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Paramagnon Effect on the BCS Transition in He^3

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The theory of Berk and Schrieffer on the paramagnon-mediated interaction in nearly ferromagnetic Fermi liquids is extended so as to include triplet as well as singlet BCS pairings. The paramagnon effect on the BCS transition in liquid He^3 is expressed in terms of the enhancement factor of the low temperature specific heat in the normal phase. If the usual estimate of the latter is accepted, all the singlet pairings become practically impossible.

§ 1. Introduction

Recent experiments[1] indicate that liquid He^3 undergoes a second order phase transition at $T_A = 2.65$ mK (under the pressure of 34 atoms) and also a first order transition at a lower temperature $T_B \cong 1.8$ mK. Let us call the new phase between T_A and T_B the phase A and the one below T_B the phase B. One is tempted to identify the phase A with the BCS superfluid state predicted many years ago.[2] In order to explain the NMR shift observed in the phase A, however, one should assume a triplet pairing[3] in the BCS theory in contradiction with the ^{1}D pairing predicted previously.[2]

In dealing with the pair of non-vanishing angular momentum, we introduce the partial wave analysis of the effective interaction potential in the BCS Hamiltonian:[2]

$$V(\boldsymbol{k},\boldsymbol{k}') = \sum_{l=0}^{\infty} (2l+1)\, V_l P_l(\cos\theta)\,. \tag{1·1}$$

Here $\boldsymbol{k}, \boldsymbol{k}'$ are on the Fermi surface ($k=k'=k_F$) and θ is the angle between them. The K-matrix calculation of Brueckner and Gammel[2),4)] shows that V_2 is most attractive near the Fermi surface in He^3 under zero pressure, and that is why the ^{1}D pair was assumed previously. When the Fermi momentum increases, however, $|V_2|$ decreases and $|V_3|$, which is next attractive, increases. We may therefore expect the ^{3}F pair at high pressures. Quantitatively, if these V_2 or V_3 are inserted into the BCS formula

$$T_c = 1.14\xi \exp[(\rho V)^{-1}_{\mathrm{BCS}}] \tag{1·2}$$

together with appropriate state density ρ at the Fermi surface and cutoff ξ ($\sim$1K), the transition point T_c is a little too high in comparison with observed T_A and T_B. Recently Soda and Yamazaki[5] have assumed the ^{3}F pairing for the phase

A and the ^{1}D pairing for the phase B, and obtained $\rho V_2 \lesssim \rho V_3 \cong -0.15$ by fitting observed T_A and T_B. A more precise K-matrix calculation might result in such values of ρV_l.

In the present paper, however, we would like to emphasize that the paramagnon effect, which has been ignored so far, has such importance that we should take it into consideration before attempting any improved calculation of V_l. As is indicated by the enhanced magnetic susceptibility, a strong exchage coupling is effective between atoms and makes liquid He^3 nearly ferromagnetic. There exist in the liquid, therefore, large spin fluctuations, which may be described as pseudo spin waves or paramagnons. In the normal phase, the atomic mass and therefore the T-linear specific heat are enhanced through the interactions with paramagnons as[6)]

$$m^*/m = 1 + \lambda . \tag{1·3}$$

If the observed enhancement of specific heat is assumed to arise entirely from the paramagnon effect, we have $\lambda \cong 2$, which is probably an overestimate. Note that (1·3) is a dynamical effect which cannot be included in the usual K-matrix calculation and similar to the electron mass enhancement through the interaction with phonons in a metal. In analogy with the phonon-mediated interaction to be added to the direct interaction between He atoms, (1·1), in the BCS Hamiltonian. In contrast to the phonon-mediated interaction, our paramagnon-mediated interaction suppresses the singlet pairing as one might intuitively expect and as was discussed by Doniach[7)] and by Berk and Schrieffer[8)] in connection with superconductivity of nearly ferromagnetic metals.

In the present paper, the argument of Berk and Schrieffer will be extended so as to include the triplet as well as the singlet pairings. A semi-quantitative estimate of the paramagnon contribution to $\rho V_{\rm BCS}$ will be made. It will thus be shown that the paramagnon effect upon the BCS transition is just as important as it is in the normal specific heat enhancement given by (1·3).

§ 2. Paramagnon-mediated interaction

Following Berk and Schrieffer,[8)] let us describe the large exchange enhancement in liquid He^3 by a simple Fermi gas model, in which we introduce a strong zero-range repulsion I acting between two atoms with antiparallel spins. In the simplest approximation, I may be identified with the S-wave part of (1·1). In order to make our argument as simple and transparent as possible, we will apply the random phase approximation (RPA) to this model, though RPA tends to overestimate paramagnon effects and therefore makes our theory rather qualitative. In our final expression (2·7), though derived on the basis of RPA, $J(\boldsymbol{k}, \boldsymbol{k}')$ may be regarded as phenomenological parameters similar to Landau parameters in Fermi liquid theory.

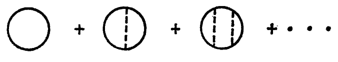

Fig. 1.

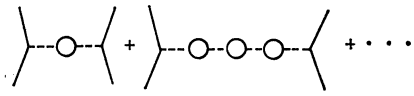

Fig. 2 (a).

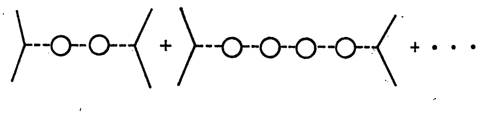

Fig. 2 (b).

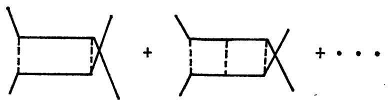

Fig. 2 (c).

Let us begin with familiar RPA expression for the dynamic susceptibility

$$\chi(q, \omega) = \chi_P[1 - \rho I u(q, \omega)]^{-1}. \tag{2·1}$$

Here χ_P is the Pauli susceptiblity and $u(q, \omega)$ is the polarization function of the Fermi gas for wave number q and frequency ω. It is normalized as $u(0, 0) = 1$. Throughout the present paper, we will assume the limit of extreme enhancement, in which the so-called Stoner factor $K_0^2 = 1 - \rho I$ satisfies

$$0 < K_0^2 \ll 1\,. \tag{2·2}$$

In the perturbational expansion in powers of I, (2·1) is the sum of the terms represented by diagrams in Fig. 1, in which the full line means the free atom propagator and the broken line means the interaction I. In accordance with this, Berk and Schrieffer explicitly took into account the terms represented by diagrams in Fig. 2(c) to obtain the paramagnon-mediated interaction for the singlet pair of atoms with opposite momenta. In order to include the case of triplet pairing and in order that our interaction be invariant under spin rotation, however, we need also to take account of the terms represented by diagrams in Figs. 2(a) and (b).

In writing down the corresponding analytic expressions, we assume the weak coupling limit of BCS theory, since the observed transition point T_A, which we wish to identify with (1·2), is much lower than ξ or even than $K_0^2\xi$. The paramagnon effect is dynamical in general, but in the weak coupling limit we can take the zero-frequency limit of the vertex function for the paramagnon-mediated interaction and regard it as an effective interaction potential to be added to (1·1) in the BCS Hamiltonian. The situation is similar to the case of weak coupling superconductors, where the zero-frequency limit of the phonon propagator plays the role of the potential for the phonon-mediated attraction between electrons.

Let us first write down the expression corresponding to Fig. 2(a), remembering that the bubble in these diagrams makes contribution $-\rho u(\boldsymbol{k} - \boldsymbol{k}')$ in the zero-frequency limit, where, $u(q) = u(q, 0)$. We thus obtain the effective potential for two atoms with opposite momenta $\boldsymbol{k}$, $-\boldsymbol{k}$ and parallel spins:

$$-I^2\rho u - I^4(\rho u)^3 - \cdots = -\frac{1}{2} I\left[\frac{1}{1-\rho I u} - \frac{1}{1+\rho I u}\right].$$

Though $u(q)$ varies from unity to 1/2 over the range $0 \leqq q \leqq 2k_F$ relevant to the scattering on the Fermi surface, we can ignore the second term on the right against the first since we have assumed (2·2). Thus the diagrams in Fig. 2(a) give the following interaction in the BCS Hamiltonian:

$$-\tfrac{1}{4} \sum J(\boldsymbol{k}-\boldsymbol{k}')\, a^+_{k\sigma} a^+_{-k\sigma} a_{-k'\sigma} a_{k'\sigma}\,. \tag{2·3}$$

Here $a_{k\sigma}$, $a^+_{k\sigma}$ are destruction and creation operators of the atom with momentum $\boldsymbol{k}$ and spin σ, and

$$J(q) = I[1-\rho I u(q)]^{-1}. \tag{2·4}$$

Similarly the diagrams in Fig. 2(b) give the interaction potential between two atoms with opposite momenta $\boldsymbol{k}$, $-\boldsymbol{k}$ and antiparallel spins.

$$\frac{I(\rho I u)^2}{1-(\rho I u)^2} = \frac{1}{2} I\left[\frac{1}{1-\rho I u} + \frac{1}{1+\rho I u} - 2\right]$$
$$\cong \frac{1}{2} J\,.$$

In the BCS Hamiltonian we thus obtain

$$\tfrac{1}{4} \sum J(\boldsymbol{k}-\boldsymbol{k}')\, a^+_{k\sigma} a^+_{-k-\sigma} a_{-k'-\sigma} a_{k'\sigma}\,. \tag{2·5}$$

In the case of Fig. 2(c), the momentum which enters into the polarization function is $\boldsymbol{k}+\boldsymbol{k}'$. Denoting $u(\boldsymbol{k}+\boldsymbol{k}')$ by u_+, we obtain the interaction potential

$$\frac{I^2 \rho u_+}{1-\rho I u_+} = I\left[\frac{1}{1-\rho I u_+} - 1\right]$$
$$\cong J_+\,.$$

By use of anticommutation rules for $a_{k\sigma}$, we may write the corresponding interaction term in the BCS Hamiltonian as

$$\tfrac{1}{2} \sum J(\boldsymbol{k}+\boldsymbol{k}')\, a^+_{k\sigma} a^+_{-k-\sigma} a_{-k'-\sigma} a_{k'\sigma}$$
$$= -\tfrac{1}{2} \sum J(\boldsymbol{k}-\boldsymbol{k}')\, a^+_{k\sigma} a^+_{-k-\sigma} a_{-k'\sigma} a_{k'-\sigma}\,. \tag{2·6}$$

The sum of (2·3), (2·5) and (2·6) is the paramagnon-mediated interaction to be added to the BCS Hamiltonian

$$\begin{aligned} H_{\mathrm{pm}} = -\tfrac{1}{4} \sum J(\boldsymbol{k}-\boldsymbol{k}')\, [&a^+_{k\uparrow} a^+_{-k\uparrow} a_{-k'\uparrow} a_{k'\uparrow} \\ &+ a^+_{k\downarrow} a^+_{-k\downarrow} a_{-k'\downarrow} a_{k'\downarrow} \\ &+ 2a^+_{k\uparrow} a^+_{-k\downarrow} a_{-k'\uparrow} a_{k'\downarrow} + 2a^+_{k\downarrow} a^+_{-k\uparrow} a^+_{-k'\downarrow} a_{k'\uparrow} \\ &- a^+_{k\uparrow} a^+_{-k\downarrow} a_{-k'\downarrow} a_{k'\uparrow} - a^+_{k\downarrow} a^+_{-k\uparrow} a_{-k'\uparrow} a_{k'\downarrow}]\,. \end{aligned} \tag{2·7}$$

With use of Pauli spin operators S_x, S_y, S_z normalized as $S_x^2 = 1/4$ etc., we can write (2·7) compactly as

$$H_{\rm pm} = -\tfrac{1}{2}\sum J(\boldsymbol{k}-\boldsymbol{k}')\,2\boldsymbol{S}_{\sigma\sigma'}\cdot\boldsymbol{S}_{\tau\tau'}a^+_{k\sigma}a^+_{-k\tau}a_{-k'\tau'}a_{k'\sigma'}\,. \tag{2·8}$$

This is manifestly invariant under spin rotation as it should be. Though derived on the basis of RPA, the final form (2·8) has such generality that $J(\boldsymbol{k},\boldsymbol{k}')$ may be regarded as phenomenological parameters if we like.

§ 3. Gap equations

Since $J>0$ in (2·4), we see from (2·8) that the paramagnon-mediated interaction is attractive for the triplet pair and repulsive for the singlet pair. In fact the operator $2\boldsymbol{S}_1\cdot\boldsymbol{S}_2$ has the eigenvalue 1/2 when $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ are coupled in triplet states and the eigenvalue $-(3/2)$ when in the singlet state. Adding the direct interaction (1·1), we thus obtain the effective potentials for triplet and singlet pairs respectively as

$$U_{\rm t} = V - \tfrac{1}{2}J\,, \qquad U_{\rm s} = V + \tfrac{3}{2}J\,. \tag{3·1}$$

We will confirm (3·1) more explicitly by deriving gap equations from the BCS Hamiltonian, which in our case takes the form

$$H_{\rm BCS} = \sum \varepsilon_k a^+_{k\sigma}a_{k\sigma} + \tfrac{1}{2}\sum\left[V(\boldsymbol{k},\boldsymbol{k}')\,\delta_{\sigma\sigma'}\delta_{\tau\tau'}\right.$$
$$\left. -2J(\boldsymbol{k},\boldsymbol{k}')\,\boldsymbol{S}_{\sigma\sigma'}\cdot\boldsymbol{S}_{\tau\tau'}\right]a^+_{k\sigma}a^+_{-k\tau}a_{-k'\tau'}a_{k'\sigma'}\,, \tag{3·2}$$

ε_k being the one-particle energy measured from the Fermi level.

We apply the Gorkov formalism[9] and write temperature Green's functions on the complex frequency plane as $\langle\!\langle a_{k\sigma}; a^+_{k\tau}\rangle\!\rangle$, etc. In our paramagnetic system without magnetic field, we can assume the form

$$\langle\!\langle a_{k\sigma}; a^+_{k\tau}\rangle\!\rangle = \delta_{\sigma\tau}G(\boldsymbol{k}, E)\,, \tag{3·3}$$

where E runs over imaginary frequencies $(2n+1)\pi iT$ with temperature T and integers n. As for anomalous Green's function, we define

$$F_1 = \langle\!\langle a^+_{-k\uparrow}; a^+_{k\uparrow}\rangle\!\rangle\,, \qquad F_{-1} = \langle\!\langle a^+_{-k\downarrow}; a^+_{k\downarrow}\rangle\!\rangle\,,$$
$$F_0 = \tfrac{1}{2}\left[\langle\!\langle a^+_{-k\downarrow}; a^+_{k\uparrow}\rangle\!\rangle + \langle\!\langle a^+_{-k\uparrow}; a^+_{k\downarrow}\rangle\!\rangle\right], \tag{3·4}$$
$$F_s = \tfrac{1}{2}\left[\langle\!\langle a^+_{-k\downarrow}; a^+_{k\uparrow}\rangle\!\rangle - \langle\!\langle a^+_{-k\uparrow}; a^+_{k\downarrow}\rangle\!\rangle\right].$$

Similarly we define anomalous amplitude $\Phi_1 = \langle a^+_{-k\uparrow}a^+_{k\uparrow}\rangle$, etc. by replacing $\langle\!\langle\cdots\rangle\!\rangle$ by corresponding expectation values $\langle\cdots\rangle$ in (3·4). These expectation values are connected with (3·4) by

$$\Phi_\alpha(\boldsymbol{k}) = T\sum_E F_\alpha(\boldsymbol{k}, E)\,, \tag{3·5}$$

where the index α runs over $1, 0, -1, s$. Obviously Φ_s is the singlet part and $\Phi_1, \Phi_0, \Phi_{-1}$ are triplet components of the pair amplitude. In fact, we have

$$\Phi_s(-\boldsymbol{k}) = \Phi_s(\boldsymbol{k})\,, \qquad \Phi_\alpha(-\boldsymbol{k}) = -\Phi_\alpha(\boldsymbol{k})\,, \tag{3·6}$$

where $\alpha = 1, 0, -1$.

We now write down Gorkov equations derived directly from (3·2)

$$(E-\varepsilon)G=1+\Delta_1{}^*F_1+(\Delta_0{}^*+\Delta_s{}^*)(F_0+F_s)$$
$$=1+\Delta_{-1}^*F_{-1}+(\Delta_0{}^*-\Delta_s{}^*)(F_0-F_s),$$
$$(E+\varepsilon)F_\alpha=\Delta_\alpha G\,. \tag{3·7}$$

We have defined order parameters

$$\Delta_\alpha(\boldsymbol{k})=\sum U_\alpha(\boldsymbol{k},\boldsymbol{k}')\Phi_\alpha(\boldsymbol{k}'), \tag{3·8}$$

where U_α is given by U_s in (3·1) for $\alpha=s$ and by U_t for $\alpha=1, 0, -1$. In the expansions of U_α similar to (1·1), we will as usual retain the single term, for which $U_{\alpha l}$ is most attractive:

$$U_\alpha(\boldsymbol{k},\boldsymbol{k}')\cong(2l+1)\,U_{\alpha l}P_l(\cos\theta). \tag{3·9}$$

Then $U_\alpha(\boldsymbol{k}-\boldsymbol{k}')=(-1)^l U_\alpha(\boldsymbol{k},\boldsymbol{k}')$. In conjunction with (3·6), we thus obtain the well-known conclusion[2)]

$$\Delta_s=0 \text{ for odd } l\,,$$
$$\Delta_\alpha=0 \text{ for even } l \text{ and } \alpha=1, 0, -1\,. \tag{3·10}$$

From (3·7), (3·5), we see that the same conclusion applies also to F_α and Φ_α.

The solution of (3·7) can always be written as

$$G=\frac{E+\varepsilon_k}{E^2-E_k^2}\,,\qquad F_\alpha=\frac{\Delta_\alpha}{E^2-E_k^2}\,. \tag{3·11}$$

Inserting in (3·8) through (3·5), we obtain gap equations

$$\Delta_\alpha(\boldsymbol{k})=-\sum U_\alpha(\boldsymbol{k},\boldsymbol{k}')\left[\mathrm{th}\frac{E_{k'}}{2T}/2E_{k'}\right]\Delta_\alpha(\boldsymbol{k}'). \tag{3·12}$$

The difference between triplet and singlet pairs appears only in the expression for the quasiparticle energy $E_k=[\varepsilon_k{}^2+\Delta^2]^{1/2}$. Thus $\Delta=|\Delta_s|$ for the singlet pair, whereas for the triplet pair

$$\Delta=[|\Delta_x|^2+|\Delta_y|^2+|\Delta_z|^2]^{1/2}. \tag{3·13}$$

We have conveniently defined

$$\Delta_x=\tfrac{1}{2}(\Delta_1+\Delta_{-1}),\quad \Delta_y=\tfrac{1}{2}(\Delta_1-\Delta_{-1}),\quad \Delta_z=\Delta_0\,. \tag{3·14}$$

They are transformed as vector components and therefore Δ is a scalar under spin rotation.

In the weak coupling limit, we may assume that order parameters depend only upon the direction Ω of k on the Fermi surface. We therefore write $\Delta_\alpha=\Delta_{m\alpha}f_\alpha(\Omega)$ and $\Delta=\Delta_m f(\Omega)$, where f_α and f are normalized on the Fermi surface. Hence $\Delta_m=\Delta_{ms}$ for the singlet pair and $\Delta_m=[\Delta_{mx}^2+\Delta_{my}^2+\Delta_{mz}^2]^{1/2}$ for the triplet. As was shown by Anderson and Morel,[2)] we can derive from (3·12) at $T=0$

$$\Delta_m=2\xi\Gamma\exp[(\rho U_{\alpha l})^{-1}],$$

$$\ln \Gamma = -\int |f|^2 \ln |f| d\Omega . \tag{3·15}$$

Under (3·9), we usually assume that the pair has the definite angular momentum l of the relative orbital motion. Hence $f_\alpha(\Omega)$ are linear combinations of spherical harmonics of the order l. The different choice of these combinations gives slightly different Γ and therefore binding energy because of the non-linear character of gap equations. The detailed analysis of this point is available only for ^{1}D and ^{3}P pairs.[2),10)] For instance, in the case of ^{3}P pair, it has been proved that the solution of the minimum free energy is given by $f_j(\Omega) \propto (k_j/k)$, where $j=x, y, z$. However, we do not know the solution of the minimum free energy for the ^{3}F pair. We will discuss the problem in a separate paper.

§ 4. Estimate of paramagnon effect

We now go back to the paramagnon effect upon the transition point (1·3). In the normal phase, it modifies the one-particle energy ε_k which is given by the pole of the Green's function, so the ε_k gives the enhanced density of states $\rho^* = (1+\lambda)\rho$. It also modifies the residue of the pole, so that the normal Green's function near the Fermi surface takes the form $[(1+\lambda)(E-\varepsilon_k)]^{-1}$. Inserting this in the Gorkov formalism[8)] to determine the transition point, we see that in (1·3) we should take

$$(\rho V)_{\rm BCS} = [1+\lambda]^{-1} \rho U_l . \tag{4·1}$$

From (3·1) and (3·11),

$$U_l = \begin{cases} V_l - \frac{1}{2} J_l , & (l \text{ odd}) \\ V_l + \frac{3}{2} J_l . & (l \text{ even}) \end{cases} \tag{4·2}$$

So we need to estimate J_l in some way.

For the scattering over the Fermi surface, q in (2·4) is given by $q = 2k_{\rm F} \sin(\theta/2)$, where θ is the scattering angle. For our purpose, it is convenient to introduce the variable $x = (q^2/4k_{\rm F})^2 = \sin^2(\theta/2)$. Then

$$\begin{aligned} J_l &= \frac{1}{2} \int_{-1}^{1} J\left(2k_{\rm F} \sin \frac{\theta}{2}\right) P_l(\cos\theta) d(\cos\theta) \\ &= \int_0^1 J(2k_{\rm F} x^{1/2}) P_l(1-2x) dx . \end{aligned} \tag{4·3}$$

In RPA, $u(q)$ in (2·4) is given by the Lindhard function.[11)] In the limit of extreme exchange enhancement, (2·2), small angle scattering makes the main contribution to (4·3), so that we may use the expansion $u(q) \cong 1 - (1/3)(q/2k_{\rm F})^2$. Quantitatively RPA may not be quite reliable, but we can always assume the low q expansion in the form $u(q) \cong 1 - \beta^{-1}(q/2k_{\rm F})^2$, where β is a numerical factor of the order of unity. Thus, in the limit (2·2)

$$\rho J \sim \beta[x+\beta K_0^2]^{-1}. \tag{4·4}$$

Similarly we take $P_l(1-2x) \cong P_l(1)=1$. Hence

$$\rho J_l \sim \beta \ln[\gamma/\beta K_0^2], \tag{4·5}$$

where γ is a cutoff parameter.

The same logarithmic singularity appears in the enhancement factor λ in (1·3). This is given by[5)]

$$\lambda = (3/4k_F^2) \int \chi(q,0)\, q dq$$

$$\sim (3\beta/2) \ln[\gamma/\beta K_0^2]. \tag{4·6}$$

Hence

$$\rho J_l \sim (2/3)\lambda, \tag{4·7}$$

$$\rho U_l \sim \begin{cases} \rho V_l - \frac{1}{3}\lambda & \text{for } l \text{ odd}, \\ \rho V_l + \lambda & \text{for } l \text{ even}. \end{cases} \tag{4·8}$$

These asymptotic relations do not depend explicitly upon the model parameters β, γ, I, and will therefore be applicable even outside the range of RPA, from which we have started.

As we have mentioned in § 1, if we assume that the observed specific heat enhancement C/C_0, where C_0 is the Fermi gas value, arises entirely from the paramagnon effect (1·3), we obtain $\lambda \cong 4$. Then all singlet pairings will be practically impossible, since $-\rho V_l$ are of the order of 0.1 even for most attractive components. It might even be possible that a triplet pairing, say ^{3}P, is stabilized almost entirely by the paramagnon effect.

This is probably an overestimate of λ, however. Apart from the paramagnon enhancement (1·3), we must also have a usual, Hartree-Fock type effective mass, so that ρ/ρ_0 differs from unity, where ρ_0 is the density of states for the free atom. Thus the observed specific heat enhancement in the normal phase is given by

$$C/C_0 = (1+\lambda)(\rho/\rho_0). \tag{4·9}$$

For example, the K-matrix calculation of Brueckner and Gammel[4)] gives $(\rho/\rho_0) \cong 2$. From (4·9), then $\lambda \cong 1.5$, which is still big enough to suppress the singlet pairing when inserted in (4·8). However, when λ becomes smaller, our asymptotic estimate is less reliable. For instance J_l will certainly depend on l. Anyway we need more precise estimates of J_l as well as of V_l to proceed further.

§ 5. Conclusion

In the present paper, we have emphasized the importance of the paramagnon effect upon the BCS transition in liquid He^3, in which we expect large spin fluctuations to exist. The paramagnon effect is just as important as it is in the enhancement of the low temperature specific heat in the normal phase, as we can clearly see from the asymptotic estimate (4·8) in the limit of extreme exchange enhancement of the normal magnetic susceptibility.

A more realistic calculation of the paramagnon effect seems difficult at present. We might rather regard ρJ_l in (4·2) as phenomenological parameters similar to Landau parameters in Fermi liquid theory. It might be possible in this way to understand the observed transitions at T_A and T_B consistently.

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During the course of publication, the work of P. W. Anderson and W. F. Brinkman on the same subject (preprint) has been brought into author's notice kindly by Professor T. Soda.

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