2Y}, E)$ *is a* p*-soft disconnected if and only if there exist two* p*-separated soft sets* (F^Y, E) *and* (K^Y, E) *in* $(Y, \eta_{1Y}, \eta_{2Y}, E)$ *such that* $(\tilde{Y}, E) = (F^Y, E) \tilde{\cup} (K^Y, E)$, *where* $(F^Y, E) = (\tilde{Y}, E) \tilde{\cap} (F, E)$, $(F, E) \in \eta_{12}$.

Proof. Straightforward.

Definition 4.3 *A property* P *of a sbts* (X, η_1, η_2, E) *is called hereditary property if every soft bitopological subspace* $(Y, \eta_{1Y}, \eta_{2Y}, E)$ *of* (X, η_1, η_2, E) *is also has the property* P.

Remark 4.3 *The pairwise soft connectedness does not hereditary property as shown by the following example.*

Example 4.5 *Let* $X = \{x, y, z\}$, $E = \{e_1, e_2, e_3\}$ *and let*

$\eta_1 = \{(\tilde{\phi}, E), (\tilde{X}, E), (G, E)\}$,
$\eta_2 = \{(\tilde{\phi}, E), (\tilde{X}, E), (H, E)\}$

Where,

$(G, E) = \{(e_1, \{x\}), (e_2, \{x\}), (e_3, \{x\})\}$,
$(H, E) = \{(e_1, \{z\}), (e_2, \{z\}), (e_3, \{z\})\}$.

Then, (X, η_1, η_2, E) is a sbts. Moreover,

$\eta_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (G, E), (H, E), (P, E)\}$

Where,

$(P, E) = \{(e_1, \{x, z\}), (e_2, \{x, z\}), (e_3, \{x, z\})\}$.

Since $(\tilde{X}, E)$ has no proper soft subset which is both p-open and p-closed soft set other than $(\tilde{\phi}, E)$, then (X, η_1, η_2, E) is a p-soft connected space.

Now, let $Y = \{x, z\}$. It is easily to seen that $(\tilde{Y}, E) = (P, E)$ and so

$\eta_{1Y} = \{(\tilde{\phi}, E), (\tilde{Y}, E), (G, E)\}$,
$\eta_{2Y} = \{(\tilde{\phi}, E), (\tilde{Y}, E), (H, E)\}$.

Then,

$(Y, \eta_{1Y}, \eta_{2Y}, E)$ is a soft bitopological subspace of (X, η_1, η_2, E). Furthermore,

$$\eta_{12Y} = \{(\tilde{\phi}, E), (\tilde{Y}, E), (G, E), (H, E)\}.$$

Since $(G, E), (H, E)$ are non-null disjoint p-open soft sets in $(Y, \eta_{1Y}, \eta_{2Y}, E)$ and $(\tilde{Y}, E) = (G, E) \tilde{\cup} (H, E)$. Hence, by Theorem 4.2 (3) we deduce that $(Y, \eta_{1Y}, \eta_{2Y}, E)$ is a p-soft disconnected space. Therefore, the pairwise soft connectedness does not hereditary property.

Theorem 4.4 *Let* $(X, \sigma_1, \sigma_2, E)$ *be a sbts finer than of a sbts* (X, η_1, η_2, E). *Then,*

(1) If (X, η_1, η_2, E) is a p-soft disconnected space, then $(X, \sigma_1, \sigma_2, E)$ is a p-soft disconnected space.

(2) If $(X, \sigma_1, \sigma_2, E)$ is a p-soft connected space, then (X, η_1, η_2, E) is a p-soft connected space.

Proof. (1): Since (X, η_1, η_2, E) is a p-soft disconnected space, then there exist $(G, E), (H, E) \in SS(X)_E$ such that $scl_{\eta_{12}}(G, E) \tilde{\cap} (H, E) = (\tilde{\phi}, E)$, $scl_{\eta_{12}}(H, E) \tilde{\cap} (G, E) = (\tilde{\phi}, E)$ and $(G, E) \tilde{\cup} (H, E) = (\tilde{X}, E)$. Now, since $(X, \sigma_1, \sigma_2, E)$ is a finer than of (X, η_1, η_2, E), then $\eta_{12} \subseteq \sigma_{12}$. It follows that for any soft set (G, E) we have $scl_{\sigma_{12}}(G, E) \tilde{\subseteq} scl_{\eta_{12}}(G, E)$ [by Theorem 2.5].Consequently, $scl_{\sigma_{12}}(G, E) \tilde{\cap} (H, E) = (\tilde{\phi}, E)$, $scl_{\sigma_{12}}(H, E) \tilde{\cap} (G, E) = (\tilde{\phi}, E)$ and $(G, E) \tilde{\cup} (H, E) = (\tilde{X}, E)$. Hence, $(X, \sigma_1, \sigma_2, E)$ is a p-soft disconnected space.

(2): Suppose that $(X, \sigma_1, \sigma_2, E)$ is a p-soft connected space. Assume that (X, η_1, η_2, E) is a p-soft disconnected space. Then, by (1), $(X, \sigma_1, \sigma_2, E)$ is a p-soft disconnected space, a contradiction.

Theorem 4.5 *Let* (X, η_1, η_2, E) *be a sbts. Then,*

If (X, η_1, E) or (X, η_2, E) is a soft disconnected space, then (X, η_1, η_2, E) is a p-soft disconnected space.

Proof. It is immediate from the fact that $scl_{\eta_{12}}(G, E) = scl_{\eta_1}(G, E) \tilde{\cap} scl_{\eta_2}(G, E)$ [Corollary 3.15 in [13]].

Remark 4.4 *Let* (X, η_1, η_2, E) *be a sbts. If* (X, η_1, E) *and* (X, η_2, E) *are both soft connected spaces, then* (X, η_1, η_2, E) *may not be a* p*-soft connected space as shown by the following example.*

Example 4.6 *Let* $X = \{a, b\}$, $E = \{e_1, e_2\}$ *and let*

$\eta_1 = \{(\tilde{\phi}, E), (\tilde{X}, E), (M, E)\}$,
$\eta_2 = \{(\tilde{\phi}, E), (\tilde{X}, E), (N, E)\}$

where,

$(M, E) = \{(e_1, \{a\}), (e_2, \{b\})\}$,
$(N, E) = \{(e_1, \{b\}), (e_2, \{a\})\}$. Then, (X, η_1, E) and (X, η_2, E) are soft connected spaces. Obvious that (X, η_1, η_2, E) is a sbts. Moreover,

$\eta_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (M, E), (N, E)\}$. Since $(M, E), (N, E)$ are non-null disjoint p-open soft sets and $(\tilde{X}, E) = (M, E) \tilde{\cup} (N, E)$. Hence, by Theorem 4.2 (3) we deduce that (X, η_1, η_2, E) is a p-soft disconnected space.

Theorem 4.6 *Let* (X,η_1,η_2,E) *be a sbts,* $Y \subseteq X$ *and let* $(Y,\eta_{1Y},\eta_{2Y},E)$ *be a* p *-soft connected space. If* (G,E) *and* (H,E) *are* p *-soft separation of* $(\tilde{X},E)$*, then* $(\tilde{Y},E)\tilde{\subseteq}(G,E)$ *or* $(\tilde{Y},E)\tilde{\subseteq}(H,E)$.

Proof. Assume that $(\tilde{Y},E)\tilde{\not\subseteq}(G,E)$ and $(\tilde{Y},E)\tilde{\not\subseteq}(H,E)$. Since (G,E) and (H,E) are p-soft separation of $(\tilde{X},E)$, then $(\tilde{Y},E)\tilde{\subseteq}(\tilde{X},E)=(G,E)\tilde{\cup}(H,E)$ implies $(\tilde{Y},E)\tilde{\cap}[(G,E)\tilde{\cup}(H,E)]=(\tilde{Y},E)$. It follows that $[(\tilde{Y},E)\tilde{\cap}(G,E)]\tilde{\cup}[(\tilde{Y},E)\tilde{\cap}(H,E)]=(\tilde{Y},E)$. On the other hand, since $(\tilde{Y},E)\tilde{\not\subseteq}(G,E)$, $(\tilde{Y},E)\tilde{\not\subseteq}(H,E)$ and $(\tilde{Y},E)\tilde{\subseteq}(G,E)\tilde{\cup}(H,E)$, then $(\tilde{\phi},E)\neq(\tilde{Y},E)\tilde{\cap}(G,E)\neq(\tilde{Y},E)$, $(\tilde{\phi},E)\neq(\tilde{Y},E)\tilde{\cap}(H,E)\neq(\tilde{Y},E)$. Now, since $(G,E)\tilde{\cap}(H,E)=(\tilde{\phi},E)$ and

$$scl_{\eta_{12Y}}(G,E)\tilde{\subseteq}(\tilde{Y},E)\tilde{\cap}scl_{\eta_{12}}(G,E),$$

then

$$scl_{\eta_{12Y}}[(\tilde{Y},E)\tilde{\cap}(G,E)]\tilde{\cap}[(\tilde{Y},E)\tilde{\cap}(H,E)]=(\tilde{\phi},E)$$

and

$$scl_{\eta_{12Y}}[(\tilde{Y},E)\tilde{\cap}(H,E)]\tilde{\cap}[(\tilde{Y},E)\tilde{\cap}(G,E)]=(\tilde{\phi},E)$$

. Therefore, $(\tilde{Y},E)\tilde{\cap}(G,E)$, $(\tilde{Y},E)\tilde{\cap}(H,E)$ are p-soft separation of $(\tilde{Y},E)$ which contradicts with that $(Y,\eta_{1Y},\eta_{2Y},E)$ is a p-soft connected space. Hence, our assumption is not true. Thus, $(\tilde{Y},E)\tilde{\subseteq}(G,E)$ or $(\tilde{Y},E)\tilde{\subseteq}(H,E)$.

Theorem 4.7 *Let* (X,η_1,η_2,E) *be a sbts. If* (X,η_1,η_2,E) *is a* p *-soft disconnected, then* $(X,\eta_{p\Lambda},E)$ *is a soft disconnected.*

Proof. Straightforward.

Theorem 4.8 *Let* (X,η_1,η_2,E) *be a sbts. If* $(X,\eta_{p\Lambda},E)$ *is a soft connected, then* (X,η_1,η_2,E) *is a* p *-soft connected.*

Proof. Straightforward.

5. PAIRWISE SOFT CONNECTED (DISCONNECTED) SOFT SETS

Definition 5.1 *A soft set* (G,E) *in a sbts* (X,η_1,η_2,E) *is said to be a pairwise disconnected soft set [briefly,* p *-disconnected soft set] if there exist two non-null* p *-open soft sets* $(O_1,E),(O_2,E)$ *such that*
$(G,E)\tilde{\cap}(O_1,E)\neq(\tilde{\phi},E)$,
$(G,E)\tilde{\cap}(O_2,E)\neq(\tilde{\phi},E)$,
$(G,E)\tilde{\subseteq}(O_1,E)\tilde{\cup}(O_2,E)$ *and*
$(O_1,E)\tilde{\cap}(O_2,E)\tilde{\subseteq}(G,E)^c$*. In this case we say that* $(O_1,E)\tilde{\cup}(O_2,E)$ *are* p *-soft disconnection of* (G,E)*. A soft set* (G,E) *is called a* p *-connected soft set if has no* p *-soft disconnection.*

Example 5.1 *From Example 3.1, let* $(F,E)=\{(e_1,\{x,y\}),(e_2,\{y,w\})\}$*. Take* $(O_1,E)=(G,E)$, $(O_2,E)=(H,E)$*. It is clear that*
$(F,E)\tilde{\cap}(O_1,E)\neq(\tilde{\phi},E)$,
$(F,E)\tilde{\cap}(O_2,E)\neq(\tilde{\phi},E)$,
$(F,E)\tilde{\subseteq}(O_1,E)\tilde{\cup}(O_2,E)=(\tilde{X},E)$ *and*
$(O_1,E)\tilde{\cap}(O_2,E)=(\tilde{\phi},E)\tilde{\subseteq}(G,E)^c$*. Hence,* (F,E) *is a* p *-disconnected soft set.*

Theorem 5.1 *Every* p *-connected soft set in a sbts* (X,η_1,η_2,E) *is a connected soft set in sbts* $(X,\eta_{p\Lambda},E)$,

Proof. Straightforward.

Lemma 5.1 *If* $(O_1,E)\tilde{\cup}(O_2,E)$ *are* p *-soft disconnection of* (G,E) *in a sbts* (X,η_1,η_2,E)*, then* $(G,E)\tilde{\cap}(O_1,E)$ *and* $(G,E)\tilde{\cap}(O_2,E)$ *are* p *-separated soft sets.*

Proof. Since $(O_1,E)\tilde{\cup}(O_2,E)$ is a p-soft disconnection of (G,E), then
$(G,E)\tilde{\cap}(O_1,E)\neq(\tilde{\phi},E)$,
$(G,E)\tilde{\cap}(O_2,E)\neq(\tilde{\phi},E)$,
$(G,E)\tilde{\subseteq}(O_1,E)\tilde{\cup}(O_2,E)$ and
$(O_1,E)\tilde{\cap}(O_2,E)\tilde{\subseteq}(G,E)^c$. We shall prove that $(O_1,E)\tilde{\cap}(G,E),(O_2,E)\tilde{\cap}(G,E)$ are p-separated soft sets.

Let $x_e\tilde{\in}scl_{12}[(O_1,E)\tilde{\cap}(G,E)]$. Then, by Theorem 2.5 $[(O_1,E)\tilde{\cap}(G,E)]\tilde{\cap}(O_{x_e},E)\neq(\tilde{\phi},E)$, $\forall\,(O_{x_e},E)\in\eta_{12}(x_e)$. Now, assume that $x_e\tilde{\in}[(O_2,E)\tilde{\cap}(G,E)]$. It follows that

$x_e \tilde{\in} (O_2,E)$. Therefore, $(O_2,E) \in \eta_{12}(x_e)$. Thus, $[(O_1,E)\tilde{\cap}(G,E)]\tilde{\cap}(O_2,E) \neq (\tilde{\phi},E)$ which a contradicts with the given $(O_1,E)\tilde{\cap}(O_2,E) \tilde{\subseteq} (G,E)^c$. Hence, $x_e \tilde{\notin} [(O_2,E)\tilde{\cap}(G,E)]$. Consequently, $scl_{12}[(O_1,E)\tilde{\cap}(G,E)]\tilde{\cap}[(O_2,E)\tilde{\cap}(G,E)] = (\tilde{\phi},E)$. Similarly, $scl_{12}[(O_2,E)\tilde{\cap}(G,E)]\tilde{\cap}[(O_1,E)\tilde{\cap}(G,E)] = (\tilde{\phi},E)$. Hence, $(O_1,E)\tilde{\cap}(G,E),(O_2,E)\tilde{\cap}(G,E)$ are p-separated soft sets.

Theorem 5.2 *A soft set (G,E) in a sbts (X,η_1,η_2,E) is a p-disconnected soft set iff there exist two p-separated soft sets $(S_1,E),(S_2,E)$ such that $(G,E) = (S_1,E)\tilde{\cup}(S_2,E)$.*

Proof. $\Rightarrow$: Suppose that (G,E) is a p-disconnected soft set in (X,η_1,η_2,E). Then (G,E) has a p-soft disconnection, say $(O_1,E)\tilde{\cup}(O_2,E)$, i.e., there exist two non-null p-open soft sets $(O_1,E),(O_2,E)$ such that $(G,E)\tilde{\cap}(O_1,E) \neq (\tilde{\phi},E)$, $(G,E)\tilde{\cap}(O_2,E) \neq (\tilde{\phi},E)$, $(G,E) \tilde{\subseteq} (O_1,E)\tilde{\cup}(O_2,E)$ and $(O_1,E)\tilde{\cap}(O_2,E) \tilde{\subseteq} (G,E)^c$. It follows by Lemma 5.1 that $(G,E)\tilde{\cap}(O_1,E)$ and $(G,E)\tilde{\cap}(O_2,E)$ are p-separated soft sets. Since $(G,E) \tilde{\subseteq} (O_1,E)\tilde{\cup}(O_2,E)$, then

$$(G,E)\tilde{\cap}[(O_1,E)\tilde{\cup}(O_2,E)] = (G,E)$$

implies $[(G,E)\tilde{\cap}(O_1,E)]\tilde{\cup}[(G,E)\tilde{\cap}(O_2,E)] = (G,E)$. Take $(S_1,E) = (O_1,E)\tilde{\cap}(G,E)$ and $(S_2,E) = (O_2,E)\tilde{\cap}(G,E)$.

$\Leftarrow$: Let $(S_1,E),(S_2,E)$ be two p-separated soft sets and let $(G,E) \in SS(X)_E$ such that $(G,E) = (S_1,E)\tilde{\cup}(S_2,E)$. Then $scl_{12}(S_1,E)\tilde{\cap}(S_2,E) = (\tilde{\phi},E)$ and $scl_{12}(S_2,E)\tilde{\cap}(S_1,E) = (\tilde{\phi},E)$. Take $(O_1,E) = [scl_{12}(S_1,E)]^c$ and $(O_2,E) = [scl_{12}(S_2,E)]^c$. So, $(O_1,E),(O_2,E)$ are non-null p-open soft sets. Since $scl_{12}(S_2,E)\tilde{\cap}(S_1,E) = (\tilde{\phi},E)$, then $(S_1,E) \tilde{\subseteq} [scl_{12}(S_2,E)]^c = (O_2,E)$. By similar we also have $(S_2,E) \tilde{\subseteq} (O_1,E)$. It follows that $(G,E) \tilde{\subseteq} (O_1,E)\tilde{\cup}(O_2,E)$. Now, since $[scl_{12}(S_1,E)]^c \tilde{\subseteq} (S_1,E)^c$, $[scl_{12}(S_2,E)]^c \tilde{\subseteq} (S_2,E)^c$, then $(O_1,E)\tilde{\cap}(O_2,E) \tilde{\subseteq} (G,E)^c$. Furthermore, since $(S_1,E),(S_2,E) \tilde{\subseteq} (G,E)$ and $(S_2,E) \tilde{\subseteq} (O_1,E)$, $(S_1,E) \tilde{\subseteq} (O_2,E)$, then

$(S_2,E) \tilde{\subseteq} (G,E)\tilde{\cap}(O_1,E)$ and $(S_1,E) \tilde{\subseteq} (G,E)\tilde{\cap}(O_2,E)$. But, $(S_1,E) \neq (\tilde{\phi},E)$, $(S_2,E) \neq (\tilde{\phi},E)$, then $(G,E)\tilde{\cap}(O_2,E) \neq (\tilde{\phi},E)$ and $(G,E)\tilde{\cap}(O_1,E) \neq (\tilde{\phi},E)$. Consequently, (G,E) is a p-disconnected soft set.

Corollary 5.1 *Let (X,η_1,η_2,E) be a sbts. If $(S_1,E),(S_2,E)$ are two p-separated soft sets, then $(S_1,E)\tilde{\cup}(S_2,E)$ is a p-disconnected soft set.*

Corollary 5.2 *A soft set (G,E) in a sbts (X,η_1,η_2,E) is said to be a p-connected soft set iff cannot expressed as a union of two p-separated soft sets.*

Proposition 5.1 *Let (X,η_1,η_2,E) be a sbts. Then*

(1) Every soft point is p-connected soft set.

(2) The null soft set is p-connected soft set.

Proof. (1): Let (x_e,E) be a soft point in $(\tilde{X},E)$ Then, for any two non-null p-open soft sets $(O_1,E),(O_2,E)$ such that $(x_e,E)\tilde{\cap}(O_1,E) \neq (\tilde{\phi},E)$, $(x_e,E)\tilde{\cap}(O_2,E) \neq (\tilde{\phi},E)$, we have $(x_e,E) \tilde{\in} (O_1,E)\tilde{\cap}(O_2,E)$. It follows that $(O_1,E)\tilde{\cap}(O_2,E)$Ú$(x_e,E)^c$. Hence, (x_e,E) is a p-connected soft set.

(2): Obvious.

Theorem 5.3 *Let (F,E) be a p-connected soft set in a sbts (X,η_1,η_2,E) and let*

$(F,E) \tilde{\subseteq} (M,E) \tilde{\subseteq} scl_{\eta_{12}}(F,E)$. Then $(M,E), scl_{\eta_{12}}(F,E)$ are also p-connected soft sets.

Proof. Let (F,E) be a p-connected soft set in a sbts (X,η_1,η_2,E) and assume that (M,E) is a p-disconnected soft set in (X,η_1,η_2,E). Then, there exist two non-null p-open soft sets $(O_1,E),(O_2,E)$ such that $(M,E)\tilde{\cap}(O_1,E)\neq(\tilde{\phi},E)$, $(M,E)\tilde{\cap}(O_2,E)\neq(\tilde{\phi},E)$, $(M,E)\tilde{\subseteq}(O_1,E)\tilde{\cup}(O_2,E)$ and $(O_1,E)\tilde{\cap}(O_2,E)\tilde{\subseteq}(M,E)^c$. Since $(F,E)\tilde{\subseteq}(M,E)$, then $(F,E)\tilde{\subseteq}(O_1,E)\tilde{\cup}(O_2,E)$ and $(O_1,E)\tilde{\cap}(O_2,E)\tilde{\subseteq}(F,E)^c$. But (F,E) is a p-connected soft set, then either $(F,E)\tilde{\cap}(O_1,E)=(\tilde{\phi},E)$ or $(F,E)\tilde{\cap}(O_2,E)=(\tilde{\phi},E)$. If we claim that $(F,E)\tilde{\cap}(O_1,E)=(\tilde{\phi},E)$, then $(O_1,E)^c$ is a p-closed soft set contains (F,E). It follows that $scl_{\eta_{12}}(F,E)\tilde{\subseteq}(O_1,E)^c$ which implies that $(M,E)\tilde{\cap}(O_1,E)=(\tilde{\phi},E)$, a contradicts with our assumption. Hence, our assumption is false. Consequently, (M,E) is a p-connected soft set. In particular, put $(M,E)=scl_{\eta_{12}}(F,E)$, then $scl_{\eta_{12}}(F,E)$ is also p-connected soft set.

Remark 5.1 *The soft subset of a p-soft connected space need not be a p-connected soft set as seen in the following example.*

Example 5.2 *Consider the p-soft connected space (X,η_1,η_2,E) in Example 4.2. Let $(F,E)=\{(e_1,\phi),(e_2,\{a\}),(e_3,\{a\})\}$. Take $(O_1,E)=(G_2,E)$ and $(O_2,E)=(H_2,E)$. Therefore,*

$(F,E)\tilde{\cap}(O_1,E)=\{(e_1,\phi),(e_1,\{a\}),(e_1,\phi)\}\neq(\tilde{\phi},E)$,

$(F,E)\tilde{\cap}(O_2,E)=\{(e_1,\phi),(e_1,\phi),(e_1,\{a\})\}\neq(\tilde{\phi},E)$,

$(F,E)\tilde{\subseteq}(O_1,E)\tilde{\cup}(O_2,E)=\{(e_1,\{b\}),(e_1,\{a,c\}),(e_1,\{a,d\})\}$ *and*

$(O_1,E)\tilde{\cap}(O_2,E)=\{(e_1,\phi),(e_1,\{c\}),(e_1,\{d\})\}\tilde{\subseteq}(F,E)^c$. *Hence, (F,E) is a p-disconnected soft subset of (X,η_1,η_2,E).*

Remark 5.2 *The union of two p-connected soft sets need not be a p-connected soft set as seen in the following example.*

Example 5.3 *In Example 5.2, it is clear by Proposition 5.1 (1) that $(a_{e_2},E),(a_{e_3},E)$ are p-connected soft sets. Nevertheless,*

$(a_{e_2},E)\tilde{\cup}(a_{e_3},E)=\{(e_1,\phi),(e_2,\{a\}),(e_3,\{a\})\}=(F,E)$

is a p-disconnected soft set.

Theorem 5.4 *Let (G,E), (H,E) be two p-connected soft sets in sbts (X,η_1,η_2,E). If $(G,E)\tilde{\cap}(H,E)\neq(\tilde{\phi},E)$, then $(G,E)\tilde{\cup}(H,E)$ is a p-connected soft set.*

Proof. Suppose that (G,E), (H,E) are p-connected soft sets and $(G,E)\tilde{\cap}(H,E)\neq(\tilde{\phi},E)$. Assume that $(G,E)\tilde{\cup}(H,E)$ is a p-disconnected soft set. Then there exist two non-null p-open soft sets $(O_1,E),(O_2,E)$ such that $[(G,E)\tilde{\cup}(H,E)]\tilde{\cap}(O_1,E)\neq(\tilde{\phi},E)$, $[(G,E)\tilde{\cup}(H,E)]\tilde{\cap}(O_2,E)\neq(\tilde{\phi},E)$, $[(G,E)\tilde{\cup}(H,E)]\tilde{\subseteq}(O_1,E)\tilde{\cup}(O_2,E)$ and $(O_1,E)\tilde{\cap}(O_2,E)\tilde{\subseteq}[(G,E)\tilde{\cup}(H,E)]^c$. Since $(G,E)\tilde{\subseteq}(G,E)\tilde{\cup}(H,E)$, then $(G,E)\tilde{\subseteq}(O_1,E)\tilde{\cup}(O_2,E)$ and $(O_1,E)\tilde{\cap}(O_2,E)\tilde{\subseteq}(G,E)^c$. But (G,E) is a p-connected soft set, then $(G,E)\tilde{\cap}(O_1,E)=(\tilde{\phi},E)$ or $(G,E)\tilde{\cap}(O_2,E)=(\tilde{\phi},E)$. Therefore, $(G,E)\tilde{\subseteq}(O_1,E)$ or $(G,E)\tilde{\subseteq}(O_2,E)$ [for $(G,E)\tilde{\subseteq}(O_1,E)\tilde{\cup}(O_2,E)$]. Similarly, $(H,E)\tilde{\subseteq}(O_1,E)$ or $(H,E)\tilde{\subseteq}(O_2,E)$. Thus, if $(G,E)\tilde{\subseteq}(O_1,E)$ and $(H,E)\tilde{\subseteq}(O_2,E)$, then

$(G,E)\tilde{\cap}(H,E)\tilde{\subseteq}(O_1,E)\tilde{\cap}(O_2,E)\tilde{\subseteq}(G,E)^c\tilde{\cap}(H,E)^c$

which implies that $(G,E)\tilde{\cap}(H,E)=(\tilde{\phi},E)$, a contradiction. Similarly when $(G,E)\tilde{\subseteq}(O_2,E)$ and $(H,E)\tilde{\subseteq}(O_1,E)$ we have a contradiction. Consequently, our assumption is not true. Hence, $(G,E)\tilde{\cup}(H,E)$ is a p-connected soft set.

Theorem 5.5 *Let f_{pu} be a p-soft continuous mapping from a sbts (X,η_1,η_2,E) into a sbts (Y,σ_1,σ_2,K). If (G,E) is a p-connected soft set in (X,η_1,η_2,E),* *then* $f_{pu}(G,E)$ *is a* p *-connected soft set in* (Y,σ_1,σ_2,K).

Proof. Let (G,E) be a p -connected soft set in (X,η_1,η_2,E). Assume that $f_{pu}(G,E)$ is not p -connected soft set in (Y,σ_1,σ_2,K). Then there exist two non-null p -open soft sets $(O_1,K),(O_2,K)$ in (Y,σ_1,σ_2,K) such that $f_{pu}(G,E)\tilde{\cap}(O_1,K)\neq(\tilde{\phi},K)$, $f_{pu}(G,E)\tilde{\cap}(O_2,K)\neq(\tilde{\phi},K)$, $f_{pu}(G,E)\tilde{\subseteq}(O_1,K)\tilde{\cup}(O_2,K)$ and $(O_1,K)\tilde{\cap}(O_2,K)\tilde{\subseteq}[(\tilde{Y},K)\setminus f_{pu}(G,E)]$. It follows by Theorems 2.2 and 2.3 that $(G,E)\tilde{\cap}f_{pu}^{-1}(O_1,K)\neq(\tilde{\phi},E)$, $(G,E)\tilde{\cap}f_{pu}^{-1}(O_2,K)\neq(\tilde{\phi},E)$, $(G,E)\tilde{\subseteq}f_{pu}^{-1}(O_1,K)\tilde{\cup}f_{pu}^{-1}(O_2,K)$ and $f_{pu}^{-1}(O_1,K)\tilde{\cap}f_{pu}^{-1}(O_2,K)\tilde{\subseteq}f_{pu}^{-1}[(\tilde{Y},K)\setminus f_{pu}(G,E)]$ $=(\tilde{X},E)\setminus(G,E)$. Since f_{pu} is a p -soft continuous, then $f_{pu}^{-1}(O_1,K),f_{pu}^{-1}(O_2,K)$ are p -open soft sets in (X,η_1,η_2,E). Hence, $f_{pu}^{-1}(O_1,K)\tilde{\cup}f_{pu}^{-1}(O_2,K)$ form a p -soft disconnection of (G,E) which contrary to the fact that (G,E) is a p -connected soft set in (X,η_1,η_2,E). Hence, $f_{pu}(G,E)$ is a p -connected soft set in (Y,σ_1,σ_2,K).

Corollary 5.3 *Let* f_{pu} *be a* p *-soft continuous mapping from a* p *-soft connected space* (X,η_1,η_2,E) *onto a sbts* (Y,σ_1,σ_2,K). *Then* (Y,σ_1,σ_2,K) *is a* p *-soft connected space.*

6. CONCLUSION

Soft bitopological spaces based on soft set which is a collection of information granules is the mathematical formulation of approximate reasoning about information systems. In this paper, we introduced and studied the notion of pairwise separated soft. Based on this notion, we defined and studied some properties and characterizations of pairwise soft connected spaces and pairwise connected soft sets in soft bitopological spaces. Some properties of such notions are obtained. We expect that the _findings in this paper can be promoted to the further study on soft bitopology to carry out general framework for the practical life applications.

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