

F. J. Ausín & Lorenzo Peña

«Paraconsistent Deontic Logic with Enforceable Rights»

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§1.— Preliminary considerations

This paper is devoted to proposing a system of gradualistic (*fuzzy*) paraconsistent deontic logic, a logic which implements the idea not just of degrees of truth (and falseness) but also that of degrees of obligatoriness and licitness. The system we propose (*transitive deontic logic*) tries to implement a correct treatment of quantified deontic and juridical propositions, chiefly those concerning positive rights, which are couched in terms of existential quantifiers (the right to have a dwelling, a job, to enjoy medical care and so on). Even negative rights are expressed through universal quantifiers. Thus, without an adequate treatment of quantifiers, no system of deontic logic is satisfactory. However, standard systems of deontic logic make it extremely difficult, if not downright impossible, to implement quantifiers in any reasonable way. In order to achieve such a treatment, new postulates are put forward, but at the same time most of the usual axioms of standard systems of deontic logic are dropped. In fact, our approach departs from the common assimilation between deontic and modal notions. We start with three preliminary considerations.

1.— Our underlying conception of obligatoriness (mandatoriness, bindingness) is as follows. First, obligatoriness is a property of states of affairs.¹ The propositional ‘o’ operator is such that $\lceil op \rceil$ is true iff the state of affairs that p has the obligatoriness-property (i.e. is obligatory). Second, applying the ‘o’ operator to a formula, $\lceil p \rceil$, with a free variable, $\lceil x \rceil$, makes it into another formula $\lceil q \rceil$ such that $\lceil q \rceil$ is satisfied by an entity, z, iff $\lceil p \rceil$ ’s being satisfied by z is an obligatory state of affairs.

Within a normative system all states of affairs such that the authorities have promulgated (and not repealed) the command to bring them about (or — which amounts to the same — claim them to be obligatory) are obligatory. Next, all states of affairs whose

¹ Many obligations, and indeed also many rights, exist because certain facts exist. While in usual modal logics we do not have theorems of the form $\lceil p \supset \Box q \rceil$, one of the main features of our current proposal is that deontic or juridical logic has little to do with modal logic, and that obligatoriness is extremely unlike necessity (whether metaphysical or physical necessity, or even practical necessity — unavoidableness). Quite often facts determine rights and obligations. Of course whenever such is the case some «more basic» principle holds, namely one to the effect that, if and when those facts exist, a certain obligation or a certain right also exists.

obligatoriness follows — according to the objectively valid juridical logic — from the obligatoriness of states of affairs which are themselves obligatory are obligatory, too.²

Obligatoriness has nothing to do with realization in ideal or optimal worlds. First, a state of affairs may be primitively obligatory while being quite undesirable. Second, many obligations exist only because the world is in fact thus or so. Many an obligation arises only when some factual circumstances are met. This is the main reason why all ideal-world approaches to juridical logic are doomed.

Nor is the situation different as regards the logic of ethical norms (as against that of juridical norms). A purely ethical code can be looked upon as nothing else but the code promulgated by the optimal legislator (natural law). Whether or not such a legislator exists (and indeed whether or not there is something like a purely ethical set of norms) is an issue which exceeds our present inquiry. But even a purely natural law (or a purely ethical normative system) contains precepts to the effect that, if some factual circumstances are met, something must be the case. Hence there are (moral) obligations in virtue of the world being as it is and not otherwise.

The same happens as regards rights (as against duties or obligations): some states of affairs become rightful or licit only because certain factual circumstances are in fact realized; both as regards ethical norms and juridical norms.

But if, consequently, the elegant idealizations provided by artificial semantical frameworks with ideal or optimal worlds are of no avail, it would be nice to propose a simple, clear-cut, intuitive idea of what closure rules hedge off the field of obligatory situations (once at least one primitive obligation has been established). Nice indeed, but fairy-like. There is no short cut to sifting the obligatory actions from the others. There is only a painstaking tacking of the inferential patterns through a quasi-inductive procedure (perhaps some sort of reflective equilibrium): given that this and that are [taken to be] obligatory, what is reasonably regarded as such that its obligatoriness follows? On the one hand the study of real reasoning in the field of law gives us at least a clue. Then a number of alternative inferential patterns are advanced and canvassed. Little by little we discard those inferential patterns clearly at odds with what, all in all, seems to emerge from the reasoning life in the world of law, and so, little by little, we hopefully come closer to something resembling the objectively valid logic of norms.

Thus the simplest idealizations are promptly shattered and crumble. The most conspicuous unsuccessful pretender is the rule of logical closure, to the effect that the necessary consequences of obligatory states of affairs are obligatory. (Such a rule is not the truism that the *deontically*-logical consequences of the obligatoriness of obligatory states of affairs are obligatory, too.) If Closure were a correct rule of inference, then, since (at least within a non-relevant logic) a necessary consequence of some one's not-committing murder

² Notice, though, that, even if we agree with juspositivism in admitting that whatever is promulgated by the authorities is legally binding, we depart from positivism in thinking that there are natural-law precepts which have been promulgated by no human authority. Otherwise the mere existence of an objectively valid logic of norms would be quite mysterious. Positivism in the end leads (as can be seen in the case of the latter Kelsen) to out and out denial of the possibility of juridical logic. In the end juspositivism only accepts as law what has been explicitly promulgated and exactly as it has been worded by the legislator. See Lorenzo Peña, «El bien común, principio básico de la ley natural», *Isegoría* N° 17 (nov. 1997), pp. 137-163 [ISSN 1130-2097]. See below, note # 29.

is that he does not square the circle, the unlawfulness of murder would entail that of squaring the circle. Calling people to square the circle would be inciting them to commit unlawful actions.

With the collapse of Closure, all possible-worlds approaches to the semantics of deontic and juridical logic are quickly discredited. Whether or not a good, simple, intuitive semantics can be found is an open question. For the time being, our task is to find out reasonably strong (but also reasonably weak) tenable inference-rules involving obligatoriness.

2.— Now, for licitness. For a state of affairs to be rightful or licit (within a certain normative system) is for its negation to be nonobligatory. If there is only one negation (classical or strong negation), then the resulting scenery is clear-cut and straightforward: for a state of affairs, p , to be licit is for its (total, downright) negation to be [utterly] non-obligatory. But what if there is a negation, ‘*not*’, admitting of degrees, a negation which does not deny absolutely but which is partly compatible with what is negated? Then, what we can say is that, to the extent that a state of affairs, *not-p*, is obligatory, p is *not*-licit (in other words: p ’s licitness exists to the extent, and to the extent only, that *not-p*’s obligatoriness does *not* exist).

We maintain that ‘*not*’ is in fact ‘not’, nothing else. The core and gist of our approach is the claim that there are degrees; degrees of truth in general, degrees of obligatoriness in particular. If a state of affairs, p , admits of degrees, so does its (simple) negation. Thus both p and *not-p* can co-exist in the world (in this world or in other worlds), each being realized to a certain extent. The higher the degree of existence of p , the lower that of *not-p*, and conversely, the extreme cases being those wherein either p or *not-p* is fully realized — the other being then utterly nonexistent. Most cases are not extreme. This is what vitiates all-or-nothing approaches to deontic logic.

3.— The main reason why we advocate degrees of licitness (and of obligatoriness) is the Leibnizian principle of justice to the effect that similar situations must receive similar treatments. Since in practice many kinds of situations (probably all relevant sorts of situations in normative contexts) are set along actual or at least possible series of very small discrepancies, any all-or-nothing approach is bound to draw a line cutting off the licit cases from the illicit ones, even if they are very very similar. Thus e.g. consider issues such as animal rights, euthanasia, abortion or positive rights (those whose *dictum* is an existential quantification). With an all-or-nothing approach, this irksome result becomes unavoidable: two cases are — as we usually say — almost the same and yet their juridical treatment is entirely different, even the opposite. For instance, an abortion is perfectly lawful on the 90th day after conception and a crime of murder a day later; or an animal may be licitly enslaved and imprisoned for life on no account while her cousin belongs to the human species and is entitled to enjoy freedom. (Imagine that there were still around creatures belonging to Neanderthal, to the *homo habilis* species etc; or imagine a juridical deliberation a few generations after our species branched off our closest relatives the chimps.) Or again, suppose euthanasia is legalized under certain conditions. As its opponents argue, there will be cases very similar to those to which it is allowed to apply and yet the line has been drawn in a crisp way (despite the fact that all relevant features come in degrees: terminalness, painfulness, unbearableness and so on.) The alternative view we advise is to stop regarding normative issues as all-or-nothing: if action A has a degree of (il)licitness,

and action B is very similar to A, then action B must have a degree of (il)licitness close to A's.

Of course it is quite possible that an Action A should be **almost** entirely illicit, but not quite, whereas a very similar action B is itself utterly illicit. (E.g. suppose that with each passing day a zygote or a foetus is less and less licitly abortable, and that on the 270th day abortion is wholly banned; take B to be an abortion on the 270th day, and A an abortion on the 269th day.) Then the degree of licitness of A is extremely small, but yet (a little) real, while the degree of licitness of B is nil; their respective degrees of illicitness being: extremely high, but not complete, as regards A; full as regards B. We do not see that such a situation undermines our gradualism. The principle of justice is not tampered with. For some purposes there is a significant difference between utter falseness and ever so small truth, but yet a very very very ... very small truth is much like complete falseness, isn't it?

Likewise, positive rights (which figure quite centrally in our current paper) are often looked upon with suspicion and misgiving on account on their colliding with other entitlements. If the shelterless have a rightful right to have a dwelling and there are no empty houses, then the shelterless seem to have a right to something or other which is owned and rightfully occupied by other people who are entitled to keep their occupation. (Later on we will sketch out the argument showing that it is indeed sound.) A common solution is to either forswear positive rights altogether or else entirely deny the right of occupancy of the legal owners. We'll advise a middle course, by way of envisaging degrees of entitlement. (But we will refrain from settling on which rights are prevalent in such cases.)

§2.— The Underlying Quantificational Calculus

Our present treatment is based on **transitive logic**, a fuzzy-paraconsistent nonconservative extension of relevant logic **E**, which has been developed in the literature.³ Transitive logic is a logic of truth-coming-in-degrees, and of truth being mixed up with falseness (the truer a proposition, the less false it is, and conversely). Thus it is mainly conceived of as a logic applicable to comparisons involving 'more', 'less' and 'as ... as'. The implicational functor, ' \rightarrow ', is here construed as a functor of alethic comparison: ' $p \rightarrow q$ ' is read as «To the extent [at least] that p, q». Since our main idea is that an action can be both (to some extent) licit and (up to a point) also illicit, and that one out of two actions, both licit (both illicit), can be more licit (less licit) than the other, our logic's intended use is to implement valid inferences involving 'more', 'less' and 'as ... as' in deontic and juridical contexts.⁴

³. Cf. Peña (1993).

⁴. One of us developed a logic of comparatives many years ago. See Lorenzo Peña: «Contribución a la lógica de los comparativos», *Lenguajes naturales y lenguajes formales II*, ed. by Carlos Martín Vide, Barcelona: University of Barcelona Press, 1987, pp. 335-50 [ISBN 84-7665-141-4.] After writing the present essay, we have come across a line of research we hitherto were completely ignorant of, namely comparative logic developed in Italy by E. Casari, Pierluigi Minari and Francesco Paoli; see of the latter author: «On Strong Comparative Logic», *Logique et Analyse* N° 155-156 (1996), pp. 271-83. An examination of the deep divergences between our respective approaches goes far beyond the scope of this paper. What is perhaps more interesting is the existence of a number of coincidences or partial agreements between the two lines of research. The main difference lies in the fact that, unlike those Italian logicians, we regard degrees of truth as (all of them, except one) also degrees of falseness, and conversely; thus we look upon degrees and comparisons as resulting from a blend of truth and

Transitive logic (system **P10**) is built up by strengthening the logic of entailment **E**. (Our notational conventions are *à la* Church: no hierarchy among connectives; a dot stands for a left parenthesis with its mate as far to the right as possible; remaining ambiguities are dispelled by associating leftwards.)

Transitive logic introduces a distinction between strong (\neg) and simple (N) negation. Simple contradictions are nothing to be afraid of, whereas contradictions involving strong negation are completely to be rejected. Alternatively, transitive logic has a primitive functor of strong assertion, ‘H’, such that $\lceil \neg p \rceil$ abbreviates $\lceil \text{HNp} \rceil$ («Hp» can be read as «It is completely (entirely, wholly, totally) the case that p»). Within transitive logic disjunctive syllogism holds for strong negation; thus we define a conditional ‘ \supset ’, such that $\lceil p \supset q \rceil$ abbreviates $\lceil \neg p \vee q \rceil$. Notice that conjunction ‘&’ is such that $\lceil p \& q \rceil$ is as true as $\lceil q \rceil$, provided $\lceil p \rceil$ is not utterly false; we read $\lceil p \& q \rceil$ as ‘It being the case that p, q’. ‘ \wedge ’ is simple conjunction, ‘and’ ($\lceil p \& q \rceil$ is defined as $\lceil \neg \neg p \wedge q \rceil$). We also define $\lceil p \setminus q \rceil$ as $\lceil p \rightarrow q \wedge \neg(q \rightarrow p) \rceil$ («p\q» means that it is less true that p than that q). Our reading of ‘ \rightarrow ’ is as follows: $\lceil p \rightarrow q \rceil$ is read: «To the extent [at least] that p, q». We add a further definition: let $\lceil \blacktriangleright p \rceil$ abbreviate $\lceil \text{N}(p \rightarrow \text{Np}) \rceil$ — $\lceil \blacktriangleright p \rceil$ meaning that $\lceil p \rceil$ is true enough.⁵ Last let ‘ α ’ be a primitive sentential constant meaning the conjunction of all truths.

Here is an axiomatization of system **P10**. Primitives: \wedge , \rightarrow , N, H, α . definitions of ‘ \setminus ’, ‘ \neg ’, ‘ \blacktriangleright ’ are as above. $\lceil p \vee q \rceil$ abbreviates $\lceil \text{N}(\text{Np} \wedge \text{Nq}) \rceil$.

AXIOMS:

- | | |
|--|--|
| P10.01 α | P10.02 $\blacktriangleright(p \rightarrow p) \rightarrow p \rightarrow p$ |
| P10.03 $\alpha \setminus \text{N}\alpha$ | P10.04 $\alpha \rightarrow p \vee p \rightarrow q$ |
| P10.05 $\text{Hp} \rightarrow q \vee \text{Np} \rightarrow r$ | P10.06 $p \rightarrow q \rightarrow r \wedge (q \rightarrow p \rightarrow r) \rightarrow r$ |
| P10.07 $p \rightarrow q \rightarrow .q \rightarrow r \rightarrow .p \rightarrow r$ | P10.08 $p \rightarrow q \rightarrow .p \rightarrow r \rightarrow .p \rightarrow .q \wedge r$ |
| P10.09 $p \rightarrow q \rightarrow .\text{NH}q \rightarrow \neg \text{Hp}$ | P10.10 $p \rightarrow (q \wedge \text{N}q) \rightarrow \text{N}p$ |
| P10.11 $\text{N}p \rightarrow q \rightarrow .\text{N}q \rightarrow p$ | P10.12 $p \rightarrow \text{N}\text{N}p$ |
| P10.13 $p \wedge q \rightarrow p$ | P10.14 $p \wedge q \rightarrow q \wedge p$ |

Sole primitive **Rule of Inference: DMP** (i.e. disjunctive *modus ponens*): for $n \leq 1$: $p_1 \rightarrow q \vee (p_2 \rightarrow q) \vee \dots \vee p_n \rightarrow q, p_1, \dots, p_n \vdash q$

(**MP** [*Modus Ponens*] for implication ‘ \rightarrow ’ is a particular case of the rule — the one wherein $n=1$. *Adjunction* is a derived inference rule.)⁶

Modus ponens for the mere conditional, ‘ \supset ’, is also a derived inference rule: from $\lceil p \supset q \rceil$ and $\lceil p \rceil$ to infer $\lceil q \rceil$. The fragment of **P10** containing only functors $\{\wedge, \vee, \supset, \neg\}$ is

falseness — in a vein which owes much to the influence of Heraclitus, Anaxagoras, and other Presocratics, as well as to Plato’s *Parmenides* and *Sophist* dialogues.

⁵. $\lceil \blacktriangleright p \rceil$ could be read ‘It is sufficiently (or amply or abundantly, or the like) true that p’, or something like that.

⁶. This axiomatic basis seems to us reasonably clear, elegant and functional. It may however contain some redundancy.

exactly classical logic (both as regards theorems and also as regards rules of inference). That's why **P10** is a conservative extension of classical logic.

Quantification:

Our quantificational extension of system **P10** is obtained by adding further axiomatic schemata plus three inference rules. We introduce universal quantifier as primitive and define $\lceil \exists xp \rceil$ as $\lceil N\forall xNp \rceil$. By $\lceil r(x) \rceil$ we mean a formula $\lceil r \rceil$ with no free occurrence of variable 'x'. Additional axiomatic schemata:

$$\begin{aligned} \exists x(\forall xq \wedge p) &\leftrightarrow \forall x(\exists xp \wedge q) & \forall x(p \wedge q) &\rightarrow (\forall xp \wedge q) & \forall xs \setminus r(x) \supset \exists x(s \setminus r) \\ \forall xp \wedge \exists xq &\rightarrow \exists x(p \wedge q) & \forall x \neg p &\rightarrow \neg \exists xp \end{aligned}$$

Additional Inference Rules:

rinfq01: universal generalization

rinfq02: Free-variables change rule

rinfq03: Alphabetic Variation

§3.— Deontic Postulates

We introduce as primitives these symbols: $\lceil ap \rceil$ is read as 'It is allowed that p' or 'It is licit that p'; $\lceil op \rceil$ abbr. $\lceil NaNp \rceil$; 'o' means obligation; $\lceil fp \rceil$ abbr. $\lceil Nap \rceil$; 'f' means interdiction (unlawfulness); \blacksquare is a modal operator meaning **concrete necessity**, practical unavailability; \blacktriangledown is defined: $\lceil \blacktriangledown p \rceil$ abbr $\lceil N\blacksquare Np \rceil$: \blacktriangledown means feasibility. \succ means a causal relation between facts, whereas f means a relation of forcing (coercing, compelling, constraining — not necessarily by violence, but in any case by material actions which make it practically impossible or very hard for the coerced person not to perform the action — or omission — she is coerced to do).⁷

Coercion is of course a particular case of causation, but while an appeal can prompt someone to act in a certain way (cause him to act), the link is not one of coercion. Coercing is causing an action (or an omission) against the coercee's will. Thus, a state of affairs, that Peter's house's door is key-locked by John, coerces (forces) another state of affairs, namely that Peter does not leave his house.⁸

We are aware that it would be nice to have axiomatic treatments of the two relations of causation and forcing, but no interesting set of such axioms has occurred to us.

We also take for granted a number of common assumptions, such as the equivalence between the obligatoriness of A and the illicitness of not-A and Bentham's postulate, namely

⁷. While feasibility and unavailability are different from mere logical or metaphysical possibility and logical or metaphysical necessity (what is feasible is also metaphysically possible, and what is metaphysically necessary is also unavailability), the logic ruling those notions may quite probably be about the same (except that unavailability is fact-sensitive: whatever actually happens in a world, w , the [metaphysically] necessary truths remain the same, whereas what is unavailability varies; thus formulae of the form $\lceil p \supset \blacksquare q \rceil$ are often true). Our favourite candidate would be something in the close neighbourhood of **S5**.

⁸. Many people will take exception at that, alleging that those are events, not states of affairs. We prefer to advocate a reductionistic approach, avoiding what seems to us an unwarranted proliferation of kinds of entities.

that what is obligatory is also licit.⁹ No normative system worthy of the name can enforce an obligation without thereby also granting people the right to perform the action which they are thereby bound to carry out. Unfortunately not everybody agrees with us on that score,¹⁰ but Bentham seems to us the most obviously true axiom in deontic and juridical logic.

2.1.— Causal closure: the causal consequences of a licit action are themselves licit, $\lceil p \supset q \ \& \ ap \rightarrow aq \rceil$. In other words, if q is a causal consequence of p , forbidding that q implies banning that p . Which means that you are deontically debarred from doing an action which causally entails a forbidden result. That is what we propose in the place of the traditional principle of closure in virtue of which the logical consequences of an obligation are also obligatory — a most irksome principle which gives rise to uncountable and intractable paradoxes, such as the Good Samaritan, the gentle murder paradox and so on.¹¹

An equivalent formulation of causal closure is the following: actions that cause a forbidden result are themselves forbidden $\lceil p \supset q \ \& \ fq \rightarrow fp \rceil$.¹² However, the causal consequences of an obligatory action may not be obligatory; $\lceil p \supset q \ \& \ op \rightarrow oq \rceil$ does not hold. It is only licit to do an action if by doing it we do not prevent any right-enjoyment. But, if it is obligatory to do something and certain facts follow from that, this does not mean that such events are also mandatory. So, if we do not comply with that obligation, we do not fail in an additional guilt by the non-happening of such facts (unless, by other reasons, we had an obligation about them).¹³

2.2.— The principle of ensuant obligation [**PEO**] that what forcibly prevents the exercise of a right is forbidden: $\lceil pfq \ \& \ ap \rightarrow oq \rceil$ — or alternatively: $\lceil pfq \supset .ap \rightarrow oq \rceil$: you are allowed to force someone to do an action only to the extent that he is anyway bound to do such an action.¹⁴ Again a third formulation: $\lceil pfNq \supset .aq \rightarrow oNp \rceil$: if by doing A you prevent someone from doing B , then, to the extent he is entitled to do B , you are forbidden to do A . What makes a right an entitlement for somebody to do some action is nothing else but the duty for everybody else not to stand in the way of his doing that action. **PEO** determines that, if doing something, A , forces someone (in the sense of compelling or constraining him) to do something which he can licitly refrain from doing, then A is forbidden: you are not entitled to force anyone not to enjoy one of his or her rights. A right — a licit course of action — is such that its owner may not be compelled or constrained to

⁹. Or the principle of licitness of obligations, **PLO**, namely: that a course of action is mandatory only to the extent that it is licit.

¹⁰. Cf. Priest (1978, p. 243); Routley and Plumwood (1989, pp. 665 and 667).

¹¹. Schotch and Jennings emphasize that closure seems a right and indeed useful principle in moral philosophy, «for by means of this axiom we may persuade moral agents that they are committed to the logical consequences of their moral principles» (Schotch & Jennings 1981, p. 151). About deontic closure and paradoxes, cf. Goble (1991) and Ausín and Peña (1993).

¹². This is why instigation to committing a crime is itself an offense.

¹³. This is an example we partly borrow from Weinberger (1991): even if paying my debts makes me broke, and I must pay my debts, I am not thereby obliged to be broke.

¹⁴. The converse does not hold. Particular citizens are not generally allowed to force others to comply with the law — which is the task of the police; only under certain circumstances may they fulfill such a role of law-keepers.

give it up. Rights imply an obligation for everybody else to respect those rights, and so a duty not to disturb the right's owners's enjoyment thereof.

2.3.— A qualified version of Kant's principle: what is unavoidably mandatory is feasible: $\lceil \blacksquare op \rightarrow \blacktriangledown p \rceil$ — or equivalently $\lceil \blacksquare p \rightarrow \blacktriangledown ap \rceil$. Notice that, thus formulated, the principle only covers obligations which the agent cannot avoid. An agent may incur unfeasible obligations as a result of their own previous choices;¹⁵ but to the extent that a course of action is unavoidable, it is feasible for the deontic or juridical system of norms to make it licit.¹⁶

2.4.—The principle of disjunctive obligation (**PDO**), namely that a disjunctive duty implies that at least one of the disjuncts is also obligatory: $\lceil o(p \vee q) \rightarrow op \vee oq \rceil$. The significance of **PDO** lies in the fact that, according to it, a disjunctive obligation can be imposed only in so much as at least one of the disjuncts is rendered obligatory. A corollary of **PDO** (in effect an alternative formulation of the same principle) is the principle of aggregative permission (**PAP**), namely: $\lceil ap \wedge aq \rightarrow a(p \wedge q) \rceil$: to the extent that two actions are both licit, it is also licit to perform them both. To the extent that you have the right to remain at home and you have the right to say what you please, you have the right to remain at home and say what you please.

Both **PAP** and **PDO** are incompatible with standard systems of deontic logic, which claim that logical consequences of obligations are also obligatory (see above 2.1.). Accordingly, such systems entail that, for every $\lceil p \rceil$, $\lceil p \vee Np \rceil$ is obligatory (within classical logic there is no difference between strong and nonstrong negation — so we can represent classical negation as 'N' or as '¬'). But of course for many — in fact most — situations, p, neither p nor not-p is obligatory. Likewise within standard deontic systems, **PAP** entails that you cannot have a right to A and also a right not to A; for then, should **PAP** obtain, you would have a right to A-and-not-A, which — within those systems — entails that everything is allowed. Thus, within standard systems of deontic logic **PAP** is incompatible with the fact that two contrary courses of actions may be both licit — you may have the right to vote and also the right not to vote, in other words you may be free as regards voting.

2.5.— The quantificational generalisation of **PDO**, namely the principle of deontic determination, **PDD**: $\lceil o \exists x p \rightarrow \exists x op \rceil$. It says that the only way of making it mandatory that something or other be such that p is to establish for some particular entity or other the duty of being such that p. No normative system may impose on obligation to the effect that at least an entity be such that p without — whether explicitly or implicitly — imposing a particular duty to fulfill it as regards some particular entity or other. (This postulate by itself serves to highlight the huge discrepancy between deontic possibility and other sorts of possibility.)

¹⁵ Bonevac and Seung also distinguish 'mere logical possibility' from 'real possibility' or feasibility, viewing the latter as the one involved in Kant's principle (Bonevac & Seung 1988, p. 329). We can find a similar argumentation in van Eck (1982, p. 268).

¹⁶ Thus, if the authorities cannot help banning a certain action, then that action is avoidable. We regard such a principle as a norm of natural law. Notice, though, that the authorities may have enacted an avoidable prohibition in virtue of which an unavoidable action is forbidden. Then natural law, without annulling or revoking the prohibition, probably adds the right not to comply with it, i.e. the right to perform what is anyway unavoidable.

An immediate result of **PDD** is its contrapositive version: to the extent that every entity is such that it may licitly have a certain feature, it is licit for all entities taken together to have that feature. ($\lceil \forall xap \rightarrow a\forall xp \rceil$). Thus, if every citizen has the right to vote for a certain party, it is licit that they all vote for that party (even if, by doing so, they are undermining multi-party democracy).¹⁷

2.6.— The rule of replacement of [provable] equivalents, or **rinf I**: if $\lceil p \leftrightarrow q \rceil$ is a theorem so is $\lceil op \leftrightarrow oq \rceil$.

2.7.— The twin principles of deontic disjunctive syllogism for strong negation, one for licitness, **PLDS**, i.e. $\lceil \neg p \wedge a(p \vee q) \rightarrow aq \rceil$, the other for obligatoriness, $\lceil \neg p \wedge o(p \vee q) \rightarrow oq \rceil$, or **PODS**. They mean that, when a situation p **completely** fails to occur, then the licitness (respectively obligatoriness) of a disjunction between p and q implies the licitness (respectively obligatoriness) of q . That is to say, when a law-giver licences a disjunction and — for whatever reason — one of the two disjuncts does not materialize at all, he is, to that extent, committed to determinately licencing the other disjunct. Likewise, when someone is under a disjunctive obligation and — for whatever reason — he is either utterly unable or fully unwilling to perform one of the disjuncts, he is, to that extent, bound determinately to fulfill the other disjunct.

2.7.1.— The second of the two twin principles of deontic disjunctive syllogism is equivalent to the principle of joint permission, **PJP**, namely: $\lceil Hp \supset aq \rightarrow a(p \wedge q) \rceil$. **PJP** determines that in a world wherein a situation, A , is fully realized, B can be allowed only if what is thereby being permitted is the joint situation A -and- B . Thus, under the assumption that, in our world, situation A is fully realized, you can forbid the joint realization of A -and- B only insomuch as you are in effect forbidding B . So, no constitution-designer can legitimately allege that he is not banning plebeians from becoming ministers if he forbids citizens to be low-born and yet become ministers. In a world like ours where certain people are in fact low-born, that prohibition implies that those low-born people are banned from becoming ministers.

2.7.2.— A result that follows from **PODS** is the principle of conditional obligation, **PCO**, viz.: $\lceil p \wedge o(p \supset q) \rightarrow oq \rceil$: to the extent that, its being the case that p , it is obligatory that q -if- p , to that extent at least it is obligatory that q . **PCO** in effect says that, when conditional duty is represented as ‘ $o(p \supset q)$ ’, such an obligation plus its being the case that p entails that it must happen that q .¹⁸ **PCO** entails that Kant’s principle — what is

¹⁷. Some constitutions make it mandatory for there to be political pluralism. Suppose everybody chooses to vote for P with all other parties receiving 0% of votes and thus being eliminated. There will be pluralism no longer. In order to prevent such a dead end it could be claimed that, even if everybody is entitled to vote for party P , it is not the case that it is legal that everybody votes for P . We disagree: what happens is that there is a conflict of duties. The situation is still worse with such constitutions as prescribe that parties failing to secure at least a certain percentage of votes — e.g. 10 % — cease to be legal political parties; for then even if not everybody, just one too many, votes for a certain party P , one by one all the other parties are done away with, which in the end brings about a one-party system.

¹⁸. A difficulty can arise in this connection. Suppose that a promise must be kept regardless (that $o(p \supset q)$, with ‘ p ’ meaning that you promise to do A and ‘ q ’ meaning that you do A), and in fact you promise. Does it follow that you must do A ? Or was the obligation to keep promises only a *prima facie* obligation, not a real one? We think that, if it is a real obligation, then, to the extent you have promised, you are bound to keep your word; on the other hand, if the obligation was not genuine, such a case is no counterexample to our principle **PCO**, which only holds for real obligations. If you are bound to pay-a-fine-if-you-have-exceeded-speed-limits and in fact you have exceeded speed limits, you must pay a fine.

obligatory is feasible — cannot be unqualifiedly espoused within our current system. For suppose it is obligatory that p-if-q, and in fact q is the case; hence, it is obligatory that p; but by the time such a situation arises, p may have become unfeasible, $\neg p$ being by then unavoidable. Thus, as a result of previous choices, an agent may put himself under obligations he can no longer discharge.

§4.— Strengthening the Transitive System of Deontic Logic

The postulates we have hitherto put forward constitute to our mind the basis for any reasonable system of norms. We now add two supplementary principles of mildness. The idea behind our two additional principles is that a system is (unduly) harsh iff it imposes a duty that an agent can carry out only by **completely** failing to enjoy one of her rights. In other words, a system is harsh iff it gives rise to either of these two situations:

- (1) $\lceil p \leftrightarrow Nq \wedge oHp \wedge aq \rceil$: $\lceil p \rceil$ is incompatible with $\lceil q \rceil$, but, while $\lceil q \rceil$ is licit, $\lceil Hp \rceil$ is obligatory.
- (2) $\lceil p \leftrightarrow Nq \wedge op \wedge aHq \rceil$: $\lceil p \rceil$ is incompatible with $\lceil q \rceil$, but, while $\lceil Hq \rceil$ is a rightful course of action, $\lceil p \rceil$ is obligatory.

The first principle of mildness, [PM#1], is the rejection of harsh situations of kind (1): $\lceil oHp \rightarrow Hop \rceil$. If there are situations of kind (1), then [PM#1] fails.

The second principle, [PM#2], is the rejection of situations of kind (2), or equivalently the principle $\lceil aHp \rightarrow Hap \rceil$.

What justifies our rejection of harsh normative systems is that they enact **stark dilemmas**, dilemmas, that is, wherein the agent cannot manage to abide by her duties (to some extent at least) by reducing the degree of her enjoying those of her rights which turn out to be in conflict with those duties; stark dilemmas constitute quite severe normative conflicts, with an agent being rightfully entitled to do an action A and yet duty-bound to do another action B, while A and B turn out to be out and out incompatible: either you perform A and then completely refrain from performing B, or conversely, with nothing in-between.

That in fact certain normative systems contain stark dilemmas is what we are going to show as regards positive rights. But whether or not our world's legislations contain stark dilemmas, it is pretty obvious that legislative systems containing them are possible. A ruler can impose upon his subjects the obligation of completely doing A while at the same time he allows them (whether explicitly or by implication) to in effect refrain from doing A somewhat or other: or perhaps such a right is not even granted by the ruler but the subjects have it in virtue of natural law. Notice that, even though it is enough for a state of affairs to be obligatory that the authorities pronounce it to be mandatory, it is neither a necessary nor a sufficient condition for a state of affairs to be entirely binding that the authorities fully claim it to be so. What is called for is the complete absence of any right to do the opposite.¹⁹

¹⁹ The reader may wonder where we stand as regards jusnaturalism vs juspositivism. It is an issue which goes far beyond the scope of this paper. According to the view proposed by one of us (see above, note # 2), all positive promulgations by human rulers are law, but they do not exhaust the law, which also encompasses natural precepts and natural rights. With such a dual source of law, conflicts, even stark conflicts, are likely to arise, and in fact do arise quite often. This kind of jusnaturalism does

We do not know whether natural law is harsh (we hope it isn't) but anyway it seems to us pretty clear that a plausible *desideratum* for our normative systems is that they should not be harsh.

§5.— Some Outstanding Results

The following proofs serve to show the juridical significance of our system:²⁰

(2)	$pfNs \& as \rightarrow oNp$	[PEO]
(3)	$\neg r_2 \& \dots \& \neg r_n \supset . pfNr_1 \rightarrow . pfN(r_1 \vee r_2 \vee \dots \vee r_n)$	assumption
(4)	$a \exists xr$	hypothesis
(5)	$\exists xr \leftrightarrow . r_1 \vee r_2 \vee \dots \vee r_n$	hypothesis
(6)	$a[r_1 \vee r_2 \vee \dots \vee r_n]$	(4), (5), replacement of equiv.
(7)	$\neg r_2 \& \dots \& \neg r_n$	hypothesis
(8)	$pfNr_1 \rightarrow . pfN(r_1 \vee r_2 \vee \dots \vee r_n)$	(3), (7), Modus Ponens
(9)	$\delta \delta \& \delta \rightarrow oNp$	(2)
(22)	$\sigma \delta \& \delta \rightarrow oNp$	(8), (9)
(23)	$\sigma \delta$	hypothesis
(24)	oNp	(23), (6)

[In the proof, 'σn' stands for the left hand of formula n, while 'δn' stands for its right hand.] We have thus proved what follows (under an extremely reasonable empirical assumption, namely, (3): when r_2, \dots, r_n completely fail to materialize, preventing r_1 implies preventing the disjunction of r_1, \dots, r_n): suppose performing a certain fact, p, would prevent an individual from enjoying some particular thing of a certain kind, while that individual is entitled to enjoy some entity or other of that kind but in fact none other is available **at all**; then, p is (to such an extent) forbidden.

Here are now two closely related proofs: first proof: to the extent that John is entitled to have a dwelling, there is some particular dwelling such that it is forbidden to prevent John from accessing it. Let 'p' stand for 'John accesses dwelling x' and 'q' stand for 'John is not prevented from accessing dwelling x' (let 'qc' abbreviate 'quantificational transitive logic'):

not claim that *lex iniusta non est lex*. What it claims is that, when a certain law is unjust, there is another law, also in operation, allowing people to do the opposite (at least to some extent). What of them is more binding? The principle of the common good is the paramount consideration.

²⁰. Our numbering is devised to avoid using digits '0' and '1', in order to save them to be used as defined sentential symbols.

- | | | |
|-----|--|--|
| (2) | $a\exists xp$ | juridical matter (John has that right) |
| (3) | $N\exists xqfN\exists xp$ | [obvious] empirical assumption ²¹ |
| (4) | $aN\exists xq\rightarrow oN\exists xp$ | (3), [PEO] |
| (5) | $NoN\exists xp$ | (2), df, qc |
| (6) | $NaN\exists xq$ | (4), (5), qc |
| (7) | $o\exists xq$ | (6), df |
| (8) | $\exists xoq$ | (7), PDD |

What we have proved is that (8) is binding at least to the extent that (2) holds, i.e. at least in so far as John is entitled to have some dwelling or other.

Now the second proof. We prove the principle of legitimate claims, **PLC**, for short: If John is entirely unable to access all other dwellings, then, to the extent he is entitled for there to be some dwelling or other he accesses, he has a legitimate claim to this particular dwelling (let '=' be a primitive predicate with usual properties for identity; let ' $x\neq z$ ' abbreviate ' $\neg(x=z)$ ').

- | | | |
|-----|---|-----------------------|
| (2) | $\exists xp\leftrightarrow \exists x(x=z\&p)\vee\exists x(x\neq z\&p)$ | qc |
| (3) | $a\exists xp\leftrightarrow a\delta 2$ | (2), rinf I |
| (4) | $\neg p\wedge a(p\vee q)\rightarrow aq$ | PLDS |
| (5) | $\neg\exists x(x\neq z\&p)\wedge a\exists xp\rightarrow a\exists x(x=z\&p)$ | (4), (3), qc |
| (6) | $\sigma 5\rightarrow ap[x/z]$ | (5), qc, =-properties |
| (7) | $\forall x(p\supset x=z)\supset a\exists xp\rightarrow ap[x/z]$ | (6), qc |

Many of our current legislative systems do nominally grant positive rights to people, but — by failing to take sufficient practical steps to ensure that those rights are really enjoyed — lead to a legal conflict. In accordance with traditional, Aristotelian standards, that conflict by itself means disaster, since it is usually admitted that a juridical system with conflicting duties is negationally inconsistent.

We show that what makes current juridical setups indefensible is not their entailing simple juridical conflicts and hence simple deontic contradictions, but their entailing harsh situations, which, if our principles of mildness are correct, entail deontic **over**contradictions.²²

On the one hand, an individual or a collectivity B may be the owners of a facility, X, and so they may be entitled to completely prevent anybody else from accessing X;²³ on

²¹. We regard it as empirical in the sense that it is through experience and induction that we ascertain that such is the case. Perhaps in a magical world (3) would not hold.

²². They are **deontic** overcontradictions, not overcontradictions. That is to say, they have the form ' $op\wedge o\neg p$ ', not the form ' $p\wedge\neg p$ '. In order to avoid confusion, we had better speak of stark dilemmas instead of using the misleading phrase 'deontic overcontradictions'.

²³. Although a referee has objected that not many people would agree, we are absolutely certain that not only do current legislations entitle the lawful occupants of a house or a flat to completely bar the shelterless from entering their home but that at least 90% of people do think that such an entitlement is right, at least to some extent. Think of yourself: if other people, whatever their plight, want to take your home as their own living place, aren't you sure you are legitimately entitled to lock them out? A different remedy (advocated by one of us [see Lorenzo Peña, *Hallazgos filosóficos*, Salamanca: Ediciones de la Universidad Pontificia de Salamanca, 1992]) is the total abolition of private ownership. However further measures would still be needed, since, even with collective ownership, there would be distributed (albeit revokable) rights of tenancy or occupation, which

the other hand it may be — and often is — the case that people D endowed with a quantificational right to some-good-or-other of the kind of goods X belongs to cannot **at all** access any other facility of that kind. Then D mustn't be forcibly prevented from accessing X (in virtue of [**PEO**]); in other words, B are in duty bound not to prevent D from accessing X (call that consequence the Non-foreclosure Duty, or **NFC**).²⁴ Suppose [PM#2] holds and B are entitled to completely debar D's eventual attempts to access X. Then, in virtue of [PM#2], B are completely allowed to prevent D from accessing X. Which is **overcontradictory** with their also being under **NFC**.²⁵ Since overcontradictions are entirely ruled out, what we have proved is that such a normative system does not comply with [PM#2].

§6.— A Remark about *Gaps* and *Gluts* in Juridical Contexts

It has been contended that, even if a code remains silent on whether or not an action, A, is allowed, and also on whether a different action, B, is allowed, we cannot jump to the conclusion that people have a legal right to do A and that they also have a right to do B (in accordance with the legal system in operation), since a person's doing A may thwart (or prevent or hamper) another person's doing B; and there is a juridical principle to the effect that nobody has the right of staying in the way of another person's exercising a right (our **PEO**).

We solve the problem by admitting degrees of duty, prohibition and permission. In many cases it may well happen that A is allowed, and so is B, but yet A is also, to some extent, forbidden — in so much as it prevents another person performing action B — at the same time that, in turn, B is also to some extent prohibited for a similar reason.

Within our approach a course of action can be both allowed, up to a point, and yet also forbidden to some extent. But our system also encompasses a strong assertion operator, 'H', read as 'It is entirely the case that', such that a person's being entirely allowed to do A completely rules out his being forbidden to do A.

still could clash with the universal right to have a home.

²⁴. Technically what we have proved is that in such legal systems as grant the positive right for everybody to have a home, there is an implied prescription granting a non-appurtenant personal easement on each particular dwelling when two conditions are met: (1) that the homeless claimant is debarred from entering all other dwellings; and (2) that there are no other available living accommodations. (We apologize for the legal jargon.)

²⁵. What emerges is a genuine conflict of interests and indeed of rights: my right to lock you out from my flat — you, homeless person out there — and your right to shelter yourself under my roof when no other dwelling is available to you. Why have you chosen precisely my home rather than any other? Because all other dwellings are unavailable, their occupants having also locked you out. Thus for any dwelling whatever, including mine, you have a legitimate claim to that dwelling on account of being prevented from entering the others. The present authors remain neutral as to which of the two conflicting rights overrides the other. What is certain is that law-givers are responsible for implementing, as far as possible, a social order wherein such stark conflicts do not arise. Thus the homeless (and in general the poor) have a priority claim against society and are entitled to demand such social subsidies and investments as are required to solve at least the most heart-rending social problems. In particular squatterism seems to us clearly justified when it concerns unoccupied buildings often destined to speculative operations; even if in some of those cases a certain right subsists for the owner to keep the facility to himself, locking all other people out, such a right seems to us to be justly overridden by the prevalent right of the homeless to shelter themselves under a roof. Of course there are lots of intermediary cases. In all of them dilemmas, even quite often stark dilemmas, arise. A just social policy is called for in order to avoid such painful situations as far as possible.

Thus, we consider that the permission principle, **PP** — whatever is not forbidden is allowed —²⁶ plays a major role in our common juridical thought, and is in fact the implicit base for many of our dearest claims against encroachment. More often than not we — rightly — feel entitled to a course of action, not in virtue of the law claiming it to be legitimate, but because of our [implicit] right to do whatever is not expressly banned, liable to penalty or punishment.

Therefore, our proposal entails that in cases of legal silence both courses of action are to some extent allowed and yet also to some extent prohibited — in so far as each of them stands in the way of the alternative course of action.²⁷

Instead of a *legal-gaps* approach, we prefer a *legal-gluts* approach — based on **PP** — when legal codes remain silent. The former maintains that courses of action not explicitly forbidden are by no means *eo ipso* implicitly allowed, so there are gaps, discontinuities, jumps in juridical contexts. The latter holds the «plenitude» of law, as defended among others by Kelsen, Ross or Lewis.²⁸

The legal-gaps approach seems to us not just fraught with unpalatable consequences but in some sense hard to understand, at least if we identify our having a certain right with other people being obliged not to stand in the way of our exercising such a right. For the approach amounts to this: (1) in such situations, neither course of action is, at all, either legally forbidden or lawful; hence, (2) nobody violates the law by preventing or thwarting such a course of action; therefore, (3) no such preventive move can be a [legal] ground for charging its performer. But (3) entails that the preventive move is not unlawful, i.e. it is within the pale of the law, and hence that: (4) it is rightful to perform such a preventive move. Which, in virtue of **PEO**, entails that what is thus prevented is forbidden. An **overcontradiction!**

§7.— Classical Deontic Principles Rejected

It is quite natural to somehow assimilate licitness to some sort of possibility, both on account of how we talk and owing to some internal features of the operators. However the similarity has been often pushed too far and thus has misled many a deontic logician. Our present approach departs from such an assimilation. A few modal principles can be kept in deontic logic by uniformly replacing '□' with 'o' and '◇' with 'a': $\lceil op \rightarrow ap \rceil$; $\lceil o(p \supset q) \wedge op \supset aq \rceil$; $\lceil op \wedge aq \rightarrow a(p \wedge q) \rceil$; $\lceil \text{NaNp} \leftrightarrow op \rceil$.

We reject, though, many principles which had been espoused on the ground of the resemblance under consideration:

²⁶. It is also known in the legal literature as Zitelmann's «negative fundamental rule».

²⁷. We thus take exception to the juridical doctrine of the 'uniquely correct solution' to any legal problem, espoused, among others, by Dworkin. He argues that for any legal case (hard or easy) there is a uniquely correct solution. The main idea which motivates that view is the belief in the ultimate coherence of legal material, in the law as a completely consistent and determinate body of norms (Dworkin 1986, p. 176). We hold that this is not the case and recognize the possibility of degrees of lawfulness as well as tensions within a legal system. Dworkin, as many legal theorists, is involved in «the ideology of consistence».

²⁸. Cf. Kelsen (1960), Ross (1968) and Lewis (1979).

- (i) Iteration, disiteration and similar ones: $\lceil op \rightarrow oop \rceil$ (there is no contradiction between being bound to do A and being allowed to be entitled to not-A); $\lceil oop \rightarrow op \rceil$ (when a Constitution makes it mandatory for a certain legislation to be promulgated, it is mandatory that something, A, should be mandatory; however, it does not follow that A is in fact mandatory: perhaps many years after the Constitution came into force no such law has yet been promulgated); $\lceil ap \rightarrow aap \rceil$; $\lceil aap \rightarrow ap \rceil$; $\lceil oap \rightarrow ap \rceil$; $\lceil aop \rightarrow op \rceil$.
- (ii) About quantification: $\lceil a\exists xp \rightarrow \exists xap \rceil$ (even if it is licit for there to be someone or other paying no taxes, it still may be utterly false that there is someone in particular entitled not to pay taxes — suppose everybody is rich); $\lceil \exists xap \rightarrow a\exists xp \rceil$ (suppose there is a bank Thomas rightfully owns; does it follow that Thomas is entitled to the existence of a bank owned by him? Suppose his bank is destroyed; is he yet entitled to the existence of some [other] bank owned by him? Or was his would-be right to such an existence disappear with the destruction of his bank? In the latter case, his purported right to the existence of a bank owned by him was nothing over and above his right to the possession of his bank); $\lceil \exists xop \rightarrow o\exists xp \rceil$ (even if there is in fact someone who must go to prison, it is not the case at all that it is mandatory for there to be a person who goes to prison; the law of course allows for there to be nobody who goes to prison — since a situation wherein no crime is committed is in accordance with what the Law mandates; the mere fact that, a crime having been committed, the criminal is bound to serve a prison sentence does not run against the legal permissibility of a situation wherein no one serves a prison sentence; suppose that a crime has been committed but the criminal dies; is it the case that someone or other must still serve a prison sentence on account of the crime having been committed? Of course not! Then, the purported obligation for there to be someone or other who goes to prison [on account of that particular crime having been committed] was nothing over and above the mere fact that there was somebody bound to go to prison [on that account]); $\lceil a\forall xp \rightarrow \forall xap \rceil$ (it is of course licit that nobody be punished — such a state of affairs is not forbidden; but, sad though it is, in real life it is not true at all — or at least not very true — that nobody is bound to be punished). The latter two are the converse to our **PDD** and its alternative formulation $\lceil \forall xap \rightarrow a\forall xp \rceil$.²⁹
- (iii) Others: $\lceil a(p\vee q) \rightarrow ap\vee aq \rceil$ (suppose you have a right to, either receive compensation or else keep your job; does it follow that either you are determinately entitled to keep your job or else you are determinately entitled to receive compensation? Hardly! Until one of the two alternatives either has been definitely denied to you or has been rejected by you, you have no right yet to the other. It is only when, for whatever reason, one of the two disjuncts completely fails to be realized that, then and only then, the other becomes determinately allowed); $\lceil ap\vee aq \rightarrow a(p\vee q) \rceil$ (even if you are entitled to receive a 100 \$ compensation, you are not entitled to either receive it or else kill your neighbour); deontic aggregation: $\lceil op\wedge oq \rightarrow o(p\wedge q) \rceil$ (even if a legislative *corpus* makes it mandatory that p and also that q — with p and q being utterly incompatible — it does not follow that such a legislation also imposes the additional burden of p-and-q; it is quite common for citizens to find themselves under conflicting duties; yet nothing within such normative systems imposes the duty of jointly performing such mutually incompatible actions); its converse, deontic simplification,

²⁹. Cf. Kanger (1971, p. 53).

or $\lceil o(p \wedge q) \rightarrow op \rceil$ (even if, in virtue of a contract, a certain firm is bound to deliver a new piano and take away an old one, the firm is not bound to take away the old one; for, suppose they fail to deliver the new piano, but they try, all the same, to take away your old one, alleging the contract compels them to perform that task; of course you'll object that their duty to perform both actions together did not entail any right, let alone duty, to perform one of them separately, since the duty to perform one of the two actions taken on its own was meant to be conditional on the other action being performed too); licitness simplification, $\lceil a(p \wedge q) \rightarrow ap \rceil$ (suppose that your late aunt Cecilia has bequeathed you her estate on condition you marry your cousin Louisa; then you are entitled to receive the inheritance and marry your cousin Louisa; are you, on that account, [unconditionally] entitled to receive the inheritance?); closure rule: from $\lceil p \supset q \rceil$ to infer $\lceil op \supset oq \rceil$. Or an implicational version thereof, replacing ' \supset ' with ' \rightarrow '.³⁰ Closure is the root of many paradoxes, such as the Good Samaritan, the gentle murder, and so on — as we have pointed out above. Notice that the result of uniformly substituting ' \supset ' for ' \rightarrow ' in any of those formulae is not a theorem of our system either.

On the other hand, we hold a number of deontic principles whose modal counterparts would be clearly wrong: $\lceil ap \wedge aq \rightarrow a(p \wedge q) \rceil$ (rights are aggregative); $\lceil \neg p \wedge a(p \vee q) \rightarrow aq \rceil$ (i.e. **PLDS**: when a disjunct entirely fails to be the case, the licitness of the disjunction makes the other disjunct a rightful one); $\lceil p \& aq \rightarrow a(p \& q) \rceil$ (to the extent that, while a certain state of affairs obtains, some action is right, it is also right for that action to be done while the state of affairs holds). The remaining similarity between licitness and possibility is very small.

§8.— Concluding Remarks

We have tried to show: (1) that a fuzzy or gradualistic approach to deontic logic is fruitful and can afford an underlying logical framework within which a non-standard system of deontic and juridical logic is developed; (2) that such a logical calculus must be paraconsistent (paraconsistency being, of course, needed — in a fuzzy or gradualistic logic — if we do not want to jettison excluded-middle and the like; in fact we embrace something far stronger than simple excluded-middle, $\lceil p \vee Np \rceil$, namely strong excluded-middle, $\lceil p \vee \neg p \rceil$: either p , or else it's not the case that p at all); (3) that with such a logic we can formalize juridical dilemmas or conflicts which arise in the real life of law; (4) that there are two different sorts of such conflicts, viz.: those which involve simple negation — which are nothing specially to be afraid of, on the one hand; and stark or strong dilemmas on the other, namely those involving strong negation or ' \neg '; the latter transgress the principles of mildness (two principles we espouse as *desiderata* or guide-lines); (5) that, in virtue of quite reasonable principles of nonstandard deontic logic, one of the fields wherein stark dilemmas tend to arise is that concerning positive rights (rights whose *dictum* is an existential quantification); the reality of such cases betokens the need for a legislative overhaul in order to make current legislation compliant with the principles of mildness.

³⁰ An enlightening assessment both of the approaches endorsing deontic simplification and of those which (rightly to our mind) waive that principle is offered in Weinberger (1991), pp. 284-307. Weinberger also rejects deontic simplification. A philosophical discussion of the issue would exceed the limits of this paper.

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