Parameter Estimation Using an Interdigital Dielectrometry Sensor with Finite-Element Software

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Abstract

A real-time software algorithm for calculating dielectric permittivity and conductivity of non-dispersive and frequency dispersive insulating materials on the basis of non-destructive measurements is presented. First, the measured response of the interdigital dielectrometry sensor is simulated off-line with two-dimensional finite-element software over the entire range of input parameters. Then, experimental data points are interpolated and material properties are determined without iterative on-line calculations. Off-line simulation lends valuable insights into sensitivity of the measurement and the uniqueness of the solution to the inverse problem of material characterization.

Introduction

This paper presents results of continuing efforts to develop fast and reliable algorithms for characterization of dielectric materials by means of interdigital dielectrometry [1,2,3]. According to the concepts of ω-k dielectrometry, application of the spatially periodic potential with wavelength $\lambda=2\pi/k$ and angular frequency $\omega=2\pi f$ to a surface of a test dielectric generates a fringing electric field which penetrates into the volume of the tested material. Consequently, the conduction and displacement currents which flow into the electrodes depend on the dielectric properties of the sample. Since many other physical properties (such as moisture content, specific density, homogeneity) can be related to the measurable dielectric properties, this technique is useful in a variety of applications.

Several approaches can be taken to relate the sensor's response to the dielectric permittivity and conductivity of the test materials. Each approach has its advantages and drawbacks, and judicious choice is needed for each specific task of the material parameter estimation.

An iterative guess method for material properties followed by simulation of the electric fields and comparison with the measured sensor's output makes possible characterization of media with complex distributions of dielectric properties. The strength of this approach is in generality of analysis as any geometry and distribution of material properties can be modeled numerically. However, extensive iterative computations are required to interpret each measurement.

Much less computation is needed for table look-up techniques. Calibration of the sensor can be conducted either by a series of measurements with known materials, including air, or by a series of computer simulations.

Sensor Design

The general design of the three wavelength sensor is shown in Figure 1. Three sets of copper electrodes are deposited on the common Teflon substrate and connected to the interface circuit through the leads shown at the bottom. Figure 2 shows the equivalent circuit of the sensor superimposed onto the schematic view of a half-wavelength cell. Note that each wavelength has an opposite conducting guard plane at the bottom of the substrate.

For each wavelength, a follower op-amp drives the guard plane at the substrate bottom at the voltage $V_G = V_S$, thus eliminating any current between the sensing and the guard electrode.

The complex voltage gain of the sensor is determined by:

$$\hat{G} = \frac{\hat{V}_S}{\hat{V}_D} = \frac{G_{12} + j\omega C_{12}}{G_{12} + j\omega C_{12} + j\omega C_L}, \qquad (1)$$

where conductance G_{12} and capacitances C_{12} and C_L are as shown in the circuit model in Figure 2.

Theoretical Background

The purpose of our study is to estimate the conductivity σ and permittivity ε of a test dielectric from measurements using an interdigital sensor. An exhaustive approach is to make measurements on a large number of known materials at different

(0-7803-3851-0) 1997 IEEE Annual Report - Conference on Electrical Insulation and Dielectric Phenomena, Minneapolis, October 19-22, 1997

frequencies f and tabulate the responses. Now, given this tabulated look up table and our response, we can identify the values of the material properties that result in the observed response.

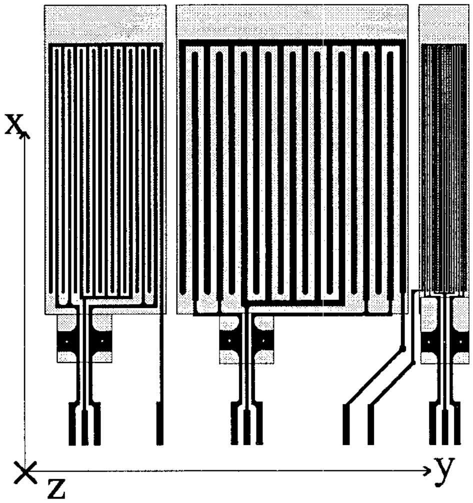

Figure 1. The latest design of the three-wavelength interdigital sensor with wavelengths of 2.5, 5.0, and 1.0 mm [4].

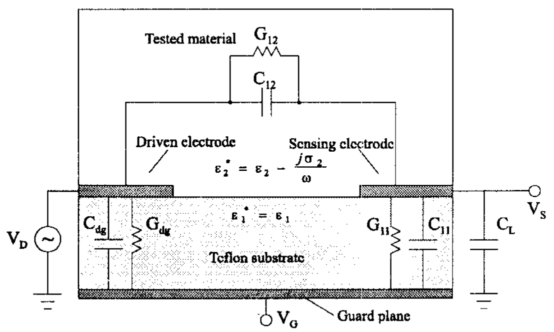

Figure 2. Floating voltage mode measurement of gain, phase, C_{12} and G_{12} with actively driven backplane voltage equal to the sensing voltage.

In practice we expedite this process by simulating the response for a wide range of parameters and frequencies using the finite-element software package Maxwell by Ansoft. One of the purposes of this paper is to demonstrate that our look up table need not be generated in terms of the three independent variables, σ, ε, and f. The response can be inferred from just two of these: σ and ε. The σ-f relation we use allows us to relate changes in frequency to an effective change in conductivity.

Consider a parallel-plate capacitor of area S and gap d in which the terminals are separated by a homogeneous material with a complex permittivity $\varepsilon^* = \varepsilon - j\sigma / 2\pi f$. Suppose a sinusoidal voltage with complex amplitude $\hat{V}$ is applied across the terminals, the steady-state current $\hat{I}$ is a function of σ, ε, and f.

$$\hat{I} = \hat{V}\, j2\pi f\,(\varepsilon - \frac{j\sigma}{2\pi f})k\,, \tag{2}$$

where the constant $k=S/d$ depends only on the geometry. The response changes with frequency in a well understood way. At applied frequency f_i, where the material conductivity is σ_i

$$\begin{aligned}\hat{I}_i &= \hat{V}\, j2\pi f_i\,(\varepsilon - \frac{j\sigma_i}{2\pi f_i})k \\ &= \hat{V}\,(\frac{f_i}{f_r}) j2\pi f_r\,(\varepsilon - j\frac{\sigma_i(f_r / f_i)}{2\pi f_r})k\end{aligned}\,, \tag{3}$$

which is simply the same response one would obtain at reference frequency f_r if the sample's conductivity is $\sigma_i(f_r/f_i)$, with the magnitude scaled by (f_i/f_r),

$$\hat{I}_r = A\,\hat{V} j2\pi f_r\,(\varepsilon - \frac{j\sigma_r}{2\pi f_r})k\,, \tag{4}$$

where $\sigma_r=\sigma_i(f_r/f_i)$ and $A=(f_i/f_r)$. This analysis allows dispersive materials where ε and σ are functions of frequency and generalizes to multiple dielectric regions.

Consider our two-dielectric interdigital sensor arrangement in Figure 2. The "sensed" complex voltage $\hat{V}_S$ across the load capacitor will depend on the known ε_1 and f, and the unknown ε_2 and σ_2. Suppose at some well-chosen reference frequency f_r the magnitude and phase of the complex gain of (1) was tabulated for all ε_2 and σ_2 keeping ε_1 constant. Then consider a test on an unknown sample whose properties are not known but are to be determined. From a practical point of view we would like to run an experiment at a drive frequency such that there is appreciable phase in the response to be able to accurately distinguish σ_2 and ε_2. Furthermore, the material may be dispersive and as such its properties will change with frequency. For these reasons the test might not be conducted at the reference frequency. Nevertheless, given the measurement frequency f_i, the response and tabulated results from computer simulation at the reference frequency f_r we can estimate the parameters σ_2 and ε_2 at frequency f_i. Specifically, for each G_{12}-C_{12} (transconductance-transcapacitance) pair determined from (1) at frequency f_i, one can identify the corresponding ε_2 and σ_2 at the reference frequency that would give this response. Note that $\sigma_i=\sigma_r(f_i/f_r)$ so that σ_2 at the frequency f_i can readily be obtained.

Let's introduce the σ-f normalization which allows one to find the values of G_{12} and C_{12} for any value frequency f and any given pair of ε and σ.

For capacitances, the normalization is:

$$C[f_r, \sigma_i f_r / f_i, \varepsilon] = C[f_i, \sigma_i, \varepsilon]\,, \tag{5}$$

where σ_i is the conductivity of interest, f_i is the measurement frequency of interest, and f_r is the reference frequency.

Similarly, for conductances, the normalization is:

$$G[f_r, \sigma_i f_r / f_i, \varepsilon] = \frac{f_r}{f_i} G[f_i, \sigma_i, \varepsilon], \qquad (6)$$

To best visualize the variation of conductance with conductivity, it is convenient to plot the conductance normalized to σ/σ_r, where σ_r can be chosen arbitrarily.

Algorithm

The following algorithm is used for estimation of complex dielectric permittivity from measurements with the interdigital sensor:

1) Convert the values of gain and phase measured at frequency f_i to G_{12} and C_{12} by solving (1).

2) Normalize to f_r by applying (5) and (6).

3) By interpolation, find the values of ε and σ on the calibration surfaces that correspond to the reference frequency values of G_{12} and C_{12}.

4) Calculate the conductivity value at measurement frequency f_i by applying $\sigma_i = \sigma_r (f_i / f_r)$.

These steps are performed for each distinct measurement frequency separately. Note that this procedure does not require any iterations and extensive computations, so, the inversion is performed nearly instantly once the look up table at the reference frequency has been computed..

The range of calibration should be selected so that the ratio $\omega\varepsilon/\sigma$ is between 10^{-3} and 10^{3}. Outside of this range the influence of one of the variables on the output is insignificant.

Calculation for the Entire Grid

The results of calculations presented below are for the 5 mm wavelength of the interdigital sensor in Figure 1 with the thickness of teflon substrate equal to 254 μm and with a 50% metallization ratio.

The variation of the normalized G_{12} and C_{12} are shown in Figure 3 and Figure 4, respectively, using a reference frequency f_r=1 Hz. For higher values of ε, the influence of the substrate on the output is reduced and the normalized transconductance is almost proportional to σ of the tested material. For a single region parallel-plate capacitor with a homogeneous lossy dielectric these plots would be flat. That is, the capacitance would be linear with ε_r and independent of σ while the normalized conductance is independent of both variables.

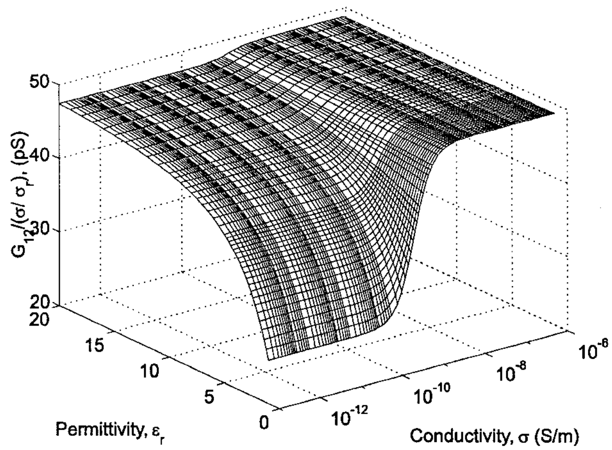

Figure 3. Variation of normalized transconductance G_{12} at frequency f_r = 1 Hz for the entire range of $\varepsilon_r = \varepsilon_2 / \varepsilon_0$ and $\sigma = \sigma_2$.

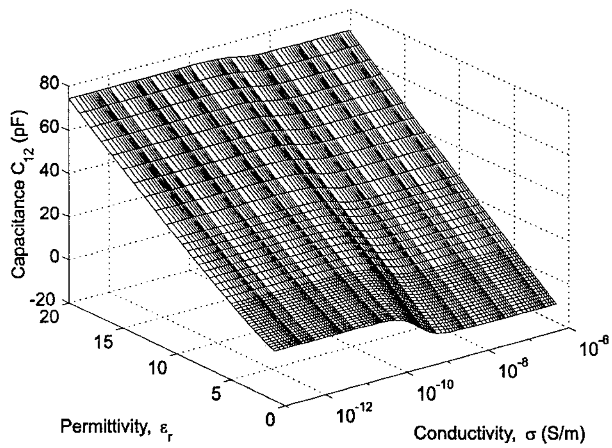

Figure 4. Variation of transcapacitance C_{12} at frequency f_r = 1 Hz for the entire range of $\varepsilon_r = \varepsilon_2 / \varepsilon_0$ and $\sigma = \sigma_2$.

These functional dependencies have relatively simple functional shapes, so, the surfaces shown can be approximated with analytical functions, if desired.

Another important observation is that each variable exhibits a monotonic behavior, it either increases or decreases smoothly with variation of ε_r or σ when the other variable (correspondingly σ or ε_r) remains constant. Such behavior guarantees that the inverse problem has a unique solution.

Solving the Inverse Problem

The solution for the forward program is stored in computer memory in the form of the table which consists of columns ($\varepsilon, \sigma, G_{12}, C_{12},$). Thus, plotting $\varepsilon[G_{12}, C_{12}]$ and $\sigma[G_{12}, C_{12}]$ displays the graphical solution of the inverse problem for the entire range of the values of interest. These plots are shown in Figure 5 and Figure 6. Uniqueness of solution is evident for all regions except for the edge regions in Figure 6 where loss of sensitivity to the variation of transconductance occurs. By the nature of the σ-f normalization, these regions correspond to very high and very low frequencies.

When the ε of the material becomes high, most of the field lines connecting the sensing and the driven electrodes go through the test material rather than through the substrate. In that case, the response of the sensor approaches that of a parallel plate sensor, and the inverted value of ε_r becomes almost independent of conductivity, as can be seen in Figure 5.

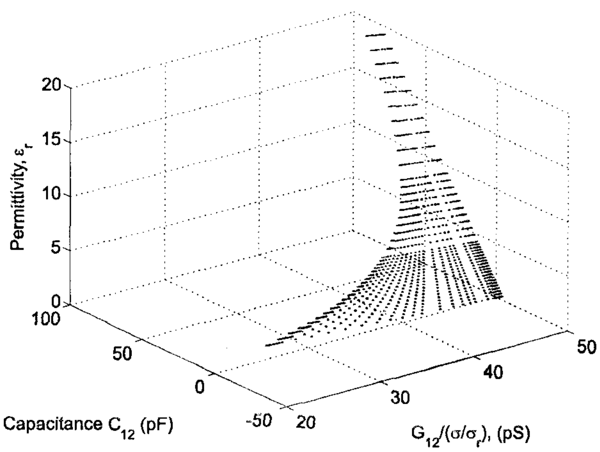

Figure 5. Inverse dependence of the relative dielectric permittivity ε_r on the transadmittance components.

Conclusions

A fast algorithm suitable for real-time implementation of interdigital dielectrometry is presented. The speed is achieved through a non-iterative calculation of material properties based on look-up tables found in advance. Simulated calibration of an interdigital sensor is performed with the finite-element software. A normalization approach that allows reduction of the number of unknown variables is proposed.

In addition, the uniqueness of the solution for the inverse problem of material characterization is demonstrated for the case of homogeneous tested material. Local function approximation is used to relate scaled values of calibration planes to the properties of tested materials. Future publications will demonstrate application of the presented algorithm to dielectrometry measurements.

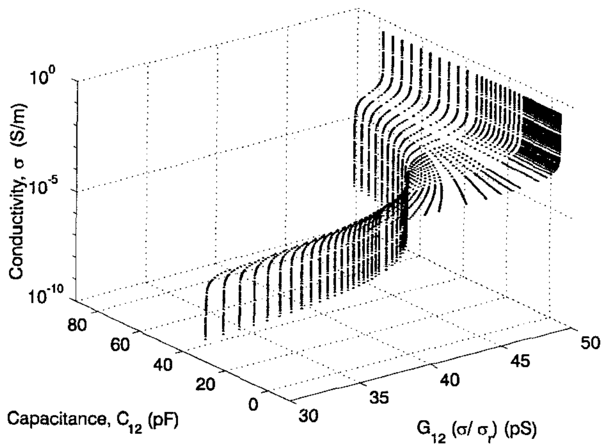

Figure 6. Inverse dependence of the electric conductivity σ on the transadmittance components.

Acknowledgments

Continuing support of the Electric Power Research Institute, under grant WO 3334-1, managed by Mr. S. Lindgren, and the National Science Foundation under Grant No. ECS-9523128 is gratefully acknowledged. Especial thanks to Ansoft Corporation, for providing the finite-element modeling software Maxwell. The authors would like to thank other graduate students in the MIT High Voltage Research Laboratory, a division of the Laboratory for Electromagnetic and Electronic Systems, namely, Darrell Schlicker, Yanqing Du, and Yanko Sheiretov who have made important contributions to this study.

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