Parallel Programming

in C with MPI and OpenMP

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Chapter 4

Message-Passing Programming

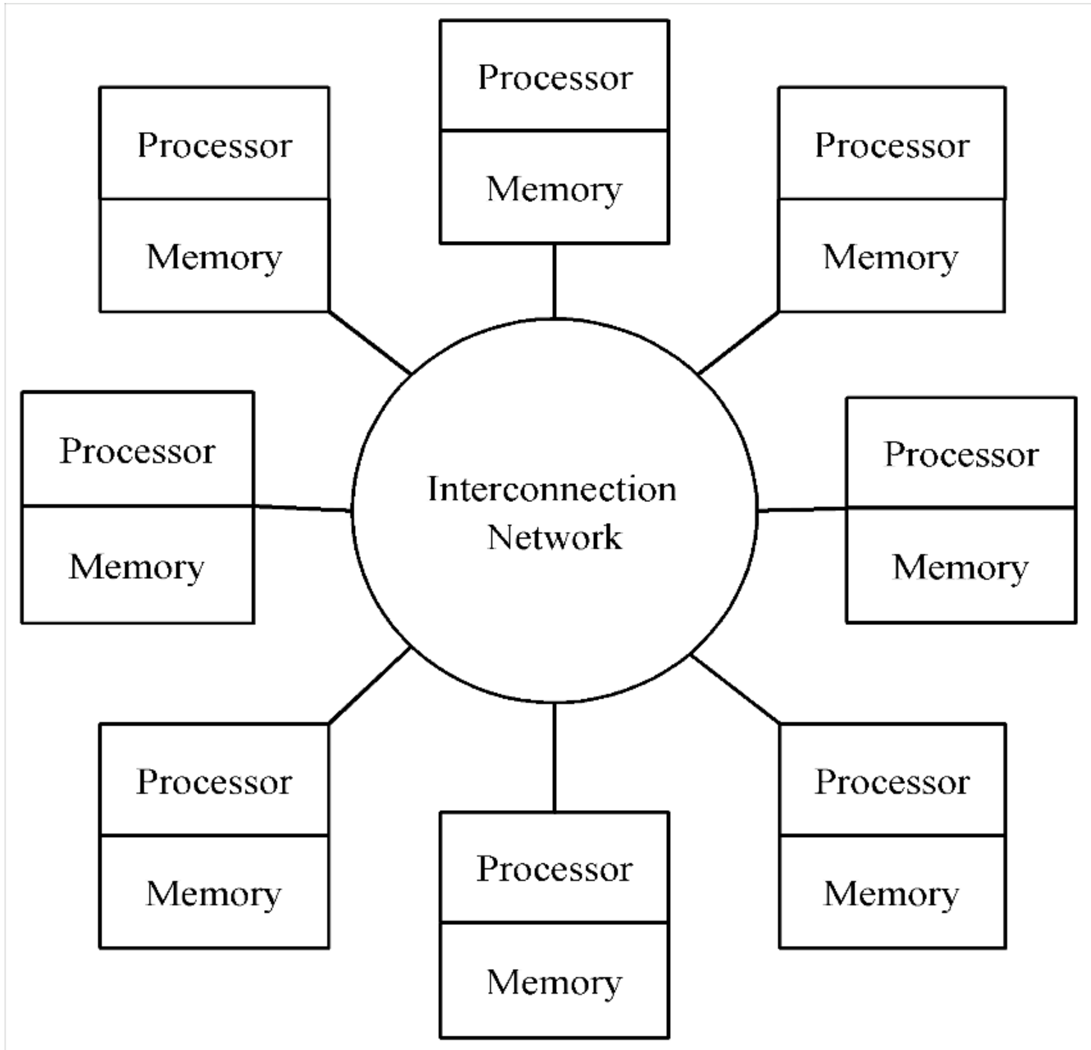
Learning Objectives

- Understanding how MPI programs execute
- Familiarity with fundamental MPI functions

Outline

- Message-passing model
- Message Passing Interface (MPI)
- Coding MPI programs
- Compiling MPI programs
- Running MPI programs
- Benchmarking MPI programs

Message-passing Model



Processes

- Number is specified at start-up time
- Remains constant throughout execution of program
- All execute same program
- Each has unique ID number
- Alternately performs computations and communicates

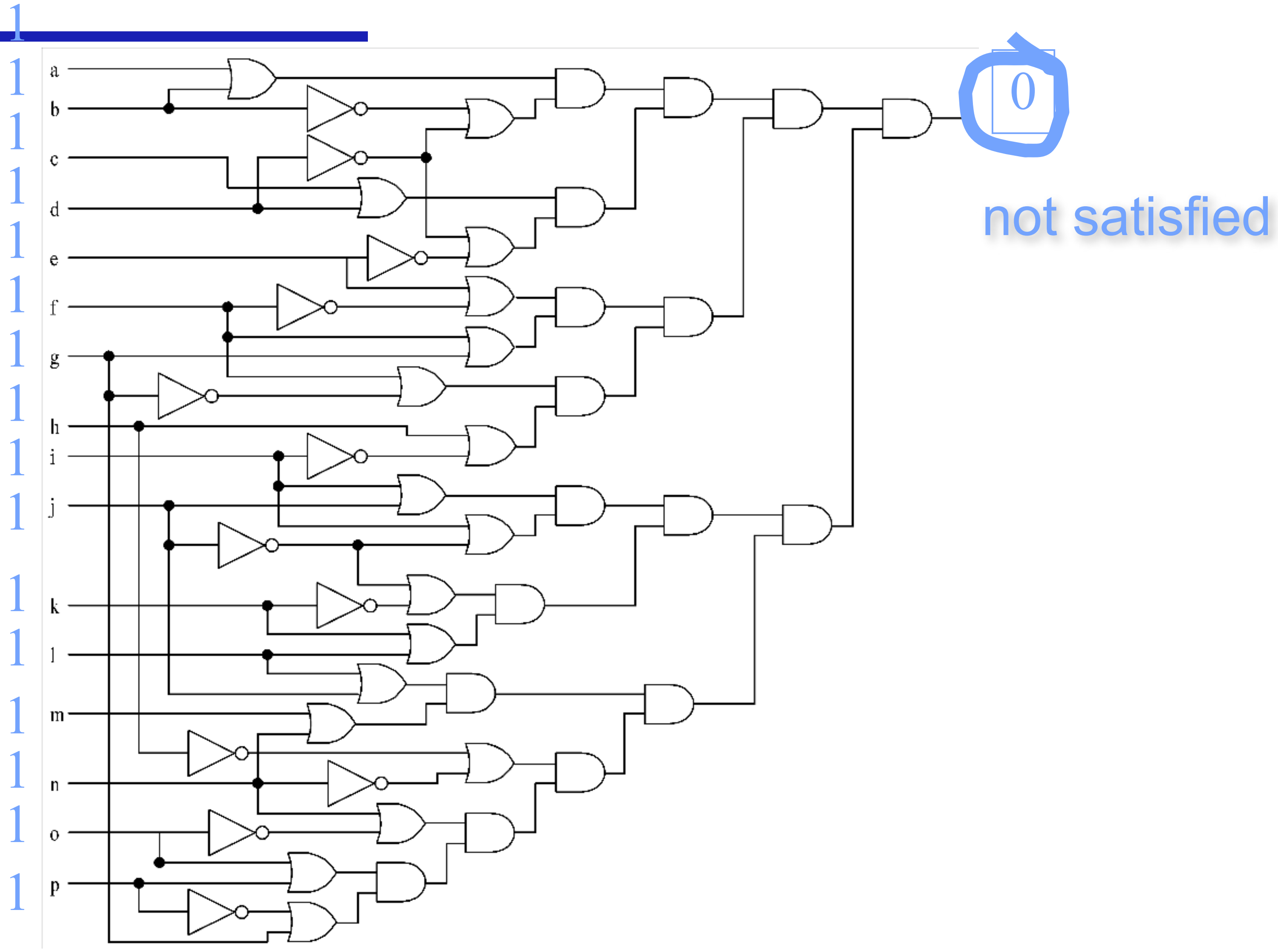
Advantages of Message-passing Model

- Gives programmer ability to manage the memory hierarchy
- Portability to many architectures
- Easier to create a deterministic program
- Simplifies debugging

The Message Passing Interface

- Late 1980s: vendors had unique libraries
- 1989: Parallel Virtual Machine (PVM) developed at Oak Ridge National Lab
- 1992: Work on MPI standard begun
- 1994: Version 1.0 of MPI standard
- 1997: Version 2.0 of MPI standard
- Today: MPI is dominant message passing library standard

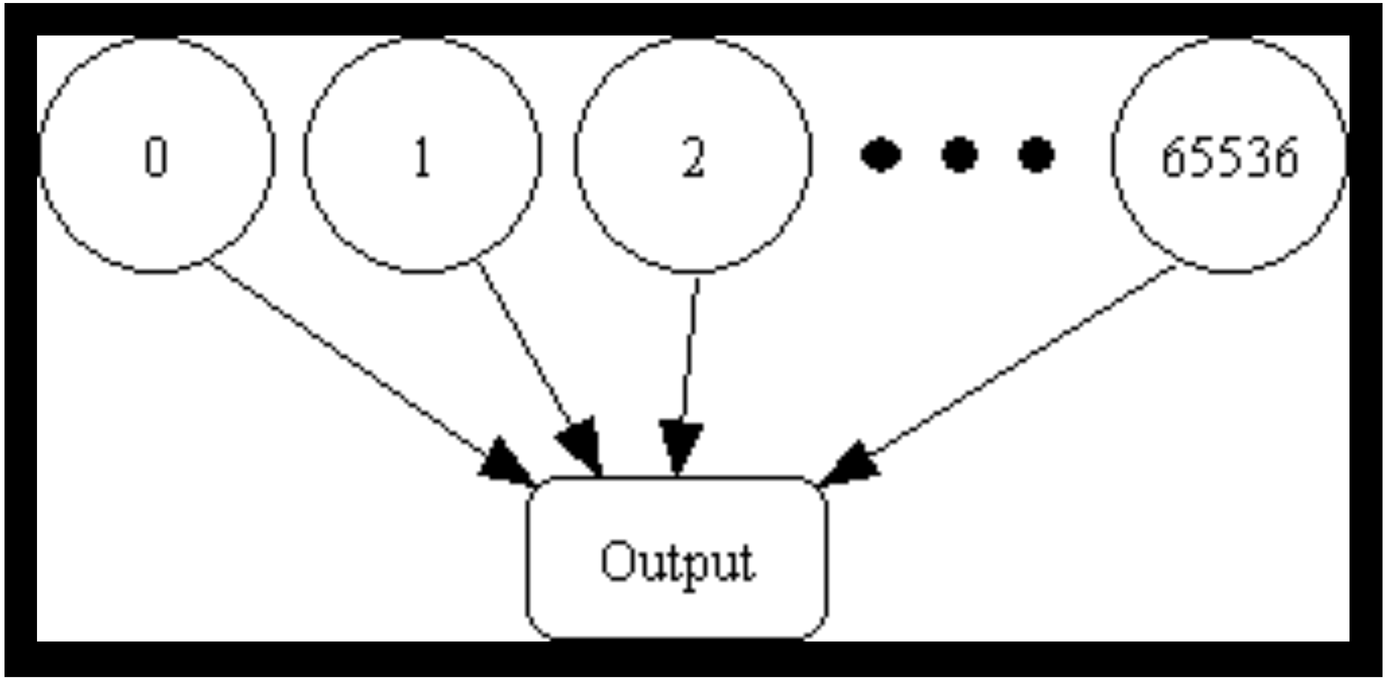
Circuit Satisfiability



Solution Method

- Circuit satisfiability is NP-complete
- No known algorithms to solve in polynomial time
- We seek all solutions
- We find through exhaustive search
- 16 inputs $\Rightarrow$ 65,536 combinations to test

Partitioning: Functional Decomposition



- Embarrassingly parallel: No channels between tasks

Agglomeration and Mapping

- Properties of parallel algorithm
  - Fixed number of tasks
  - No communications between tasks
  - Time needed per task is variable
- Map tasks to processors in a cyclic fashion

Cyclic (interleaved) Allocation

- Assume *p* processes
- Each process gets every *p*th piece of work
- Example: 5 processes and 12 pieces of work
  - P_0: 0, 5, 10
  - P_1: 1, 6, 11
  - P_2: 2, 7
  - P_3: 3, 8
  - P_4: 4, 9

Pop Quiz

- Assume *n* pieces of work, *p* processes, and cyclic allocation
- What is the most pieces of work any process has?
- What is the least pieces of work any process has?
- How many processes have the most pieces of work?

Summary of Program Design

- Program will consider all 65,536 combinations of 16 boolean inputs
- Combinations allocated in cyclic fashion to processes
- Each process examines each of its combinations
- If it finds a satisfiable combination, it will print it

Include Files

```
#include <mpi.h>
```

- MPI header file

```
#include <stdio.h>
```

- Standard I/O header file

Local Variables

```
int main (int argc, char *argv[]) {
    int i;
    int id; /* Process rank */
    int p;  /* Number of processes */
    void check_circuit (int, int);
```

- Include **argc** and **argv**: they are needed to initialize MPI
- One copy of every variable for each process running this program

Initialize MPI

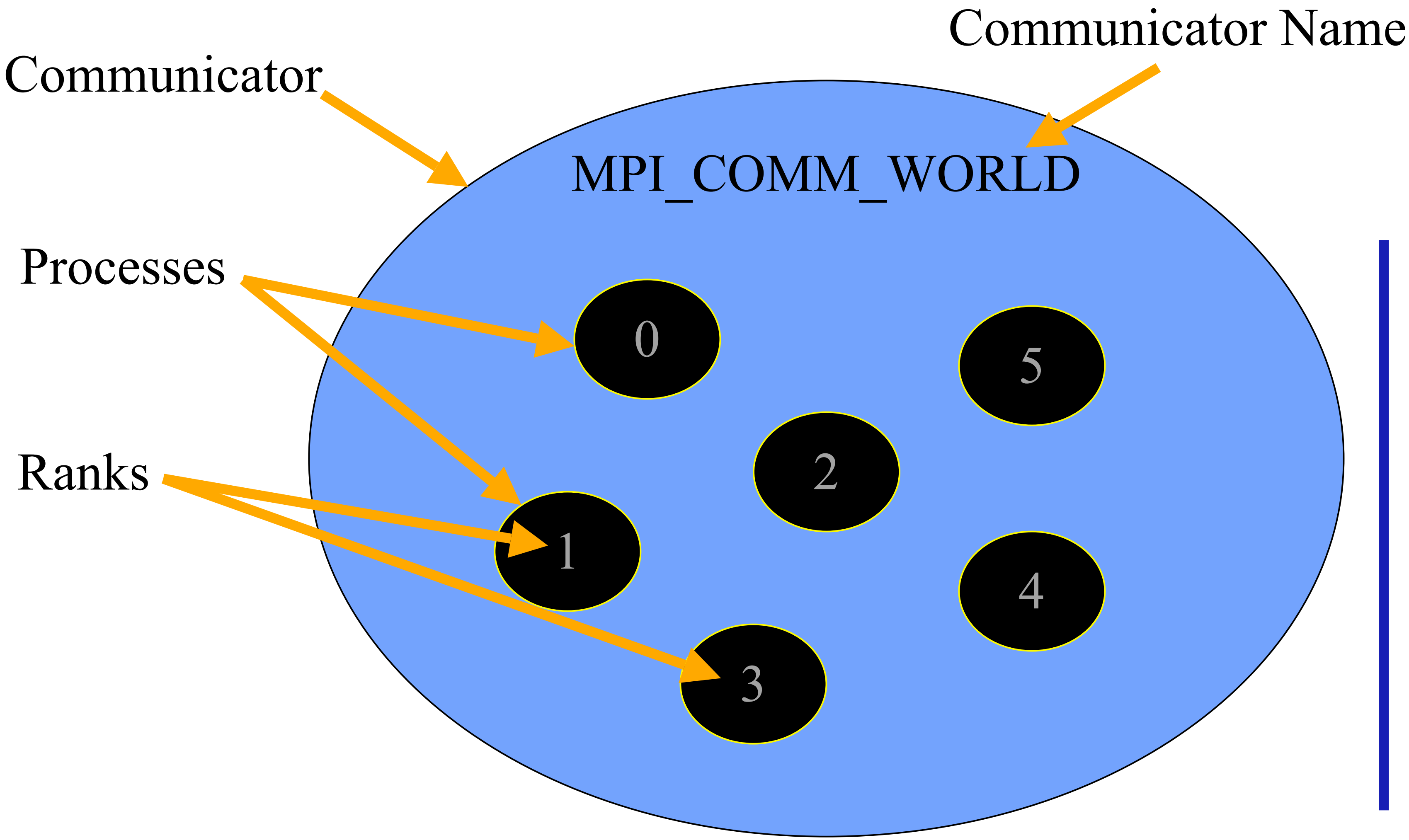
```
MPI_Init (&argc, &argv);
```

- First MPI function called by each process
- Not necessarily first executable statement
- Allows system to do any necessary setup

Communicators

- Communicator: opaque object that provides message-passing environment for processes
- MPI_COMM_WORLD
  - Default communicator
  - Includes all processes
- Possible to create new communicators
  - Will do this in Chapters 8 and 9

Communicator



Determine Number of Processes

```
MPI_Comm_size (MPI_COMM_WORLD, &p);
```

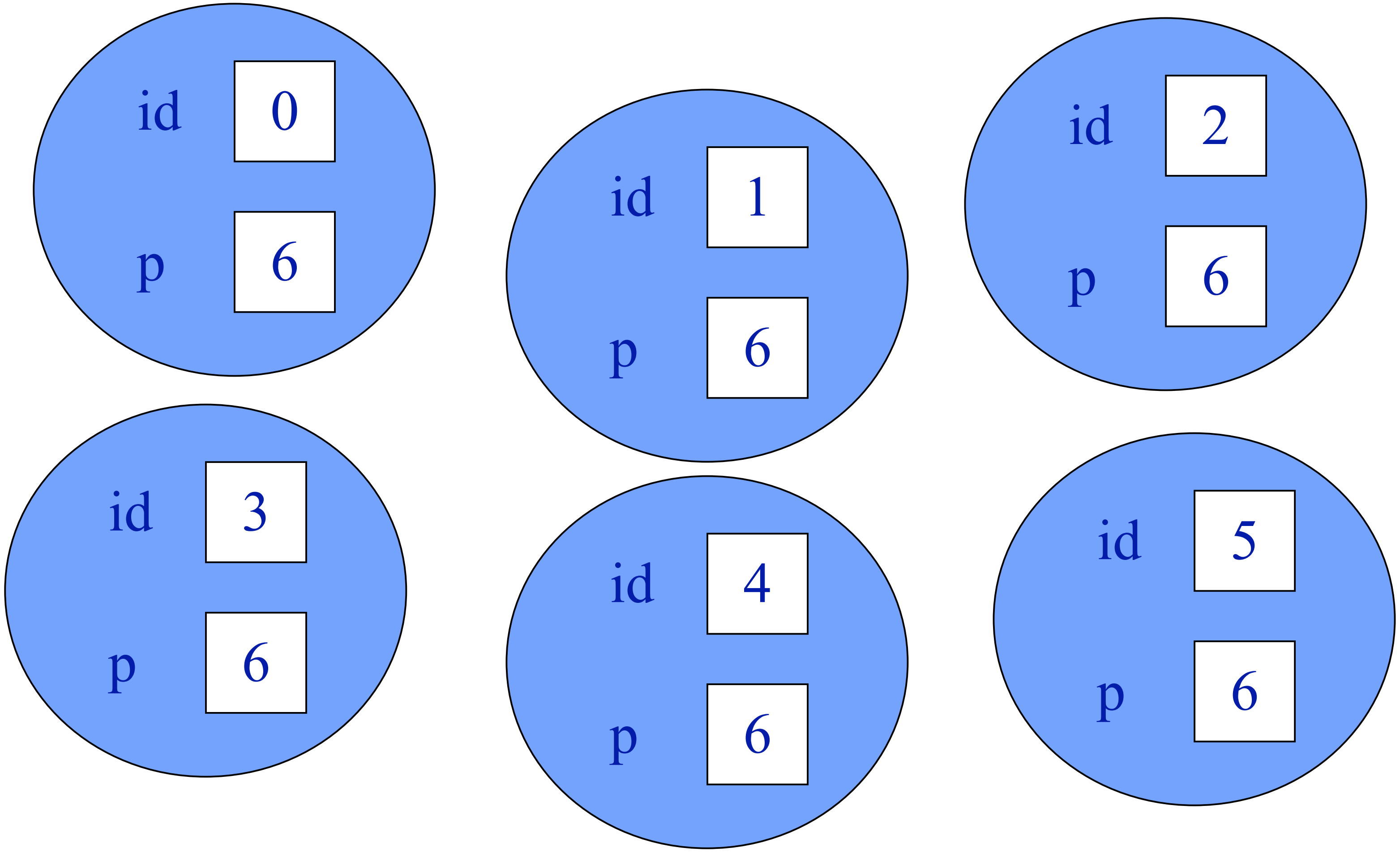
- First argument is communicator
- Number of processes returned through second argument

Determine Process Rank

```
MPI_Comm_rank (MPI_COMM_WORLD, &id);
```

- First argument is communicator
- Process rank (in range 0, 1, …, *p*-1) returned through second argument

Replication of Automatic Variables



What about External Variables?

```
int total;

int main (int argc, char *argv[]) {
    int i;
    int id;
    int p;
    …
```

- Where is variable **`total`** stored?

Cyclic Allocation of Work

```
for (i = id; i < 65536; i += p)
   check_circuit (id, i);
```

- Parallelism is outside function **`check_circuit`**
- It can be an ordinary, sequential function

Shutting Down MPI

```
MPI_Finalize();
```

- Call after all other MPI library calls
- Allows system to free up MPI resources

```
#include <mpi.h>
#include <stdio.h>

int main (int argc, char *argv[]) {
   int i;
   int id;
   int p;
   void check_circuit (int, int);

   MPI_Init (&argc, &argv);
   MPI_Comm_rank (MPI_COMM_WORLD, &id);
   MPI_Comm_size (MPI_COMM_WORLD, &p);

   for (i = id; i < 65536; i += p)
      check_circuit (id, i);

   printf ("Process %d is done\n", id);
   fflush (stdout);
   MPI_Finalize();
   return 0;
}
```

Put fflush() after every printf()

```
/* Return 1 if 'i'th bit of 'n' is 1; 0 otherwise */
#define EXTRACT_BIT(n,i) ((n&(1<<i))?1:0)

void check_circuit (int id, int z) {
   int v[16];        /* Each element is a bit of z */
   int i;

   for (i = 0; i < 16; i++) v[i] = EXTRACT_BIT(z,i);

   if ((v[0] || v[1]) && (!v[1] || !v[3]) && (v[2] || v[3])
      && (!v[3] || !v[4]) && (v[4] || !v[5])
      && (v[5] || !v[6]) && (v[5] || v[6])
      && (v[6] || !v[15]) && (v[7] || !v[8])
      && (!v[7] || !v[13]) && (v[8] || v[9])
      && (v[8] || !v[9]) && (!v[9] || !v[10])
      && (v[9] || v[11]) && (v[10] || v[11])
      && (v[12] || v[13]) && (v[13] || !v[14])
      && (v[14] || v[15])) {
      printf ("%d) %d%d%d%d%d%d%d%d%d%d%d%d%d%d%d%d\n", id,
         v[0],v[1],v[2],v[3],v[4],v[5],v[6],v[7],v[8],v[9],
         v[10],v[11],v[12],v[13],v[14],v[15]);
      fflush (stdout);
   }
}
```

Compiling MPI Programs

```
mpicc -O -o foo foo.c
```

- **mpicc**: script to compile and link C+MPI programs
- Flags: same meaning as C compiler
  - `-O` — optimize
  - `-o <file>` — where to put executable

Running MPI Programs

- `mpirun -np <p> <exec> <arg1> …`
  - `-np <p>` — number of processes
  - `<exec>` — executable
  - `<arg1> …` — command-line arguments

Specifying Host Processors

- File **`.mpi-machines`** in home directory lists host processors in order of their use
- Example **`.mpi_machines`** file contents

```
band01.cs.ppu.edu
band02.cs.ppu.edu
band03.cs.ppu.edu
band04.cs.ppu.edu
```

Enabling Remote Logins

- MPI needs to be able to initiate processes on other processors without supplying a password
- Each processor in group must list all other processors in its **.rhosts** file; e.g.,

```
band01.cs.ppu.edu student
band02.cs.ppu.edu student
band03.cs.ppu.edu student
band04.cs.ppu.edu student
```

Execution on 1 CPU

```
% mpirun -np 1 sat
0) 1010111110011001
0) 0110111110011001
0) 1110111110011001
0) 1010111111011001
0) 0110111111011001
0) 1110111111011001
0) 1010111110111001
0) 0110111110111001
0) 1110111110111001
Process 0 is done
```

Execution on 2 CPUs

```
% mpirun -np 2 sat
0) 0110111110011001
0) 0110111111011001
0) 0110111110111001
1) 1010111110011001
1) 1110111110011001
1) 1010111111011001
1) 1110111111011001
1) 1010111110111001
1) 1110111110111001
Process 0 is done
Process 1 is done
```

Execution on 3 CPUs

```
% mpirun -np 3 sat
0) 0110111110011001
0) 1110111111011001
2) 1010111110011001
1) 1110111110011001
1) 1010111111011001
1) 0110111110111001
0) 1010111110111001
2) 0110111111011001
2) 1110111110111001
Process 1 is done
Process 2 is done
Process 0 is done
```

Deciphering Output

- Output order only partially reflects order of output events inside parallel computer
- If process A prints two messages, first message will appear before second
- If process A calls `printf` before process B, there is no guarantee process A's message will appear before process B's message

Enhancing the Program

- We want to find total number of solutions
- Incorporate sum-reduction into program
- Reduction is a collective communication

Modifications

- Modify function `check_circuit`
  - Return 1 if circuit satisfiable with input combination
  - Return 0 otherwise
- Each process keeps local count of satisfiable circuits it has found
- Perform reduction after `for` loop

New Declarations and Code

```
int count;  /* Local sum */

int global_count; /* Global sum */

int check_circuit (int, int);


count = 0;

for (i = id; i < 65536; i += p)

   count += check_circuit (id, i);
```

Prototype of `MPI_Reduce()`

```
int MPI_Reduce (
   void          *operand,
                 /* addr of 1st reduction element */
   void          *result,
                 /* addr of 1st reduction result */
   int           count,
                 /* reductions to perform */
   MPI_Datatype type,
                 /* type of elements */
   MPI_Op        operator,
                 /* reduction operator */
   int           root,
                 /* process getting result(s) */
   MPI_Comm      comm
                 /* communicator */
)
```

MPI_Datatype Options

- MPI_CHAR
- MPI_DOUBLE
- MPI_FLOAT
- MPI_INT
- MPI_LONG
- MPI_LONG_DOUBLE
- MPI_SHORT
- MPI_UNSIGNED_CHAR
- MPI_UNSIGNED
- MPI_UNSIGNED_LONG
- MPI_UNSIGNED_SHORT

`MPI_Op` Options

- `MPI_BAND`
- `MPI_BOR`
- `MPI_BXOR`
- `MPI_LAND`
- `MPI_LOR`
- `MPI_LXOR`
- `MPI_MAX`
- `MPI_MAXLOC`
- `MPI_MIN`
- `MPI_MINLOC`
- `MPI_PROD`
- `MPI_SUM`

Our Call to `MPI_Reduce()`

```
MPI_Reduce (&count,
            &global_count,
            1,
            MPI_INT,
            MPI_SUM,
            0,
            MPI_COMM_WORLD);
```

Only process 0 will get the result

```
if (!id) printf ("There are %d different solutions\n",
   global_count);
```

Execution of Second Program

```
% mpirun -np 3 seq2
0) 0110111110011001
0) 1110111111011001
1) 1110111110011001
1) 1010111111011001
2) 1010111110011001
2) 0110111111011001
2) 1110111110111001
1) 0110111110111001
0) 1010111110111001
Process 1 is done
Process 2 is done
Process 0 is done
There are 9 different solutions
```

Benchmarking the Program

- `MPI_Barrier` — barrier synchronization
- `MPI_Wtick` — timer resolution
- `MPI_Wtime` — current time

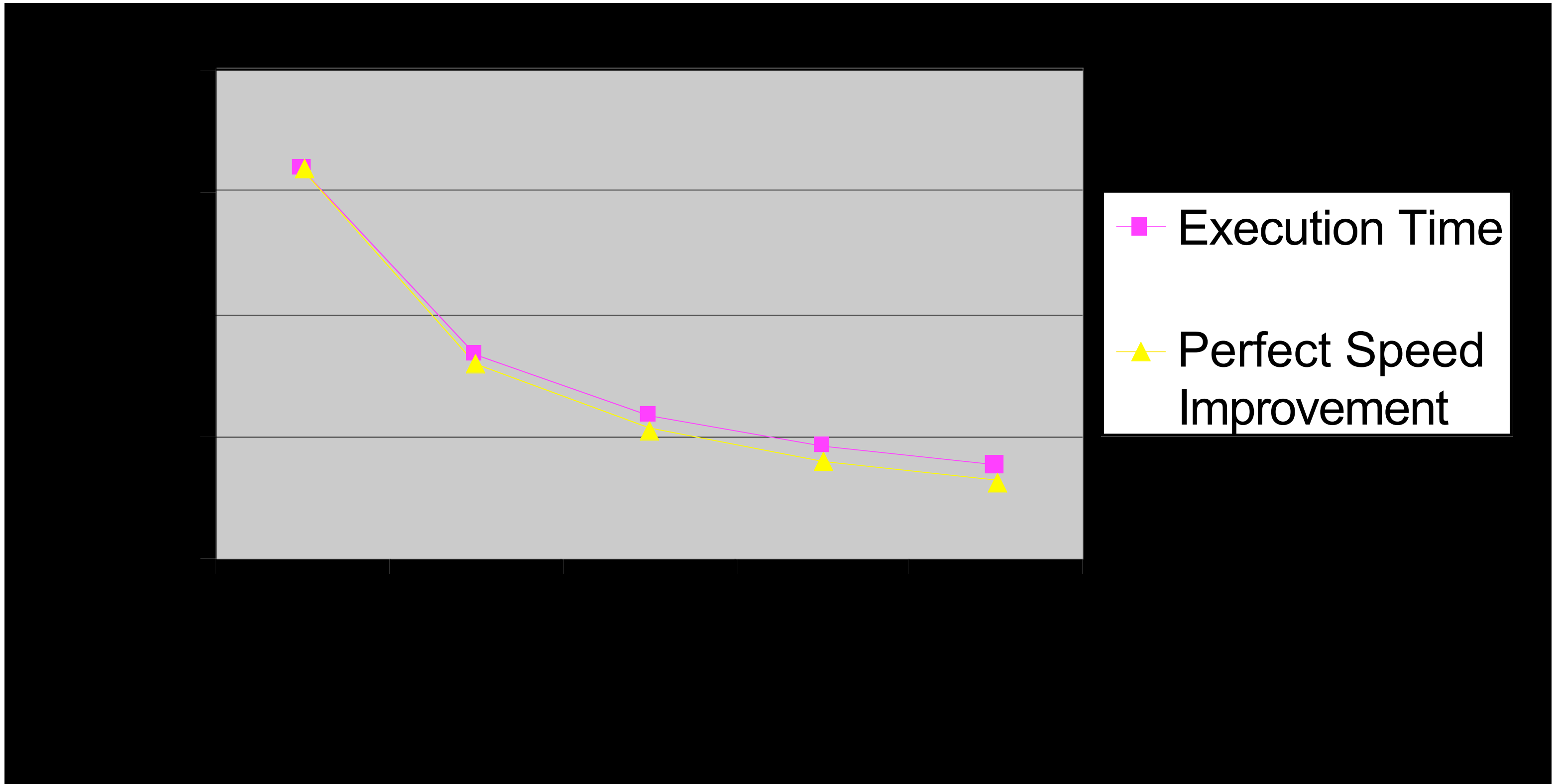
Benchmarking Code

```
double elapsed_time;
…
MPI_Init (&argc, &argv);
MPI_Barrier (MPI_COMM_WORLD);
elapsed_time = - MPI_Wtime();
…
MPI_Reduce (…);
elapsed_time += MPI_Wtime();
```

Benchmarking Results

Processors	Time (sec)
1	15.93
2	8.38
3	5.86
4	4.60
5	3.77

Benchmarking Results



Summary (1/2)

- Message-passing programming follows naturally from task/channel model
- Portability of message-passing programs
- MPI most widely adopted standard

Summary (2/2)

- MPI functions introduced
  - `MPI_Init`
  - `MPI_Comm_rank`
  - `MPI_Comm_size`
  - `MPI_Reduce`
  - `MPI_Finalize`
  - `MPI_Barrier`
  - `MPI_Wtime`
  - `MPI_Wtick`

Chapter 6

Floyd's Algorithm

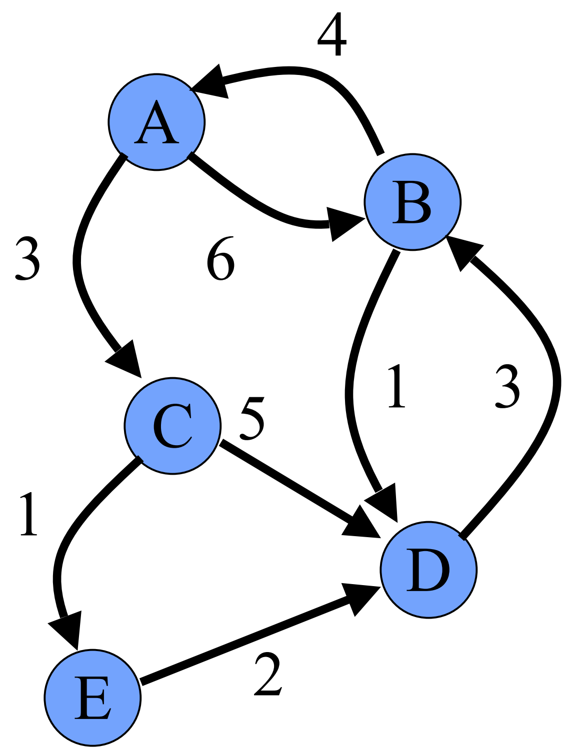
Chapter Objectives

- Creating 2-D arrays
- Thinking about "grain size"
- Introducing point-to-point communications
- Reading and printing 2-D matrices
- Analyzing performance when computations and communications overlap

Outline

- All-pairs shortest path problem
- Dynamic 2-D arrays
- Parallel algorithm design
- Point-to-point communication
- Block row matrix I/O
- Analysis and benchmarking

All-pairs Shortest Path Problem



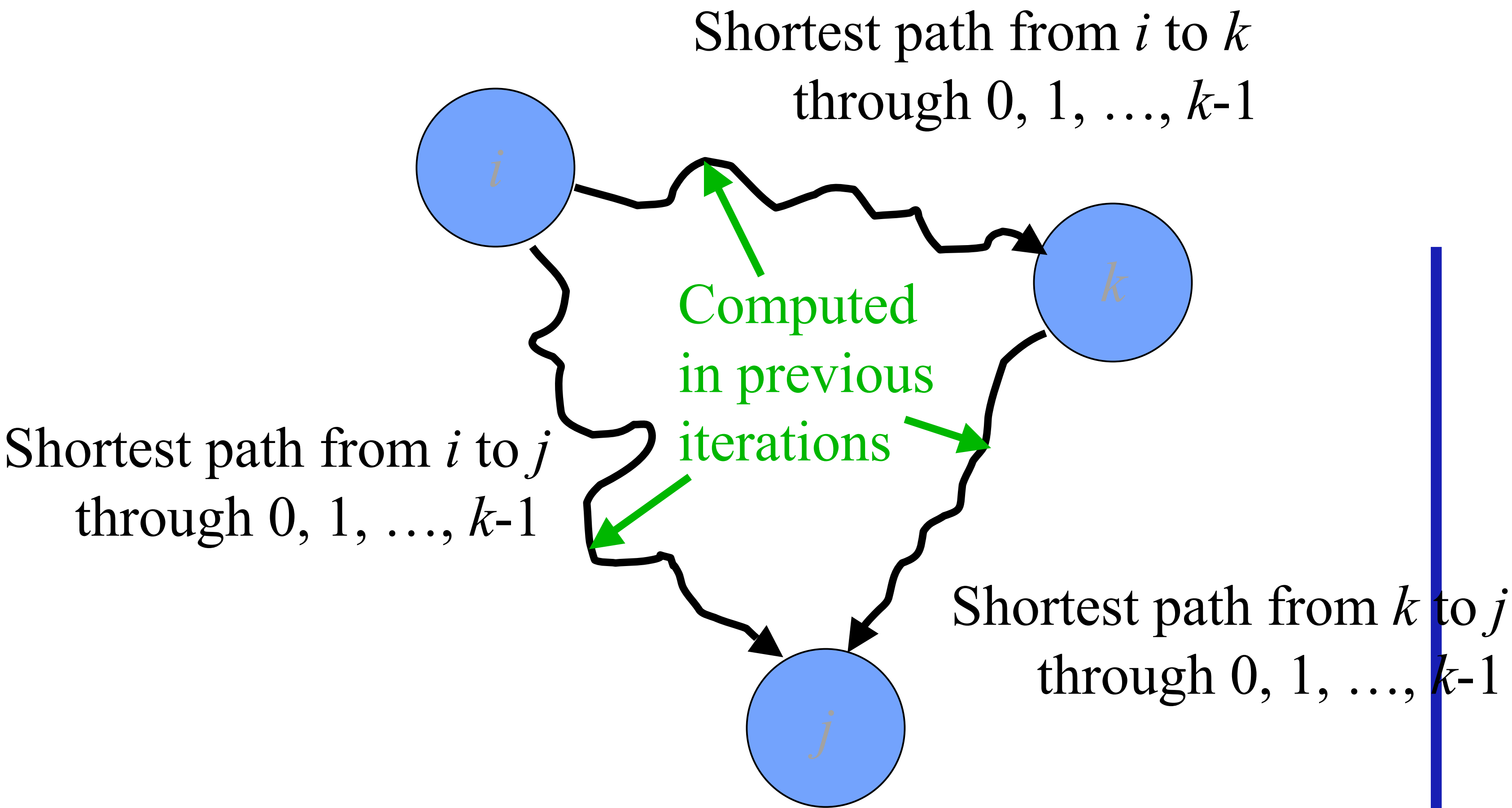
	A	B	C	D	E
A	0	6	3	6	4
B	4	0	7	10	8
C	12	6	0	3	1
D	7	3	10	0	11
E	9	5	12	2	0

Resulting Adjacency Matrix Containing Distances

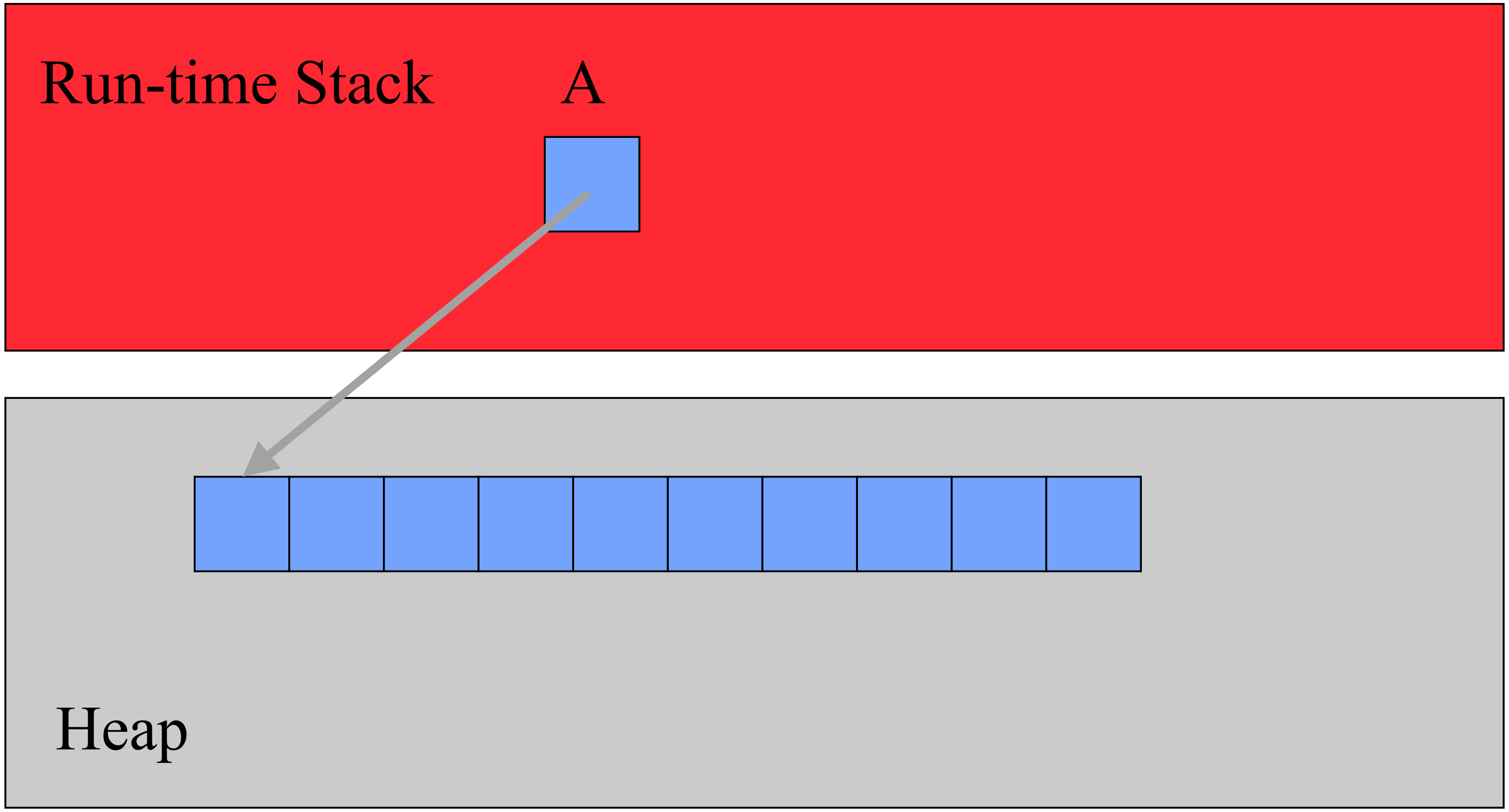
Floyd's Algorithm

for $k \leftarrow 0$ to n-1
    for $i \leftarrow 0$ to n-1
        for $j \leftarrow 0$ to n-1
            $a[i,j] \leftarrow \min(a[i,j], a[i,k] + a[k,j])$
        endfor
    endfor
endfor

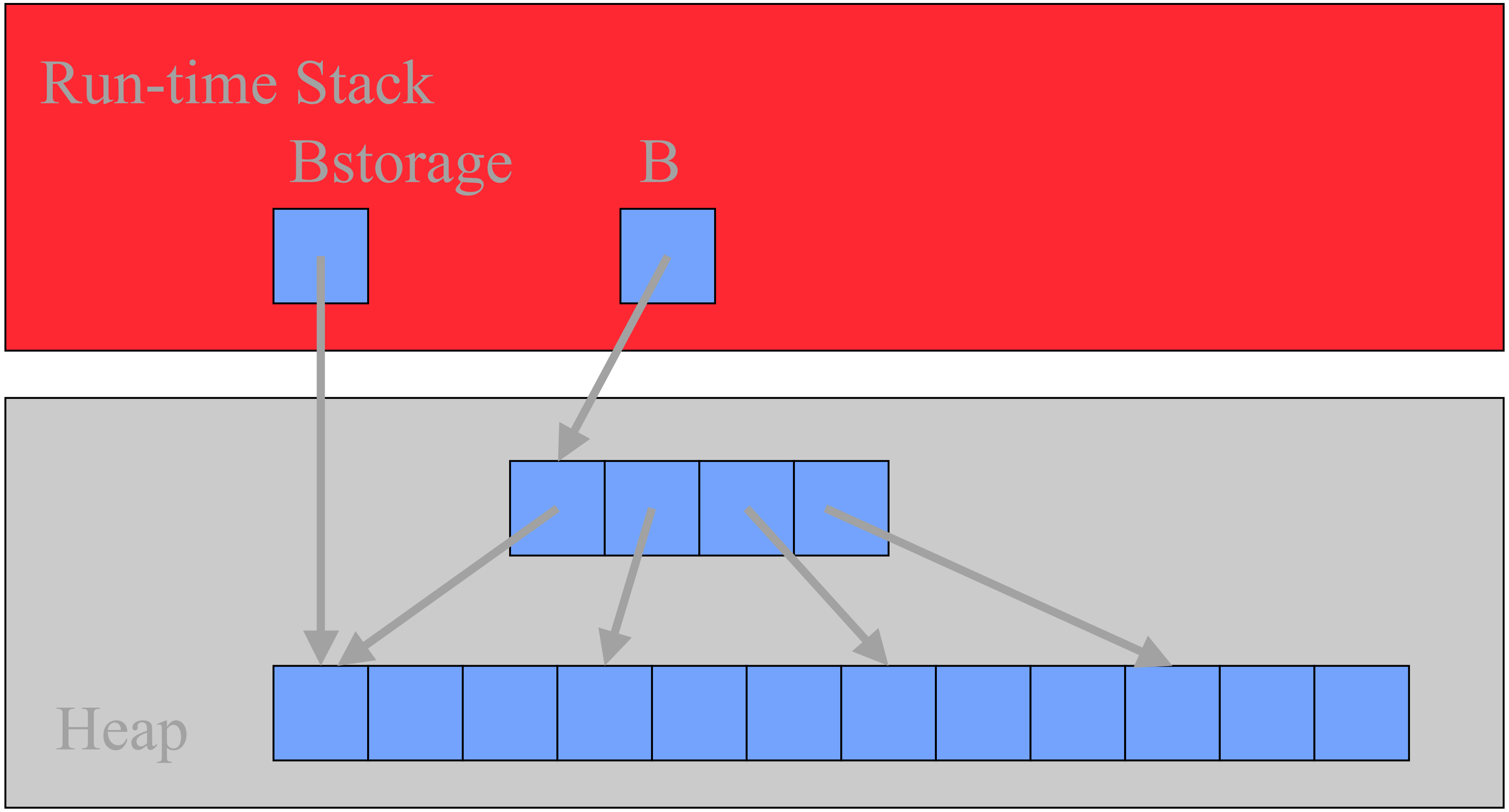
Why It Works


Shortest path from *i* to *k*
through 0, 1, …, *k*-1
i
k
Computed
in previous
iterations
Shortest path from *i* to *j*
through 0, 1, …, *k*-1
Shortest path from *k* to *j*
through 0, 1, …, *k*-1
j

Dynamic 1-D Array Creation



Dynamic 2-D Array Creation



Designing Parallel Algorithm

- Partitioning
- Communication
- Agglomeration and Mapping

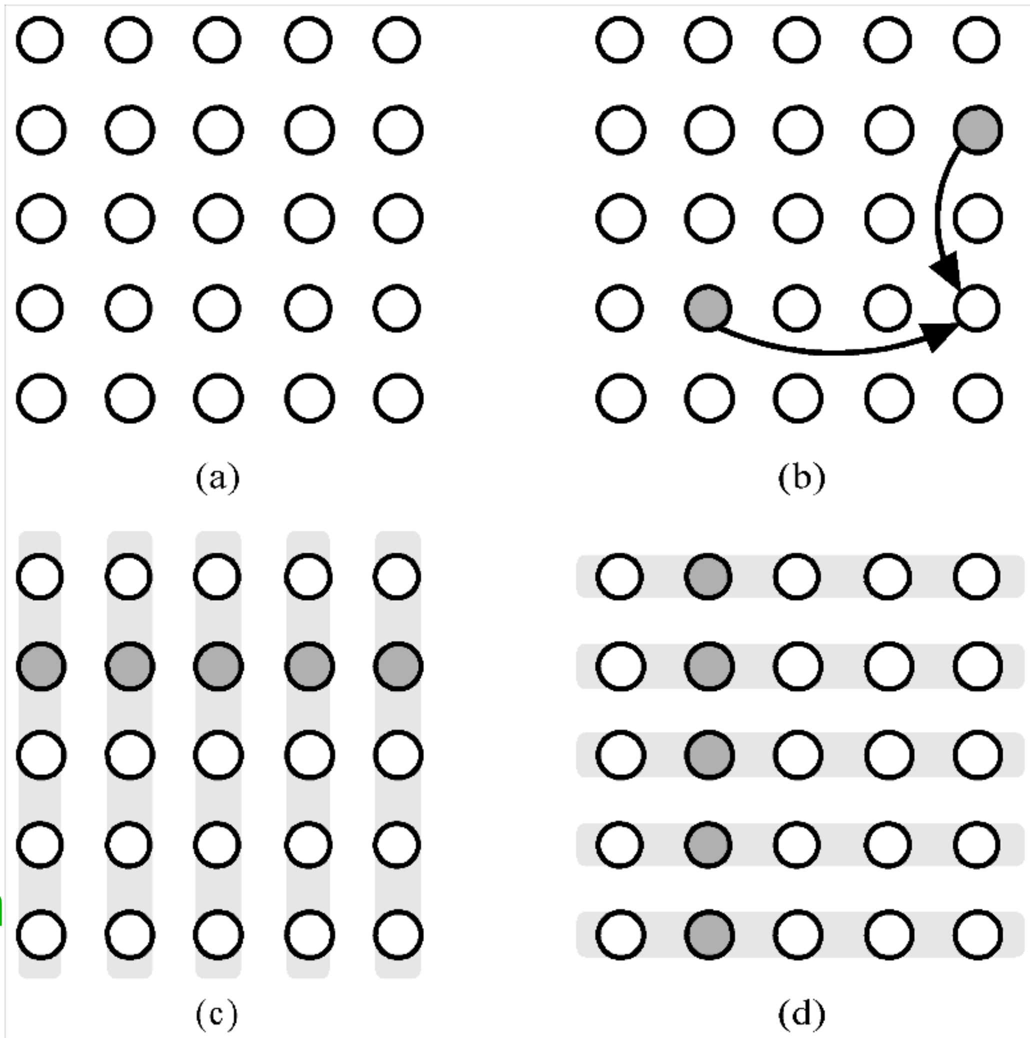
Partitioning

- Domain or functional decomposition?
- Look at pseudocode
- Same assignment statement executed n^3 times
- No functional parallelism
- Domain decomposition: divide matrix **A** into its n^2 elements

Communication

Primitive tasks

Updating *a[3,4]* when *k* = 1



Iteration *k*: every task in row *k* broadcasts its value w/in task column

Iteration *k*: every task in column *k* broadcasts its value w/in task row

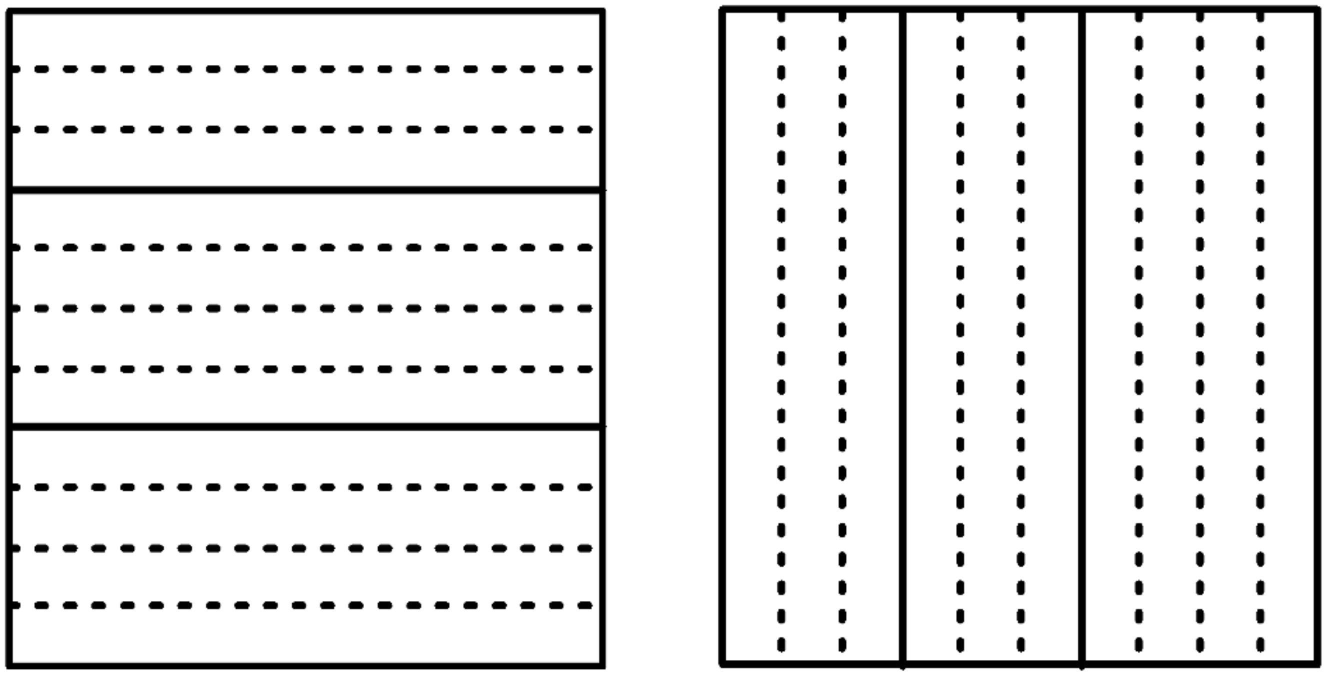
Agglomeration and Mapping

- Number of tasks: static
- Communication among tasks: structured
- Computation time per task: constant
- Strategy:
  - Agglomerate tasks to minimize communication
  - Create one task per MPI process

Two Data Decompositions

Rowwise block striped

Columnwise block striped



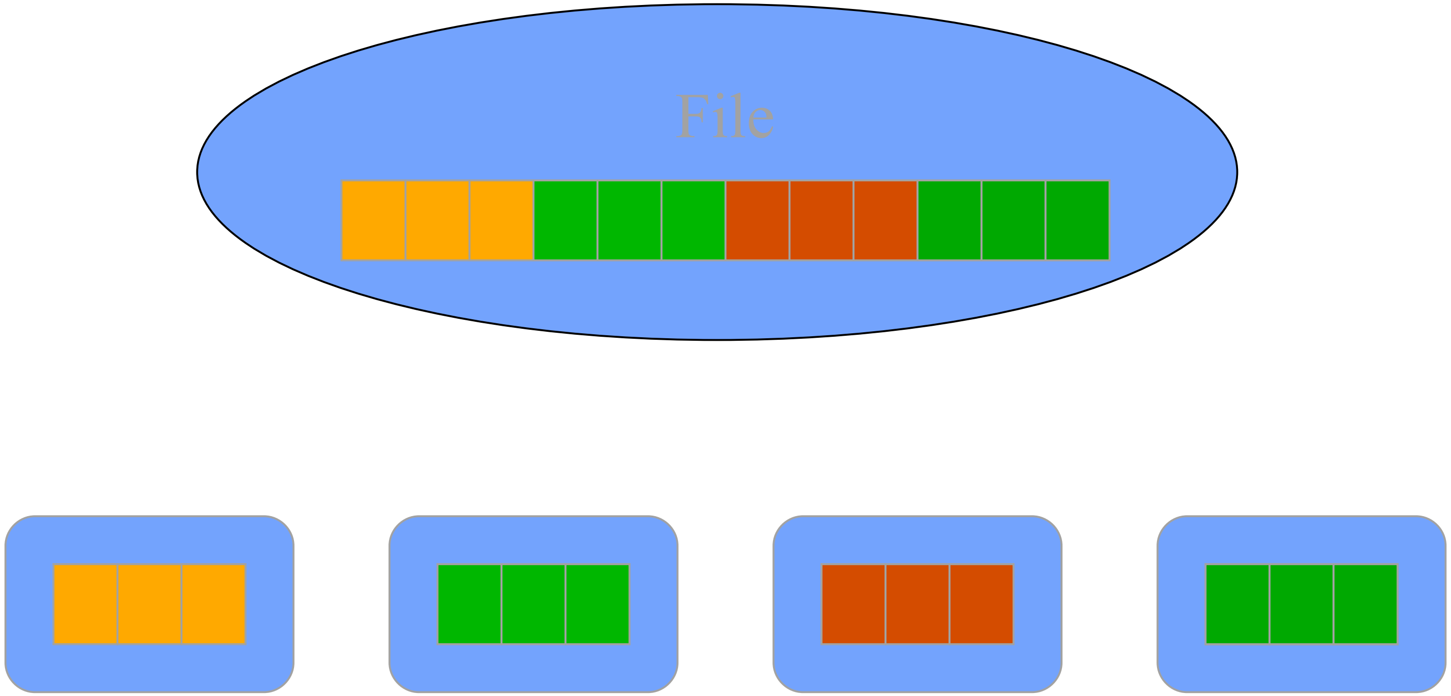
(a)

(b)

Comparing Decompositions

- Columnwise block striped
  - Broadcast within columns eliminated
- Rowwise block striped
  - Broadcast within rows eliminated
  - Reading matrix from file simpler
- Choose rowwise block striped decomposition

File Input



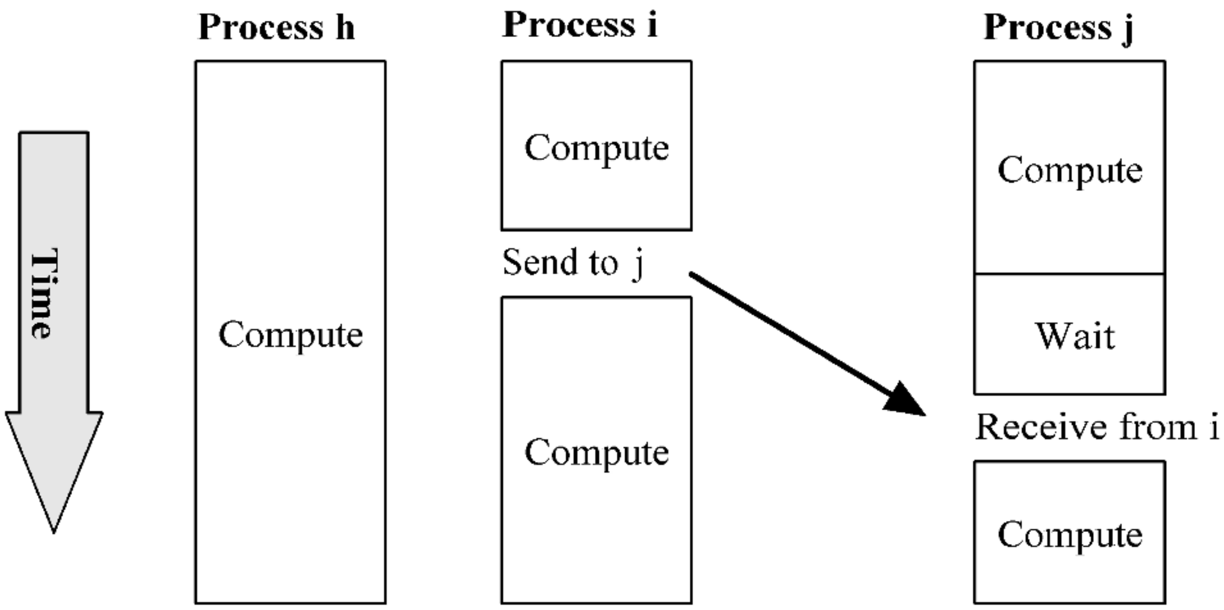
Pop Quiz

Why don’t we input the entire file at once and then scatter its contents among the processes, allowing concurrent message passing?

Point-to-point Communication

- Involves a pair of processes
- One process sends a message
- Other process receives the message

Send/Receive Not Collective



Function MPI_Send

```
int MPI_Send (
      void           *message,
      int             count,
      MPI_Datatype    datatype,
      int             dest,
      int             tag,
      MPI_Comm        comm
)
```

Function MPI_Recv

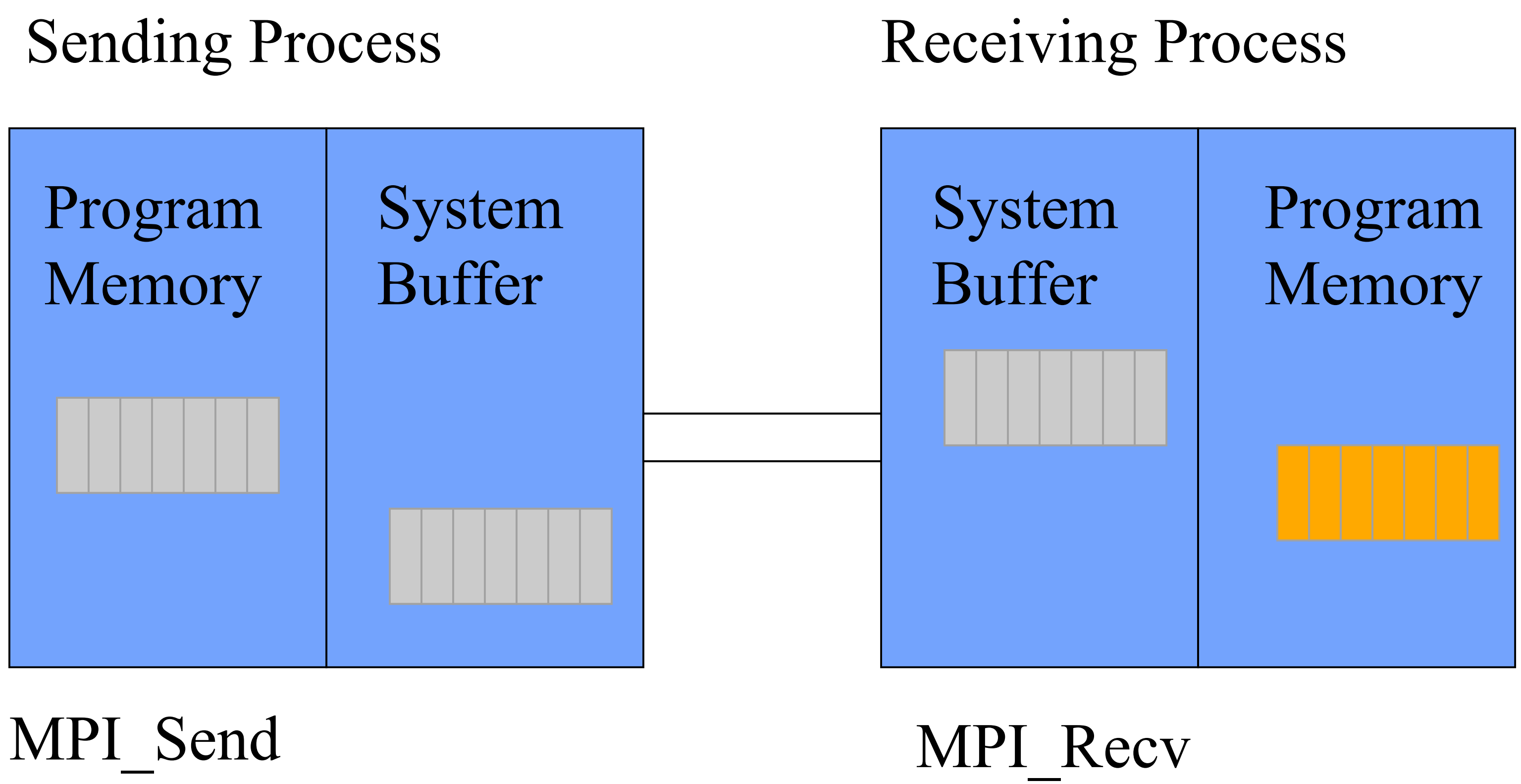
```
int MPI_Recv (
      void           *message,
      int             count,
      MPI_Datatype    datatype,
      int             source,
      int             tag,
      MPI_Comm        comm,
      MPI_Status     *status
)
```

Coding Send/Receive

```
…
if (ID == j) {
    …
    Receive from I
    …
}
…
if (ID == i) {
    …
    Send to j
    …
}
…
```

Receive is before Send.
Why does this work?

Inside MPI_Send and MPI_Recv



Return from MPI_Send

- Function blocks until message buffer free
- Message buffer is free when
  - Message copied to system buffer, or
  - Message transmitted
- Typical scenario
  - Message copied to system buffer
  - Transmission overlaps computation

Return from MPI_Recv

- Function blocks until message in buffer
- If message never arrives, function never returns

Deadlock

- Deadlock: process waiting for a condition that will never become true
- Easy to write send/receive code that deadlocks
  - Two processes: both receive before send
  - Send tag doesn't match receive tag
  - Process sends message to wrong destination process

Function MPI_Bcast

```
int MPI_Bcast (
   void *buffer, /* Addr of 1st element */
   int count,    /* # elements to broadcast */
   MPI_Datatype datatype, /* Type of elements */
   int root,     /* ID of root process */
   MPI_Comm comm)  /* Communicator */
```

```
MPI_Bcast (&k, 1, MPI_INT, 0, MPI_COMM_WORLD);
```

Computational Complexity

- Innermost loop has complexity $\Theta(n)$
- Middle loop executed at most $\lceil n/p \rceil$ times
- Outer loop executed n times
- Overall complexity $\Theta(n^3/p)$

Communication Complexity

- No communication in inner loop
- No communication in middle loop
- Broadcast in outer loop — complexity is $\Theta(n \log p)$
- Overall complexity $\Theta(n^2 \log p)$

Execution Time Expression (1)

$$n\lceil n/p \rceil n\chi + n\lceil \log p \rceil(\lambda + 4n/\beta)$$

Message-passing time

Messages per broadcast

Iterations of outer loop

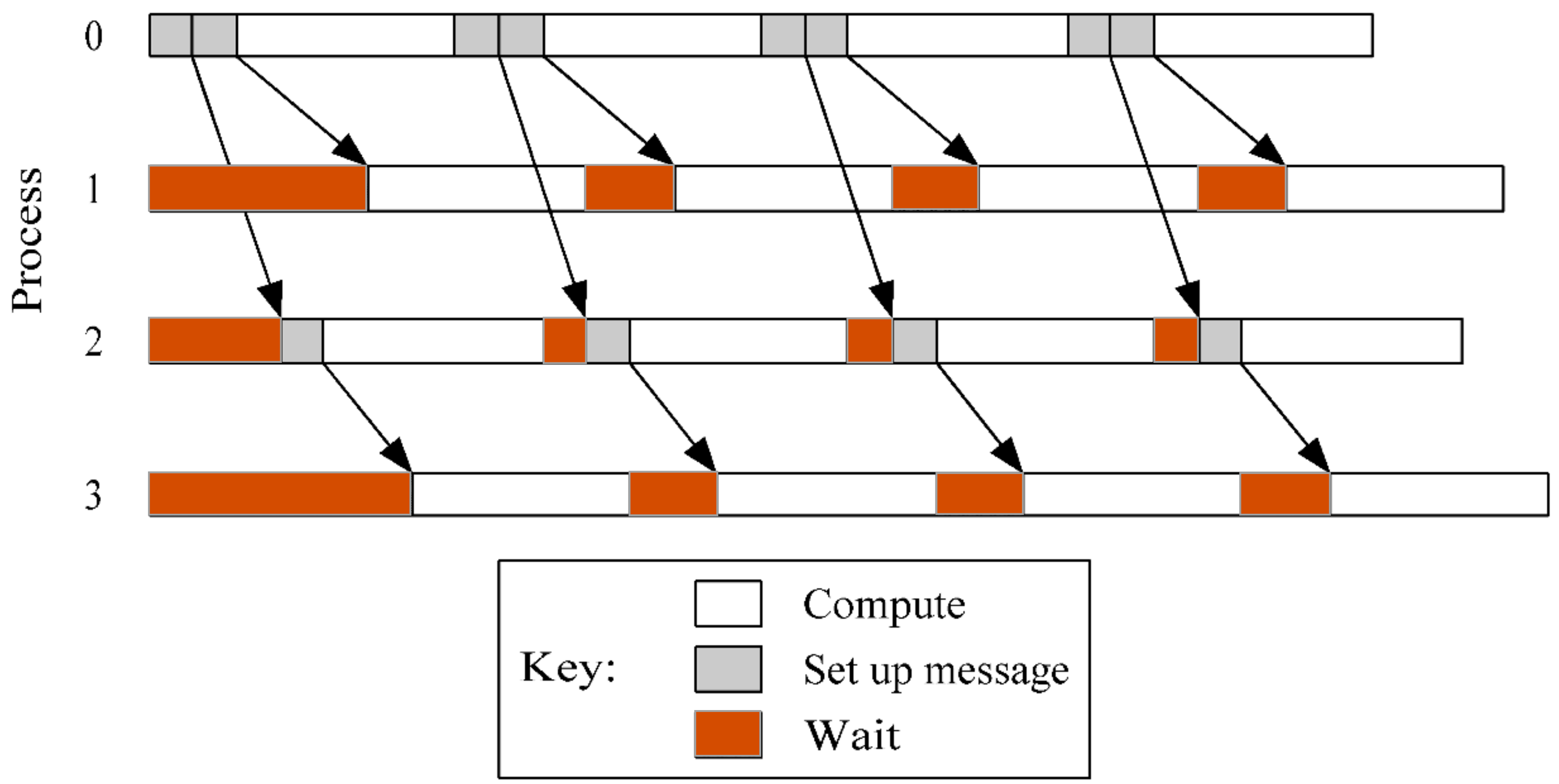
Cell update time

Iterations of inner loop

Iterations of middle loop

Iterations of outer loop

Computation/communication Overlap



Execution Time Expression (2)

$$n\lceil n/p\rceil n\chi + n\lceil \log p\rceil \lambda + \lceil \log p\rceil 4n/\beta$$

Message transmission

Message-passing time

Messages per broadcast

Iterations of outer loop

Cell update time

Iterations of inner loop

Iterations of middle loop

Iterations of outer loop

Predicted vs. Actual Performance

	Execution Time (sec)	
Processes	Predicted	Actual
1	25.54	25.54
2	13.02	13.89
3	9.01	9.60
4	6.89	7.29
5	5.86	5.99
6	5.01	5.16
7	4.40	4.50
8	3.94	3.98

Summary

- Two matrix decompositions
  - Rowwise block striped
  - Columnwise block striped
- Blocking send/receive functions
  - MPI_Send
  - MPI_Recv
- Overlapping communications with computations