# Parallel OWL 2 RL Materialisation in Centralised, Main-Memory RDF Systems

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**Abstract.** We present a novel approach to parallel *materialisation* (i.e., fixpoint computation) of OWL RL Knowledge Bases in centralised, main-memory, multi-core RDF systems. Our approach comprises a datalog reasoning algorithm that evenly distributes the workload to cores, and an RDF indexing data structure that supports efficient, 'mostly' lock-free parallel updates. Our empirical evaluation shows that our approach parallelises computation very well so, with 16 physical cores, materialisation can be up to 13.9 times faster than with just one core.

## 1 Introduction

The OWL 2 RL profile forms a fragment of datalog so that reasoning over OWL 2 RL knowledge bases (KB) can straightforwardly be rendered into answering datalog queries. These datalog queries are either the OWL 2 RL/RDF rules [18, Section 4.3] applied to the KB or the datalog translations [10] of the OWL 2 RL axioms of the KB applied to the data portion (ABox) of the KB. Answering datalog queries can be solved by backward chaining [2, 23], or one can *materialise* all consequences of the rules and the data so that subsequent queries can be answered without the rules. Materialisation supports efficient querying, so it is commonly used in practice, but it is also very expensive. We show that materialisation can be efficiently parallelised on modern multi-core systems. In addition, main-memory databases have been gaining momentum in academia and practice [16] due to the decreasing cost of RAM, so we focus on centralised, main-memory, multi-core RDF systems. We present a new materialisation algorithm that evenly distributes the workload to cores, and an RDF data indexing scheme that supports efficient 'mostly' lock-free data insertion. Our techniques are complementary to the ones for shared-nothing distributed RDF systems with nontrivial communication cost between the nodes [23, 20, 25]: each node can parallelise computation and/or store RDF data using our approach.

Materialisation has P-complete data complexity and is thus believed to be inherently sequential. Nevertheless, many practical parallelisation techniques have been developed; we discuss these using the following OWL 2 RL example axioms and their translation into datalog rules.

$$A \sqsubseteq B \qquad A(x,y) \rightarrow B(x,y) \tag{R1}$$

$$C \circ E \sqsubseteq D \qquad C(x,y) \wedge E(y,z) \rightarrow D(x,z) \tag{R2}$$

$$D \circ E \sqsubseteq E \qquad D(x,y) \wedge E(y,z) \rightarrow C(x,z) \tag{R3}$$

*Interquery parallelism* identifies rules that can be evaluated in parallel. For example, rules (R2) and (R3) must be evaluated jointly since $C$ and $D$ are mutually dependent, but rule (R1) is independent since $B$ is independent from $C$ and $D$. Such an approach does not guarantee a balanced workload distribution: for example, the evaluation of (R2) and (R3) might be more costly than of (R1); moreover, the number of independent components (two in our example) limits the degree of parallelism. *Intraquery parallelism* assigns distinct rule instantiations to threads by constraining variables in rules to domain subsets [7, 21, 9, 29, 27]. For example, with $N$ threads and assuming that all objects are represented as integers, the $i$th thread can evaluate (R1)–(R3) with $(x \bmod N = i)$ added to the rules' antecedents. Such a *static* partitioning does not guarantee an even workload distribution due to data skew. Systems such as WebPIE [23], Marvin [20], C/MPI [25]; DynamiTE [24], and [13] use variants of these approaches to support OWL 2 RL fragments such as RDFS or pD$^*$ [22].

In contrast, we handle general, recursive datalog rules using a parallel variant of the seminaïve algorithm [2]. Each thread extracts a fact from the database and matches it to the rules; for example, given a fact $E(a, b)$, a thread will match it to atom $E(y, z)$ in rule (R2) and evaluate subquery $C(x, b)$ to derive the rule's consequences, and it will handle rule (R3) analogously. We thus obtain independent subqueries, each of which is evaluated on a distinct thread. The difference in subquery evaluation times does not matter because the number of queries is proportional to the number of tuples, and threads are fully loaded. We thus partition rule instantiations *dynamically* (i.e., as threads become free), unlike static partitioning which is predetermined and thus susceptible to skew.

To support this idea in practice, an RDF storage scheme is needed that (i) supports efficient evaluation of subqueries, and (ii) can be efficiently updated in parallel. To satisfy (i), indexes over RDF data are needed. Hexastore [26] and RDF-3X [19] provide six-fold sorted indexes that support merge joins and allow for a high degree of data compression. Such approaches may be efficient if data is static, but data changes continuously during materialisation so maintaining sorted indexes or re-compressing data can be costly and difficult to parallelise. Storage schemes based on columnar databases [15] with vertical partitioning [1] suffer from similar problems.

To satisfy both (i) and (ii), we use hash-based indexes that can efficiently match all RDF atoms (i.e., RDF triples in which some terms are replaced with variables) and thus support the index nested loops join. Hash table access can be easily parallelised, which allows us to support 'mostly' *lock-free* [14] updates: most of the time, at least one thread is guaranteed to make progress regardless of the remaining threads; however, threads do occasionally resort to localised locking. Lock-free data structures are resilient to adverse thread scheduling and thus often parallelise better than lock-based ones. Compared to the sort-merge [3] and hash join [4] algorithms, the index nested loops join with hash indexes exhibits random memory access which is potentially less efficient than sequential access, but our experiments suggest that hyperthreading and a high degree of parallelism can compensate for this drawback.

We have implemented our approach in a new system called *RDFox* and have evaluated its performance on several synthetic and real-world datasets. Parallelisation was beneficial in all cases, achieving a speedup in materialisation times of up to 13.9 with 16 physical cores, rising up to 19.3 with 32 virtual cores obtained by hyperthreading.

Our system also proved competitive with OWLIM-Lite (a commercial RDF system) and our implementation of the seminaïve algorithm without parallelisation on top of PostgreSQL and MonetDB, with the latter systems running on a RAM disk. We did not independently evaluate query answering; however, queries are continuously answered during materialisation, so we believe that our results show that our data indexing scheme also supports efficient query answering over RDF data.

## 2 Preliminaries

A *term* is a *resource* (i.e., a constant) or a variable; we denote terms with $t$, and variables with $x$, $y$, and $z$. An *(RDF) atom* is a triple $\langle s, p, o \rangle$ of terms called the *subject*, *predicate*, and *object*, respectively. A *fact* is a variable-free atom. A *rule* $r$ has the form (1), where $H$ is the *head atom*, and $B_1, \ldots, B_n$ are *body atoms*; we let $\mathsf{h}(r) := H$ and $\mathsf{b}_i(r) := B_i$.

$$B_1 \wedge \ldots \wedge B_n \rightarrow H \tag{1}$$

Rules must be *safe*: each variable in $H$ must occur in some $B_i$. A *program* $P$ is a finite set of possibly recursive rules. The *materialisation* (i.e., the fixpoint) $P^\infty(I)$ of a finite set of facts $I$ with $P$, a *substitution* $\sigma$ and its application $A\sigma$ to an atom $A$, and the composition $\tau\theta$ of substitutions $\tau$ and $\theta$ are defined as usual [2].

Our RDF system does not use the fixed OWL 2 RL/RDF rule set [18, Section 4.3]. Instead, we translate the OWL 2 RL axioms of the given KB into a datalog program; such programs generally contain simpler rules with fewer body atoms and are thus easier to evaluate. Our approach is, however, also applicable to OWL 2 RL/RDF rules.

## 3 Parallel Datalog Materialisation

We now present our algorithm that, given a datalog program $P$ and a finite set of facts $I$, computes $P^\infty(I)$ on $N$ threads. The data structure storing these facts (which we identify with $I$) must support several abstract operations: $I.\mathsf{add}(F)$ should check whether $I$ contains a fact $F$ and add it to $I$ if not; moreover, $I$ should provide an iterator $\mathsf{facts}_I$ where $\mathsf{facts}_I.\mathsf{next}$ returns a not yet returned fact or $\varepsilon$ if such a fact does not exist, and $\mathsf{facts}_I.\mathsf{hasNext}$ returns $\mathsf{true}$ if $I$ contains a not yet returned fact. These operations need not enjoy the ACID[1] properties, but they must be *linearisable* [14]: each asynchronous sequence of calls should appear to happen in a sequential order, with the effect of each call taking place at an instant between the call's invocation and response. Accesses to $I$ thus does not require external synchronisation via locks or critical sections.

Furthermore, $I$ must support an interface for answering conjunctive queries constrained to a subset of $I$; the latter will be used to prevent repeated derivations by a rule. To formalise this, we assume that $I$ can be viewed as a vector; then, for $F \in I$ a fact, $I^{<F}$ contains all facts that come before $F$, and $I^{\leq F} := I^{<F} \cup \{F\}$. We make

[1] Atomicity, Consistency, Isolation, Durability; These four properties are assumed to hold true in database transactions and ensure their reliable execution.

no assumptions on the order in which $\mathsf{facts}_I$ returns the facts; the only requirement is that, once $\mathsf{facts}_I.\mathsf{next}$ returns a fact $F$, further additions should not change $I^{\leq F}$—that is, returning $F$ should 'freeze' $I^{\leq F}$. An *annotated query* is a conjunction of RDF atoms $Q = A_1^{\diamond_1} \wedge \ldots \wedge A_k^{\diamond_k}$ where $\diamond_i \in \{<, \leq\}$. For $F$ a fact and $\sigma$ a substitution, $I.\mathsf{evaluate}(Q, F, \sigma)$ returns the set containing each substitution $\tau$ such that $\sigma \subseteq \tau$ and $A_i\tau \in I^{\diamond_i F}$ for each $1 \leq i \leq k$. Such calls are valid only when set $I^{\leq F}$ is 'frozen' and hence does not change via additions.

Finally, for $F$ a fact, $P.\mathsf{rulesFor}(F)$ is the set containing all $\langle r, Q_i, \sigma \rangle$ where $r$ is a rule in $P$ of the form (1), $\sigma$ is a substitution with $B_i\sigma = F$ for some $1 \leq i \leq n$, and

$$Q_i = B_1^{<} \wedge \cdots \wedge B_{i-1}^{<} \wedge B_{i+1}^{\leq} \wedge \ldots \wedge B_n^{\leq}. \tag{2}$$

To implement this operation efficiently, we index the rules of $P$ using two hash tables $H_1$ and $H_2$ that map resources to sets. Each body atom $B_i$ in a rule $r$ obtained as in [10] is of the form (i) $\langle t, \textit{rdf:type}, C \rangle$, in which case we add $\langle r, i, Q_i \rangle$ to $H_1[C]$, or (ii) $\langle t_1, R, t_2 \rangle$ with $R \neq \textit{rdf:type}$, in which case we add $\langle r, i, Q_i \rangle$ to $H_2[R]$. Then, to compute $P.\mathsf{rulesFor}(\langle s, p, o \rangle)$, we iterate over $H_1[o]$ if $p = \textit{rdf:type}$ or $H_2[p]$ otherwise, and for each $\langle r, i, Q_i \rangle$ we determine whether a substitution $\sigma$ exists such that $B_i\sigma = \langle s, p, o \rangle$. This strategy can be adapted to the OWL 2 RL/RDF rules.

To compute $P^\infty(I)$, we initialise a global counter $W$ of waiting threads to 0 and let each of the $N$ threads execute Algorithm 1. In lines 2–5, a thread acquires an unprocessed fact $F$ (line 2), iterates through each rule $r$ and each body atom $B_i$ that can be mapped to $F$ (line 3), evaluates the instantiated annotated query (line 4), and, for each query answer, instantiates the head of the rule and adds it to $I$ (line 5). This can be seen as a fact-at-a-time version of the seminaïve algorithm [2]: set $\{F\}$ plays the role of the 'delta-old' relation; atoms $B_k^{<}$ in (2) are matched to 'old' facts (i.e., facts derived before $F$); and atoms $B_k^{\leq}$ are matched to the 'current' facts (i.e., facts up to and including $F$). Like the seminaïve algorithm, our algorithm does not repeat derivations: each rule instantiation is considered at most once (but both algorithms can rederive the same fact using different rule instantiations).

A thread failing to extract a fact waits until either new facts are derived or all other threads reach the same state. This termination check is performed in a critical section (lines 7–15) implemented via a mutex $m$; only idle threads enter the critical section, so the overhead of mutex acquisition is small. Counter $W$ is incremented in line 7 and decremented in line 14 so, inside the critical section, $W$ is equal to the number of threads processing lines 7–13. We increment $W$ before acquiring $m$ since, otherwise, termination will rely on fairness of mutex acquisition: the operating system may spuriously wake the threads waiting in line 13, forcing them, in this way, to continuously acquire the mutex $m$, and hence potentially preventing $W = N$ form becoming true. If $W = N$ holds in line 9, then all other threads are waiting in lines 7–13 and cannot produce more facts, so termination is indicated (line 10) and all waiting threads are woken up (line 11). Otherwise, a thread waits in line 13 for another thread to either produce a new fact or detect termination. The loop in lines 8–13 ensures that a thread stays inside the critical section and does not decrement $W$ even if it is woken up but no work is available. Theorem 1 captures the correctness of our algorithm, and its proof is given in Appendix A.

**Algorithm 1** The Parallel Materialisation Algorithm

```
1:  while run do
2:      while F := facts_I.next and F ≠ ε do
3:          for all ⟨r, Q_i, σ⟩ ∈ P.rulesFor(F) do
4:              for all τ ∈ I.evaluate(Q_i, F, σ) do
5:                  I.add(h(r)τ)
6:      increment W atomically
7:      acquire m
8:          while facts_I.hasNext = false and run do
9:              if W = N then
10:                 run := false
11:                 notify all waiting threads
12:             else
13:                 release m, await notification, acquire m
14:         decrement W atomically
15:     release m
```

**Algorithm 2** $I.\mathsf{nestedLoops}(Q, F, \tau, j)$

```
1: if j is larger than the number of atoms in Q then
2:     output τ
3: else
4:     let B_j^◇j be the j-th atom of Q
5:     for all θ such that B_j τθ ∈ I^◇j do
6:         I.nestedLoops(Q, F, j + 1, τ ∪ θ)
```

**Theorem 1** *Algorithm 1 terminates and computes $P^\infty(I)$; moreover, each combination of $r$ and $\tau$ is considered in line 5 at most once, so derivations are not repeated.*

Procedure $I.\mathsf{evaluate}(Q_i, F, \sigma)$ in line 3 can use any join method, but our system uses a left to right evaluation of the body atoms (index nested loops). To this end, we reorder the atoms of each $Q_i$ to obtain an efficient left-to-right join order $Q_i'$ and then store $Q_i'$ in our rule index; we use a simple greedy strategy, but any known planning algorithm can be used too. We then implement line 4 by calling $I.\mathsf{nestedLoops}(Q_i', F, \sigma, 1)$ defined in Algorithm 2. The latter critically depends on efficient matching of atoms in $I$, which we discuss in Section 4.

## 4 RAM-Based Storage of RDF Data

We next describe a main-memory RDF indexing scheme that (i) can efficiently match RDF atoms in line 5 of Algorithm 2, but also (ii) supports concurrent updates. The RDF stores presened in [26] and [19] satisfy (i) using ordered, compressed indexes, but maintaining the ordering can be costly when the data changes continuously and concurrently. Instead, we index the data using hash tables, which allows us to make insertions 'mostly' lock-free.

As is common in RDF systems, we encode resources as integers using a dictionary. We store encoded triples in a six-column *triple table* shown in Figure 1. Columns $R_s$, $R_p$, and $R_o$ contain the integer encodings of the subject, predicate, and object of each triple. Each triple participates in three linked lists: an $sp$-list connects all triples with the same $R_s$ grouped (but not necessarily sorted) by $R_p$, an $op$-list connects all triples with the same $R_o$ grouped by $R_p$, and a $p$-list connects all triples with the same $R_p$ without any grouping; columns $N_{sp}$, $N_{op}$, and $N_p$ contain the next-pointers. Triple pointers are implemented as offsets into the triple table.

We next discuss RDF atom matching and the indexes used. Note that RDF atoms have eight different 'binding patterns'. We can match $\langle x, y, z\rangle$ (only variables) by iterating over the triple table; if, for example, $x = y$, we skip triples with $R_s \neq R_p$. For the remaining seven patterns, we maintain six indexes of pointers into the $sp$-, $op$- and $p$-lists. Index $I_{spo}$ contains each triple in the table, and so it can match RDF facts $\langle s, p, o\rangle$. Index $I_s$ maps each $s$ to the head $I_s[s]$ of the respective $sp$-list; to match an RDF atom

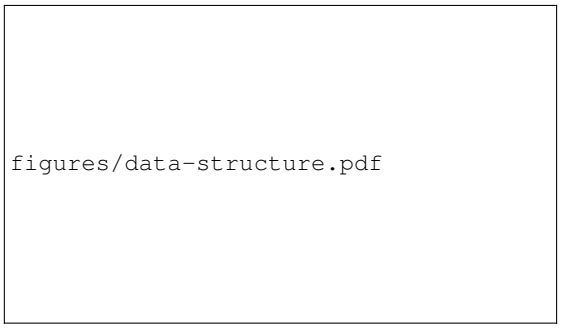



**Fig. 1:** Data Structure for Storing RDF Triples

$\langle s, y, z\rangle$ in $I$, we look up $I_s[s]$ and traverse the $sp$-list to its end; if $y = z$, we skip triples with $R_p \neq R_o$. Index $I_{sp}$ maps each $s$ and $p$ to the first triple $I_{sp}[s, p]$ in an $sp$-list with $R_s = s$ and $R_p = p$; to match an RDF atom $\langle s, p, z\rangle$ in $I$, we look up $I_{sp}[s, p]$ and traverse the $sp$-list to its end or until we encounter a triple with $R_p \neq p$. We could match the remaining RDF atoms analogously using indexes $I_p$ and $I_{po}$, and $I_o$ and $I_{os}$; however, in our experience, RDF atoms $\langle s, y, o\rangle$ occur rarely in queries, and $I_{os}$ can be as big as $I_{spo}$ since RDF datasets rarely contain more than one triple connecting the same $s$ and $o$. Therefore, we use instead indexes $I_o$ and $I_{op}$ to match $\langle x, y, o\rangle$ and $\langle x, p, o\rangle$, and an index $I_p$ to match $\langle x, p, z\rangle$. Finally, we match $\langle s, y, o\rangle$ by iterating over the $sp$- or $op$-list skipping over triples with $R_s \neq s$ or $R_o \neq o$; we keep in $I_s[s]$ and $I_o[o]$ the sizes of the two lists and choose the shorter one. To restrict any of these matches to $I^{<F}$ or $I^{\leq F}$, we compare the pointer to $F$ with the pointer to each matched tuple, and we skip the matched tuple if necessary.

Indexes $I_s$, $I_p$, and $I_o$ are realised as arrays. Indexes $I_{sp}, I_{op}$, and $I_{spo}$ are realised as open addressing hash tables storing triple pointers, and they are doubled in size when the fraction of used buckets exceeds some factor $f$. To determine worst-case memory usage per triple (excluding the dictionary), let $n$ be the number of triples; let $p$ and $r$ be the numbers of bytes used for pointers and resources; and let $d_{sp}$ and $d_{op}$ be the numbers of distinct $sp$- and $op$-groups divided by $n$. Each triple uses $3(r + p)$ bytes in the triple table. Index $I_{spo}$ holds up to size$(I_{spo}) \cdot f = n$ many triples and uses most memory per triple just after resizing: size$(I_{spo}) = 2n/f$ buckets then require $2p/f$ bytes per triple. Analogously, worst-case per-triple memory usage for $I_{sp}$ and $I_{op}$ is $d_{sp} \cdot 2p/f$ and $d_{op} \cdot 2p/f$. Finally, $I_s$, $I_p$, and $I_o$ are usually much smaller than $n$ so we disregard them. Thus, for the common values of $r = 4$, $p = 8$, $f = 0.7$, $d_{sp} = 0.5$, and $d_{po} = 0.4$, we need at most 80 bytes per triple; this drops to 46 bytes for $p = 4$ (but then we can store at most $2^{32}$ triples).

### 4.1 'Mostly' Lock-Free Insertion of Triples

Lock-freedom is usually achieved using compare-and-set: $\mathsf{CAS}(loc, exp, new)$ loads the value stored at location $loc$ into a temporary variable $old$, stores $new$ into $loc$ if $old = exp$, and returns $old$; hardware ensures that these steps are atomic (i.e., without thread interference). Inserting triples lock-free is difficult as one must atomically query $I_{spo}$, add the triple to the table, and update $I_{spo}$. CAS does not directly support atomic modification of multiple locations, so descriptors [8] or multiword-CAS [12] are

**Algorithm 3** add−triple$(s, p, o)$

**Input:**
$s$, $p$, $o$ : the components of the triple to be inserted

```
1:  i := hash(s, p, o) mod |I_spo.buckets|
2:  do
3:      If needed, handle resize and recompute i
4:      while T := I_spo.buckets[i] and T ≠ null do
5:          if T = INS then continue
6:          if ⟨T.R_s, T.R_p, T.R_o⟩ = ⟨s, p, o⟩ then
7:              return
8:          i := (i + 1) mod |I_spo.buckets|
9:  while CAS(I_spo.buckets[i], null, INS) ≠ null
10: Let T_new point to a fresh triple in the triple table
11: T_new.R_s := s,   T_new.R_p := p,   T_new.R_o := o
12: I_spo.buckets[i] := T_new
13: Update all remaining indexes
```

**Algorithm 4** insert−sp−list$(T_{new}, T)$

**Input:**
$T_{new}$ : pointer to the newly inserted triple
$T$ : pointer to the triple that $T_{new}$ comes after

```
1: do
2:     T_next := T.N_sp
3:     T_new.N_sp := T_next
4: while CAS(T.N_sp, T_next, T_new) ≠ T_next
```

needed. The latter techniques can be costly, so we instead resort to localised locking, cf. Algorithm 3. Here, $I_{spo}.buckets$ is the bucket array of the $I_{spo}$ index; $|I_{spo}.buckets|$ is the array's length; and, for $T$ a triple pointer, $T.R_s$ is the subject component of the triple that $T$ points to, and $T.R_p$, $T.R_o$, $T.N_{sp}$, $T.N_p$, and $T.N_{op}$ are defined analogously.

Lines 1–12 of Algorithm 3 follow the standard approach for updating hash tables with open addressing: we determine the first bucket index (line 1), and we scan the buckets until we find an empty one (lines 4–8) or encounter the triple being inserted (line 7). The main difference to the standard approach is that, once we find an empty bucket, we need to lock it so that we can allocate a new triple. This is commonly done by introducing a separate lock that guards access to a range of buckets [14]. We, however, avoid the overhead of separate locks by storing into the bucket a special marker INS (line 9), and we make sure that other threads do not skip the bucket until the marker is removed (line 5). We lock the bucket using CAS so only one thread can claim it, and we reexamine the bucket if CAS fails as another thread could have just added the same triple. If CAS succeeds, we allocate a new triple $T_{new}$ (line 10); if the triple table is big enough this requires only an atomic increment and is thus lock-free. Finally, we initialise the new triple (line 11), we store $T_{new}$ into the bucket (line 12), and we update all remaining indexes (line 13).

To resize the bucket array (line 3), a thread locks the index, allocates a new array, raises a resize flag, and unlocks the index. Any thread that accesses the index first checks whether the resize flag is raised; if so, it keeps transferring blocks of 1024 buckets from the old array into the new array (which can be done lock-free since triples already exist in the table) until all buckets have been transferred; the thread processing the last block deallocates the old bucket array and resets the resize flag. Resizing is thus divided among threads and is lock-free, apart from the array allocation step.

To update the $I_{sp}$ index in line 13, we scan its buckets as in Algorithm 3. If we find a bucket containing some $T$ with $T.R_s = s$ and $T.R_p = p$, we insert $T_{new}$ into the $sp$-list after $T$, which can be done lock-free as shown in Algorithm 4: we identify the triple $T_{next}$ that follows $T$ in the $sp$-list (line 2), we modify $T_{new}.N_{sp}$ so that $T_{next}$ comes after $T_{new}$ (line 3), and we update $T.N_{sp}$ to $T_{new}$ (line 4); if another thread modifies $T.N_{sp}$ in the meantime, we repeat the process. If we find an empty bucket while scanning $I_{sp}$, we store $T_{new}$ into the bucket and make $T_{new}$ the head of $I_s[s]$; since this requires multiword-CAS, we again use local locks: we store INS into the

bucket of $I_{sp}$, we update $I_s$ lock-free analogously to Algorithm 4, and we store $T_{new}$ into the bucket of $I_{sp}$ thus unlocking the bucket.

We update $I_{op}$ and $I_o$ analogously, and we update $I_p$ lock-free as in Algorithm 4. Updates to all indexes are independent, which promotes concurrency.

### 4.2 Reducing Thread Interference

Each processor/core in modern systems has its own cache so, when core $A$ writes to a memory location cached by core $B$, the cache of $B$ is invalidated. If $A$ and $B$ keep writing into a shared location, cache synchronisation can significantly degrade the performance of parallel algorithms. Our data structure exhibits two such bottlenecks: each triple $\langle s, p, o \rangle$ is added at the end of the triple table; moreover, it is added after the first triple in the *sp*- and *op*-groups, which changes the next-pointer of the first triple.

To address the first bottleneck, each thread reserves a block of space in the triple table. When inserting $\langle s, p, o \rangle$, the thread writes $\langle s, p, o \rangle$ into a free location $T_{new}$ in the reserved block, and it updates $I_{spo}$ using a variant of Algorithm 3: since $T_{new}$ is known *beforehand*, one can simply write $T_{new}$ into $I_{spo}.buckets[i]$ in line 9 using CAS; moreover, if one detects in line 7 that $I_{spo}$ already contains $\langle s, p, o \rangle$, one can simply reuse $T_{new}$ later. Different threads thus write to distinct portions of the triple table, which reduces memory contention; moreover, allocating triple space in advance allows Algorithm 3 to become fully lock-free.

To address the second bottleneck, each thread $i$ maintains a 'private' hash table $I^i_{sp}$ holding 'private' insertion points for $s$ and $p$. To insert triple $\langle s, p, o \rangle$ stored at location $T_{new}$, the thread determines $T := I^i_{sp}[s, p]$. If $T = \mathit{null}$, the thread inserts $T_{new}$ into the global indexes $I_{sp}$ and $I_s$ as usual and sets $I^i_{sp}[s, p] := T_{new}$; thus, $T_{new}$ becomes a 'private' insertion point for $s$ and $p$ in thread $i$. If $T \neq \mathit{null}$, the thread adds $T_{new}$ after $T$; since $T$ is 'private' to thread $i$, updates are interference-free and do not require CAS. Furthermore, for each $s$ the thread counts the triples $\langle s, p, o \rangle$ it derives, and it uses $I^i_{sp}[s, p]$ only once this count exceeds 100. 'Private' insertion points are thus maintained only for commonly occurring subjects, which keeps the size of $I^i_{sp}$ manageable. Thread $i$ analogously maintains a 'private' index $I^i_{op}$.

The former optimisation introduces a problem: when $\mathsf{facts}_I$ eventually reaches a reserved block, the block cannot be skipped since further additions into the reserved space would invalidate the assumptions behind Theorem 1. So, when $\mathsf{facts}_I$ reaches a reserved block, all empty rows in the reserved blocks are invalidated, and later ignored by $\mathsf{facts}_I$, and from this point onwards all triples are added at the end of the table.

## 5 Evaluation

We implemented and evaluated a new system called **RDFox**; all datasets, systems, scripts, and test results are available online.[2] We evaluated RDFox in three ways: we investigated how materialisation scales with the number of threads; we compared a sequential version (i.e., without CAS) of RDFox with the concurrent version on a single thread to estimate the overhead of concurrency support; and we compared RDFox

[2] http://www.cs.ox.ac.uk/isg/tools/RDFox/tests/

**Table 1:** Test Datalog Programs and RDF Graphs

| | LUBM | | | UOBM | | DBpedia | | Claros | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Ontology | $\mathcal{O}_L$ | $\mathcal{O}_{LE}$ | $\mathcal{O}_U$ | $\mathcal{O}_L$ | $\mathcal{O}_U$ | $\mathcal{O}_L$ | $\mathcal{O}_{LE}$ | $\mathcal{O}_L$ | $\mathcal{O}_{LE}$ | $\mathcal{O}_U$ |
| Rules | 98 | 107 | 122 | 407 | 518 | 7,258 | 7,333 | 2,174 | 2,223 | 2,614 |
| RDF Graph | 01K | | 05K | 010 | 01K | | | | | |
| Resources | 32.9M | | 164.3M | 0.4M | 38.4M | 18.7M | | 6.5M | | |
| Triples | 133.6M | | 691.1M | 2.2M | 254.8M | 112.7M | | 18.8M | | |

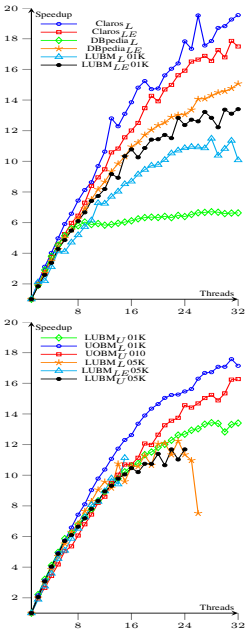
with other RDF systems. Unfortunately, no system we know of targets our setting exactly: many do not support materialisation of recursive rules [1, 26, 19, 31] and many are disk-based [6]. Hence, we compared RDFox with the following systems that, we believe, cover a wide range of approaches.

**OWLIM-Lite**, version 5.3, is a commercial RDF system developed by Ontotext. It stores triples in RAM, but keeps the dictionary on disk, and we configured it to evaluate our custom datalog programs, rather than the fixed OWL 2 RL/RDF rule set. Ontotext have confirmed that this is a fair way to use their system.

**DBRDF** is an RDF store we implemented on top of PostgreSQL (PG) 9.2 and MonetDB (MDB), Feb 2013-SP3 release. DBRDF can use either the vertical partitioning (VP) [1] or the triple table (TT) [5, 28] scheme for storing RDF data in a relational database. Neither PG nor MDB support recursive datalog rules, so DBRDF implements the seminaïve algorithm by translating datalog rules into DDL and SQL update statements and executing them sequentially; parallelising the latter is out of scope of this paper. More detail is given in Appendix B.

Table 1 summarises our test datasets. LUBM [11] and UOBM [17] are synthetic datasets so, for a parameter $n$, one can generate RDF graphs LUBM$n$ and UOBM$n$. The DBPedia dataset contains information about Wikipedia entities. Finally, Claros integrates cultural heritage data using a common vocabulary. The ontologies of LUBM, UOBM, and Claros are not in OWL 2 RL. In [30] such an ontology $\mathcal{O}$ is converted into programs $\mathcal{O}_L$ and $\mathcal{O}_U$ such that $\mathcal{O}_U \models \mathcal{O}$ and $\mathcal{O} \models \mathcal{O}_L$; thus, $\mathcal{O}_L$ ($\mathcal{O}_U$) captures a *lower* (*upper*) bound on the consequences of $\mathcal{O}$. The former is a natural test case since most RDF systems consider only $\mathcal{O}_L$, and the latter is interesting because of its complex rules. In order to make reasoning more challenging and to enlarge the materialised data set, we enriched $\mathcal{O}_L$ to $\mathcal{O}_{LE}$ with complex chain rules that encode relations specific to the domain of $\mathcal{O}$; for example, we defined in DBPedia$_{LE}$ teammates as pairs of football players playing for the same team. We identify each test by combining the names of the datalog program and the RDF graph, such as LUBM$_{LE}$ 01K.

We tested RDFox on a Dell computer with 128 GB of RAM, 64-bit Red Hat Enterprise Linux Server 6.3 kernel version 2.6.32, and two Xeon E5-2650 processors with 16 physical cores, extended to 32 virtual cores via *hyperthreading* (i.e., when cores maintain separate state but share execution resources). In our comparison tests, we used a Dell computer with 128 GB of RAM, 64-bit CentOS 6.4 kernel version 2.6.32, and two Xeon E5-2643 processors with 8/16 cores. Each test involved importing an RDF graph and materialising a datalog program, and we recorded the wall-clock times for import and materialisation, the final number of triples, and the memory usage before and after materialisation. RDFox was allowed to use at most 100 GB of RAM, and it stored triple pointers using four bytes. To mitigate the overhead of disk access, we stored the databases of MDB and PG on a 100 GB RAM disk. For OWLIM-Lite, we kept the




| | Claros$_L$ | | Claros$_{LE}$ | | DBpedia$_L$ | | DBpedia$_{LE}$ | | LUBM$_L$ 01K | | LUBM$_{LE}$ 01K | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Time | Spd | Time | Spd | Time | Spd | Time | Spd | Time | Spd | Time | Spd |
| seq | 1907 | 1.3 | 4128 | 1.2 | 158 | 1.0 | 8457 | 1.1 | 68 | 1.1 | 913 | 1.0 |
| 1 | 2477 | 1.0 | 4989 | 1.0 | 161 | 1.0 | 9075 | 1.0 | 73 | 1.0 | 947 | 1.0 |
| 2 | 1177 | 2.1 | 2543 | 2.0 | 84 | 1.9 | 4699 | 1.9 | 35 | 2.1 | 514 | 1.8 |
| 8 | 333 | 7.4 | 773 | 6.5 | 28 | 5.8 | 1453 | 6.2 | 14 | 5.2 | 155 | 6.1 |
| 16 | 179 | 13.9 | 415 | 12.0 | 26 | 6.1 | 828 | 11.0 | 8 | 8.7 | 88 | 10.8 |
| 24 | 139 | 17.8 | 313 | 15.9 | 25 | 6.4 | 695 | 13.1 | 7 | 10.9 | 77 | 12.4 |
| 32 | 127 | 19.5 | 285 | 17.5 | 24 | 6.6 | 602 | 15.1 | 7 | 10.1 | 71 | 13.4 |
| Seq. imp. | 61 | | 57 | | 331 | | 335 | | 421 | | 408 | |
| Par. imp. | 58 | -4.9% | 58 | 1.7% | 334 | 0.9% | 367 | 9.5% | 415 | 1.4% | 415 | 1.7% |
| Triples | 95.5 M | | 555.1 M | | 118.3 M | | 1529.7 M | | 182.4 M | | 332.6 M | |
| Memory | 4.2 GB | | 18.0 GB | | 6.1 GB | | 51.9 GB | | 9.3 GB | | 13.8 GB | |
| Active | 93.3% | | 98.8% | | 28.1% | | 94.5% | | 66.5% | | 90.3% | |

| | LUBM$_U$ 01K | | LUBM$_L$ 05K | | LUBM$_{LE}$ 05K | | LUBM$_U$ 05K | | UOBM$_U$ 010 | | UOBM$_L$ 01K | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Time | Spd | Time | Spd | Time | Spd | Time | Spd | Time | Spd | Time | Spd |
| seq | 122 | 1.1 | 422 | 1.0 | 4464 | 1.1 | 580 | 1.1 | 2381 | 1.2 | 476 | 1.1 |
| 1 | 135 | 1.0 | 442 | 1.0 | 4859 | 1.0 | 635 | 1.0 | 2738 | 1.0 | 532 | 1.0 |
| 2 | 61 | 2.2 | 221 | 2.0 | 2574 | 1.9 | 310 | 2.1 | 1400 | 2.0 | 247 | 2.2 |
| 8 | 20 | 6.8 | 65 | 6.8 | 745 | 6.5 | 96 | 6.6 | 451 | 6.1 | 72 | 7.4 |
| 16 | 13 | 10.6 | 42 | 10.6 | Mem | | 60.7 | 10.5 | 256 | 10.7 | 42 | 12.6 |
| 24 | 11 | 12.7 | 39 | 11.3 | Mem | | 54.3 | 11.7 | 188 | 14.6 | 34 | 15.5 |
| 32 | 10 | 13.4 | Mem | | Mem | | Mem | | 168 | 16.3 | 31 | 17.1 |
| Seq. imp. | 410 | | 2610 | | 2587 | | 2643 | | 6 | | 798 | |
| Par. imp. | 415 | 1.2% | 2710 | 3.8% | 3553 | 37% | 2733 | 3.4% | 6 | 0.0% | 783 | -1.9% |
| Triples | 219.0 M | | 911.2 M | | 1661.0 M | | 1094.0 M | | 35.6 M | | 429.6 M | |
| Memory | 11.1 GB | | 49.0 GB | | 75.5 GB | | 53.3 GB | | 1.1 GB | | 20.2 GB | |
| Active | 85.3% | | 66.5% | | 90.3% | | 85.3% | | 99.7% | | 99.1% | |

**Fig. 2:** Scalability and Concurrency Overhead of RDFox (All Times Are in Seconds)

**Table 2:** Comparison of RDFox with DBRDF and OWLIM-Lite (All Times Are in Seconds)

| | RDFox | | | PG-VP | | MDB-VP | | PG-TT | MDB-TT | OWLIM-Lite | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Import | | Materialise | Import | Materialise | Import | Materialise | Import | Import | Import | Materialise |
| | T | B/t | T B/t | T B/t | T B/t | T B/t | T B/t | T B/t | T B/t | T B/t | T B/t |
| Claros$_L$ | 48 | 89 | 2062 60 | 1138 165 | 25026 97 | 883 58 | Mem | 1174 217 | 896 94 | 300 54 | 14293 28 |
| Claros$_{LE}$ | | | 4218 44 | | Mem | | Mem | | | | Time |
| DBpedia$_L$ | 274 | 69 | 143 67 | 5844 148 | Mem | 15499 51 | 354 49 | 5968 174 | 4492 92 | 1735 171 | 14855 164 |
| DBpedia$_{LE}$ | | | 9538 49 | | Mem | | Mem | | | | Time |
| LUBM$_L$ 01K | | | 71 65 | | 948 127 | | 27 41 | | | | 316 36 |
| LUBM$_{LE}$ 01K | 332 | 74 | 765 54 | 7736 144 | 15632 112 | 6136 56 | Mem | 7947 187 | 5606 104 | 2409 40 | Time |
| LUBM$_U$ 01K | | | 113 67 | | Time | | 138 44 | | | | Time |
| UOBM$_U$ 010 | 5 | 68 | 2501 43 | 116 154 | Time | 96 50 | Mem | 120 203 | 96 85 | 32 69 | 11311 24 |
| UOBM$_L$ 01K | 632 | 64 | 467 63 | 13864 119 | 6708 107 | 11063 41 | 358 33 | 14901 176 | 10892 101 | 4778 38 | Time |

dictionary on a 50 GB RAM disk; since the system materialises rules during import, we loaded each RDF graph once with the test program and once with no program and we subtracted the two times; Ontotext confirmed that this yields a good materialisation time estimate. Each test was limited to 10 hours, and we report averages over three runs.

Figure 2 shows the speedup of RDFox with the number of used threads. The middle part of the tables shows the sequential and parallel import times, and the percentage slowdown for the parallel version. The lower part of the tables shows the number of triples and memory consumption after materialisation (the number of triples before is given in Table 1), and the percentage of *active* triples (i.e., triples to which a rule was applied). Import times differ by at most 5% between the sequential and parallel version. For materialisation, the overhead of lock-free updates is between 10% and 30%, so parallelisation pays off already with two threads. With all 16 physical cores, RDFox achieves a speedup of up to 13.9; this increases to 19.5 with 32 virtual cores, suggesting that hyperthreading and a high degree of parallelism can mitigate the effect of CPU

stalls due to random memory access. The flattening of the speedup curves is due to the limited capabilities of virtual cores, and the fact that each thread contributes to system bus congestion. Per-thread indexes (see Section 4) proved very effective at reducing thread interference, although they did cause memory exhaustion in some tests; however, the comparable performance of the sequential version of RDFox (which does not use such indexes) suggests that the cost of maintaining them is not high. The remaining source of interference is in the calls to $\mathsf{facts}_I.\mathsf{next}$, which are more likely to overlap with many threads and few active triples. The correlation between the speedup for 32 threads and the percentage of active triples is 0.9, explaining the low speedup on $\text{DBpedia}_L$. The excessive parallel import time observed for $\text{LUBM}_{LE}$05K is due to a glitch and should be in the same range as $\text{LUBM}_L$05K and $\text{LUBM}_U$05K. Since every thread keeps its private insertion points (cf. Section 4.2), the memory intensive $\text{LUBM}_*$05K test cases run out of memory for higher numbers of threads.

Table 2 compares RDFox with OWLIM-Lite and DBRDF on PG and MDB with the VP or TT scheme. Columns T show the times in seconds, and columns B/t show the number of bytes per triple. Import in DBRDF is about 20 times slower than in RDFox, but half of this time is used by the Jena RDF parser. VP is 33% more memory efficient than TT, as it does not store triples' predicates, and MDB-VP can be up to 34% more memory-efficient than RDFox; however, MDB-TT is not, which is surprising since RDFox does not compress data. We do not know how OWLIM-Lite splits the dictionary between the disk and RAM, so the RAM consumption in Table 2 is a 'best-case' estimate. On materialisation tests, both MDB-TT and PG-TT ran out of time in all but one case (MDB-TT completed $\text{DBpedia}_L$ in 11,958 seconds): as observed in [1], self-joins on the triple table are notoriously difficult for RDBMSs. In contrast, although it implements TT, RDFox successfully completed all tests. MDB-VP was faster than RDFox on two tests ($\text{LUBM}_L$01K and $\text{UOBM}_L$01K); however, it was slower on the others, and it ran out of memory on many tests. PG-VP was always much slower, and it could not complete many tests.

## 6 Conclusion & Outlook

We presented a novel and very efficient approach to parallel materialisation of datalog in centralised, multi-core, main-memory RDF systems. However, when equality is axiomatised and thus explicated in the materialised data, equality cliques cause a quadratic blowup. We would like to address this challenge by using native equality reasoning in which reasoning is performed over a factorised representation of the data. We further intend to combine native equality reasoning with incremental reasoning techniques. Our goals also include adapting the RDF indexing scheme to secondary storage, the main difficulty of which will be to reduce random access.

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## A Proof of Theorem 1

Let $I$ be a finite set of facts, and let $P$ be a datalog program; we next recapitulate the definition of $P^\infty(I)$. Given a rule $r$, let $r(I)$ be the smallest set containing $\mathsf{h}(r)\sigma$ for each substitution $\sigma$ such that, for each $1 \leq i \leq \mathsf{len}(r)$, we have $B_i\sigma \in I$. Given a datalog program $P$, let $P(I) := \bigcup_{r \in P} r(I)$. Furthermore, let $P^0(I) := I$; for each $i > 0$, let $P^i(I) := P^{i-1}(I) \cup P(P^{i-1}(I))$; and let $P^\infty(I) := \bigcup_i P^i(I)$.

**Theorem 1** *Algorithm 1 terminates and computes $P^\infty(I)$; moreover, each combination of $r$ and $\tau$ is considered in line 5 at most once, so derivations are not repeated.*

*Proof.* Our discussion of the termination condition (lines 6–15) in Section 3 shows that all threads terminate only when $\mathsf{facts}_I.\mathsf{hasNext}$ returns $\mathsf{false}$. Moreover, the number of different facts that our algorithm can derive is finite since it is determined by the number of resources in $I$ and $P$, and duplicates are eliminated eagerly. Finally, $\mathsf{facts}_I.\mathsf{next}$ and $I.\mathsf{add}(F)$ are linearisable, so let $\mathcal{F} = F_1, \ldots, F_m$ be the sequence of distinct facts extracted in line 2; we show that $\mathcal{F} = P^\infty(I)$.

That $F_i \in P^\infty(I)$ holds for each $1 \leq i \leq m$ follows by a straightforward induction on $i$: each $F_i$ is either contained in $I$, or it is obtained by applying a rule $r \in P$ to $I^{\leq F_i}$.

We next show that, for each $i$ and each $F' \in P^i(I)$, we have $F' \in \mathcal{F}$. This is obvious for $i = 0$, so assume that the claim holds for some $i$ and consider an arbitrary fact $F' \in P^{i+1}(I) \setminus P^i(I)$. By the definition of $P^{i+1}(I)$, a rule $r \in P$ exists that produces $F'$ by matching atom $\mathsf{b}_j(r)$ to some $F^j \in P^i(I)$ for each $1 \leq j \leq \mathsf{len}(r)$; from the induction assumption, we have $F^j \in \mathcal{F}$ for each $1 \leq j \leq \mathsf{len}(r)$. Now let $F$ be the fact in $\{F^j | 1 \leq j \leq \mathsf{len}(r)\}$ with the largest index in $I$, and let $k$ be the smallest integer such that $F^k = F$. Fact $F$ is returned at some point in line 2, so the algorithm considers in line 3 the rule $r$, annotated query $Q_k$, and substitution $\sigma$ such that $\mathsf{b}_k(r)\sigma = F$. But then, $I.\mathsf{evaluate}(Q_k, F, \sigma)$ returns a substitution $\tau$ such that $\mathsf{b}_j(r)\tau \in I^{\leq F}$ for each $1 \leq j \leq \mathsf{len}(r)$, and hence the algorithm derives $F' = \mathsf{h}(r)\tau$ in line 5, as required.

Finally, to show that derivations are not repeated, assume that two (not necessarily distinct) facts, $F$ and $F'$, are extracted in line 2, and let $Q_i$ and $Q'_{i'}$ be annotated queries for the same rule $r$ and substitution $\tau$ considered in line 5. By the definition of $Q_i$ and $Q'_{i'}$, we have $\mathsf{b}_i(r)\tau = F$ and $\mathsf{b}_{i'}(r)\tau = F'$. We now consider two cases.

- Assume $F = F'$, and without loss of generality assume $i \leq i'$. For $i = i'$, we have a contradiction since $F$ is extracted in line 2 only once and $Q_i = Q'_{i'}$ is considered in line 3 only once. For $i < i'$, we have a contradiction since we have $\diamond_i = <$ in annotated query $Q_i$, so atom $\mathsf{b}_i(r)$ cannot be matched to $F'$ (i.e., we cannot have $\mathsf{b}_i(r)\tau = F'$) due to $F' \notin I^{<F}$.
- Assume $F \neq F'$. Without loss of generality assume that $F'$ occurs after $F$ in $I$; thus, $F' \notin I^{\leq F}$. But then, $\mathsf{b}_{i'}(r)\tau = F'$ gives us a contradiction since atom $\mathsf{b}_{i'}(r)$ cannot be matched to $F'$ when fact $F$ is extracted in line 2.

## B Implementation of DBRDF

DBRDF is our implementation of the seminaïve algorithm, where RDF data is stored in an RDBMS. It works with PostgreSQL 9.2 or MonetDB, Feb 2013-SP3 release, and so

it emulates RDF systems based on row and column stores such as [1, 5]. In both cases we stored the data on a RAM disk drive; while this clearly does not mimic the performance of a purely RAM-based system, it nevertheless mitigates the overhead of disk access. On PostreSQL, we created all tables as `UNLOGGED` to mitigate the overhead due to fault recovery, and we configured the database to aggressively use memory using the following options:

```
fsync = off,                    synchronous_commit = off,
full_page_writes = off,         bgwriter_lru_pages = 0,
shared_buffers = 16GB,          work_mem = 16GB,
effective_cache_size = 16GB.
```

DBRDF can store RDF data using one of the two well-known storage schemes summarised below. Data is *not* clustered explicitly in either version: MonetDB manages data disposition internally, and clustering in PostgreSQL makes no sense due to continuous data updates during materialisation.

- In the vertical partitioning variant [1], a triple of the form $\langle s, \textit{rdf:type}, C\rangle$ is stored as tuple $\langle s\rangle$ in a unary relation $C$, and a triple of the form $\langle s, R, o\rangle$ such that $R \neq \textit{rdf:type}$ is stored as tuple $\langle s, o\rangle$ in a binary relation $R$. Moreover, each unary relation $C(s)$ is indexed on $s$, while each binary relation $R(s, o)$ is indexed on $\langle s, o\rangle$, and $\langle o\rangle$.
- In the triple table variant [6, 5], each triple of the form $\langle s, p, o\rangle$ is stored directly in a common ternary table, and the latter is indexed on $\langle s, p, o\rangle$, $\langle p, o\rangle$, and $\langle o, s\rangle$.

Given a datalog program $P$, DBRDF uses the well-known notion of a rule dependency graph [2] to partition $P$ into a sequence of programs $(P_1, \ldots, P_n)$ such that $P^\infty(I) = P_n^\infty(\ldots P_1^\infty(I)\ldots)$; then, DBRDF applies the seminaïve algorithm [2] to each $P_i$. The 'old', 'delta-old', and 'new' relations from the seminaïve algorithm are implemented as temporary tables. The rules in $P_i$ are translated naturally into `INSERT INTO ... SELECT ...` statements; for example, with vertical partitioning, rule $\langle x, R, y\rangle \wedge \langle y, R, z\rangle \rightarrow \langle x, S, z\rangle$ is translated as follows.

```
INSERT INTO S(s,o) SELECT DISTINCT t1.s, t2.o
FROM R t1, R t2 WHERE t1.o = t2.s AND NOT EXISTS
(SELECT 1 FROM S ex WHERE ex.s = t1.s AND ex.o = t2.o)
```

Here, `DISTINCT` and `NOT EXISTS` eliminate duplicates, which is essential for both termination and to not repeat derivations. All statements are evaluated using the 'read uncommitted' transaction isolation level. On PostgreSQL, SQL statements are combined into a PL/pgSQL script that is evaluated sequentially inside the database; all scripts used in our evaluation are available online. On MonetDB, SQL statements are issued sequentially from a Java program. MonetDB can exploit intraquery parallelism, but for fairness we restricted the number of threads to one. Devising ways to exploit both inter- and intraquery parallelism is out of scope of this paper.