



Palatini quadratic gravity: spontaneous breaking of gauged scale symmetry and inflation

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Abstract We study quadratic gravity $R^2 + R_{[\mu\nu]}^2$ in the Palatini formalism where the connection and the metric are independent. This action has a *gauged* scale symmetry (also known as Weyl gauge symmetry) of Weyl gauge field $v_\mu = (\tilde{\Gamma}_\mu - \Gamma_\mu)/2$, with $\tilde{\Gamma}_\mu$ (Γ_μ) the trace of the Palatini (Levi-Civita) connection, respectively. The underlying geometry is non-metric due to the $R_{[\mu\nu]}^2$ term acting as a gauge kinetic term for v_μ . We show that this theory has an elegant spontaneous breaking of gauged scale symmetry and mass generation in the absence of matter, where the necessary scalar field (ϕ) is not added ad-hoc to this purpose but is “extracted” from the R^2 term. The gauge field becomes massive by absorbing the derivative term $\partial_\mu \ln \phi$ of the Stueckelberg field (“dilaton”). In the broken phase one finds the Einstein–Proca action of v_μ of mass proportional to the Planck scale $M \sim \langle \phi \rangle$, and a positive cosmological constant. Below this scale v_μ decouples, the connection becomes Levi-Civita and metricity and Einstein gravity are recovered. These results remain valid in the presence of non-minimally coupled scalar field (Higgs-like) with Palatini connection and the potential is computed. In this case the theory gives successful inflation and a specific prediction for the tensor-to-scalar ratio $0.007 \leq r \leq 0.01$ for current spectral index n_s (at 95% CL) and $N = 60$ e-folds. This value of r is mildly larger than in inflation in Weyl quadratic gravity of similar symmetry, due to different non-metricity. This establishes a connection between non-metricity and inflation predictions and enables us to test such theories by future CMB experiments.

1 Introduction

At a fundamental level gravity may be regarded as a theory of connections. An example is the “Palatini approach” to gravity due to Einstein [1, 2], hereafter called EP approach [3–5].

In this case the “Palatini connection” ($\tilde{\Gamma}$) is a priori independent of the metric ($g_{\alpha\beta}$) and is actually determined by its equations of motion, from the action considered. For simple actions, $\tilde{\Gamma}$ plays an auxiliary role only, with no dynamics. For example, for an Einstein action in the EP approach the variation principle gives that $\tilde{\Gamma}$ is actually equal to the Levi-Civita connection (Γ). With this solution for $\tilde{\Gamma}$, one then recovers Einstein gravity – the metric formulation and EP approach are equivalent.

However, this equivalence is not true in general, for complicated actions, with matter present, etc. [6–21]. For example, for quadratic gravity actions of the type studied here in the EP approach, the equations of motion of $\tilde{\Gamma}$ become complicated *second-order* differential equations; further, some components of $\tilde{\Gamma}$ even become dynamical in a sense discussed shortly, etc. The question remains, however, if such general actions in the EP formalism and *in the absence of matter* can recover *dynamically* the Levi-Civita connection and Einstein gravity. If true, this would be similar to the original Weyl quadratic gravity theory [22–25] as we showed recently in [26, 27]. The main goal of this paper is to answer this question.

To address this question we study a gravity action in the EP approach with *gauged* scale symmetry also called Weyl gauge symmetry, see [26, 27] for an example.¹ This symmetry, first present in Weyl gravity [22–24] is important for mass generation, hence our interest. This symmetry demands us to consider quadratic gravity actions, with no dimensionful parameters. For such action we shall: (1) explain the spontaneous breaking of this symmetry and the emergence of Levi-Civita connection, Einstein gravity and Planck scale in the broken phase, even in the absence of matter. This answers the above question; (2) study the relation of this action to Weyl theory [22–24] of similar symmetry; (3) study its inflation predictions.

¹ See e.g. [22–47] for models with gauged scale symmetry and [48–72] for conformal or global scale symmetry.

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In Sect. 2 we first review the $R(\tilde{\Gamma}, g)^2$ gravity in the EP approach, where $R(\tilde{\Gamma}, g)$ denotes the scalar curvature in this formalism. This action is local scale invariant. The connection is shown to be conformally related to the Levi-Civita connection. When “fixing the gauge” of this symmetry, the “auxiliary” scalar field ϕ introduced to “linearise” the R^2 term decouples. As a result, one finds that $\tilde{\Gamma} = \Gamma$ and Einstein action is obtained.

In Sects. 3 and 4 we study the quadratic action $R(\tilde{\Gamma}, g)^2 + R_{[\mu\nu]}(\tilde{\Gamma})^2$ in the EP approach, hereafter called “EP quadratic gravity”. Here we used the notation $R_{[\mu\nu]} \equiv (R_{\mu\nu} - R_{\nu\mu})/2$. In this action the trace $\tilde{\Gamma}_\mu$ of the Palatini connection (assumed symmetric) is *dynamical* in the sense that $R_{[\mu\nu]}(\tilde{\Gamma})^2$ is a gauge kinetic term for $\tilde{\Gamma}_\mu$ or, more exactly, for the vector field² $v_\mu \sim \tilde{\Gamma}_\mu - \Gamma_\mu$, ($\tilde{\Gamma}_\mu \equiv \tilde{\Gamma}_{\mu\alpha}^\alpha$, $\Gamma_\mu \equiv \Gamma_{\mu\alpha}^\alpha$). With $\tilde{\Gamma}$ independent of $g_{\mu\nu}$, one notices that the local scale symmetry of this action is actually a *gauged* scale symmetry, of gauge field v_μ .

A consequence of the gauged scale symmetry is that EP quadratic gravity is non-metric³ i.e. $\tilde{\nabla}_\mu g_{\alpha\beta} \neq 0$. This is due to a dynamical $v_\mu \sim \tilde{\Gamma}_\mu$ and v_μ is the non-metricity field, also called Weyl gauge field. Further, we find that the equations of motion for $\tilde{\Gamma}$ are *second-order* differential equations. In this case the usual EP approach in $f(R)$ theories to solve algebraically for $\tilde{\Gamma}$ [4, 5] does not work, due to local scale symmetry and non-metricity. Nevertheless, we compute $\tilde{\Gamma}$ and find that EP quadratic gravity with $\tilde{\Gamma}$ *onshell* is equivalent to a ghost-free second-order gauged scale invariant theory with an *additional* dynamical field (“dilaton”). (Expressed in terms of this field the differential equations of $\tilde{\Gamma}$ simplify considerably and this is how they are solved).

The main result of this work (Sect. 3) is that the gauged scale invariance of the above action is broken spontaneously by a new mechanism [26, 27] valid even in the absence of matter; in this, the necessary scalar field (ϕ) is *not* added ad-hoc to this purpose (as usually done), but is “extracted” from the R^2 term in the action; ϕ is thus of geometric origin. After a Stueckelberg mechanism [74–76] the gauge field v_μ becomes massive, of mass m_v near the Planck scale $M \sim \langle \phi \rangle$, by “absorbing” the derivative $\partial_\mu \ln \phi$ of the Stueckelberg field (also referred to as “dilaton”). Near the Planck scale we obtain the Einstein–Proca action of v_μ . Further, below the scale $m_v \propto M$, the field v_μ decouples and we recover metricity, Levi-Civita connection and Einstein gravity; the Planck scale M is then an *emergent* scale where this sym-

metry is broken. These results remain true if the theory also has matter fields (Higgs, etc.) non-minimally coupled with *Palatini connection*, while respecting gauged scale invariance (Sect. 4). Briefly, the EP quadratic gravity is a gauged scale invariant theory broken à la Stueckelberg, even in the absence of matter, to an Einstein–Proca action with a positive cosmological constant and a potential for the scalar fields – if present. This answers the main goal of the paper.

Another theory where the connection is not determined by the metric itself is the original Weyl quadratic gravity of gauged scale invariance [22–24] (also [25]). With hindsight, it is then not too surprising that the above results are similar to those in [26–28] for Weyl theory. This theory came under early criticism from Einstein [22] for its non-metricity implying e.g. changes of the atomic spectral lines, in contrast to experiment; however, if the Weyl “photon” (v_μ) of non-metricity is actually massive (mass $\sim M$) by the same Stueckelberg mechanism, metricity and Einstein gravity are recovered below its decoupling scale (\sim Planck scale). Non-metricity effects are then strongly suppressed by a large M (their current lower bound seems low [77, 78]). Hence, the long-held criticisms that have implicitly assumed v_μ be massless are actually avoided and Weyl gravity is then viable [26–28]. As outlined, in this work we obtain similar results in EP quadratic gravity, up to different non-metricity effects.

We also study inflation in EP quadratic gravity (Sect. 5). We consider this theory with an extra scalar field (Higgs-like) with perturbative non-minimal coupling and Palatini connection, that plays the role of the inflaton. We compute the potential after the gauged scale symmetry breaking. With the Planck scale a simple phase transition scale in our theory, field values above M are natural. Interestingly, the inflaton potential is similar to that in Weyl quadratic gravity [28], up to couplings and field redefinitions (due to a different non-metricity of the theory). Inflation in EP quadratic gravity has a specific prediction for the tensor-to-scalar ratio $0.007 \leq r \leq 0.010$ for the current spectral index n_s at 95% CL. This range of r is distinct from that predicted by inflation in Weyl gravity [28, 47] and will soon be reached by CMB experiments [79–81]. The conclusions are presented in Sect. 6 followed by an Appendix.

2 Palatini R^2 gravity

For later reference we first review R^2 gravity in the EP formalism [82, 83]. As discussed below, the action is local scale invariant (unlike its Riemannian counterpart):

$$L_1 = \sqrt{g} \frac{\xi_0}{4!} R(\tilde{\Gamma}, g)^2, \quad \xi_0 > 0, \quad (1)$$

² Unlike $\tilde{\Gamma}_\mu$ and Γ_μ , $v_\mu \propto \tilde{\Gamma}_\mu - \Gamma_\mu$ is indeed a vector (see Appendix). We assume $\tilde{\Gamma}_{\mu\nu}^\alpha = \tilde{\Gamma}_{\nu\mu}^\alpha$ (no torsion).

³ Non-metricity means that under parallel transport along a curve a vector changes its norm i.e. is path dependent; therefore, in realistic theories with matter present (as here) it must be suppressed by a large scale (e.g. Planck) to avoid atomic spectral lines changes that it would otherwise induce [22]. In the absence of matter non-metricity can be traded for torsion in related R^2 theory [73].

where

$$R(\tilde{\Gamma}, g) = g^{\mu\nu} R_{\mu\nu}(\tilde{\Gamma}),$$

$$R_{\mu\nu}(\tilde{\Gamma}) = \partial_\lambda \tilde{\Gamma}^\lambda_{\mu\nu} - \partial_\mu \tilde{\Gamma}^\lambda_{\lambda\nu} + \tilde{\Gamma}^\lambda_{\rho\lambda} \tilde{\Gamma}^\rho_{\mu\nu} - \tilde{\Gamma}^\lambda_{\rho\mu} \tilde{\Gamma}^\rho_{\nu\lambda} \quad (2)$$

$R_{\mu\nu}(\tilde{\Gamma})$ is the metric-independent Ricci tensor in the EP formalism. Our conventions are as in [84] with metric $(+, -, -, -)$, $g \equiv |\det g_{\mu\nu}|$ and we assume there is no torsion i.e. $\tilde{\Gamma}^\rho_{\mu\nu} = \tilde{\Gamma}^\rho_{\nu\mu}$.

There is an equivalent “linearised” version of L_1 , found by using an auxiliary field ϕ

$$L_1 = \sqrt{g} \frac{\xi_0}{4!} \{-2\phi^2 R(\tilde{\Gamma}, g) - \phi^4\}. \quad (3)$$

Indeed, (1) is recovered if we use in (3) the solution $\phi^2 = -R(\tilde{\Gamma}, g)$ of the equation of motion of the scalar field ϕ . With the connection $\tilde{\Gamma}$ independent of the metric, (3) and (1) have local scale symmetry i.e. are invariant under a Weyl transformation $\Omega = \Omega(x)$ with⁴

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \sqrt{\hat{g}} = \Omega^4 \sqrt{g},$$

$$\hat{\phi} = \frac{1}{\Omega} \phi, \quad \hat{R}(\tilde{\Gamma}, \hat{g}) = \frac{1}{\Omega^2} R(\tilde{\Gamma}, g). \quad (4)$$

Unlike in the metric case, $R_{\mu\nu}(\tilde{\Gamma})$ is invariant under (4) while $R(\tilde{\Gamma}, g)$ transforms covariantly, hence (1) and (3) are invariant. L_1 has a shift symmetry: $\ln \phi \rightarrow \ln \phi - \ln \Omega$. In global cases $\ln \phi$ is the dilaton field generating a mass scale from its vev (assumed to be non-zero); here, $\ln \phi$ is similar to a would-be Goldstone, as seen if we “gauge” symmetry (4) (see later, Eq. (18)).

Let us solve the equation of motion for $\tilde{\Gamma}$, then find the action for $\tilde{\Gamma}$ onshell.⁵ The change of $R_{\mu\nu}(\tilde{\Gamma})$ under a variation of the connection is $\delta R_{\mu\nu}(\tilde{\Gamma}) = \tilde{\nabla}_\lambda (\delta \tilde{\Gamma}^\lambda_{\mu\nu}) - \tilde{\nabla}_\nu (\delta \tilde{\Gamma}^\lambda_{\mu\lambda})$, where the operator $\tilde{\nabla}$ is defined with connection $\tilde{\Gamma}$. Then from (3) the equation of motion of $\tilde{\Gamma}^\lambda_{\mu\nu}$ gives

$$\tilde{\nabla}_\lambda (\sqrt{g} g^{\mu\nu} \phi^2) - \frac{1}{2} \left[\tilde{\nabla}_\rho (\sqrt{g} g^{\rho\mu} \phi^2) \delta_\lambda^\nu + (\mu \leftrightarrow \nu) \right] = 0. \quad (5)$$

Setting $\nu = \lambda$ and then summing over, then⁶

$$\tilde{\nabla}_\rho (\sqrt{g} g^{\rho\mu} \phi^2) = 0. \quad (6)$$

⁴ From $\phi^2 = -R(\tilde{\Gamma}, g)$, ϕ^2 transforms under metric rescaling like $R(\tilde{\Gamma}, g)$, as expected for a scalar field.

⁵ Obviously, with $\Omega^2 = \xi_0 \phi^2 / (6M^2)$ (with M the Planck scale), one can set ϕ to a constant (fix the “gauge” of local scale symmetry). L_1 becomes $L_1 = \sqrt{-g} \{- (1/2) M^2 R(\tilde{\Gamma}, g) - 3 / (2\xi_0) M^4\}$. This is the Palatini formulation of Einstein action; via equations motion then $\tilde{\nabla}_\mu g_{\alpha\beta} = 0$ where $\tilde{\nabla}_\mu$ is computed with $\tilde{\Gamma}$. Hence $\tilde{\Gamma}$ is a Levi-Civita connection. However, this approach obscures the role of local scale symmetry, relevant later.

⁶ If we use $\phi^2 = -R(\tilde{\Gamma}, g)$ in (5) and (6), one recovers the equation of motion of $\tilde{\Gamma}$ found directly from (1) [82].

To simplify notation, introduce an auxiliary dimensionful “metric” $h_{\mu\nu} \equiv \phi^2 g_{\mu\nu}$, then

$$\tilde{\nabla}_\lambda (\sqrt{h} h^{\mu\nu}) = 0. \quad (7)$$

This means that in terms of $h_{\mu\nu}$, the connection is Levi-Civita⁷

$$\tilde{\Gamma}^\alpha_{\mu\nu}(h) = (1/2) h^{\alpha\lambda} (\partial_\mu h_{\lambda\nu} + \partial_\nu h_{\lambda\mu} - \partial_\lambda h_{\mu\nu}), \quad (9)$$

or, in terms of $g_{\mu\nu}$

$$\tilde{\Gamma}^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu}(g) + (1/2) (\delta_\nu^\alpha u_\mu + \delta_\mu^\alpha u_\nu - g^{\alpha\lambda} g_{\mu\nu} u_\lambda),$$

$$u_\mu \equiv \partial_\mu \ln \phi^2, \quad (10)$$

with Levi-Civita $\Gamma^\alpha_{\mu\nu}(g) = (1/2) g^{\alpha\lambda} (\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu})$. Next, if we use the equation of motion of ϕ of solution $\phi^2 = -R(\tilde{\Gamma}, g)$, Eq. (10) for $\tilde{\Gamma}$ (also (5), (6)) becomes a second-order differential equation since $\partial \phi^2 \sim \partial R \sim \partial^2 \tilde{\Gamma}$, and it is difficult to solve (and since solution $\tilde{\Gamma}$ of (10) involves $\partial g_{\mu\nu}$ from $\Gamma(g)$ then for $\tilde{\Gamma}$ onshell action (1) is a four-derivative theory in $g_{\mu\nu}$). An easy way out is to keep ϕ an independent variable hereafter (no use of its equation of motion), then Eqs. (5) and (6) have solution $\tilde{\Gamma}$ given by the rhs of (10). For this solution, then

$$R(\tilde{\Gamma}, g) = R(g) - 3 \nabla_\mu u^\mu - \frac{3}{2} g^{\mu\nu} u_\mu u_\nu, \quad (11)$$

with the Ricci scalar $R(g)$ for $g_{\mu\nu}$ while ∇ is defined with the Levi-Civita connection (Γ). Using (11) in (3) of the same metric, we find for $\tilde{\Gamma}$ onshell⁸

$$L_1 = \sqrt{g} \left\{ \frac{\xi_0}{2} \left[-\frac{1}{6} \phi^2 R(g) - (\partial_\mu \phi)^2 - \frac{1}{12} \phi^4 \right] \right\}. \quad (12)$$

L_1 is a second order theory with an additional dynamical variable demanded by symmetry (4) and is equivalent to action (1) which for $\tilde{\Gamma}$ onshell is a four-derivative theory, as noticed.

Lagrangian (12) has local scale symmetry so one may like to “fix the gauge”. We choose the Einstein or unitarity gauge reached by a ϕ -dependent transformation $\Omega^2 = \phi^2 / \langle \phi \rangle^2$ that is gauge-fixing ϕ to a constant (vev); in this gauge $M^2 = \xi_0 \langle \phi \rangle^2 / 6$ is the Planck mass. From (12)

$$L_1 = \sqrt{\hat{g}} \left\{ \frac{-1}{2} M^2 \hat{R}(\hat{g}) - \frac{3}{2\xi_0} M^4 \right\}. \quad (13)$$

⁷ One shows $\tilde{\nabla} h_{\mu\nu} = 0$ by using

$$\tilde{\nabla}_\lambda h^{\mu\nu} = -h^{\mu\sigma} h^{\nu\rho} \tilde{\nabla}_\lambda h_{\sigma\rho}, \quad \text{and} \quad \tilde{\nabla}_\lambda \sqrt{h} = (1/2) \sqrt{h} h^{\alpha\beta} \tilde{\nabla}_\lambda h_{\alpha\beta}. \quad (8)$$

⁸ From (12) the equation of motion for $g^{\mu\nu}$ and its trace give that on the ground state $\phi^2 = -R$ and $R(R_{\mu\nu} - 1/4 g_{\mu\nu} R) = 0$ which is similar to that found directly from equivalent action (1), see also footnote 6.

Hence Einstein action (13) is recovered as a gauge fixed form of (12); symmetry (4) is now spontaneously broken and ϕ decouples⁹ [85]; this may be expected since the local scale symmetry current of (12) is vanishing [86–88] (this will change in Sect. 3.3). With ϕ “gauge fixed” to a constant, Eqs. (7) and (10) give $h_{\mu\nu} \propto g_{\mu\nu}$ and $\tilde{\Gamma} = \Gamma$ so the theory is metric.^{10,11}

3 Palatini quadratic gravity with gauged scale symmetry

3.1 The Lagrangian and its expression for onshell $\tilde{\Gamma}$

Consider now the following EP quadratic gravity, with $\alpha = \text{constant}$ and $R_{[\mu\nu]} \equiv (R_{\mu\nu} - R_{\nu\mu})/2$

$$L_2 = \sqrt{g} \left\{ \frac{\xi_0}{4!} R(\tilde{\Gamma}, g)^2 - \frac{1}{4\alpha^2} R_{[\mu\nu]}(\tilde{\Gamma}) R^{\mu\nu}(\tilde{\Gamma}) \right\}. \quad (14)$$

With $R_{\mu\nu}(\tilde{\Gamma})$ from Eq. (2) and $\tilde{\Gamma}_{\mu\nu}^\alpha$ symmetric in (μ, ν) , L_2 has a more intuitive form

$$L_2 = \sqrt{g} \left\{ \frac{\xi_0}{4!} R(\tilde{\Gamma}, g)^2 - \frac{1}{4\alpha^2} F_{\mu\nu}(\tilde{\Gamma}) F^{\mu\nu}(\tilde{\Gamma}) \right\}. \quad (15)$$

This is a natural extension of L_1 of Eq. (1), with the second term above indicating we now have a dynamical trace ($\tilde{\Gamma}_\mu$) of the Palatini connection, as seen from the notation below:

$$F_{\mu\nu}(\tilde{\Gamma}) = \tilde{\nabla}_\mu v_\nu - \tilde{\nabla}_\nu v_\mu; \quad v_\mu = (1/2)(\tilde{\Gamma}_\mu - \Gamma_\mu(g)), \quad (16)$$

with $\tilde{\Gamma}_\mu \equiv \tilde{\Gamma}_{\mu\lambda}^\lambda$ and $\Gamma_\mu \equiv \Gamma_{\mu\lambda}^\lambda$. Since $\tilde{\Gamma}_{\mu\nu}^\alpha = \tilde{\Gamma}_{\nu\mu}^\alpha$ and $\tilde{\nabla}_\mu v_\nu = \partial_\mu v_\nu - \tilde{\Gamma}_{\mu\nu}^\alpha v_\alpha$, then we have $F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu = (\partial_\mu \tilde{\Gamma}_\nu - \partial_\nu \tilde{\Gamma}_\mu)/2 = -R_{[\mu\nu]}$, and Eqs. (14) and (15) are equivalent. While $\Gamma_\mu(g)$ does not contribute to $F_{\mu\nu}(\tilde{\Gamma})^2$, it is needed to ensure that v_μ is a vector under coordinate transformation (which is not true for $\tilde{\Gamma}_\mu$ or Γ_μ , see Appendix). v_μ is the Weyl field¹² and measures the trace of the deviation of the Palatini connection $\tilde{\Gamma}$ from Levi-Civita connection $\Gamma(g)$. L_2 is quadratic in R but for $\tilde{\Gamma}$ offshell resembles a second order theory.

⁹ There is no gauge field here to “eat” would-be Goldstone $\ln \phi$ but in Sect. 3, v_μ of (18) will “eat” $\ln \phi$.

¹⁰ This result is also valid for Palatini $f(R)$ action instead of (1); we do not consider it here since it violates Weyl scale symmetry, but (unlike here) the trace of the equation of motion of $g^{\mu\nu}$ is non-trivial, giving $f'(R) = \text{constant}$, which fixes R , then $h_{\mu\nu} \propto g_{\mu\nu}$, $\tilde{\Gamma} = \Gamma$ so metricity/Einstein action is recovered [3–5].

¹¹ This situation is different from the Riemannian $R(g)^2$ gravity [89] (Eq. 2.11), also [90], where in the Einstein frame a kinetic term for ϕ remains present in (13) (being absent in Eq. (12)).

¹² Definition (16) of gauge field v_μ is general, it also applies to Weyl gravity of similar symmetry (Appendix).

As in previous section, write L_2 in an equivalent “linearised” form useful later on

$$L_2 = \sqrt{g} \left\{ -\frac{\xi_0}{12} \phi^2 R(\tilde{\Gamma}, g) - \frac{1}{4\alpha^2} F_{\mu\nu}(\tilde{\Gamma})^2 - \frac{\xi_0}{4!} \phi^4 \right\}. \quad (17)$$

The equation of motion for ϕ has solution $\phi^2 = -R(\tilde{\Gamma}, g)$ which replaced in L_2 recovers (15).

Since $\tilde{\Gamma}$ does not transform under (4) and with $\Gamma_\mu(g) = \partial_\mu \ln \sqrt{g}$ that follows from the definition of Levi-Civita connection, then L_2 is invariant under (4) extended by

$$\hat{v}_\lambda = v_\lambda - \partial_\mu \ln \Omega^2. \quad (18)$$

The invariance of L_2 under transformations (4) and (18), is referred to as *gauged* scale invariance or Weyl gauge symmetry, with a (dilatation) group isomorphic to \mathbf{R}^+ , as in Weyl gravity.

Let us then compute the connection $\tilde{\Gamma}_{\mu\nu}^\lambda$ from its equation of motion which is

$$\tilde{\nabla}_\lambda(\sqrt{g} g^{\mu\nu} \phi^2) - \left\{ \frac{1}{2} \delta_\lambda^\nu \left[\tilde{\nabla}_\rho(\sqrt{g} g^{\mu\rho} \phi^2) - \frac{6\sqrt{g}}{\alpha^2 \xi_0} \nabla_\rho F^{\rho\mu} \right] + (\mu \leftrightarrow \nu) \right\} = 0. \quad (19)$$

Here $\tilde{\nabla}_\mu$ and ∇_μ are evaluated with the Palatini ($\tilde{\Gamma}$) and Levi-Civita (Γ) connections, respectively. Setting $\lambda = \nu$ and summing over gives (compare against Eq. (6))

$$\tilde{\nabla}_\rho(\sqrt{g} g^{\mu\rho} \phi^2) = \frac{10}{\alpha^2} \frac{1}{\xi_0} \sqrt{g} \nabla_\rho F^{\rho\mu}, \quad (20)$$

which is an equation of motion for the trace $\tilde{\Gamma}_\mu \sim v_\mu$. Replacing (20) back in (19) leads to

$$\tilde{\nabla}_\lambda(\sqrt{g} g^{\mu\nu} \phi^2) - \frac{1}{5} \left\{ \delta_\lambda^\nu \tilde{\nabla}_\rho(\sqrt{g} g^{\rho\mu} \phi^2) + (\mu \leftrightarrow \nu) \right\} = 0. \quad (21)$$

Therefore, the set of Eq. (19) is equivalent to the combined set of Eqs. (21) and (20).¹³

Let us find $\tilde{\Gamma}_{\mu\nu}^\lambda$ from (21). Note that if one used the equation of motion of ϕ of solution $\phi^2 = -R(\tilde{\Gamma}, g)$, then (21) would be a *second-order* differential equation for $\tilde{\Gamma}_{\mu\nu}^\alpha$, since $\tilde{\nabla}_\lambda \phi^2 \sim \partial \phi^2 \sim \partial R(\tilde{\Gamma}, g) \sim \partial^2 \tilde{\Gamma}$, with further complications. It is however easier to simply regard ϕ hereafter as an independent variable¹⁴ (i.e. no use of its equation of motion) in terms of which one then easily computes $\tilde{\Gamma}$ algebraically,

¹³ Unlike in (19), setting $\lambda = \nu$ in (21) brings no information – this was “moved” into 4 equations in (20).

¹⁴ The consequence of doing so is that ϕ acquires a kinetic term and becomes dynamical (see also Sect. 2).

as we do below. To find a solution to (21) we first introduce, based on an approach of [8]:

$$\tilde{\nabla}_\lambda \left(\sqrt{g} g^{\mu\nu} \phi^2 \right) = (-2) \sqrt{g} \phi^2 (\delta_\lambda^\mu V^\nu + \delta_\lambda^\nu V^\mu), \tag{22}$$

where V_μ is some arbitrary vector field (to be determined later). V_μ is introduced since, due to underlying symmetry, Eq. (21) with $\lambda = \nu$ summed over is automatically respected for fixed μ ($= 0, 1, 2, 3$); this is leaving four undetermined components, accounted for by V_μ . Further, if in Eq. (21) one replaces $\tilde{\nabla}(\dots)$ terms by the rhs of (22) one easily shows that (21) is indeed verified. Hence, instead of finding $\tilde{\Gamma}$ from (21), it is sufficient to compute $\tilde{\Gamma}$ from (22),¹⁵ which is easier. To this end, multiply (22) by $g_{\mu\nu}$ and use $g_{\mu\nu} \tilde{\nabla}_\lambda g^{\mu\nu} = -2 \tilde{\nabla}_\lambda \ln \sqrt{g}$, to find that

$$V_\lambda = -(1/2) \tilde{\nabla}_\lambda \ln \left(\sqrt{g} \phi^4 \right). \tag{23}$$

From (22) and (23)

$$\tilde{\nabla}_\lambda (\phi^2 g_{\mu\nu}) = (-2) (g_{\mu\nu} V_\lambda - g_{\mu\lambda} V_\nu - g_{\nu\lambda} V_\mu) \phi^2, \tag{24}$$

so the theory is non-metric. From (24) we find the solution¹⁶ $\tilde{\Gamma}$ to (21) in terms of V_λ :

$$\begin{aligned} \tilde{\Gamma}_{\mu\nu}^\alpha &= \Gamma_{\mu\nu}^\alpha (\phi^2 g) - (3 g_{\mu\nu} V_\lambda - g_{\nu\lambda} V_\mu - g_{\lambda\mu} V_\nu) g^{\lambda\alpha}, \\ &\text{with } \Gamma_{\mu\nu}^\alpha (\phi^2 g) = \Gamma_{\mu\nu}^\alpha (g) \\ &+ 1/2 (\delta_\nu^\alpha \partial_\mu + \delta_\mu^\alpha \partial_\nu - g^{\alpha\lambda} g_{\mu\nu} \partial_\lambda) \ln \phi^2. \end{aligned} \tag{25}$$

$\Gamma_{\mu\nu}^\alpha (g)$ is Levi-Civita connection of $g_{\mu\nu}$. From (25), $\tilde{\Gamma}_\lambda = \Gamma_\lambda(\phi^2 g) + 2V_\lambda$ and with (16) and (23)

$$v_\lambda = -(1/2) \tilde{\nabla}_\lambda \ln \sqrt{g}, \tag{26}$$

and finally, $V_\lambda = v_\lambda - \partial_\lambda \ln \phi^2$. With this relation between V_λ and v_λ , the solution $\tilde{\Gamma}$ in (25) is finally expressed as a function of v_λ, ϕ , and will be used shortly to compute the action for $\tilde{\Gamma}$ onshell (see Eq. (29) below¹⁷). Notice that solution $\tilde{\Gamma}$ of (25) and also (24), are invariant under transformations (4) and (18) for any $\Omega(x)$ since $\phi^2 g_{\mu\nu}, V_\lambda, \sqrt{g} \phi^4$ are invariant.

¹⁵ One cannot solve algebraically (22) as done in Palatini $f(R)$ theories [3–5] due to non-vanishing rhs (dynamical $\tilde{\Gamma}_\mu$) and to the conformal symmetry of L_2 , absent in $f(R)$ theories, see discussion in [91], p.5–6.

¹⁶ Use that $\tilde{\nabla}_\lambda g_{\mu\nu} = \partial_\lambda g_{\mu\nu} - \tilde{\Gamma}_{\mu\lambda}^\rho g_{\rho\nu} - \tilde{\Gamma}_{\nu\lambda}^\rho g_{\mu\rho}$, for cyclic permutations of indices and combine them.

¹⁷ As a remark, recall that Eq. (19) for $\tilde{\Gamma}$ were shown to be equivalent to the combined set of (21), (20) and we solved (21) with solution $\tilde{\Gamma}_{\mu\nu}^\alpha$ in (25) expressed in terms of $V_\mu \sim \tilde{\Gamma}_\mu$. We have four remaining equations in (20) for $\tilde{\Gamma}_\mu$ itself, that could in principle be used to also “fix” $\tilde{\Gamma}_\mu \sim V_\mu$ or equivalently v_μ , since $V_\mu = v_\mu - \partial_\mu \ln \phi^2$; however we do not do this step since v_μ is a massless, dynamical gauge field enforcing gauged scale symmetry (18) of initial action (14), (17). Then what information does (20) bring? With Eq. (22) for $\lambda = \nu$, Eq. (20) is actually $\tilde{\nabla}_\rho F^{\rho\mu} + \alpha^2 \xi_0 \phi^2 V^\mu = 0$, which is just the equation of motion of v_λ ; this may be seen from final Lagrangian (29) which has all $\tilde{\Gamma}_{\mu\nu}^\alpha$ onshell (expressed in terms of $\tilde{\Gamma}_\mu$) but $\tilde{\Gamma}_\mu$ is kept offshell, for the reason mentioned; v_λ may be integrated out after becoming massive, see later.

As expected, v_λ is the Weyl field of non-metricity defined as $Q_{\lambda\mu\nu} \equiv \tilde{\nabla}_\lambda g_{\mu\nu}$, since from (26) the trace $Q_{\lambda\mu}^\mu = -4 v_\lambda$. Non-metricity is a consequence of the dynamical v_λ , see (20). Equation (26) is similar to that in Weyl quadratic gravity of same symmetry (e.g. [33]).

Finally, from solution (25) and (2) we compute $R_{\mu\nu}(\tilde{\Gamma})$ and scalar curvature¹⁸ $R(\tilde{\Gamma}, g)$

$$\begin{aligned} R(\tilde{\Gamma}, g) &= R(g) - 6g^{\mu\nu} \nabla_\mu \nabla_\nu \ln \phi - 6(\nabla_\mu \ln \phi)^2 \\ &\quad - 12(\nabla_\lambda V^\lambda + V^\lambda \partial_\lambda \ln \phi^2) - 6V_\mu V^\mu. \end{aligned} \tag{28}$$

$R(g)$ is here the usual Ricci scalar and $V_\lambda = w_\lambda - \partial_\lambda \ln \phi^2$. Using (28) in (17), then finally

$$\begin{aligned} L_2 &= \sqrt{g} \left\{ -\frac{\xi_0}{12} [\phi^2 R(g) + 6(\partial_\mu \phi)^2] \right. \\ &\quad \left. + \frac{\xi_0}{2} \phi^2 (v_\mu - \partial_\mu \ln \phi^2)^2 - \frac{1}{4\alpha^2} F_{\mu\nu}^2 - \frac{\xi_0}{4!} \phi^4 \right\}. \end{aligned} \tag{29}$$

This is the “onshell” Lagrangian of EP quadratic gravity of Eq. (14) and is gauged scale invariant. L_2 is a second-order scalar–vector–tensor theory of gravity which is ghost-free according to [92] for a torsion-free connection as here (this is also obvious from (30) below). This is relevant since initial action (14) which (offshell) was of second order is actually a four-derivative theory in the metric¹⁹ for $\tilde{\Gamma}$ onshell; indeed, $R(\tilde{\Gamma}, g)^2$ in (14) with replacement (28) contains the higher derivative term $R^2(g) + \dots$; this four-derivative theory has an equivalent second-order formulation with additional ϕ , as shown in Eq. (29). Finally, if $v_\mu = \partial_\mu \ln \phi^2$ (“pure gauge”), the model is Weyl integrable and (29) recovers (12).

Lagrangian (29) (also initial (15)) is similar to that of Weyl quadratic gravity [26,27], up to a Weyl tensor-squared term not included here. However, unlike in Weyl theory, here $\tilde{\Gamma}$ is ϕ -dependent; also, in Weyl theory non-metricity follows from the underlying Weyl conformal geometry, while here it emerges after we determine $\tilde{\Gamma}$ from its equation of motion.

3.2 Stueckelberg breaking to Einstein–Proca action

Given L_2 in (29) with gauged scale symmetry we would like to “fix the gauge”. We choose the Einstein gauge obtained from (29) by transformations (4) and (18) of a special $\Omega^2 =$

¹⁸ $R_{\mu\nu}(\tilde{\Gamma})$ has the following expression (which by contraction with $g^{\mu\nu}$ gives $R(\tilde{\Gamma}, g)$ of (28)):

$$\begin{aligned} R_{\mu\nu}(\tilde{\Gamma}) &= R_{\mu\nu}(g) - 3g_{\mu\nu} (\nabla^\lambda V_\lambda + V^\lambda \partial_\lambda \theta) \\ &\quad - (\nabla_\mu V_\nu - \nabla_\nu V_\mu) - 6V_\mu V_\nu + 1/2 (\partial_\mu \theta)(\partial_\nu \theta) \\ &\quad - 1/2 g_{\mu\nu} g^{\alpha\beta} (\partial_\alpha \theta)(\partial_\beta \theta) - 1/2 g_{\mu\nu} \nabla^\lambda \partial_\lambda \theta \\ &\quad + 1/2 \nabla_\nu \partial_\mu \theta - 3/2 \nabla_\mu \partial_\nu \theta, \quad \theta \equiv \ln \phi^2. \end{aligned} \tag{27}$$

¹⁹ This agrees with e.g. [19] that in general in a Palatini model its metric part leads to a fourth order theory.

$\xi_0 \phi^2 / (6M^2)$ fixing ϕ to a constant ($\langle \phi \rangle \neq 0$). After removing the hats ($\hat{\cdot}$) on transformed g, v_μ, R , we find

$$L_2 = \sqrt{g} \left\{ -\frac{1}{2} M^2 R(g) + 3 M^2 v_\mu v_\nu g^{\mu\nu} - \frac{1}{4\alpha^2} F_{\mu\nu}^2 - \frac{3}{2\xi_0} M^4 \right\}. \tag{30}$$

This is the Einstein–Proca action for the gauge field v_μ with a positive cosmological constant, in which we identified M with the Planck scale (M) as seen from Eq. (29)

$$M^2 \equiv \xi_0 \langle \phi \rangle^2 / 6. \tag{31}$$

The initial gauged scale invariance is broken by a gravitational Stueckelberg mechanism [74–76]: the massless ϕ is not part of the action anymore, but v_μ has become massive, after “absorbing” the derivative $\partial_\mu(\ln \phi)$ of the Stueckelberg field (dilaton) in Eq. (29). Note that $\partial_\mu(\ln \phi)$ is actually the Goldstone of special conformal symmetry – this Goldstone is not independent but is determined by the derivative of the dilaton [93]. The number of degrees of freedom (dof) other than graviton is conserved in going from (29) to (30), as it should be for spontaneous breaking: massless v_μ and dynamical ϕ are replaced by massive v_μ (dof = 3). The mass of v_μ is $m_v^2 = 6\alpha^2 M^2$ which is near Planck scale M (unless one fine-tunes $\alpha \ll 1$).

Using the same transformation Ω , from (24)

$$\tilde{\nabla}_\lambda g_{\mu\nu} = (-2)(g_{\mu\nu} v_\lambda - g_{\mu\lambda} v_\nu - g_{\nu\lambda} v_\mu). \tag{32}$$

This has a solution $\tilde{\Gamma}$ that is immediate from (25) for ϕ constant and V_λ replaced by v_λ . Finally, after the massive field v_μ decouples, metricity is recovered below m_v , so $\tilde{\nabla}_\lambda g_{\mu\nu} = 0$ and $\tilde{\Gamma} = \Gamma(g)$. Briefly, Einstein action is a “low energy” limit of Einstein–Palatini quadratic gravity, and the Planck scale $M \sim \langle \phi \rangle$ is a phase transition scale (up to coupling α).²⁰

For comparison, in Weyl quadratic gravity e.g. [26,27], non-metricity is different²¹

$$\tilde{\nabla}_\lambda g_{\mu\nu} = -g_{\mu\nu} v_\lambda. \tag{33}$$

Interestingly the different non-metricity of these theories (giving different $\tilde{\Gamma}$) has phenomenological impact, see Sect. 5. In both theories the non-metricity scale is $m_v \sim$ Planck scale and is large enough (current bounds [77,78] are low \sim TeV) to suppresses unwanted effects e.g. atomic spectral lines spacing. Past critiques of non-metricity assumed a massless v_μ .

²⁰ A special case: consider (17) with $\phi^2 = 6M^2/\xi_0 = \text{constant}$, i.e. a different initial action with no symmetry! then (28) and (29) simplify; we still find (30) but there is no dynamical ϕ and thus no Stueckelberg mechanism.

²¹ Contracting (32) and (33) by $g^{\mu\nu}$ gives the same non-metricity trace, justifying our normalization of v_μ Eq. (16).

Finally, let us remark that the above spontaneous symmetry breaking mechanism for initial action (14) is special since it takes place *in the absence of matter*. Indeed, the necessary scalar (Stueckelberg) field $\ln \phi$ was not added ad-hoc to this purpose, as usually done in the literature; instead, this field was “extracted” from the R^2 term in the initial, symmetric action (14) and is thus of geometric origin. This situation is similar to Weyl quadratic gravity where this mechanism was first noticed [26,27].

3.3 Conserved current

Equations (20) and (22) show there is now a non-trivial current due to dynamical $v_\mu \sim \tilde{\Gamma}_\mu$

$$J^\mu = \sqrt{g} g^{\rho\mu} \phi (\partial_\rho - 1/2 v_\rho) \phi, \quad \nabla_\mu J^\mu = 0, \tag{34}$$

This is conserved since $F_{\mu\nu}$ in (20) is anti-symmetric. To obtain (34) we used that the lhs of (20) and of (22) (with $\lambda = \nu$) are equal and replaced $V_\lambda = v_\lambda - \partial_\lambda \ln \phi^2$. The current J^μ is the same as that in Weyl quadratic gravity [26] (Eq. 18) which has similar symmetry but different non-metricity. The presence of this conserved current extends to the case of the *gauged* scale symmetry a similar conservation for a global scale symmetry [64]. For a Friedmann–Robertson–Walker metric with ϕ only t -dependent such current conservation in the global case naturally leads to $\phi = \text{constant}$ [64] and a breaking of scale symmetry. In our case, since Eq. (30) has $\phi = \text{constant}$ (assumed $\langle \phi \rangle \neq 0$), then from (34) one has $\nabla_\mu v^\mu = 0$ which is a condition similar to that for a Proca (massive) gauge field, leaving 3 degrees of freedom for v_μ in (30).

4 Palatini quadratic gravity: adding matter

In this section we re-do the previous analysis in the presence of a scalar χ which can be the SM Higgs, with non-minimal coupling with *Palatini connection* to the EP quadratic gravity.

The general Lagrangian of the field χ , with gauged scale invariance, Eqs. (4) and (18) is

$$L_3 = \sqrt{g} \left[\frac{\xi_0}{4!} R(\tilde{\Gamma}, g)^2 - \frac{1}{4\alpha^2} F_{\mu\nu}^2 - \frac{1}{12} \xi_1 \chi^2 R(\tilde{\Gamma}, g) + \frac{1}{2} (\tilde{D}_\mu \chi)^2 - \frac{\lambda_1}{4!} \chi^4 \right], \tag{35}$$

with the potential dictated by this symmetry and with

$$\tilde{D}_\mu \chi = (\partial_\mu - 1/2 v_\mu) \chi. \tag{36}$$

Under (4) and (18) the Weyl-covariant derivative transforms as $\hat{D}_\mu \hat{\chi} = (1/\Omega) \tilde{D}_\mu \chi$. As in previous sections, replace $R(\tilde{\Gamma}, g)^2 \rightarrow -2\phi^2 R(\tilde{\Gamma}, g) - \phi^4$ to find an equivalent “lin-

earised” L_3

$$L_3 = \sqrt{g} \left[-\frac{1}{2} \rho^2 R(\tilde{\Gamma}, g) - \frac{1}{4\alpha^2} F_{\mu\nu}^2 + \frac{1}{2} (\tilde{D}_\mu \chi)^2 - \mathcal{V}(\chi, \rho) \right], \tag{37}$$

where

$$\mathcal{V}(\chi, \rho) \equiv \frac{1}{4!} \left[\frac{1}{\xi_0} (6\rho^2 - \xi_1 \chi^2)^2 + \lambda_1 \chi^4 \right],$$

and $\rho^2 = \frac{1}{6} (\xi_1 \chi^2 + \xi_0 \phi^2)$. (38)

Notice that we also replaced the scalar field ϕ by the new, radial direction field ρ ; $\ln \rho$ transforms as $\ln \rho \rightarrow \ln \rho - \ln \Omega$ and acts as the (would-be) Goldstone of the symmetry.

The equation of motion for $\tilde{\Gamma}_{\mu\nu}^\lambda$ is similar to (19) but with a replacement $\phi \rightarrow \rho$ and with an additional contribution from the kinetic term of χ . Following the same steps as in the previous section, we eliminate the contributions of the kinetic terms of χ and v_μ to the equation of $\tilde{\Gamma}$ and find an equation similar to (21) with $\phi \rightarrow \rho$:

$$\tilde{\nabla}_\lambda (\sqrt{g} g^{\mu\nu} \rho^2) - \frac{1}{5} \left\{ \delta_\lambda^\nu \tilde{\nabla}_\sigma (\sqrt{g} g^{\sigma\mu} \rho^2) + (\mu \leftrightarrow \nu) \right\} = 0. \tag{39}$$

This gives (see previous section):

$$\tilde{\nabla}_\lambda (\rho^2 g_{\mu\nu}) = (-2)\rho^2 (g_{\mu\nu} V_\lambda - g_{\mu\lambda} V_\nu - g_{\nu\lambda} V_\mu), \tag{40}$$

where $V_\mu = (-1/2)\tilde{\nabla}_\mu \ln(\sqrt{g}\rho^4) = v_\mu - \partial_\mu \ln \rho^2$. From (40) one finds the solution for Palatini connection $\tilde{\Gamma}_{\mu\nu}^\alpha$ in terms of $v_\mu \sim \tilde{\Gamma}_\mu$, with a result similar to (25) but with $\phi \rightarrow \rho$. We use this solution for the connection back in the action and find for $\tilde{\Gamma}$ onshell²²

$$L_3 = \sqrt{g} \left\{ \frac{-1}{2} [\rho^2 R(g) + 6(\partial_\mu \rho)^2] + 3\rho^2 (v_\mu - \partial_\mu \ln \rho^2)^2 - \frac{1}{4\alpha^2} F_{\mu\nu}^2 + \frac{1}{2} (\tilde{D}_\mu \chi)^2 - \mathcal{V}(\chi, \rho) \right\}. \tag{41}$$

L_3 has a gauged scale symmetry and extents (29) in the presence of scalar field χ .

Finally, we choose the Einstein gauge by using transformation (4) and (18) of a particular $\Omega = \rho/M$ which essentially sets $\hat{\rho}$ to a *constant* (vev). In terms of the new variables (with a hat) we find

$$L_3 = \sqrt{\hat{g}} \left\{ -\frac{1}{2} M^2 R(\hat{g}) + 3M^2 \hat{v}_\mu \hat{v}^\mu - \frac{1}{4\alpha^2} \hat{F}_{\mu\nu}^2 + \frac{1}{2} (\hat{D}_\mu \hat{\chi})^2 - \mathcal{V}(\hat{\chi}, M) \right\}, \tag{42}$$

²² As in Sect. 3, the trace $\tilde{\Gamma}_\mu \sim v_\mu$ is kept offshell since we do not integrate out *massless* dynamical v_μ .

with $\hat{D}_\mu \hat{\chi} = (\partial_\mu - 1/2 \hat{v}_\mu) \hat{\chi}$ and we identify M with the Planck scale ($M = \langle \hat{\rho} \rangle$). As in the absence of matter, we obtained the Einstein–Proca action of a gauge field that became massive after Stueckelberg mechanism of “absorbing” the derivative term $\partial_\mu (\ln \rho)$. A canonical kinetic term of $\hat{\chi}$ remained present in the action, since only one degree of freedom (radial direction ρ) was “eaten” by v_μ . The mass of v_μ is $m_v^2 = 6\alpha^2 M^2$. The potential becomes

$$\mathcal{V} = \frac{3M^4}{2\xi_0} \left[1 - \frac{\xi_1 \hat{\chi}^2}{6M^2} \right]^2 + \frac{\lambda_1}{4!} \hat{\chi}^4. \tag{43}$$

For a “standard” kinetic term for $\hat{\chi}$, similar to a “unitary gauge” in electroweak case, we remove the coupling $\hat{v}^\mu \partial_\mu \hat{\chi}$ in the Weyl-covariant derivative in (42) by a field redefinition

$$\hat{v}'_\mu = \hat{v}_\mu - \partial_\mu \ln \cosh^2 \left[\frac{\sigma}{2M\sqrt{6}} \right],$$

$$\hat{\chi} = 2M\sqrt{6} \sinh \left[\frac{\sigma}{2M\sqrt{6}} \right], \tag{44}$$

which replaces $\chi \hat{\rightarrow} \sigma$. After some algebra, we find the final Lagrangian

$$L_3 = \sqrt{\hat{g}} \left\{ -\frac{1}{2} M^2 \hat{R} + 3M^2 \cosh^2 \left[\frac{\sigma}{2M\sqrt{6}} \right] \hat{v}'_\mu \hat{v}'^\mu - \frac{1}{4\alpha^2} \hat{F}_{\mu\nu}^2 + \frac{\hat{g}^{\mu\nu}}{2} \partial_\mu \sigma \partial_\nu \sigma - \hat{\mathcal{V}}(\sigma) \right\} \tag{45}$$

with

$$\hat{\mathcal{V}}(\sigma) = \hat{\mathcal{V}}_0 \left\{ \left[1 - 4\xi_1 \sinh^2 \frac{\sigma}{2M\sqrt{6}} \right]^2 + 16 \lambda_1 \xi_0 \sinh^4 \frac{\sigma}{2M\sqrt{6}} \right\}, \quad \hat{\mathcal{V}}_0 \equiv \frac{3}{2} \frac{M^4}{\xi_0}. \tag{46}$$

In (45) one finally rescales $\hat{v}'_\mu \rightarrow \alpha \hat{v}'_\mu$ for a canonical gauge kinetic term.

For *small* field values, $\sigma \ll M$, then $\hat{\chi} \approx \sigma$ (up to $\mathcal{O}(\sigma^3/M^2)$) and a SM Higgs-like potential is recovered,²³ see Eq. (43). For $\xi_1 > 0$ it has spontaneous breaking of the symmetry carried by σ i.e. electroweak (EW) symmetry if σ is the Higgs; this is triggered by the non-minimal coupling to gravity ($\xi_1 \neq 0$) and Stueckelberg mechanism. The negative mass term originates in (38) due to the ϕ^4 term (itself induced by \tilde{R}^2). The mass $m_\sigma^2 \propto \xi_1 M^2/\xi_0$ may be small enough, near the EW scale by tuning $\xi_1 \ll \xi_0$. It may be interesting to study if the gauged scale symmetry brings some “protection” to m_σ at the quantum level.

L_3 of (45) is similar to that in Weyl quadratic gravity with a non-minimally coupled scalar/Higgs field [26–28],²⁴ up

²³ We shall see shortly that inflation “prefers” ultraweak or vanishing values for λ_1 in (38) and (43).

²⁴ For comparison to Weyl gravity Lagrangian see e.g. eqs. (39)–(41) in [26] and eqs. (21), (22) in [27].

to a rescaling of the couplings (ξ_1, λ_1) and fields (σ) . This difference is due to the different non-metricity of the two theories, Eqs. (32) and (33). Both cases provide a gauged scale invariant theory of quadratic gravity coupled to matter. They both recover Einstein gravity in their broken phase, see Eq. (45), and also metricity below the scale $m_v \sim \alpha M$ ($\alpha \leq 1$). This result may be more general – it may apply to other theories with this symmetry and can be used for model building.

To conclude, mass generation (Planck scale, v_μ mass) and Einstein gravity emerge naturally from spontaneous breaking of gauged scale symmetry in Einstein–Palatini theories, even in the absence of matter. Actions (14) and (35) were inspired by Weyl quadratic gravity of similar breaking [27]; but in a more general case, additional operators may be present in (14) and (35); for a list of all quadratic operators and a complementary study see [14]. The mechanism of symmetry breaking should remain valid in their presence if one includes the terms in (14): R^2 that ‘supplied’ the scalar field and $R^2_{[\mu\nu]}$ generating the symmetry and non-metricity. However, in such general case it is unclear that one can still solve algebraically the second-order differential equations of motion of $\tilde{\Gamma}$ (Eq. (19)) without simplifying assumptions, since these equations acquire new terms of different indices structure and new states will be present (ghosts, etc.).

5 Palatini R^2 inflation

In this section we consider an application to inflation of the action in the previous section.

For large field values, the potential in (46) can also be used for inflation (hereafter Palatini R^2 inflation), with σ as the inflaton.²⁵ For a Friedmann–Robertson–Walker metric (FRW) $(1, -a^2(t), -a^2(t), -a^2(t))$ and compatible background $v_\mu(t) = (v_0(t), 0, 0, 0)$ the gauge fixing condition $\nabla_\mu v^\mu = 0$ gives that $v_\mu(t)$ redshifts to zero $v_\mu(t) \sim 1/a^3(t)$. Then the coupling $v_\mu - \sigma$ in (45) is vanishing and therefore $v_\mu(t)$ cannot affect inflation; this means we have single-field inflation of potential (46) and standard slow-roll formulae can be used. Further, since M is just a phase transition scale, field values $\sigma \geq M$ are natural. $\hat{V}(\sigma)$ is similar to that in Weyl gravity R^2 -inflation, see [28,47] for a detailed analysis²⁶; however, as mentioned, the couplings and field normalization in the potential differ (for same initial couplings and non-metricity trace); hence the spectral index n_s and tensor-to-scalar ratio r are different, too, and need to be analyzed separately.

²⁵ Unlike in Starobinsky models, there is no scalaron here, its counterpart was “eaten” by massive v_μ .

²⁶ For related works on inflation in the Einstein–Palatini formalism see e.g. [95–104].

The potential is shown in Fig. 1 for perturbative values of the couplings relevant for successful inflation. This demands $\lambda_1 \xi_0 \ll \xi_1^2 \ll 1$, with the first relation from demanding that the initial energy be larger than at the end of inflation $\hat{V}_0 > \hat{V}_{\min}$, respected by choosing a small enough λ_1 for given $\xi_{0,1}$. Therefore, we shall work in the leading order in $(\lambda_1 \xi_0)$.

The slow-roll parameters are:

$$\epsilon = \frac{M^2}{2} \left\{ \frac{\hat{V}'(\sigma)}{\hat{V}(\sigma)} \right\}^2 = \frac{4}{3} \xi_1^2 \sinh^2 \frac{\sigma}{M\sqrt{6}} + \mathcal{O}(\xi_1^3) \tag{47}$$

$$\eta = M^2 \frac{\hat{V}''(\sigma)}{\hat{V}(\sigma)} = -\frac{2}{3} \xi_1 \cosh \frac{\sigma}{M\sqrt{6}} + \mathcal{O}(\xi_1^2). \tag{48}$$

Then

$$n_s = 1 + 2\eta_* - 6\epsilon_* = 1 - \frac{4}{3} \xi_1 \cosh \frac{\sigma_*}{M\sqrt{6}} + \mathcal{O}(\xi_1^2), \tag{49}$$

where σ_* is the value of σ at the horizon exit. With $r = 16\epsilon_*$ we have²⁷

$$r = 12(1 - n_s)^2 + \mathcal{O}(\xi_1^2). \tag{50}$$

The contribution of ϵ is subleading for small ξ_1 considered here. The slope of the curves in the plane (n_s, r) , shown in leading order in (50), is steeper than in Weyl R^2 inflation [28] (or Starobinsky model) where $r = 3(1 - n_s)^2 + \mathcal{O}(\xi_1^2)$.

The exact numerical results for (n_s, r) in our model, for different e-folds number N , are shown in Fig. 1. From experimental data $n_s = 0.9670 \pm 0.0037$ (68% CL) and $r < 0.07$ (95% CL) from Planck 2018 (TT, TE, EE + low E + lensing + BK14 + BAO) [105]. Using this data, Fig. 1 (right plot) shows that a specific, small range for r is predicted in our model for the current range for n_s at 95% CL:

$$N = 60, \quad 0.007 \leq r \leq 0.010, \quad [\text{Palatini } R^2 \text{ inflation}]. \tag{51}$$

Similar values for r can be read from Fig. 1 for $55 \leq N \leq 65$. The lower bound on r comes from that for n_s while the upper one corresponds to a saturation limit, $\xi_1 \rightarrow 0$, with values $\xi_1 < 10^{-3}$ having similar (n_s, r) . One should also respect the constraint $\lambda_1 \leq \xi_1^2/\xi_0$, giving $\lambda_1 \sim 10^{-12}$ or smaller (with the CMB anisotropy constraint $\xi_0 \geq 6.89 \times 10^8$).

For comparison, in Weyl R^2 -inflation for same n_s at 95% CL one has a smaller r [28,47]

$$N = 60, \quad 0.00257 \leq r \leq 0.00303, \quad [\text{Weyl } R^2 \text{ inflation}]. \tag{52}$$

²⁷ There is also a constraint on the parametric space from the normalization of CMB anisotropy $\mathcal{V}_0/(24\pi^2 M^4 \epsilon_*) = \kappa_0, \kappa_0 = 2.1 \times 10^{-9}$ and $r = 16\epsilon_*$ with $r < 0.07$ [105] then $\xi_0 = 1/(\pi^2 r \kappa) \geq 6.89 \times 10^8$. The aforementioned condition $\lambda_1 \xi_0 \ll \xi_1^2$ is then respected for perturbative $\xi_1, 1/\xi_0$ by choosing small $\lambda_1 \ll \xi_1^2/\xi_0$.

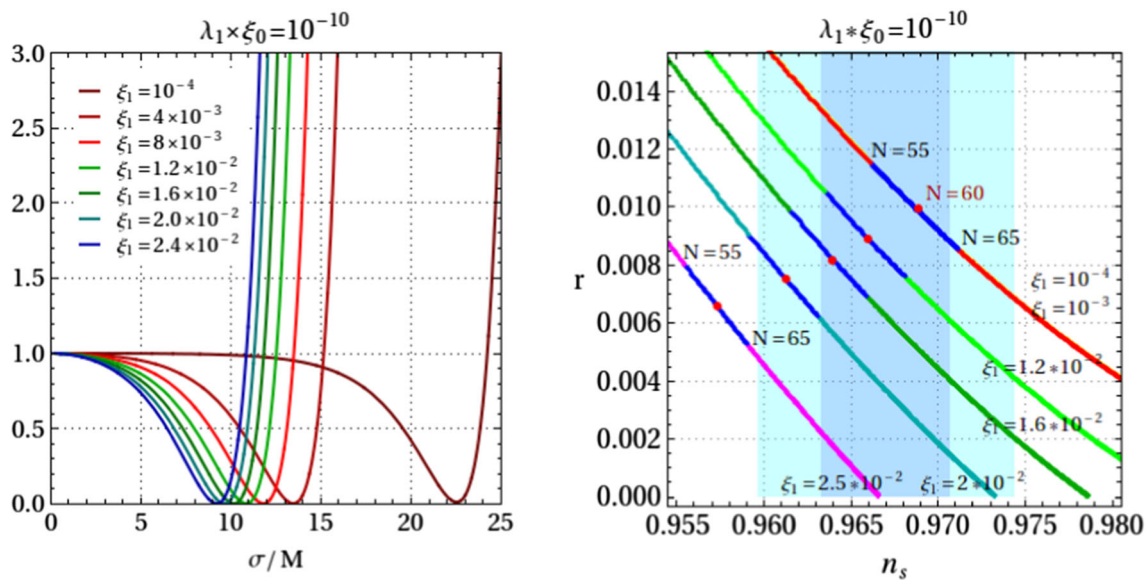


Fig. 1 Left plot: The potential $\hat{V}(\sigma)/\hat{V}_0$ for $\lambda_1 \xi_0 = 10^{-10} \ll \xi_1^2$ with different $\xi_1 \ll 1$. For larger $\lambda_1 \xi_0$ the curves move to the left while the minimum of the rightmost ones is lifted. Larger values of $\lambda_1 \xi_0$ are allowed, but inflation becomes less likely when $\lambda_1 \xi_0 \sim \xi_1^2$. The flat region is wide for a large range of σ , with the width controlled by $1/\sqrt{\xi_1}$ while its height is $\hat{V}_0 \propto 1/\xi_0$. We have $\hat{V}/\hat{V}_{\min} \propto \xi_1^2/(\lambda_1 \xi_0)$.

Right plot: The values of (n_s, r) for different values of ξ_1 that enable values of $n_s = 0.9670 \pm 0.0037$ at 68% CL (blue band) and 95% CL (light blue region). For each curve $N = 60$ e-folds is marked by a red point and the dark blue interval corresponds to $55 \leq N \leq 65$. Curves of $\xi_1 < 10^{-3}$ are degenerate with the red one while those with $\xi_1 > 2.5 \times 10^{-2}$ have $N > 65$

The different range for r in Eq. (51) versus Eq. (52) is important since it enables us to distinguish these two inflation models based on gauged scale invariance, and is due to their different non-metricity.^{28,29} Such values for $r \sim 10^{-3}$ will soon be reached by various CMB experiments [79–81] that will then be able to test both models. This establishes an interesting connection between non-metricity and testable inflation predictions.

Similar values for r were found in other recent inflation models in Palatini R^2 gravity [102–104] but these are not gauged scale invariant. In the absence of this symmetry, other successful models (e.g. Starobinsky model [106]) have corrections to r from higher curvature operators (R^4 , etc.) of unknown coefficients [108]. Such operators (and their corrections) are not allowed here because they must be suppressed by some effective scale whose presence would violate scale invariance.³⁰ Another advantage is that due to the *gauged* scale symmetry Palatini R^2 inflation is allowed by black-hole

physics (similarly for Weyl R^2 inflation [28]), in contrast to models of inflation with *global* scale symmetry.³¹

6 Conclusions

At a fundamental level gravity may be regarded as a theory of connections. An example is the Einstein–Palatini (EP) approach to gravity where the connection ($\tilde{\Gamma}$) is a priori independent of the metric, and is determined by its equation of motion, from the action. For simple actions $\tilde{\Gamma}$ plays an auxiliary role (no dynamics) and can be solved algebraically. In particular, for Einstein action in the EP approach one finds that the connection is actually equal to the Levi-Civita connection (of the metric formulation); then Einstein gravity is recovered, so the metric and EP approaches are equivalent. However, this equivalence is not true in general, for complicated actions, etc. In this work we considered quadratic gravity actions in the EP approach, with the goal to show that, while this equivalence does not hold true, one can still find actions that recover *dynamically* the Levi-Civita connection, metricity, Einstein gravity and Planck mass in some “low-energy” limit, even in the *absence* of matter.

We studied EP quadratic gravity given by $R(\tilde{\Gamma}, g)^2 + R_{[\mu\nu]}(\tilde{\Gamma})^2$ which has local scale symmetry. $R_{[\mu\nu]}(\tilde{\Gamma})^2$ can

²⁸ For a more detailed comparison of Einstein–Palatini R^2 -inflation to Weyl R^2 -inflation see [94].

²⁹ In the Starobinsky model [106] for similar n_s one has $r \sim \mathcal{O}(10^{-3})$, e.g. $r = 0.0034$ ($N = 55$) [107].

³⁰ The dilaton field cannot suppress them itself since it is “eaten” to all orders by the gauge field v_μ .

³¹ A global symmetry is broken since global charges can be eaten by black holes which then evaporate [109].

be regarded as a gauge kinetic term for the vector field $v_\mu \sim \tilde{\Gamma}_\mu - \Gamma_\mu$ where $\tilde{\Gamma}_\mu$ (Γ_μ) denotes the trace of the Palatini (Levi-Civita) connections, respectively. Hence this theory actually has a *gauged* scale symmetry, with v_μ the Weyl gauge field. A consequence of this symmetry is that the theory is non-metric i.e. $\tilde{\nabla}_\mu g_{\alpha\beta} \neq 0$ (due to dynamical $v_\mu \sim \tilde{\Gamma}_\mu$). At the same time, the equations of motion of the connection ($\tilde{\Gamma}$) become complicated second-order differential equations and we showed how to solve them *algebraically* in terms of an auxiliary scalar ϕ that “linearises” the $R(\tilde{\Gamma}, g)^2$ term. While initially the action appears to be of second order, for $\tilde{\Gamma}$ onshell it is a *higher derivative* theory since $R(\tilde{\Gamma}, g)^2$ contains a (four-derivative) metric contribution $R(g)^2 + \dots$. We showed that for $\tilde{\Gamma}$ onshell, the action is equivalent to a second-order theory in which the initial auxiliary field ϕ has become dynamical, while preserving the symmetry of the theory.

The main result is that our EP quadratic gravity action has an elegant spontaneous breaking mechanism of gauged scale invariance and mass generation valid even in the absence of matter; in this, the necessary scalar field (ϕ) was not added ad-hoc to this purpose (as usually done), but was “extracted” from the R^2 term, as mentioned, being of geometric origin. The derivative $\partial_\mu \ln \phi$ of this field acting as a Stueckelberg field is “eaten” by v_μ which becomes massive, of mass m_v proportional to the Planck scale $M \sim \langle \phi \rangle$. One obtains the Einstein–Proca action for the gauge field v_μ and a positive cosmological constant. This is a “low-energy” broken phase of the initial action. Below the scale $m_v \sim M$, the Proca field v_μ decouples and metricity and the Einstein action are recovered. Non-metricity effects are strongly suppressed by a large scale ($\propto M$), which is important for the theory to be viable.

The above results remain valid in the presence of scalar matter (Higgs, etc.) with a (perturbative) non-minimal coupling to this theory with a *Palatini connection*; in such case and following the Stueckelberg mechanism, the scalar potential also has a breaking of the symmetry under which this scalar is charged, e.g. electroweak symmetry in the Higgs case. This is relevant for building models with this symmetry for physics beyond the SM.

To summarise, Einstein–Palatini quadratic gravity $R(\tilde{\Gamma}, g)^2 + R_{[\mu\nu]}^2(\tilde{\Gamma})$ is a gauged theory of scale invariance that is spontaneously broken to the Einstein–Proca action for the Weyl field with a positive cosmological constant; if initial action also contains (non-minimally coupled) scalar fields with Palatini connection, a scalar potential is also present.

This picture is similar to a recent analysis for the original Weyl quadratic gravity, despite the different non-metricity of these two theories. With hindsight, this is not too surprising, since in both theories there is a gauged scale symmetry and the connection is not fixed by the metric, except that in Weyl gravity non-metricity is present from the onset (due to under-

lying Weyl conformal geometry) while here it emerges for $\tilde{\Gamma}$ onshell. It is worth studying further the relation of these two theories, by including any remaining operators (on the Einstein–Palatini side) that can have this symmetry.

There are also interesting predictions from inflation. While the scalar potential is Higgs-like for small field values ($\ll M$), for large field values it can be used for inflation. With the Planck scale M a simple phase transition scale, field values above M are natural. The inflaton potential is similar to that in Weyl quadratic gravity, up to couplings and field redefinitions (due to different non-metricity of the two theories). We find a specific prediction for the tensor-to-scalar ratio, $0.007 \leq r \leq 0.01$, for the current value of the spectral index at 95% CL. This value of r is mildly larger than that predicted by inflation in Weyl gravity. This enables us to distinguish and test these two theories by future CMB experiments that will reach such values of r . It also establishes an interesting connection between non-metricity and inflation predictions.

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Appendix

For a self-contained presentation, we include here basic aspects of the Palatini formalism used in the text. In the (pseudo-)Riemannian geometry, the Levi-Civita connection $\Gamma(g)$ is determined by the metric. In general, however, the connection can be introduced without reference to $g_{\mu\nu}$. In the Palatini approach the connection $\tilde{\Gamma}$ is a priori independent of the metric and is determined by the equations of motion. To ensure the covariant derivatives transform under coordinate change ($x \rightarrow x'(x)$) as true tensors, Palatini connection has a transformation law

$$\tilde{\Gamma}'^{\lambda}_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial x'^\lambda}{\partial x^\rho} \tilde{\Gamma}^{\rho}_{\alpha\beta} + \frac{\partial x'^\lambda}{\partial x^\sigma} \frac{\partial^2 x^\sigma}{\partial x'^\mu \partial x'^\nu} \quad (53)$$

with $\tilde{\Gamma}' = \tilde{\Gamma}'(x')$, $\tilde{\Gamma} = \tilde{\Gamma}(x)$. In the text we also assumed $\tilde{\Gamma}'_{\mu\nu} = \tilde{\Gamma}'_{\nu\mu}$ (no torsion). The Levi-Civita connection $\Gamma^\lambda_{\mu\nu}(g)$ has a similar transformation

$$\Gamma'^\lambda_{\mu\nu}(g) = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial x'^\lambda}{\partial x^\rho} \Gamma^\rho_{\alpha\beta}(g) + \frac{\partial x'^\lambda}{\partial x^\sigma} \frac{\partial^2 x^\sigma}{\partial x'^\mu \partial x'^\nu}. \tag{54}$$

Note that the difference of these connections transforms as a tensor

$$\tilde{\Gamma}'^\lambda_{\mu\nu} - \Gamma'^\lambda_{\mu\nu}(g) = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial x'^\lambda}{\partial x^\rho} (\tilde{\Gamma}^\rho_{\alpha\beta}(x) - \Gamma^\rho_{\alpha\beta}(g)). \tag{55}$$

Setting $\lambda = \nu$ and with the notation $\tilde{\Gamma}'_\mu \equiv \tilde{\Gamma}'^\nu_{\mu\nu}$, $\Gamma_\mu \equiv \Gamma^\nu_{\mu\nu}$, etc., then

$$\tilde{\Gamma}'_\mu - \Gamma'_\mu(g) = \frac{\partial x^\alpha}{\partial x'^\mu} (\tilde{\Gamma}'_\alpha - \Gamma_\alpha(g)) \tag{56}$$

and therefore v_μ introduced in Sect. 3 transforms as a covariant vector

$$v'_\mu(x') = \frac{\partial x^\alpha}{\partial x'^\mu} v_\alpha(x) \tag{57}$$

Further, the covariant derivatives used in the text are

$$\tilde{\nabla}_\nu v_\mu = \partial_\nu v_\mu - \tilde{\Gamma}^\lambda_{\mu\nu} v_\lambda, \quad \tilde{\nabla}_\nu v^\mu = \partial_\nu v^\mu + \tilde{\Gamma}^\mu_{\lambda\nu} v^\lambda. \tag{58}$$

One also has $\tilde{\nabla}_\lambda g_{\mu\nu} = \partial_\lambda g_{\mu\nu} - \tilde{\Gamma}^\rho_{\mu\lambda} g_{\rho\nu} - \tilde{\Gamma}^\rho_{\nu\lambda} g_{\rho\mu}$, also used in the text.

In Sect. 3 we introduced the gauge field v_μ in Eq. (16). This is general. For example, in Weyl gravity of similar gauged scale symmetry, an identical formula exists for the gauge field. To see this, note that in Weyl gravity [26,27], see also Eq. (33) in the text, non-metricity is different from EP quadratic gravity: $\tilde{\nabla}_\lambda g_{\mu\nu} = -v_\lambda g_{\mu\nu}$. Contracting this equation with $g^{\mu\nu}$ and using $\tilde{\nabla}_\lambda \sqrt{g} = (1/2)\sqrt{g} g^{\mu\nu} \tilde{\nabla}_\lambda g_{\mu\nu}$ we find $v_\lambda = (-1/2)\tilde{\nabla}_\lambda \ln \sqrt{g}$. This is similar to Palatini case, Eq. (26) in the text, although the connection is different. From these last two equations, by writing the action of $\tilde{\nabla}_\lambda$ on $g_{\mu\nu}$ one immediately finds

$$v_\mu = (1/2)(\tilde{\Gamma}'_\mu - \Gamma_\mu(g)) \tag{59}$$

with the trace of Levi-Civita connection $\Gamma_\mu(g) = \partial_\mu \ln \sqrt{g}$. Equation (59) was used as a definition for the gauge field in Einstein–Palatini quadratic gravity, Eq. (16).

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