Parameter Estimation STAR(1;1) Model Using Binary Weight

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Abstract. *Space Time Auto Regressive*(1;1) Model or STAR(1;1) model is a form of model that involves location and time. The STAR(1;1) model is a stationary *space time* model in mean and variance. The STAR model can be used to forecast future observations at these locations by involving the effects of observations at other nearby locations in spatial lag 1 and lag time 1 [2]. The STAR model can be written as a linear model assuming that *error* is normally distributed with zero mean and constant variance. In this research, the parameter estimation procedure for STAR model using binary weight, MKT method and STAR model for the estimation of petroleum production in 3 wells is assumed to be in a homogeneous reservoir.

Keyword: Parameter, Model or STAR(1;1), Binary Weight, stationary *space time,* Estimation

1. Introduction

Stationary time series of stochastic processes are sequences of random variables that are given time sequences. Problems related to actual time are often found in daily life, for example in the production of petroleum wells, petroleum production, cement production, rainfall and so forth.

Data problems that are observed and related to time are called *time series* data. Production from PT Persero Pertamina's oil wells, Mundu, Jatibarang is a time series problem, which can be modeled with the Order 1 A*uto Regression* model or AR(1), which is a simple model, so it is better known and often used today [3]. This model can be developed if there are more than one petroleum wells through the Order 1 Auto *Regression* Vector model or VAR(1) [2]. In the VAR(1) model a correlation can be drawn between petroleum wells [4]. Further location characteristics can be illustrated through a weight matrix. The special case of the VAR(1) model is the *Space Time Auto Regression* or STAR(1;1) model [1].

2. Methods used

This study is secondary data from petroleum production, namely data of 3 petroleum production wells in PT Persero Pertamina, Mundu, Jatibarang, which was taken through collaboration between the research team of the Mathematics Department of FMIPA Unpad and the Data Section of PT Pertamina, Mundu Field, Jatibarang.

Research diagram as follows:

The flow/diagram below is the flow/diagram for research in the FMIPA Unpad Independent Research in the Mathematics Department in cooperation with PT Persero Pertamina, Mundu, Jatibarang: First start by determining the data to be processed, to process the data it is necessary to study the theory of stochastic processes; The time series, starting from the univariate; AR(1) model, ACF (Autocorrelation Function), PACF (Partial Autocorrelation Function), to establish AR(1) model. Then determine the uniform matrix, weight matrix of location W and binary matrix, parallel matrix, then; study the STAR(1;1) model, apply the STAR(1;1) model, estimate the parameters of the STAR(1;1) model with the MKT method, then check diagnosis, if there is an obstacle it will return to the next data, if it is done The diagnosis check is successful, then the process will be continued namely forecast based on the estimated STAR(1;1) model on secondary data from oil production, namely data on 3 petroleum production wells at PT Persero Pertamina, Mundu, Jatibarang, which was taken through collaboration between the Department's research team Mathematics with PT Pertamina Data Section, Mundu Field, Jatibarang.
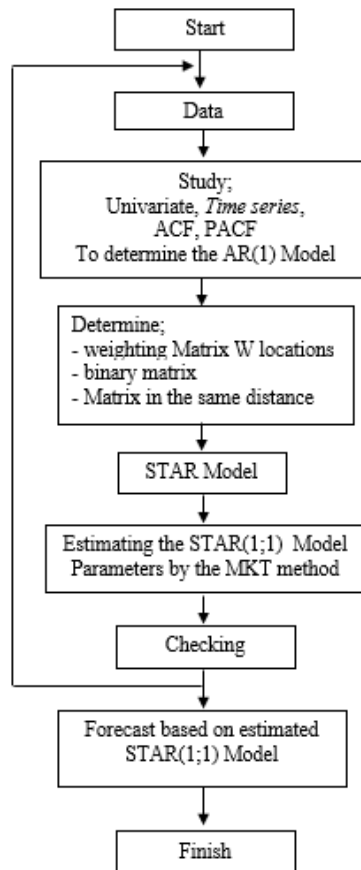
3. Results and Discussion

Model AR(1) is the simplest univariate time series model, because it states that current time observations are influenced by observations of one time before and the *error* element [2] is written as follows:

$$z(t) = \phi(1)\ z(t-1) + e(t),\ e(t) \overset{iid}{\sim} N(0, \sigma^2) \tag{1}$$

The STAR(1;1) model states that the current time observations at a particular location are influenced by an observation of a previous time at that location and the surrounding locations in a study group. For simplicity of the model, the study focused on time lag 1 and spatial lag 1 for the STAR(1;1) model in several locations [5].

The STAR(1;1) model is stated:

$$z(t) = \phi_{01}\, z(t-1) + \phi_{11} Wz(t) + e(t) \tag{2}$$

and *error* vectors:

$$e(t) \overset{iid}{\sim} \underline{N}(0,\sigma^2 I_N)$$

and uniform weights:

$$\underline{W} = \begin{pmatrix} 0 & 0{,}5 & 0{,}5 \\ 0{,}5 & 0 & 0{,}5 \\ 0{,}5 & 0{,}5 & 0 \end{pmatrix}$$

The STAR(1;1) model equation for 3 locations can be presented in the following form:

$$\begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} = \phi_{01} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \\ z_3(t-1) \end{bmatrix} + \phi_{11} \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \\ z_3(t-1) \end{bmatrix} + \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} \tag{3}$$

Equation (1) can be expressed in the form of:

$$z(t) = \left[\phi_{01}I + \phi_{11}W\right]z(t-1) + e(t)$$

$$z(t) = \Phi z(t-1) + e(t) \tag{4}$$

with

$$\Phi = \left[\phi_{01}I + \phi_{11}W\right]$$

Estimating the STAR(1;1) Model Parameters

The parameter estimation of the STAR(1;1) model can be done using the MKT method, because the STAR(1;1) model can be expressed as a linear model:

$$y = X\beta + e(t), \; e(t) \overset{iid}{\sim} N(0,\sigma^2) \tag{5}$$

Estimating the linear model parameters by the MKT method for equation (5) yields:

$$\hat{\beta} = (X' \quad X)^{-1} X' y. \tag{6}$$

In this study, it will be shown that the parameter estimation of the STAR(1;1) model by the MKT method will give results in accordance with the equation (6).

Data of 3 petroleum production wells of PT Persero Pertamina, Mundu, Jatibarang, taken through collaboration between the Mathematics Department research team and PT Pertamina Data Section, Mundu Field, Jatibarang and stated with well 1 (V1), well 2 (V2) and well 3 (V3), from each of the data wells taken 80 data are presented in the following table;

Data Table 3 Wells, namely well 1 and well 3 of petroleum production.

	V1	V2	V3		V1	V2	V3		V1	V2	V3		V1	V2	V3
1	1162	1597	2590	21	2038	2230	1152	41	1108	1819	837	61	1350	1054	770
2	1358	759	155	22	2998	1310	1014	42	553	1874	818	62	898	1004	2887
3	1406	672	2092	23	1866	1864	1022	43	884	1532	589	63	943	1637	809
4	923	881	1956	24	1695	2283	975	44	1151	1401	708	64	874	1114	698
5	429	1028	1649	25	1533	2197	1224	45	446	2232	593	65	774	1155	750
6	1064	1862	2007	26	1035	1659	1007	46	268	1155	1320	66	817	896	1075
7	1349	1301	2003	27	1068	1737	1046	47	249	843	522	67	880	1252	709
8	1448	1658	2536	28	1086	1227	1088	48	666	986	397	68	1119	1065	949
9	2008	1607	1527	29	1020	1615	946	49	833	1227	324	69	759	1423	1082
10	1511	1618	1480	30	1192	1679	957	50	915	1079	921	70	858	1284	913
11	2261	1320	1793	31	1203	966	979	51	685	792	610	71	1223	1322	961
12	1446	874	1625	32	881	1938	957	52	617	944	565	72	895	1285	872
13	1517	1017	1548	33	1435	1304	877	53	457	589	713	73	581	1372	809
14	1361	690	1569	34	1363	1121	549	54	789	753	797	74	565	1463	895
15	1460	1053	2415	35	1645	1272	1057	55	1075	854	792	75	568	1811	955
16	1550	1373	2260	36	1450	793	693	56	665	951	765	76	525	1320	828
17	1237	2203	2577	37	1769	1054	1059	57	672	997	701	77	574	1105	618
18	1353	2386	1630	38	1290	1909	894	58	654	912	729	78	584	939	698
19	1346	2767	1600	39	1085	1450	1011	59	826	1450	776	79	676	966	770
20	1905	3107	1502	40	1490	1541	884	60	633	1274	570	80	657	1054	726

> summary

	V1	V2	V3
Min.:	249.0	589	324.0
1st Qu.:	573.8	966	693.8
Median:	880.5	1142	854.5
Mean:	977.6	1290	1031.0
3rd Qu.:	1349.0	1534	1084.0
Max.:	2998.0	3107	2887.0

> cor

	V1	V2	V3
V1	1.0000000	0.3912407	0.4744414
V2	0.3912407	1.0000000	0.2368895
V3	0.4744414	0.2368895	1.0000000

From the data of 3 petroleum production wells of PT Persero Pertamina, Mundu Field, Jatibarang, which were taken through collaboration between the Mathematics Department research team with the Data Section of PT Pertamina Pertamina, Mundu Field, Jatibarang and stated with wells 1 (V1), wells (V2) and wells 3 (V3), from each of the data wells taken 80 data generated graph images from each well and combined graphs from the data of the three wells and ACF and PACF graphic images of oil production data for each well, namely Well 1 (V1), Well 2 (V2) and Well 3 (V3) of petroleum production are presented as follows:
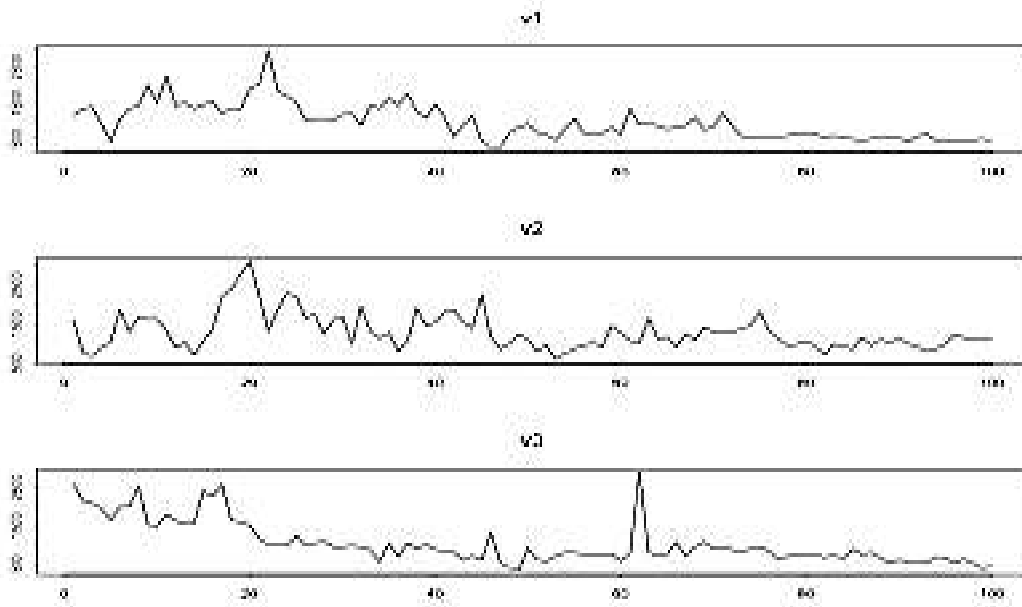

Fig 1. Oil Production Data for each Well, namely Well 1 (V1), Well 2 (V2) and Well 3 (V3)
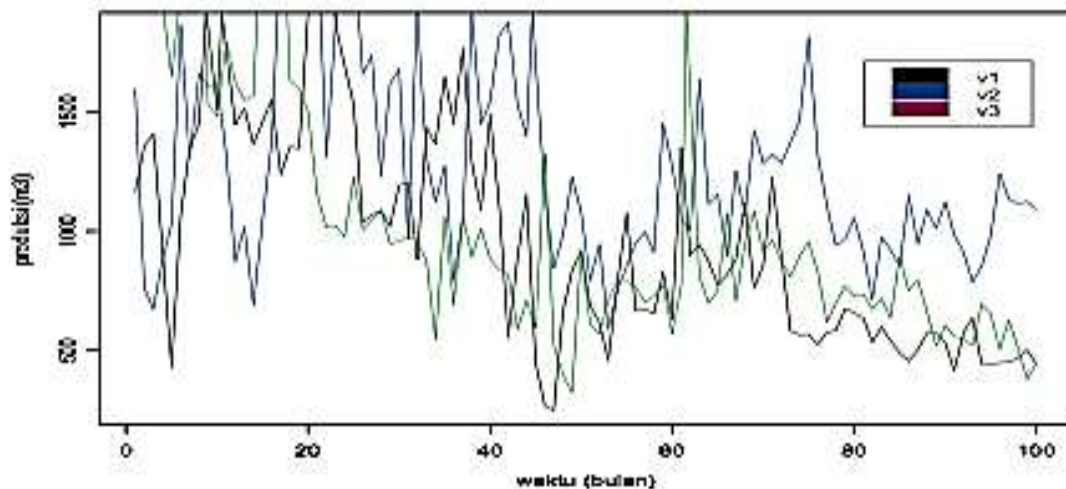

Fig 2. Combined Graph of Oil Production Data for Well 1 (V1), Well 2 (V2) and Well 3 (V3)
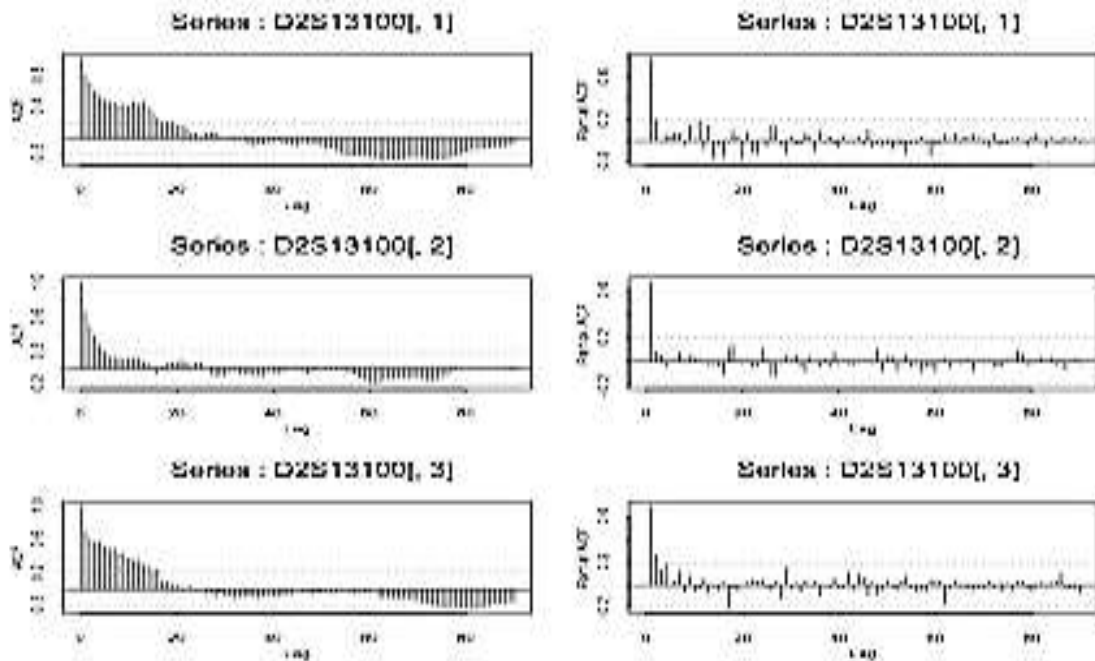
Fig 3. The ACF and PACF Graph of Oil Production Data for Well 1
(V1), Well 2 (V2) and Well 3 (V3)

MODEL STAR (TIME SERIES)

STAR Model Data

1162	1358	1406	923	429	1064	1349	1448
1597	759	672	881	1028	1862	1301	1658
2590	155	2092	1956	1649	2007	2003	2536

Weight Matrix

0	0,5	0,5
0,5	0	0,5
0,5	0,5	0

b		=		x1		x2		
	1162		=		0		0	. matrix phi

X' / transpose matrix X

0	0	0	1162	1597	2590	1358	759
0	0	0	2093,5	1876	1379,5	457	756,5

X'X, product of matrices X' and X

4,63E+08	3,65E+08
3,65E+08	4,14E+08

matrix phi

0,474817

0,455369

Estimated matrix z

0	953,3151	208,1037	1181,057	1404,223	1839,287	1525,713	1112,656
0	1406,009	1574,262	1156,826	1323,096	1466,445	1117,533	966,8949
0	1857,957	555,6047	1466,445	1339,484	1114,709	1619,162	1554,422

Matrix Error (z reduced by estimated z)

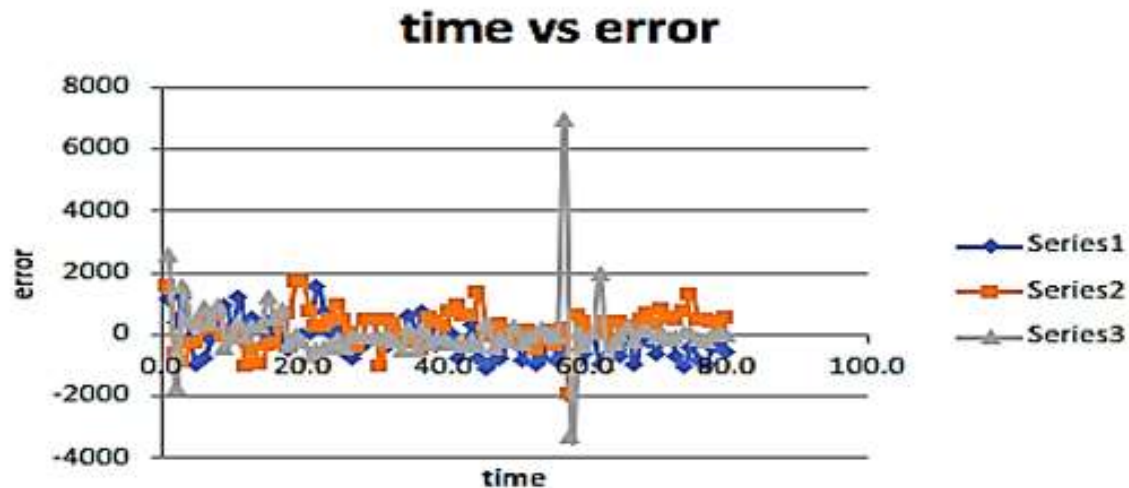| 1	2	3	4	5	6	7	8
1162	404,6849	1197,896	-258,057	-975,223	-775,287	-176,713	335,3444
1597	-647,009	-902,262	-275,826	-295,096	395,5549	183,4673	691,1051
2590	-1702,96	1536,395	489,5549	309,5156	892,2908	383,8379	981,5781

MSE

1,51E+08

Fig 4. Star Model (Time Series) Data 3 Earth Oil Wells

Graphic Image Descriptions:

1. The blue color is a graph of Well 1 Petroleum data
2. The red color is a graph of Well 2 Petroleum data
3. The purple/grey color is a graph of Well 3 Petroleum data

4. Conclusion

The STAR(1;1) model is a multivariate time series model using a weight matrix that describes location characteristics. The STAR(1;1) model parameter $\left|\phi_{10}\right| + \left|\phi_{11}\right| < 1$. The STAR(1;1) model can be applied to various real phenomena, such as data on petroleum production. For data on STAR(1;1) oil production, it can also be used to forecast oil production at a location involving other locations around it.

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