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Parameter Identification in an Estimated New Keynesian Open Economy Model*

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Abstract

In this paper, we use Monte Carlo methods to study the small sample properties of the classical maximum likelihood (ML) estimator in artificial samples generated by the New-Keynesian open economy DSGE model estimated by Adolfson et al. (2008) with Bayesian techniques. While asymptotic identification tests show that some of the parameters are weakly identified in the model and by the set of observable variables we consider, we document that ML is unbiased and has low MSE for many key parameters if a suitable set of observable variables are included in the estimation. These findings suggest that we can learn a lot about many of the parameters by confronting the model with data, and hence stand in sharp contrast to the conclusions drawn by Canova and Sala (2009) and Iskrev (2008). Encouraged by our results, we estimate the model using classical techniques on actual data, where we use a new simulation based approach to compute the uncertainty bands for the parameters. From a classical viewpoint, ML estimation leads to a significant improvement in fit relative to the log-likelihood computed with the Bayesian posterior median parameters, but at the expense of some the ML estimates being implausible from a microeconomic viewpoint. We interpret these results to imply that the model at hand suffers from a substantial degree of model misspecification. This interpretation is supported by the DSGE-VAR(λ) analysis in Adolfson et al. (2008). Accordingly, we conclude that problems with model misspecification, and not primarily weak identification, is the main challenge ahead in developing quantitative macromodels for policy analysis.

Keywords: Identification; Bayesian estimation; Monte-Carlo methods; Maximum Likelihood estimation; New-Keynesian DSGE Model; Open economy.

JEL Classification Numbers: C13; C51; E30.

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1. Introduction

Following the seminal papers by Christiano, Eichenbaum and Evans (2005), and Smets and Wouters (2003), the interest in building and estimating dynamic stochastic general equilibrium (DSGE) models for welfare and policy analysis have increased sharply in both academic and policy surroundings.

Most of the papers in the recent literature on estimated New-Keynesian type of DSGE models have used Bayesian estimation techniques. The choice of applying this approach can partly be explained by compelling arguments of why Bayesian methods are appropriate when thinking about macroeconomic models and policy applications, see e.g. the discussions in Sims (2007, 2008). But there is also a possibility that Bayesian methods have been applied because "they work". If a given set of variables in the data is not informative about some particular parameters in the model, i.e. if all parameters in the model are not identified by the data, the priors provide curvature for the posterior and thus enable "successful" estimation of the model.

The ideas above have been well articulated in the recent papers by Canova and Sala (2009) and Iskrev (2008, 2010), who suggest that it is difficult to ensure identification of parameters in DSGE models, casting doubts on the reliability of the empirical results in the literature on estimated DSGE models.[1] The models considered by Canova and Sala, and Iskrev are standard New Keynesian models closely related to the model estimated e.g. in the seminal paper by Smets and Wouters (2003), so their findings are clearly a matter of great concern for the literature.

In this paper, we provide a study of the small sample properties of the classical maximum likelihood (ML) estimator in order to examine identification issues in the state-of-the-art New-Keynesian open-economy DSGE model of Adolfson et al. (2008). A log-linearized version of this DSGE model is used to generate artificial samples using Adolfson et al.'s posterior median parameters, and the estimation strategy is identical with the exception that classical ML methods are used instead of Bayesian techniques. To put our small sample results into proper perspective, we also apply the asymptotic tests for identification developed by Iskrev (2010) on our model.

A limitation of our analysis is that it is restricted to one baseline model. So even if this particular model is identified, it does not allow us to draw general conclusions about identification in New Keynesian DSGE models. There are however four distinct reasons why we think our analysis should be of interest nevertheless. First, we work with an empirically plausible model that has well-documented good empirical properties (see e.g. Adolfson et al. 2008).[2] One could probably figure out examples of other, less empirically anchored, models that would lead to a different conclusion than the one drawn here. Second, many models in the open-economy literature are similar in spirit (see e.g. Cristadoro et al., 2008, Justiniano and Preston, 2008, Rabanal and Tuesta, 2006 and Smets and Wouters, 2002), and several central banks are also currently working with comparable models, e.g., the Federal Reserve Board's SIGMA model (Erceg et al., 2006), the European Central Bank's New Area Wide model (Christoffel et al.,

[1] Identification has to do with the ability to do inference about a particular set of model parameters given an observed set of variables. Following Canova and Sala (2009), we define a DSGE model to suffer from observational equivalence if different parameterizations of the model are indistinguishable with respect to the likelihood. Another, more relevant case in practice, is a situation where the DSGE model is plauged by weak identfication, i.e. where the likelihood function has a unique but weak curvature for (some of) the parameters that the econometrician tries to estimate. In the former case, the ML estimator will be inconsistent, wheras in the latter case, the ML estimator will be consistent but a very large sample may be required to learn from data about (some of) the parameters of the DSGE model.

[2] With the exception of the uncovered interest rate parity condition, this model is essentially identical to the model originally developed by Adolfson et al. (2007). Sims (2007) acknowledges that this is the first estimated fully-fledged DSGE model that is in full operational use as core model in the policy process at an inflation targeting central bank (Sveriges Riksbank).

2008), and the International Monetary Fund's GEM model (Pesenti, 2008). Third, the structure of the domestic part of the model resembles very closely the structure in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003), and the set of observed variables used in estimation span the variables employed by Smets and Wouters (2003, 2007). Fourth and finally, many of the parameter values used in the model to generate artificial samples are similar to the ones estimated elsewhere in the DSGE literature, with a few important exceptions that will be further discussed below.

Relative to the recent work by Canova and Sala (2006), which focuses on limited information methods (i.e. the minimum distance estimator used by e.g. Christiano, Eichenbaum and Evans, 2005), we add to their analysis by considering full information methods instead. Relative to the work by Iskrev (2008, 2010), who also indeed consider full information methods, we add value by quantifying the economic significance of problems with weak identification in terms of small sample distributions of individual parameters. A potential drawback with the tests developed by Iskrev is that they can give misleading results if some parameters are highly correlated and weakly identified only within a small subset of the parameter space.[3] Hence, although the asymptotic tests suggest that problems with weak identification are important, the effects on mean square errors can be very limited for (some of) the parameters even in small samples. Our Monte Carlo analysis allows us to quantify this potential weakness of the local asymptotic tests in Canova and Sala (2009) and Iskrev (2010), and examine to what extent the data can be informative about the parameters in small samples.

Our results document that the ML estimator is unbiased for nearly all parameters. This finding differs from Canova and Sala (2009) who report sizeable small sample biases for many of their estimated parameters. Moreover, when our sample size increases from 100 to 400 observations, the small sample bias almost disappears for the few parameters plagued by small sample bias and the marginal distributions start to collapse markedly around the true parameter values. However, in line with Canova and Sala (2009) and Iskrev (2008, 2010), both our small sample analysis and asymptotic tests for identification lend clear support to the view that many of the parameters are weakly identified from the aggregate quantities and prices that are used as observables in our model. But a key insight from our small sample analysis is that although both the asymptotic tests and small sample results show that many of the parameters are weakly identified, their absolute mean square error (MSE) is often low and we can hence learn a lot by estimating them.

An implication of the problems with weak identification is that the median standard deviations computed with the inverse Hessian are substantially lower than the standard deviation (i.e. the MSE) in the marginal distributions. According to our results, the most severe problems with weak identification pertain to some of the parameters in the policy rule. This finding is not surprising given that many papers in the empirical DSGE literature have documented that the posterior for, for example, the long-run response coefficients for inflation and the output gap typically move very little from the prior (see e.g. Smets and Wouters, 2003). Given that the interest rate smoothing coefficient is estimated to be quite high for Euro area and Swedish data, this result can easily be obtained as the effective short-run coefficients (i.e. the long-run coefficients times one minus the smoothing coefficient) become rather small and is little affected by movements in the long-run coefficients when the interest rate smoothing coefficient is high.

[3] To examplify, assume two parameters θ and η are that highly correlated and jointly have weak a impact on the likelihood function in the ranges $\theta_0 < \tilde{\theta}\hat{<}\theta_1$ and $\eta_0 < \hat{\eta} < \eta_1$. But outside these ranges, the parameters are less correlated and have a strong impact on the likelihood function. If the information matrix is computed at the point $\left\{\hat{\theta}, \hat{\eta}\right\}$ inside these ranges, then the methodology outlined in Iskrev can erroneously suggest that we cannot learn much by estimating these parameters, which is incorrect unless the ranges (θ_0, θ_1) and (η_0, η_1) are large.

One of our most troublesome finding is that the parameter governing the degree of nominal wage stickiness has a rather high MSE in small samples. We document that this result is driven by the persistence properties of the labour supply shock in the model. In our estimated model, which is used as the data generating process, the AR(1) labour supply shock process is characterized by a low persistence coefficient and very volatile innovations. This mainly reflects that our measured real wage series is very erratic and display much less persistence than the real wage series for the Euro area and the US. Accordingly, labour supply shocks are estimated to be substantially more persistent on Euro area and US data, see e.g. Adolfson et al. (2007) and Smets and Wouters (2007). When we increase the degree of persistence in the data generating process, following e.g. the estimation results in Smets and Wouters (2007), we find that the MSE for the sticky wage parameter narrows by 50 percent and that the dispersion of the marginal parameter distributions for many of the other deep parameters shrink considerably as well. We therefore draw the conclusion that the weak identification problem pertaining to the sticky wage parameter can be a specific feature of the Swedish data, and may not carry over to other countries where better measures of the real wage series are available. Furthermore, this experiment also makes clear that the performance of ML estimation is not invariant w.r.t. the parameterization of the model, which stresses the importance of using an empirically vetted calibration of the model.

Our results above stand in sharp contrast with the findings in Canova and Sala (2009) and Iskrev (2008). One important reason why our results differ to theirs is that they attempt to estimate more parameters than we do. Following Christiano, Eichenbaum and Evans (2005), we calibrate (i.e. use strict priors) for some of the parameters that we have good prior information about, and that we *a priori* believe are not well identified by the set of variables included in the estimation. For instance, we keep the parameters determining e.g. the steady state wage markup, labour share of income, depreciation rate, the household's labour supply elasticity, risk aversion and discount factor fixed at their true values. According to the results in Iskrev (2008) (Table 3.6), many of these parameters along with the policy rule parameters are the source of problems with weak identification in his analysis of the Smets and Wouters model. Some of these parameters could be well identified from aggregate quantities and prices if a larger set of observed variables and first-order moments were included in the estimation. For instance, by including the average labour and investment to output shares as observable variables, we would be able to pin down the labour share of income and the depreciation rate in the estimation. Other parameters, like e.g. the labour supply elasticity is better identified by micro data, see e.g. Domeij and Flodén (2006), and we therefore fix this parameter at a reasonable value when we estimate the model on aggregate data. In principle, our model implies a distribution and history of households with different nominal wages and hours worked, and under the standard maintained assumption that the data generating process is not misspecified, the information in these distributions in conjunction with aggregate hours worked and the aggregate real wage could be used to efficiently estimate the labour supply elasticity and the steady state markup. As a consequence, the fact that these parameters are weakly identified when aggregate data are used exclusively to estimate DSGE models is not an identification problem for the models per se, it merely reflects a limitation of what can be achieved with aggregate data only. Of course, it may be the case that the DSGE model does not represent a reasonable approximation of actual wage-setting behaviour, so that estimating the first-order conditions right off the micro data would yield implausible results, but this is a problem with misspecification and not with identification. A final important reason for why we obtain more favorable results in terms of MSEs is that we assume that the markup shocks to be white noise following the empirical work of Adolfson et al. (2007) and Smets and Wouters (2003). In contrast, Iskrev (2008) shows that the parameters governing the degree of price stickiness are very weakly identified in the Smets and Wouters (2007) model in which the markup shocks follow an ARMA(1,1) process. Inflation persistence needs to be intrinsic under the assumption of white noise markup shocks while it can both be intrinsic and inherited by the markup shocks when these are allowed to be highly correlated. The work of Adolfson et al. (2005) and Del Negro and Schorfheide (2008) document that there is a strong negative relationship between the estimated degree of price stickiness and the persistence coefficient of the markup shocks. When the markup shocks are allowed to be correlated, the posterior median and the uncertainty bands for the price stickiness parameters increase substantially.[4] Taken together, the arguments above stress the need to carefully select the parameters and functional form of the shock processes when bringing DSGE models to the data. Strict or very tight priors should be used for parameters that we have good information about from microeconomic data and previous studies, and that can be expected to be less well identified from the particular set of aggregate quantities and prices used in the estimation.

Moreover, we document, for a given set of shocks and estimated parameters, that the dispersion in the small sample marginal distributions are strongly moderated and small sample biases reduced when a more informative set of observable variables are used in the estimation. This finding stresses the importance that great care needs to be taken in selecting how many and which variables to include among the set of observable variables in order to enhance identification of the estimated parameters.

Finally, we use the lessons learned in the Monte Carlo analysis and estimate the model with classical ML estimation techniques and compare the estimation results with the Bayesian estimation results. As anticipated from our previous exercises, we find support that the standard deviations based on the inverse Hessian in many cases strongly underestimate the uncertainty about the parameter estimates, due to problems with weak identification, and we therefore simulate 90-percent confidence bands for the ML estimates using a novel approach based on the Metropolis-Hastings algorithm where we accept all parameter draws that cannot be statistically rejected from the ML estimates according to a standard likelihood ratio test. Consistent with the small sample simulation results, we find that the data is very informative about many of the parameters, but that some of the point estimates are driven to implausible values in the ML estimation. In particular, this finding pertains to the sticky price parameters and the implied variances of the markup shocks. Another key finding is that there is significant increase in the likelihood for the ML estimates of the model in comparison with the log-likelihood associated with the Bayesian posterior median parameters. We interpret these findings to suggest that the model suffers from problem with misspecification, an interpretation consistent with the findings of Adolfson et al. (2008) who apply the DSGE-VAR methodology of Del Negro and Schorfheide (2004) and find that there is a large improvement in log marginal likelihood when the cross-equation restrictions implied by the DSGE model are relaxed. Del Negro et al. (2007) report similar findings for a Smets and Wouters (2003) style model of the US economy.

The remainder of the paper is organized as follows. In the next section, we describe the open economy DSGE model that we use as the data generating process, and briefly describe

[4] In our analysis, we therefore make the assumption that the markup shocks are white noise processes. Allowing for correlated markup shocks, like e.g. in Smets and Wouters (2007) would enable the model to fit the data about equally well for the given set of variables but with substantially lower price stickiness parameters. However, there are two problems with allowing for correlated markup shocks when estimating DSGE models. First, as discussed in detail by Chari, Kehoe and McGrattan (2008), the correlated markup shocks in Smets and Wouters (2007) result in implausibly highly volatile markup shocks. Second, the high inflation outcome in the 1970s is to a large extent driven by positive markup shocks according to the analysis in Smets and Wouters (2007). However, real profits were not very high or rising in the 1970s (see e.g. the price-earnings data for all S&P 500 firms collected by Shiller, 2005), and given this fact we argue that large and correlated markup shocks is not a compelling explanation of inflation inertia.

how the model has been estimated on actual data with Bayesian techniques. We then apply the asymptotic tests for identification developed in Iskrev (2010) on our model in Section 3. In Section 4, we describe the setup in the Monte Carlo simulation exercise and present the benchmark results of this exercise. The simulation results for the alternative parameterisation with more persistent labour supply shocks are discussed in Section 5. In Section 6, we take the lessons from the previous analysis into account and estimate the DSGE model with classical ML techniques and offer a comparison with the Bayesian estimation results. Finally, we provide some concluding remarks in Section 7.

2. The DGP - an Open Economy New Keynesian DSGE model

The model is an open economy DSGE model identical to the model presented and estimated in Adolfson et al. (2008). It shares its basic closed economy features with many recent new Keynesian models, including the benchmark models of Christiano, Eichenbaum and Evans (2005), Altig, Christiano, Eichenbaum and Lindé (2004), and Smets and Wouters (2003). To provide an explanation of the role various parameters play, this section gives an overview of the model and presents the key equations of it. To motivate the benchmark parameterisation and the set of parameters we include in the subsequent Monte Carlo simulations, we also discuss how it has been estimated on Swedish data by Adolfson et al. (2008) with Bayesian techniques.

2.1. The Model

The model economy includes four different categories of operating firms. These are domestic goods firms, importing consumption, importing investment, and exporting firms, respectively. Within each category there is a continuum of firms that each produces a differentiated good and set prices. The domestic goods firms produce their goods using capital and labour inputs, and sell them to a retailer which transforms the intermediate products into a homogenous final good that in turn is sold to the households. The final domestic good is a composite of a continuum of i differentiated goods, each supplied by a different firm, which follows the constant elasticity of substitution (CES) function

$$Y_t = \left[\int_0^1 (Y_{i,t})^{\frac{1}{\lambda_t^d}} \, di \right]^{\lambda_t^d}, 1 \leq \lambda_t^d < \infty, \tag{1}$$

where λ_t^d is a stochastic process that determines the time-varying flexible-price markup in the domestic goods market. The demand for firm i's differentiated product, $Y_{i,t}$, follows

$$Y_{i,t} = \left(\frac{P_{i,t}^d}{P_t^d} \right)^{-\frac{\lambda_t^d}{\lambda_t^d - 1}} Y_t. \tag{2}$$

The production function for intermediate good i is given by

$$Y_{i,t} = z_t^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} H_{i,t}^{1-\alpha} - z_t \phi, \tag{3}$$

where z_t is a unit-root technology shock capturing world productivity, ϵ_t is a domestic covariance stationary technology shock, $K_{i,t}$ the capital stock and $H_{i,t}$ denotes homogeneous labour hired by the i^{th} firm. A fixed cost $z_t \phi$ is included in the production function. We set this parameter so that profits are zero in steady state, following Christiano et al. (2005).

We allow for working capital by assuming that the intermediate firms' wage bill has to be financed in advance through loans from a financial intermediary. Cost minimization then yields the following nominal marginal cost for intermediate firm i:

$$mc_t^d = \frac{1}{(1-\alpha)^{1-\alpha}} \frac{1}{\alpha^\alpha} (R_t^k)^\alpha \left(W_t R_{t-1}\right)^{1-\alpha} \frac{1}{(z_t)^{1-\alpha}} \frac{1}{\epsilon_t}, \tag{4}$$

where R_t^k is the gross nominal rental rate per unit of capital, R_{t-1} the gross nominal (economy wide) interest rate, and W_t the nominal wage rate per unit of aggregate, homogeneous, labour $H_{i,t}$.

Each of the domestic goods firms is subject to price stickiness through an indexation variant of the Calvo (1983) model. Since we have a time-varying inflation target in the model we allow for partial indexation to the current inflation target, but also to last period's inflation rate in order to allow for a lagged pricing term in the Phillips curve. Each intermediate firm faces in any period a probability $(1-\xi_d)$ that it can reoptimize its price.[5] In each period, the price is set so that the firms maximize a future stream of marginal utility discounted period-profits, taking into account that they might not be able to optimally change their price in each future period.

Log-linearization of the first-order condition of the profit maximization problem yields the following log-linearized Phillips curve:

$$\begin{aligned}\left(\widehat{\pi}_t^d - \widehat{\bar{\pi}}_t^c\right) \quad = \quad & \frac{\beta}{1+\kappa_d\beta}\left(\mathrm{E}_t\widehat{\pi}_{t+1}^d - \rho_{\bar{\pi}}\widehat{\bar{\pi}}_t^c\right) + \frac{\kappa_d}{1+\kappa_d\beta}\left(\widehat{\pi}_{t-1}^d - \widehat{\bar{\pi}}_t^c\right) \\ & - \frac{\kappa_d\beta\left(1-\rho_{\bar{\pi}}\right)}{1+\kappa_d\beta}\widehat{\bar{\pi}}_t^c + \frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\left(1+\kappa_d\beta\right)}\left(\widehat{mc}_t^d + \widehat{\lambda}_t^d\right),\end{aligned} \tag{5}$$

where $\hat{\pi}_t^d$ denotes inflation in the domestic sector (a hat denotes percent deviation from steady state, i.e., $\hat{X}_t = dX_t/X \approx \ln X_t - \ln X$) and $\widehat{\bar{\pi}}_t^c$ the time-varying inflation target of the central bank. β is the discount factor and $\rho_{\bar{\pi}}$ the persistence coefficient in the AR(1)-process for $\widehat{\bar{\pi}}_t^c$.

We now turn to the import and export sectors. There is a continuum of importing consumption and investment firms that each buys a homogenous good at price P_t^* in the world market, and converts it into a differentiated good through a brand naming technology. The exporting firms buy the (homogenous) domestic final good at price P_t^d and turn this into a differentiated export good through the same type of brand naming. The nominal marginal cost of the importing and exporting firms are thus $S_tP_t^*$ and P_t^d/S_t, respectively, where S_t is the nominal exchange rate (domestic currency per unit of foreign currency). The differentiated import and export goods are subsequently aggregated by an import consumption, import investment and export packer, respectively, so that the final import consumption, import investment, and export good is each a CES composite according to the following:

$$C_t^m = \left[\int_0^1 \left(C_{i,t}^m\right)^{\frac{1}{\lambda_t^{mc}}} di\right]^{\lambda_t^{mc}}, \qquad I_t^m = \left[\int_0^1 \left(I_{i,t}^m\right)^{\frac{1}{\lambda_t^{mi}}} di\right]^{\lambda_t^{mi}}, \qquad X_t = \left[\int_0^1 \left(X_{i,t}\right)^{\frac{1}{\lambda_t^{x}}} di\right]^{\lambda_t^{x}}, \tag{6}$$

where $1 \leq \lambda_t^j < \infty$ for $j = \{mc, mi, x\}$ is the time-varying flexible-price markup in the import consumption (mc), import investment (mi) and export (x) sector. By assumption the continuum of consumption and investment importers invoice in the domestic currency and exporters in the foreign currency. To allow for short-run incomplete exchange rate pass-through to import

[5] For the firms that are not allowed to reoptimize their price P_{t+1}^d, we adopt the indexation scheme $P_{t+1}^d = \left(\pi_t^d\right)^{\kappa_d}\left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_d} P_t^d$ where κ_d is an indexation parameter.

and export prices we introduce nominal rigidities in the local currency price, modeled through the same type of Calvo setup as above. The price setting problems of the importing and exporting firms are completely analogous to that of the domestic firms, and the demand for the differentiated import and export goods follow similar expressions as to equation (2). In total there are thus four specific Phillips curve relations determining inflation in the domestic, import consumption, import investment and export sectors.

There is a continuum of j households, whose preferences are given by

$$\mathrm{E}_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \ln\left(C_{j,t} - bC_{j,t-1}\right) - \zeta_t^h A_L \frac{(h_{j,t})^{1+\sigma_L}}{1+\sigma_L} + A_q \frac{\left(\frac{Q_{j,t}}{z_t P_t^d}\right)^{1-\sigma_q}}{1-\sigma_q} \right], \tag{7}$$

where $C_{j,t}$, $h_{j,t}$ and $Q_{j,t}/\left(z_t P_t^d\right)$ denote the j^{th} household's levels of aggregate consumption, labour supply and stationarized real cash holdings, respectively. Consumption is subject to habit formation through $bC_{j,t-1}$. ζ_t^c and ζ_t^h are persistent preference shocks to consumption and labour supply, respectively. Aggregate consumption is assumed to be given by the following CES function:

$$C_t = \left[(1-\omega_c)^{1/\eta_c} \left(C_t^d\right)^{(\eta_c-1)/\eta_c} + \omega_c^{1/\eta_c} \left(C_t^m\right)^{(\eta_c-1)/\eta_c} \right]^{\eta_c/(\eta_c-1)}, \tag{8}$$

where C_t^d and C_t^m are consumption of the domestic and imported good (provided by the domestic and importing consumption firms, respectively). ω_c is the share of imports in consumption, and η_c is the elasticity of substitution across consumption goods.

The households invest in a basket of domestic and imported investment goods (I_t) to form the capital stock (K_t), and decide how much capital to rent to the domestic firms given costs of adjusting the investment rate. The capital accumulation equation is given by

$$K_{t+1} = (1-\delta)K_t + \Upsilon_t \left(1 - \tilde{S}\left(I_t/I_{t-1}\right)\right) I_t, \tag{9}$$

where $\tilde{S}\left(I_t/I_{t-1}\right)$ determines the investment adjustment costs through the estimated parameter $\tilde{S}''$, and Υ_t is a stationary investment-specific technology shock. Total investment is assumed to be given by a CES of aggregate domestic and imported investment goods (I_t^d and I_t^m, respectively) according to

$$I_t = \left[(1-\omega_i)^{1/\eta_i} \left(I_t^d\right)^{(\eta_i-1)/\eta_i} + \omega_i^{1/\eta_i} \left(I_t^m\right)^{(\eta_i-1)/\eta_i} \right]^{\eta_i/(\eta_i-1)}, \tag{10}$$

where ω_i is the share of imports in investment, and η_i is the elasticity of substitution across investment goods.

Following Erceg, Henderson and Levin (2000), each household is a monopoly supplier of a differentiated labour service which implies that they can set their own wage. After having set their wage, households supply the firms' demand for labour at the going wage rate. Each household sells its labour to a firm which transforms household labour into a homogenous good that is demanded by each of the domestic goods producing firms. Wage stickiness is introduced through the Calvo (1983) setup, with partial indexation to last period's CPI inflation rate, the current inflation target and the technology growth. Household j reoptimizes its nominal wage rate $W_{j,t}^{new}$ according to the following

$$\begin{gathered} \max_{W_{j,t}^{new}} \quad \mathrm{E}_t \textstyle\sum_{s=0}^{\infty} \left(\beta\xi_w\right)^s \left[-\zeta_{t+s}^h A_L \frac{(h_{j,t+s})^{1+\sigma_L}}{1+\sigma_L} + \right. \\ \left. \upsilon_{t+s} \frac{\left(1-\tau_{t+s}^y\right)}{\left(1+\tau_{t+s}^w\right)} \left(\left(\pi_t^c \ldots \pi_{t+s-1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c \ldots \bar{\pi}_{t+s}^c\right)^{(1-\kappa_w)} \left(\mu_{z,t+1} \ldots \mu_{z,t+s}\right) W_{j,t}^{new} \right) h_{j,t+s} \right], \end{gathered} \tag{11}$$

where ξ_w is the probability that a household is not allowed to reoptimize its wage, τ_t^y a labour income tax, τ_t^w a pay-roll tax (paid for simplicity by the households), and $\mu_{z,t} = z_t/z_{t-1}$ is the growth rate of the permanent technology level.[6]

The save in domestic and foreign bonds, and the choice between domestic and foreign bond holdings balances into an arbitrage condition pinning down expected exchange rate changes (i.e., an uncovered interest rate parity condition). To ensure a well-defined steady-state in the model, we assume that there is a premium on the foreign bond holdings which depends on the aggregate net foreign asset position of the domestic households, following, e.g. Lundvik (1992), and Schmitt-Grohé and Uribe (2001). Our specification of the risk premium also includes the expected change in the exchange rate $E_t S_{t+1}/S_{t-1}$ which is based on the vast empirical evidence of a forward premium puzzle in the data (i.e., that risk premia are strongly negatively correlated with the expected depreciation of the exchange rate), see e.g. Fama (1984) Duarte and Stockman (2005). The risk premium is given by:

$$\Phi(a_t, S_t, \tilde{\phi}_t) = \exp\left(-\tilde{\phi}_a(a_t - \bar{a}) - \tilde{\phi}_s\left(\frac{E_t S_{t+1}}{S_t}\frac{S_t}{S_{t-1}} - 1\right) + \tilde{\phi}_t\right), \tag{12}$$

where $a_t \equiv (S_t B_t^*)/(P_t z_t)$ is the net foreign asset position, and $\tilde{\phi}_t$ is a shock to the risk premium.[7] The UIP condition in its log-linearized form is given by:

$$\widehat{R}_t - \widehat{R}_t^* = \left(1 - \widetilde{\phi}_s\right) E_t \Delta\widehat{S}_{t+1} - \widetilde{\phi}_s \Delta\widehat{S}_t - \widetilde{\phi}_a \widehat{a}_t + \widehat{\widetilde{\phi}}_t. \tag{13}$$

By setting $\tilde{\phi}_s = 0$ we obtain the UIP condition typically used in small open economy models (see e.g. Adolfson et al., 2007).

Following Smets and Wouters (2003), monetary policy is approximated with a generalized Taylor-type rule:

$$\begin{aligned} \widehat{R}_t &= \rho_R \widehat{R}_{t-1} + (1-\rho_R)\left[\widehat{\bar{\pi}}_t^c + r_\pi\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) + r_y \hat{y}_{t-1} + r_x \hat{x}_{t-1}\right] \\ &\quad + r_{\Delta\pi}\Delta\hat{\pi}_t^c + r_{\Delta y}\Delta\hat{y}_t + \varepsilon_{R,t}, \end{aligned} \tag{14}$$

where $\hat{\pi}_t^c$ denotes CPI inflation, $\hat{y}_t$ the output gap (actual minus trend output), $\hat{x}_{t-1}$ the lagged real exchange rate $\left(\hat{x}_t \equiv \hat{S}_t + \hat{P}_t^* - \hat{P}_t^c\right)$, and $\varepsilon_{R,t}$ an uncorrelated monetary policy shock.

To clear the final goods market, the foreign bond market, and the loan market for working capital, the following three constraints must hold in equilibrium:

$$C_t^d + I_t^d + G_t + C_t^x + I_t^x \leq z_t^{1-\alpha}\epsilon_t K_t^\alpha H_t^{1-\alpha} - z_t\phi, \tag{15}$$

$$S_t B_{t+1}^* = S_t P_t^x\left(C_t^x + I_t^x\right) - S_t P_t^*\left(C_t^m + I_t^m\right) + R_{t-1}^*\Phi(a_{t-1}, \widetilde{\phi}_{t-1}) S_t B_t^*, \tag{16}$$

$$\nu W_t H_t = \mu_t M_t - Q_t, \tag{17}$$

where G_t is government expenditures, C_t^x and I_t^x are the foreign demand for export goods, and $\mu_t = M_{t+1}/M_t$ is the monetary injection by the central bank. When defining the demand for export goods, we introduce a stationary asymmetric (or foreign) technology shock $\tilde{z}_t^* =$

[6] For the households that are not allowed to reoptimize, the indexation scheme is $W_{j,t+1} = (\pi_t^c)^{\kappa_w}\left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w)}\mu_{z,t+1}W_{j,t}^{new}$, where κ_w is an indexation parameter.

[7] Notice that we consider a steady state where $\bar{a}$ is zero. Combined with our assumption that foreign and domestic inflation targets both equal $\bar{\pi}$ in the steady-state, the change in the nominal exchange rate is nil in steady state, so that $\frac{\bar{S}_{t+1}}{\bar{S}_t}\frac{\bar{S}_t}{\bar{S}_{t-1}} - 1 = 0$. We have also verified that the solution exists and is unique for our augmented specification of the risk-premium in (13) in the joint prior distribution for $\left\{\tilde{\phi}_a, \tilde{\phi}_a\right\}$.

z_t^*/z_t, where z_t^* is the permanent technology level abroad, to allow for temporary differences in permanent technological progress domestically and abroad.

The log-linearized shock processes are given by the univariate representation

$$\hat{\varsigma}_t = \rho_\varsigma \hat{\varsigma}_{t-1} + \varepsilon_{\varsigma,t}, \qquad \varepsilon_{\varsigma,t} \overset{iid}{\sim} N\left(0, \sigma_\varsigma^2\right)$$

where $\varsigma_t = \{\ \mu_{z,t}, \epsilon_t, \lambda_t^j, \zeta_t^c, \zeta_t^h, \Upsilon_t, \tilde{\phi}_t, \varepsilon_{R,t}, \bar{\pi}_t^c, \tilde{z}_t^*\}$ and $j = \{d, mc, mi, x\}$.

The government spends resources on consuming part of the domestic good, and collects taxes from the households. The resulting fiscal surplus/deficit plus the seigniorage are assumed to be transferred back to the households in a lump sum fashion. Consequently, there is no government debt. The fiscal policy variables - taxes on capital income, labour income, consumption, and the pay-roll, together with (HP-detrended) government expenditures - are assumed to follow an identified VAR model with two lags.

To simplify the analysis we adopt the assumption that the foreign prices, output (HP-detrended) and interest rate are exogenously given by an identified VAR model with four lags. Both the foreign and the fiscal VAR models are being estimated, using uninformative priors, ahead of estimating the structural parameters in the DSGE model.[8]

To compute the equilibrium decision rules, we proceed as follows. First, we stationarize all quantities determined in period t by scaling with the unit root technology shock z_t. Then, we log-linearize the model around the constant steady state and calculate a numerical (reduced form) solution with the AIM algorithm developed by Anderson and Moore (1985).

2.2. Parameterisation of the model

We start the empirical analysis by estimating the DSGE model on actual data, using a Bayesian approach and placing a prior distribution on the structural parameters. We use quarterly Swedish data for the period $1980Q1 - 2004Q4$ and match the following 15 variables: the GDP deflator, the real wage, consumption, investment, the real exchange rate, the short-run interest rate, hours worked per capita (total hours divided by working age population), GDP, exports, imports, the consumer price index (CPI), the investment deflator, foreign output, foreign inflation and the foreign interest rate. The data is identical to the Adolfson et al. (2008), and the interested reader is kindly referred to this paper for further details.

As in Altig et al. (2004), the unit root technology shock induces a common stochastic trend in the real variables of the model. To make these variables stationary we use first differences and derive the state space representation for the following vector of observed variables

$$\tilde{Y}_t = \begin{array}{ccccccccc} [\ \pi_t^d & \Delta\ln(W_t/P_t) & \Delta\ln C_t & \Delta\ln I_t & \hat{x}_t & R_t & \hat{H}_t & \Delta\ln Y_t ... \\ & \Delta\ln\tilde{X}_t & \Delta\ln\tilde{M}_t & \pi_t^{cpi} & \pi_t^{def,i} & \Delta\ln Y_t^* & \pi_t^* & R_t^* \ ]'. \end{array} \qquad (18)$$

[8]The scaled level of foreign output $\hat{y}_t^*$ $\left(d\ln\frac{Y_t^*}{z_t^*}\right)$ enters the stationarized log-linear represenation of the DSGE model (via in the aggregate resource constraint, eq. 15 due to our assumption that total export demand follows $C_t^x + I_t^x = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} Y_t^*$). In order to avoid joint estimation of the parameters in the VAR and the deep parameters in the model, we use the HP-filter to compute $\hat{y}_t^*$, defined as $\ln Y_t^* - \overline{\ln Y_t^*}$ where $\overline{\ln Y_t^*}$ is the HP-trend in logs with the smoothing coefficient set to 1600, that is used in the pre-estimated VAR model. While this procedure is admittedly is not perfect from a model viewpoint, it does not seem to have any serious implications for the estimations on actual data for the following reasons. First, the computed process for $\hat{y}_t^*$ is quite reasonable with a first-order autocorrelation coefficient of 0.97, while $\overline{\ln Y_t^*}$ has largely similar statistical properties as the process for the estimated stochastic unitroot shock. Second, in Appendix A (see Figures A2a-A2c), we report results when we use exactly the same approach on simulated data samples as on actual data, with the key finding being that our approach does not distort the estimation results to any greater extent.

The growth rates are computed as quarter to quarter log-differences, while the inflation and interest rate series are measured as annualized quarterly rates.[9] It should be noted that the stationary variables $\hat{x}_t$ and $\hat{H}_t$ are measured as deviations around the mean, i.e. $\hat{x}_t = (x_t - x)/x$ and $\hat{H}_t = (H_t - H)/H$, respectively.

In comparison with other papers in the open economy literature, such as for example Justiniano and Preston (2004) and Lubik and Schorfheide (2005), we have chosen to work with a large number of variables because it facilitates identification of the parameters and shocks processes we estimate.[10] We estimate 13 structural shocks of which 5 are assumed to be identically independently distributed and 8 follow AR(1) processes. In addition to these shocks, there are eight additional shocks provided by the exogenous (pre-estimated) fiscal and foreign VARs, whose parameters are kept fixed at their posterior mean estimates throughout the estimation of the DSGE model parameters. The shocks enter in such a way that there is no stochastic singularity in the likelihood function.[11] To compute the likelihood function, the reduced form solution of the model is transformed into a state-space representation mapping the unobserved state variables into the observed data. We apply the Kalman filter to calculate the likelihood function of the observed variables, where the period 1980Q1-1985Q4 is used to form a prior on the unobserved state variables in 1985Q4 and the period 1986Q1-2004Q4 for inference.

We choose to calibrate those parameters which we think are weakly identified by the set of observables we match. These parameters are stated in Table 1. Most of these parameters are related to the steady-state values of permutations of the observables, and would be identified by inclusion of the following set of first-order moments: C/Y, $\tilde{M}/Y$, I/Y and G/Y (identifies ω_c, ω_i, δ, and g_r); π and R (identify μ and β when μ_z is estimated); $1 - WH/Y$ (identifies α); average income, pay-roll and VAT rates (identifies τ^y, τ^w, and τ^c). However, as the sample period used to form the likelihood is fairly short, we preferred to impose strict priors on (i.e. calibrate) these parameters to ensure that they are consistent with the corresponding moments in the long-run. Christiano, Eichenbaum and Evans (2005) pursue the same approach in their paper. Four of the parameters in Table 1, $\{\eta_c, \sigma_L, \lambda_w, \rho_{\bar{\pi}}\}$, are not pinned down by first-order moments. As discussed in the introduction and in further detail in Section 3, two of these parameters, σ_L and λ_w are very weakly identified from the aggregate observables we consider,

[9] In the state-space representation of the model, which links the theoretical model to the observed data in (18), we measure world output growth as $\Delta \ln Y_t^* = \mu_{z,t} + \Delta \tilde{z}_t^* + \Delta \hat{y}_t^*$ where $\mu_{z,t}$ is the common world productivity growth rate (unit-root in levels, but difference stationary), $\tilde{z}_t^*$ the stationary asymmetric (or foreign) technology shock to the business cycle component $\hat{y}_t^*$ of foreign output. This enables us to identify the stationary asymmetric technology shock, since $\hat{y}_t^*$ is identified from the pre-estimated VAR and $\mu_{z,t}$ is partly identified from domestic quantities. We include $\tilde{z}_t^*$ in the model to allow capture differences persistent, but not permanent differences in growth rates between Sweden and the rest of the world.

[10] We show in Section 4.3 that the small sample distributions are more dispersed when we consider a smaller set of observables in the estimation. Boivin and Giannoni (2006) argues that a large set of observables is advantageous when allowing for measurement errors like we do. Guerron-Quintana (2010) finds that around 7 observables appears preferable for a closed economy version of the model considered, but suggests in the conclusions that a larger set of observables may be warranted in an open economy framework as ours and when allowing for measurement errors like we do.

[11] Even if there is no stochastic singularity in the model we include measurement errrors in the 12 domestic variables, since we know that the data series used are not perfectly measured and at best only approximations of the 'true' series. In particular it was hard to remove the seasonal variation in the series, and there are still spikes in for example the inflation series, perhaps due to changes in the collection of the data. The variance of the white noise measurement errors is set to 0 for the foreign variables and the domestic interest rate, 0.1 percent for the real wage, consumption and output, and 0.2 percent for all the variables. This implies that the fundamental shocks explain about 90-95% of the variation in most of the variables. It should also be noted that the measurement errors mostly captures some of the high frequency movements in the data and little of the business cycle fluctuations.

and we therefore adapt their values from Christiano, Eichenbaum and Evans (2005). The two parameters η_c and $\rho_{\bar{\pi}}$ are shown in Section 3 to be identified in our model, but we nevertheless decided to calibrate them to have our setup consistent with Adolfson et al. (2008).[12]

The parameters we choose to estimate pertain mostly to the nominal and real frictions in the model as well as the exogenous shock processes. Table 2 shows the assumptions for the prior distribution of the estimated parameters. The location of the prior distribution of the 43 estimated parameters with no break in the monetary policy rule corresponds to a large extent to those in Adolfson et al. (2007) on Euro area data, and are more thoroughly discussed in Adolfson et al. (2008).

The joint posterior distribution of the estimated parameters is obtained in two steps. First, the posterior mode and Hessian matrix evaluated at the mode is computed by standard numerical optimization routines. Second, the Hessian matrix is used in the Metropolis-Hastings algorithm to generate a sample from the posterior distribution (see Smets and Wouters (2003), and the references therein, for details). Table 2 reports the median estimates based on a sample of 500,000 post burn-in draws from the posterior distribution.

3. Asymptotic tests of identification

As a starting point we test the DSGE model for identification using the asymptotic diagnostic tests developed in Iskrev (2010). This provides an assessment of the asymptotic identification properties of the model parameters, and are useful for putting the results in our subsequent small sample Monte Carlo simulation exercise into proper perspective.

Following Iskrev (2010), we let $m_T(\theta)$ denote all first- and second-order moments for the l series with observed variables ($l = 15$ in our benchmark case). If the second-order moments are computed for contemporaneous and $T-1$ lags of observed data, then $m_T(\theta)$ is a $(T-1)\,l^2 + l\,(l+3)\,/2$ sized vector with moments in the model determined by the k estimated parameters in θ. Now, if m_T is a continuously differentiable function of θ, then θ_0 is locally identifiable if the Jacobian matrix $J(T) = \partial m_T/\partial\theta$ has rank k at θ_0. Moreover, by applying the chain rule, Iskrev (2010) shows that the Jacobian matrix $J(T)$ can be conveniently expressed as

$$J(T) = \frac{\partial m_T}{\partial\tau}\frac{\partial\tau}{\partial\theta} \equiv J_1(T)\,J_2, \tag{19}$$

where τ is a vector with all reduced-form coefficients in the DSGE model. The decomposition in (19) is very useful, as it implies that if the matrix $J_2 = \partial\tau/\partial\theta$ has rank less than k at θ_0, then some of the parameters in θ are not locally identified at θ_0 regardless of which dataseries are used in the estimation as this matrix is independent of the set of observables. For this reason, Iskrev (2010) suggests to study the rank of both matrices J_2 and $J(T)$ to learn about identification. To have full column rank (k) of J_2 is necessary for identification, whereas the rank of $J(T)$ will show if θ is locally identified for the particular set of observables and contemporaneous and lagged moments under consideration.

Iskrev (2008, 2010) emphasizes that although both $J(T)$ and J_2 have rank k, they can be close to rank deficient and hence poorly conditioned. Poorly conditioned $J(T)$ and J_2 matrices suggest that identification of (at least) some parameters is weak. A parameter θ_i can

[12] The rationale for calibrating $\rho_{\bar{\pi}}$ to 0.975 was to ensure that the inflation target is a highly persistent process and preferably high before the introduction of the inflation target regime in 1995 and then gradually declining to the new inflation target of 2 percent. η_c was calibrated to 5 because this parameter was otherwise driven towards a very high number (around 20) due to a slightly better match of the low volatility of consumption and high volatility of imports; see Adolfson et al. (2007, 2008) for further discussion.

be weakly identified because it has little impact on the reduced-form coefficients of the model, i.e. $\partial\tau/\partial\theta_i \approx 0$, or because there is a high degree of substitutability between the parameter and (possibly a linear combination of) other parameters, so that $\partial\tau/\partial\theta_i \approx \Sigma_{j\neq i}a_j\left(\partial\tau/\partial\theta_j\right)$ where $\Sigma_{j\neq i}a_j$ is a suitable linear combination of the other deep parameters in θ. The problem with parameter interdependence arises when different parameter combinations play a very similar role in the model, and can be assessed by computing multiple collinearity coefficients for each parameter in θ. The multiple collinearity coefficient for θ_i measures how well θ_i:s effect in the model can be mimicked by the other parameters in θ, and values close to unity (or -1) implies a very strong degree of multiple collinearity, and accordingly problems with weak identification.

In Table 3a below, we report the multiple collinearity coefficients for J_2 and $J\left(T\right)$ for the benchmark parameterisation (posterior median) of the model that is used to generate the simulated samples in the Monte Carlo simulation exercise in Section 4.[13] The two right-most columns show the corresponding results when using only a subset of the observed variables, i.e. the seven variables matched by Smets and Wouters (2003) (see Section 4.3 for further details). We see that the multiple collinearity coefficients are ranging from 0.0117 ($\sigma_{\lambda_{m,i}}$) to 0.9950 (r_π) for J_2, and that they are identical for both sets of observables as they should be (as the structure and parameterisation of the model is identical irrespective of which observables are used). The second and fourth columns with figures in the table shows results for $J\left(T\right)$. Following Iskrev (2010), we set $T = 2$ (i.e. consider all moments for contemporaneous and one lags of data). By comparing J_2 and $J\left(2\right)$ for the full set of observables in the estimation, we see that the multiple collinearity coefficients are generally much higher for $J\left(2\right)$ than J_2. Indeed, many of the multiple collinearity coefficients in $J\left(2\right)$ are very close to unity, indicating that identification is indeed week for many of the parameters in the model given this set of observables. Not surprisingly, the multiple collinearity coefficients are even higher given a smaller set of observables (b and μ_z being the exceptions), suggesting very weak identification of many parameters at this point in the parameter space (see the right-most column).

However, the results in Table 3a only pertains to the benchmark parameterisation of the model. To explore the properties of $J\left(T\right)$ and J_2 in a larger region of the parameter space, we sample 5,000 draws from the prior distributions in Table 2. In Table 3b we report the results as the share of draws for which $J\left(2\right)$ and J_2 have full column rank along with their (median) condition number, the (median) smallest singular value and (median) tolerance value. The conditioning number is the ratio between the largest and smallest singular value of $J\left(2\right)$ and J_2, which have full rank if their smallest singular value is larger than the tolerance value. Hence, a large conditioning number is a strong indication of weak identification.

Table 3b makes clear that the model is locally identified in essentially all parts of the parameter space that we explore, but the large conditioning number and comparison of the low smallest singular value with the tolerance value indicates that problems with weak identification are typical as argued by Canova and Sala (2009) and Iskrev (2010), and even more important when only a subset of observables are considered.

[13] We use Dynare 4.3 which has the routines that compute $J\left(T\right)$ and J_2 numerically. To examine the robustness of the numerical derivates, we have checked and verified that the results are not sensitive to setting all elements in $J\left(T\right)$ and J_2 which are smaller than $1e-7$ equal to nil. To be consistent with the theoretical structure of the model, we assume that the shocks in the real wage, consumption, investment and Phillips curve equations are multiplied by the friction parameters (i.e. $\frac{(1-\beta)(1-\beta\kappa)}{(1+\beta\kappa)\xi}$ in equation 5), and do not use a coefficient of unity as we assume in the estimations in Sections 4 and 6 following our previous empirical work (Adolfson et al. 2007, 2008). In the empirical work, a unit coefficient on the shocks were convenient when testing for the empirical relevance of various nominal and real frictions. In any event, our conclusions regarding the the rank of $J\left(T\right)$ and J_2 are not affected if the shocks enter with a coefficient of unity or with multiplicative coefficient as in eq. (5), given that the prior on the standard deviation for each these shocks are suitably adjusted.

Although not reported, we have also computed the statistics in Tables 3a and 3b when the parameters $\{\eta_c, \sigma_L, \lambda_w, \rho_{\bar{\pi}}\}$ are included among the set of estimated parameters. In this case, we find that $J(2)$ and J_2 still have full rank, but that (as expected) λ_w and σ_L are very weakly identified by the reduced form coefficients in the model (J_2). In addition, inclusion of both λ_w and σ_L among the set of estimated parameters also exacerbates the weak identification problem for ξ_w, which is essentially completely collinear to the other parameters in the model (J_2) in this case, regardless of which aggregate dataseries that are used in the estimation.[14]

4. Maximum likelihood estimation in small samples: A Monte Carlo simulation exercise

In this section, we first describe in detail how the small sample ML distributions have been generated with the DSGE model, and present the results for the baseline set of observables. Then, we report results when only a subset of the observed variables are included in the estimation.

4.1. The setup

To assess the performance of Maximum Likelihood in small samples simulated with the DSGE model, the following steps are conducted:

1. Solve the DSGE model using the calibrated parameters (see Table 1) and the posterior median of the estimated parameters (see Table 2).

2. Generate an artificial sample of length T by simulating the model $1000+T$ periods initiated from the steady state. The first 1000 observations are discarded as burn-ins. The innovations in the shock series were drawn from the normal distribution, where we set the seed for each sample to $i = 1, ..., N$ where N is the number of artificial samples considered.[15]

3. The calibrated parameters in Table 1 are kept fixed at the 'true' values used to generate simulated data. As a consequence, the ML estimation results will not reflect any uncertainty stemming from these parameters.

4. Given the simulated data (and the calibrated parameters), we estimate the parameters in Table 2 by maximizing the likelihood function using the same set of observable variables as on the actual data (see eq. 18). We use Chris Sims' optimizer CSMINWEL to perform the estimation.[16,17]

[14] If only either λ_w or σ_L are additionally included among the set of estimated parameters, then identification of these parameters improves and while ξ_w is still weakly identified in the model (J_2) it then has a multicollinearity coefficient below unity. Moreover, notice that we do not include the parameters $\{\beta, \alpha, \mu, \delta, \omega_i, \omega_c, \tau^y, \tau^w, \tau^c, g_r\}$ among the set of estimated parameters for reasons discussed in Section 2.2.

[15] An alternative to sample from the normal distribution would be to sample the innovations in the shock processes from the empirical distribution of the 2-sided estimates. But given that the purpose of the paper is to examine whether ML estimation can retrieve the true parameters used in the underlying data generating process under the null hypothesis that the model is correctly specified, this approach is not appealing because the 2-sided estimates of the shock innovations are likely to be heteroscedastic, autocorrelated and cross-correlated, which is at odds with the assumptions in the DSGE model.

[16] In the estimations, we impose the lower (b_l) and upper bounds (b_u) that are reported in the last two columns in Table 2. In cases where the solution algorithm failed to solve the model, the log-likelihood function is set to $-200,000$. We use the following smooth mapping function $p_{\text{mod}} = b_u - \frac{b_u - b_l}{1+e^{p_{\text{opt}}}}$ between the model parameters (p_{mod}) and the parameters that we optimize over (p_{opt}). Notice that p_{mod} converges to b_u as p_{opt} approaches ∞, and that p_{mod} converges to b_l as p_{opt} approaches $-\infty$.

[17] In recent work, Bastani and Guerrieri (2008) shows that more reliable convergence is obtained when automatic

5. We store the resulting parameter estimates along with the likelihood information, inverse Hessian, seed number used to generate the sample, and convergence diagnostics.

6. We repeat Step 1 to 5 a sufficiently large number of times to obtain a parameter distribution that is stable. In practice it took between $1,000$ and $1,500$ samples to obtain approximate convergence in mean and variance in the distribution for each estimated parameter. We therefore decided to use $N = 1,500$.

We consider two sample sizes. As a benchmark, we set $T = 100$, which is equivalent to the size of our actual data sample. In order to examine potential small sample problems, we also generate distributions when we set $T = 400$. The results in the tables and figures below are based on the convergent estimations only, but we will provide information about the fraction of simulations that did not converge. We define a convergent estimation to one where the optimizer CSMINWEL terminates without an error message and where the inverse Hessian has full rank and is positive definite. Dropping non-convergent optimizations reflects our belief that the econometrician would not be satisfied with an estimation that led to a non-convergent estimation, and would redo the estimation by perturbing the starting values of the optimization until a satisfactory convergence was found. Here, however, we instead decided to draw a new sample and continue. Given that very few samples are plagued with convergence problems, our approach do not seem to be critical.

One difference with respect to how the model was estimated on actual data with Bayesian techniques, is that we do not include measurement errors in the ML estimation in Step 3 above. Also, we decided to fix the parameters of the exogenous foreign and fiscal policy VARs at their true values throughout the analysis. The reason for this is to simplify the interpretation of the results, and focus on the key model parameters in Table 2. As a robustness check we have, however, also conducted ML estimations when we add measurement errors to the artificial model data in line with how they were calibrated on Swedish data. In this case, we reestimated the VAR(4) and VAR(2) models for the foreign and fiscal variables respectively (where the foreign output gap variable and government expenditure series are computed using the HP-filter) for each artificial sample. This alternative approach of incorporating measurement errors and estimated fiscal and foreign VARs did not change the bias and consistency properties of the ML estimation results, but it somewhat widened the dispersion in the distributions for some parameters. These results are available in Appendix A.2.

4.2. Benchmark results

In Table 4, we report results from the benchmark parameterisation.[18] Results for two sample sizes are reported, $T = 100$ and $T = 400$. As can be seen from the table, almost every parameter's mean and median are equal or close to the true value already for a sample size of $T = 100$. So the ML estimator appears to be an unbiased estimator for almost every parameter in the model. Two important exceptions are the coefficients in the policy rule, r_π and r_y, which both have mean estimates that are much higher than their true values. However, the median for the two parameters is of the right magnitude, indicating that the parameter distributions are skewed to

differentiation methods are used in favor of the finite-difference based derivatives utilized by CSMINWEL, but for ease of comparison with the existing empirical literature, we decided to use a standard optimization routine.

[18] In addition to matching variables in first differences as in equation 18, we have also studied the properties of the ML estimator when imposing the true co-integrating vectors among the set of observed variables. These results are reported in Appendix A.3. The results show that there are rather small efficiency gains to be made in ML estimation by exploiting the true cointegrating vectors relative to matching the quantities in first differences.

the right. Given the specification of the instrument rule, where ρ_R multiplies the coefficients in the policy rule (see eq. 14), it is perhaps not surprising that the distributions for these two parameters can be skewed to the right. In samples when ρ_R is driven close to unity, the values of r_π and r_y can easily end up at very high values without affecting the short-run coefficients much. The fourth column of Table 4, labeled "Std. of distribution", shows the standard deviation of the parameter distributions, i.e. the mean square errors (MSEs), and not surprisingly the standard deviations are very high for these two parameters. The MSEs are also relatively high for the investment adjustment cost parameter, $\tilde{S}''$, and the persistence coefficient for the asymmetric technology shock, $\rho_{\tilde{z}^*}$, suggesting that also these parameters are sometimes driven to very high and low values, respectively. Interestingly, the MSEs for the key parameters pertaining to the nominal rigidities in the model reveal that the marginal distributions are much tighter for the sticky price parameters (ξ_d, ξ_{mc}, ξ_{mi} and ξ_x), relative to the parameter governing nominal wage stickiness, ξ_w, indicating that the data should be much more informative about the degree of price stickiness relative to the estimated degree of nominal wage stickiness.

In addition to the standard deviations of the resulting marginal parameter distributions, the fifth column with figures in Table 4 reports the relative MSEs, i.e. the standard deviations of the distribution divided by the median standard deviation of the estimates in each sample computed from the inverse Hessian matrix.[19] As discussed in further detail in Iskrev (2008, 2010), weak identification induces poor conditioning of the Hessian (analogous with the poor conditioning of the Jacobians reported in Table 3b), and leads to standard deviations based on the inverse Hessian to underestimate the true sampling uncertainty. The relative MSEs are hence very useful indicators of weak identification problems. As is clear from the table, the relative MSEs are often substantially higher than 1, and above 2 for 10 of the 43 parameters, confirming the asymptotic results in Tables 3a and 3b. However, even if the relative MSEs are substantially higher than 1, it is clear from the MSEs that the data is still very informative about the key parameters. For instance, while the relative MSEs are around two for the sticky price parameters (ξ_d, ξ_{mc}, ξ_{mi} and ξ_x), the (absolute) MSEs show that the likelihood is very informative about these parameters even when $T = 100$. Hence, although there are problems with weak identification, our results for $T = 100$ suggest that we can learn a lot from the data about many parameters as the regions in the parameter space where there is a high degree of parameter substitutability is quite limited.

Turning to the results for $T = 400$, we see that the mean and median parameter estimates are getting more similar in general, and for $\tilde{S}''$ and r_π and r_y in particular. Both the mean and median are now also very similar to the true parameter values, with the exception of r_π which still has too high mean relative to the true parameter value (but the median is virtually identical to the true parameter value). In addition, it is clear that the distributions start to collapse around the true values as the standard deviations of the marginal distributions have been reduced by at least a factor of 2, consistent with square-root sample size convergence properties of the ML estimator. As can be seen from the last column, the relative MSEs are reduced for this larger sample size, but there is still a clear tendency that the median standard deviations computed from the inverse Hessians underestimate the true degree of uncertainty in the marginal parameter distributions for some parameters.

In Figures 1a-1c, we complement the information in the table by plotting the kernel density

[19] The inverse Hessian has full rank and is positive definite with the exception of a few simulations (22 cases) in the benchmark estimations for $T = 100$. Notice that since the parameter optimizations are done in the transformed parameter space (see Footnote 16), the standard deviations are computed by assuming normality of the estimated parameters in each optimization and using the inverse Hessian and point estimates in the unbounded space to form a distribution in the bounded parameter space, for which the covariance matrix is computed.

estimates of the marginal parameter distributions.[20] The figure confirms the picture in Table 4 and shows that the distributions for $\tilde{S}''$ and r_π and r_y are clearly skewed to the right. Notice that the marginal distribution in Figure 1c is reported in logs for r_π in order to improve the visibility of the results. Despite the high multiple collinearity coefficients in Tables 3a and 3b, the figures make it clear that this set of data suffices for identification of the parameters in the notion of Rothenberg (1971): as the sample size increases, the parameter distributions start to collapse around the true values. So conditional on this number of observed variables and estimated parameters, the ML estimator is consistent.[21]

To examine if there are any bivariate parameter combinations which account for the weak identification problems, we report all the pairwise parameter combinations with correlation coefficients above 0.5 in Figure 2. The figure gives clear support for the idea that in certain regions of the parameter space there is a large but not perfect degree of substitutability between some of the model parameters. Some parameter combinations imply a certain degree of partial identification. In particular, Figure 2 suggests that this problem pertains to three sets of parameters.

First, consistent with the multiple collinearity coefficients for J_2 and $J(2)$ in Table 3a, we see that many of the parameters in the policy rule are highly correlated with each other. For example, there is a clear positive and non-linear relationship between ρ_R and $\{r_\pi, r_y\}$ and negative correlation between ρ_R and r_x, which is not surprising given that these coefficients enter multiplicative in the Taylor rule (14).

The second set of parameters which exhibit a high degree of pairwise correlation are some of the persistence and standard deviation parameters of the shock processes. This feature pertains to the unobserved AR(1) shock process for the unit root technology shock ($\mu_{z,t}$), the investment specific technology shock (Υ_t), the exchange risk-premium shock ($\tilde{\phi}_t$) and the labour supply shock (ζ_t^h). Quite naturally, there is a negative correlation between these parameters, suggesting that the ML estimator has difficulties in distinguishing whether it is high persistence/low variance of the innovations or low persistence/high variance of the innovations that is most plausible for these latent shock processes.

The third set of parameters which exhibit a high degree of linear dependence is a set of parameters pertaining to the open economy aspects of the model. In particular, some of the markup parameters on imported consumption and investment goods, and the elasticity of substitution between domestically and imported investment goods are highly correlated. Especially the pairwise correlation between λ_{mi} and η_i is very high, suggesting that both of them are very weakly identified and that one of them should not be included in the estimation. However, while this is true locally, it is not the right interpretation in more a global perspective. The high degree of linear dependence between some of the markup and import/export elasticity parameters appears only locally in the parameter space. For instance, the data is strongly informative that λ_{mi} and λ_{mc} should be in the range of $1.55 - 1.65$ and $1.05 - 1.25$, respectively, as is evident from Figures 1a and 2. But within these narrow ranges the ML estimates are highly correlated with η_i, so they are imprecise in small samples and also classified as weakly identified with the asymptotic identification tests.

[20] As indicated by the red cross in Figures 1a-1c, the starting values in all the optimizations are set to the true parameter values. It is imperative to notice, however, that the marginal parameter distributions in Figures 1a-1c and the results in Table 4 are essentially unaffected by the choice of starting values. In Appendix A.1, we examine the robustness of the results when instead sampling starting values from the prior distributions in Table 2, and show that the results are essentially unaffected for $T = 100$ already.

[21] The consistency of the ML estimator is confirmed by results reported in Appendix A.4, where we report results when increasing the number of observations in each simulated sample to $1,600$ and $6,400$, respectively.

To sum up, while the small sample results above confirms the problems with weak identification suggested by the asymptotic identification tests conducted in Section 3, they indicate that the implications of such problems are quite limited in small samples for many of the parameters. The marginal distributions are satisfactory from a frequentistic perspective in the sense that the ML estimator is unbiased for nearly all parameters and consistent for all parameters (with square-root sample size convergence, as the MSEs are reduced by about a factor of 2 when the sample size is increased by a factor of 4). By and large, our results thus paint a somewhat different picture than the one by Canova and Sala (2009), who question the ability to achieve identification in DSGE models and the usefulness of estimating them to learn about the parameters.

4.3. Results for a subset of observable variables

In Section 4.2 above, we used all the 15 variables in eq. (18) as observables when taking the model to the data. To understand how the performance of the ML estimator depends on the choice of observed variables, we now assume that, for some reason, the econometrician only includes 7 variables when estimating the model, but still tries to estimate all 43 parameters in Table 2. More specifically, we assume that the the following subset of variables in (18) is used:

$$\tilde{Y}_t^{subset} = \begin{bmatrix} \pi_t^d & \Delta \ln(W_t/P_t) & \Delta \ln C_t & \Delta \ln I_t & R_t & \hat{H}_t & \Delta \ln Y_t \end{bmatrix}' . \tag{20}$$

The variables in (20) are the "closed economy" variables used by Smets and Wouters (2003). Thus, we anticipate that the marginal distributions for the parameters particularly pertaining to the open-economy aspects of the model will be much more dispersed. This is also confirmed by the asymptotic test results in Table 3b, which show that the conditioning numbers are worse for the subset of observables, and that $J(2)$ is rank deficient in 0.46 percent of the prior draws whereas $J(2)$ is always full rank for the full set of observables in (18). However, some of the multiple collinearity coefficients in Table 3a are in fact reduced when using the subset of observables in contrast to the full set of variables. In particular this is noticeable for the degree of habit persistence in consumption preferences (b) and the steady-state technology growth (μ_z), so the results for some of the parameters in θ are not clear cut according to the asymptotic tests.

For the sample size $T = 100$, we plot the resulting marginal distributions in Figures 3a-3c based on equation (20) along with the distributions that are obtained when all 15 variables are used as observables (i.e. the benchmark results for $T = 100$ reported in Table 3 and Figures 1a-1c). The results are based on samples where the estimations converged in both cases.

As can be seen from Figures 3a-3c, restricting the set of observable variables from (18), solid line, to the ones in (20), dashed line, is associated with substantially more dispersion in the parameter distributions. As expected, this is particularly the case for parameters related to the open economy aspects of the model. For instance, the uncertainty about $\xi_{m,c}$, $\xi_{m,i}$ and ξ_x as measured by the standard deviation in the parameter distributions is now substantially higher. It is also the case that the number of convergent estimations fall from $1,452$ to $1,376$, and in around 50 times the inverse Hessian has reduced rank, suggesting that (some of) the parameters in the DSGE model estimated on the subset of variables in (20) suffer from a stronger degree of weak identification in larger parts of the parameter space, consistent with the findings in Table 3b.[22] Moreover, and at odds with the asymptotic results in Table 3a, Figures 3a-3c reveal

[22] However, in Appendix A.4, we examine if there is information in the likelihood function to identify the parameters asymptotically for the limited set of observables in (20) by reporting results for $T = 1,600$ and $T = 6,400$ observations in each sample. The results in Appendix A.4 demonstrate that the ML estimator is consistent even if only the subset of variables in (20) are used, although the ML estimates are converging to the

that the marginal distributions for the other parameters (e.g. the habit persistence parameter, b, and the steady state growth rate parameter, μ_z) are more dispersed as well. Thus, the decision to narrow down the set of observable variables implies that the marginal distributions for parameters not directly linked to the dropped variables tend to be more dispersed as well.

This exercise thus demonstrates that the econometrician needs to be very careful when selecting the number of variables in estimating the model. If classical estimation techniques are applied, it is imperative to think hard about the structure of the model and which variables that need to be included in order to ensure identification of a given set of parameters in small samples. All else equal, the results in Figures 3a-3c and Tables 3a and 3b show that a larger set of observables will tend to facilitate identification and lower the MSEs (under the presumption that problems with model misspecification are not too severe). Hence, we conclude that a larger set of variables than those in (20) are useful when estimating the model, consistent with the conclusions in Boivin and Giannoni (2006) and Guerron-Quintana (2010).

5. Weak identification of nominal wage stickiness and the labour supply shock

In Section 4, we documented that while the ML estimator is unbiased for nearly all parameters, quite a few of the parameters were plagued by weak identification. One key parameter that is more weakly identified and associated with a larger MSE than the other nominal stickiness parameters is ξ_w, the parameter governing nominal wage stickiness. In this section, we will examine the reasons behind the larger parameter uncertainty of this parameter. The first possible explanation we will consider is the highly volatile labour supply shocks.[23] As can be seen in Table 2, the labour supply shock process is not very persistent ($\rho_{\zeta_h} = 0.27$) but the innovations has a high estimated standard deviation of about 0.40. Even if nominal wages are estimated to be quite sticky (around 4 quarters) and prices are sticky, the labour market setup in the model will imply that the large high-frequency movements in the labour supply shocks will tend to shift the labour supply curve substantially over time.

The large high-frequency movements in the real wage will also imply that the serial correlation in real wages and the cross-correlation between real wages and hours worked per capita are not very high. This is visualized in Figure 4. In panels a, b and c, we plot the real wage ($\widehat{\bar{w}}_t$) as deviation around steady state (in percent) against the percentage deviation of hours worked per capita ($\hat{H}_t$) around steady state for different degrees of nominal wage stickiness and parameterisation of the labour supply shock process. Panels a, b, c are based on a random sample of 200 observations from the model, and the colorbar to the right indicates the period in the sample, i.e. $t = 1, 2, ..., 200$. In the lower right panel, we also plot the real wage against hours worked per capita on actual data $1985Q1 - 2004Q4$.[24] Notice that the actual data panel thus only contains 100 observations. As can be seen from the upper left panel, the estimated

true parameters at a slower rate compared to the case when the larger set of observables in (18) are used in the estimations. Thus, the likelihood function is informative about all the parameters in the model even when only the closed economy variables are matched, but only weakly so in small samples.

[23] While it is certainly possible that identification of ξ_w can depend on other parameter figurations as well, this particular deviation from the benchmark parameterisation is of special interest as the estimated persistence of the labor supply shock is low relative to evidence for the US (Smets and Wouters, 2007) and Euro area (Adolfson et al., 2007).

[24] To compute a measure of the scaled real wage in the actual data panel, we apply the Hodrick-Prescott filter on the actual real wage (in logs) where the smoothing coefficient is set to $10,000$. We use a high smoothing coefficient in order to get a smooth trend with about the same variance as the trend real wage (i.e. the variance of the unit-root technology shock) in the model. The results in the panel are thus very similar if the two-sided smoothed Kalman estimates of the real wage (computed at the posterior median) is used, but we preferred to make use of a less model dependent measure of the gap.

benchmark parameterisation of the model does not imply a strong negative correlation between the real wage and hours worked, and little persistence in real wages. A priori, we expect a negative correlation between the real wage and hours worked per capita in the model due to the fact that supply shocks are the predominant source of business cycle fluctuations in the model. As is standard in estimated sticky price and wage models, our model implies that stationary but persistent technology shocks raise real wages but induce hours worked to fall. Stationary technology shocks are the most important source of output fluctuations according to our model, and thus contribute significantly to an unconditional negative correlation between real wages and hours. In addition, a positive labour supply shock (i.e. a negative $\hat{\zeta}_{h,t}$ shock), will induce hours to rise, but since the marginal productivity of labour falls when hours rise, real wages fall. Panel a also reveals that hours tend to change quite a bit relative to the variations in the real wage. A change in the location in the panel is also less persistent, in the sense that the distance between a coordinate $\left\{\hat{H}_t, \widehat{\bar{w}}_t\right\}$ and $\left\{\hat{H}_{t+j}, \widehat{\bar{w}}_{t+j}\right\}$ for some $j > 1, 2, ..., J$ tends to be rather large. In other words, the Euclidean norm is on average rather high between the coordinates in panel a. In contrast, assuming the labour supply shocks to be more persistent and less volatile would imply much more persistence in the real wage and hours worked, and is associated with a sharp fall in the Euclidean norm between the pairs $\left\{\hat{H}_t, \widehat{\bar{w}}_t\right\}$ and $\left\{\hat{H}_{t+1}, \widehat{\bar{w}}_{t+1}\right\}$. In addition, panel b reveals that this alternative parameterisation of the labour supply shock process induces a strong negative correlation between fluctuations in the real wage and hours worked. According to the bottom right panel, this negative correlation and low Euclidean norm is not a distinct feature of the data that the model is set to match, so this alternative parameterisation is clearly not supported by the data. On the other hand, panel c reveals that completely flexible wages are not supported by the data either, as flexible wages induce too high volatility in the real wage. Thus, judging from the panels in Figure 4, it is not surprising that the estimation procedure resulted in a relatively high degree of nominal wage stickiness and less persistent but volatile labour supply shocks.

We now explore the role more persistent labour supply shocks play for the weak identification problems pertaining to the nominal wage stickiness parameter ξ_w, even if Swedish data are not supportive of such a setup. To do this, we change the parameterisation of the labour supply shocks in the DGP in order to make the innovations less volatile but more persistent. In the alternative specification for the labour supply shock process, we adopt the parameters used in Figure 4 and thus raise ρ_{ξ_h} from 0.27 (see Table 2) to 0.95 and lower the standard deviation σ_{ζ_h} of the innovations from 0.386 (see Table 2) to 0.125. This combination of parameters ensures that the unconditional variance of the labour supply shock (wage markup shock) process, ζ_t^h , remains unchanged.

Table 5 shows the mean, median and standard deviation of the parameter distributions (MSE), as well as the relative MSE (i.e. MSE divided by the standard deviation according to the (median) inverse Hessian) for the alternative DGP with persistent labour supply shocks. To put the results into perspective, we also report the parameter distributions for the benchmark parameterisation in Section 4.2. The results are based on $1,339$ samples for $T = 100$ where the estimations converged in both cases. From the table we see that more persistent labour supply shocks strongly shrink the MSE for ξ_w by a factor of 1.5. Moreover, it is also evident from Table 5 that the alternative labour supply shock process also strongly improves the precision of the estimation of many of the other parameters, as the resulting MSEs are generally lower than for the benchmark DGP (cf. standard deviations). In particular, this is true for the investment adjustment cost parameter, $\tilde{S}''$, that were found to have a large MSE in the benchmark parameterisation. Second, it is clear that the alternative specification of the labour supply process has little consequences for identifying the shock processes per se. It is the deep parameters that govern the propagation of the labour supply shocks which benefits the most from the less erratic labour supply shocks. This feature is also obvious for the policy parameters, as they are important for shaping the propagation of the shocks in the model. However, an interesting feature of Table 5 is that the *relative* MSEs are roughly unchanged or even higher for many of the parameters. This feature of the small sample results suggests that the problems with weak identification are not mitigated at all in this alternative parameterisation, a finding that is confirmed by applying the asymptotic tests in Tables 3a and 3b which shows that the multiple collinearity coefficients and the condition numbers for $J(2)$ are equally high. But the much more narrow MSEs suggest that the identification problems are much more locally contained. In our view, these results testifies the importance of assessing the quantitative implications of the weak identification problems in small samples.

To sum up, we have documented that the erratic real wage growth series have led to an estimated labour supply process characterized by low persistence and volatile innovations. Given the other parameters, the erratic labour supply process cause the degree of nominal wage stickiness to be imprecisely estimated, and increase the dispersion in the marginal distributions for some other key parameters. The economic intuition behind this result is that the erratic labour supply shocks only temporary drive the wage setting households alongside the labour demand curve, and therefore generates a real wage series with little autocorrelation over time. With more persistent labour supply shocks, households are more persistently driven alongside the labour demand curve as it takes longer time for the real wage to adjust to their frictionless supply curve in our model where both nominal wages and prices are sticky. This enhances the precision in the estimation of the sticky wage parameter as it causes the real wage to be more negatively correlated with hours worked per capita, and more serially correlated over time. As the substantial high-frequency movements in the real wage series seem to be a particular characteristic of the Swedish labour market and is possibly related to the construction and seasonal adjustment of the real wage series, there are less reasons to believe that this particular feature of the DGP we study here carries over to other estimated DSGE models. For instance, Adolfson et al. (2007) and Smets and Wouters (2007) report much more persistent labour supply shock processes in their estimations on data for the Euro area and the US, respectively. This suggest that the likelihood function may be more informative about the degree of nominal wage stickiness when estimating New Keynesian DSGE models on Euro area and US data

6. ML estimation on actual data

From the small sample Monte Carlo exercise, we conclude that the likelihood function should be quite informative about many of the key parameters in the model under the null hypothesis that the model is correctly specified. In this section, we therefore estimate the model using classical ML techniques on actual data. The setting in the estimation is identical to the setting that was employed in the Bayesian estimation procedure that resulted in the posterior median estimates reported in Table 2, with the exception that the policy parameters r_π, r_y and r_x are estimated as short-run coefficients in an attempt to reduce the large uncertainty bands stemming from trying to directly estimate the long-coefficients in eq. (14). Our motivation for adopting this slight change in the estimation procedure is driven by the simulation results in Table 4 and Figures 1c and 2, which documented that the long-term coefficients were highly correlated with the interest smoothing parameter ρ_R.[25]

[25] Thus, we estimate $\tilde{r}_\pi = (1-\rho_R)\, r_\pi$, $\tilde{r}_y = (1-\rho_R)\, r_y$, and $\tilde{r}_x = (1-\rho_R)\, r_x$ directly instead of r_π, r_y and r_x. The ML point estimates are invariant with respect to the approach taken here, but this led to more plausible

To find the classical ML point estimates, we impose the lower and upper bounds reported in Table 2 and perform $3,000$ optimizations with CSMINWEL by sampling starting values from the prior distribution. The ML estimates are the vector of parameters $\hat{\theta}$ out of all vectors with optimized parameters $\theta_i,\ i = 1, 2, ..., 3,000$, that returned the highest log-likelihood. To assess the uncertainty about the point estimates, i.e. how much we can learn from the log-likelihood function about the parameters, we report two pieces of information. First, we use the standard deviations based on the inverse Hessian to compute 5 and 95 confidence bands associated with the ML estimates. Second, because the simulation results in Table 4 documented that the standard deviations based on the inverse Hessian are likely to underestimate the true degree of uncertainty associated with the ML estimates due to problems with weak identification, we also report 90 % simulated confidence bands. These bands were computed as follows. First, the ML point estimates and the associated inverse Hessian matrix were used to generate draws from the joint parameter distribution using the Metropolis-Hastings algorithm. The proposal distribution is taken to be the multivariate normal density centered at the previous draw with a covariance matrix proportional to the inverse Hessian. Second, all draws that could not be differentiated from the highest log-likelihood according to a standard likelihood ratio (LR-) test at the 10 percent significance level were accepted in the chain. A chain with $1,000,000$ accepted draws was simulated, and from this chain the lower and upper confidence bands were computed as the minimum and maximum values for each parameter in the chain.[26] The robustness of the simulated confidence bands were checked by simulating and computing the confidence bands for an additional chain of $1,000,000$ draws. Finally, it is important to notice that none of the draws in either chain resulted in a higher log-likelihood than the one associated with $\hat{\theta}$, which is a good robustness check that $\hat{\theta}$ indeed is the ML estimate.

In Table 6, we report the classical ML estimation results along with the Bayesian posterior median and 5 and 95 percent posterior uncertainty bands. Compared to the prior distribution and the Bayesian posterior median in Table 2, we see that the classical ML estimate moves in the same direction from the prior as the posterior median, but typically a bit more. Also, and in line with the results on artificial samples, the data appear to be highly informative about the sticky price parameters ξ_d, $\xi_{m,c}$, $\xi_{m,i}$ and ξ_x which are estimated to be very high. The estimated degree of price stickiness appears to be implausibly high according to the ML estimate in relation to the microeconomic evidence, the median estimate of the four $\xi's$ equals 0.979 which implies an unrealistically high average duration between price reoptimizations of about 47 quarters under the assumption of economy-wide capital markets. It is important to point out that the finding of very high degree of price stickiness with classical methods is not specific to the model at hand. Smets and Wouters (2003) report a very high degree of price stickiness in their model with i.i.d. markup shocks, and to reduce the degree of price stickiness in more recent work (Smets and Wouters, 2007), they assume that the markup shocks in the pricing equations follow an ARMA(1,1) process (where the AR term is estimated to be very high) and use the Kimball (1995) aggregator whereby the price elasticity of demand is increasing in the relative price

confidence intervals for the parameters. Notice that the Bayesian posterior median results presented in Table 6 have been transformed to short-run parameters for these parameters, although the priors used in the Bayesian estimation are still for the long-run coefficients. The standard deviations for the composite Bayesian posterior short-term parameters have been appropriately adjusted by sampling $100,000$ parameter combinations using the inverse Hessian matrix under a joint normality assumption and computing the standard deviations from this simulated distribution.

[26] As we estimate 43 parameters, $2\left[\ln L(\hat{\theta}) - \ln L(\theta_i)\right]$ follows the χ^2-distribution with 43 degrees of freedom and a particular parameter draw θ_i is rejected in favor of the best fitting parameter configuration $\hat{\theta}$ associated with $\ln L_{\max}$ at the 10 percent level if the χ^2-statistic exceeds 55.23. We plot the parameters in the simulated chains in Appendix B.

difference between the firm and its competitors, which implies that the firms will respond less to movements in marginal costs for a given level of price stickiness. In the setup here, we obtain a larger degree of price stickiness because we assume that the markup shocks are i.i.d. and that the price elasticity of demand is constant. To reduce the implausible degree of price stickiness obtained here, we could use the Kimball aggregator and/or allow for correlated markup shocks. An additional assumption that would reduce the implied degree of price stickiness would be to follow Altig et al. (2004) and assume that capital is specific to the firm, instead of being rented by the intermediate firms in economy-wide capital markets each period.[27] The analysis in Altig et al. (2004), Adolfson et al. (2005) and Smets and Wouters (2007) suggest that these three modifications together would substantially reduce the estimated degree of price stickiness implied by the model. However, an unappealing feature of the introduction of the correlated markup shocks in Smets and Wouters (2007) is that positive markup shocks account for a substantial part of the great inflation of the 1970s (see their Figure 4 on page 600), implying that firm profits should have risen substantially in the 1970s. But aggregate firm profits did not rise in the 1970s, and we therefore argue that there is still a tension between accounting for inflation persistence and obtaining a plausible degree of price stickiness not only on Swedish data but also on US data.[28] Additionally, Chari, Kehoe and McGrattan (2008) have argued that the implied variance of the correlated markup shocks is implausibly high. Among the other parameters, we notice that the ML estimate of the habit persistence parameter and investment adjustment costs are notably higher, and that the markup shock parameters in the import sector are implausibly high. The ML point estimate for the parameters governing the risk premium in the UIP condition is also estimated to be substantially higher. Although the shock process parameters and policy rule parameters are arguably less affected, the overall impression from Table 4 is hence that the ML point estimate have changed substantially relative to the Bayesian posterior median.

However, before drawing to firm conclusions about the point estimates, we need to consider the possibility that the large changes in some of the parameters (e.g. the price stickiness parameters) merely reflects large small-sample uncertainty due to weak identification problems associated with the ML estimator in small sample properties. As can be seen from Table 6, the 90 percent confidence interval based on the inverse Hessian suggest that many parameters are very tightly estimated, with the exception of the investment adjustment cost $\tilde{S}''$ which has a high standard deviation of about 4. However, by comparison to the simulated 90-percent confidence bands (last two columns in the table), we see that the standard deviations based on the inverse Hessian severely underestimate the true degree of sampling uncertainty about the ML point estimate. Despite the fact that the simulated confidence bands are much larger than the ones based on the inverse Hessian, it is clear from the last two columns in Table 6 that the log-likelihood function is very informative about the sticky price parameters in the model. For all these parameters except ξ_x, the Bayesian posterior median lies outside the simulated 90 percent ML confidence band. Therefore, we conclude that the higher ML estimate relative to the Bayesian estimate of the sticky price parameters cannot be explained by small sample properties of the ML estimator. It is also clear that habit formation and investment adjustment costs are empirically important frictions; the lower bound for these coefficients are well above nil

[27] Another potential reason for why the estimated slope of the Phillips curve may not necessarily imply an implausible degree of price stickiness is aggregation problems where the persistence of aggregated price series becomes higher than the persistence in the underlying disaggregated price series, see Mumtaz et al. (2009). In our analysis below, we proceed under the maintained assumption that aggregation problems in the construction of prices indices are of second-order importance.

[28] See e.g. the real earnings data on Robert Shiller's website http://www.econ.yale.edu/~shiller/data.htm or Shiller (2000).

in both cases. The uncertainty band for the degree of nominal wage stickiness is substantially higher than the corresponding ones for the price stickiness parameters, but the lower bound is well above nil suggesting that the model needs a fair amount of sticky nominal wages in order to maximize the empirical coherence of the model, consistent with the results in Figure 4.

These findings raise the issue of why the classical ML estimate differ so much relative the Bayesian posterior estimate. The obvious candidate explanation why the ML estimate differs so much relative to the Bayesian estimate is model misspecification. There are two pieces of evidence in favor of this explanation. First, the maximum log-likelihood in the classical estimations equals $-2022,2$. From a classical viewpoint, this number is significantly higher than the log-likelihood $(-2128,6)$ associated with the Bayesian posterior median parameters in Table 2. According to the classical LR-test, the posterior median - here viewed as an arbitrary parameter vector - is thus statistically rejected in favor of the ML estimate at the one percent level. Second, the misspecification interpretation of the contrasting Bayesian and classical estimation results is also in line with the evidence reported in Adolfson et al. (2008), who show that when estimating a DSGE-VAR(λ) model following Del Negro et al. (2007), they obtain an estimate of the hyper-parameter λ that is lower than infinity (7), implying that the best fitting VAR wants to relax the cross–equation restrictions implied by the estimated open economy DSGE model. Del Negro et al. (2007) also obtain a λ less than infinity (0.5) in their closed economy model on US data, suggesting that the standard Smets and Wouters (2003) type of closed economy DSGE models is plagued by misspecification problems as well.

We therefore interpret the evidence reported in this section to suggest that while classical ML methods can be used to estimate DSGE models, the application of ML methods on actual data may lead to implausible estimation results due to problems associated with model misspecification. An important implication of this finding is thus that the advantage for using Bayesian methods is not primarily related to problems with weak identification. If the confidence bands for the point estimates are appropriately computed, classical ML techniques will provide the econometrician with a correct answer to which extent the data is informative about the estimated parameters. Rather, the motivation for using Bayesian methods is that it allows the researcher to explore if a theoretical model can match the data well for parameter regions that are supported by microeconomic evidence and prior empirical evidence.

7. Concluding remarks

In this paper we have analyzed the properties of maximum likelihood estimation in a state-of-the-art open economy new Keynesian DSGE model. Our analysis suggests that our open economy DSGE model is identifiable in the notion of Rothenberg (1971): if an appropriate set of variables are used to estimate the DSGE model, the ML distributions collapse at the true parameter values as the sample size is increased. However, our asymptotic and small sample results show that many of the parameters are plagued by weak identification problems, and in this sense the results in this paper also lend some support to the findings in Canova and Sala (2009) and Iskrev (2008, 2010). But, a key lesson from our study is that even if problems with weak identification are present, the implications for the MSE of the ML estimator is often quite limited, and the econometrician can hence learn a lot about many parameters in the DSGE by estimating them. This finding stands in sharp contrast to the conclusions in Canova and Sala (2009), who argue that problems with weak identification are wide-spread in large portions of the parameter space. With the exception of the coefficients in the policy rule, we find that only a few key parameters are weakly identified in large regions of the parameter space. In our benchmark parameterisation of the model, one such problematic parameter is the degree of nominal wage stickiness. As this is a key parameter in the new generation of DSGE models, we explored the reason for the weak identification and resulting high MSE pertaining to this parameter in greater detail, and found that it could be explained by the erratic labour supply shock. When the persistence of the labour supply shock is increased to be in line with Adolfson et al. (2007) and Smets and Wouters (2007), we find that the MSE of the sticky wage parameter is reduced by a factor of two and the MSEs for the other deep parameters are substantially reduced as well. An important lesson is that the small sample properties of ML is not invariant w.r.t the calibration of the model.

Taking the lessons in the Monte Carlo analysis into account, we estimated the model with classical ML techniques. As the Monte Carlo analysis revealed that the inverse Hessian is likely to underestimate the uncertainty associated with the ML estimates due to problems with weak identification, we used a simulation-based approach to compute confidence bands for the ML estimates with the Metropolis-Hastings algorithm where we accepted all draws with an associated likelihood ratio statistic lower than the critical value for a given significance level. Although the simulated confidence bands for the ML estimate are substantially larger than the ones implied by the inverse Hessian, the movements in the ML estimate relative to the Bayesian posterior median are substantial (and statistically significant from a classical viewpoint) in some cases. Relative to the Bayesian posterior median, the ML estimate is also associated with a strong and significant increase in the log likelihood of above 100 units. By itself, this improvement is evidence against the notion that aggregate data is not informative about many parameters in DSGE models as argued by Canova and Sala (2009) and Iskrev (2008). However, given the setup of the model, some of the ML estimates are in contrast with the microeconomic evidence. In particular, the ML estimate of the slope of the Phillips curve (i.e., the coefficient multiplying marginal cost) implies an extremely high degree of price stickiness through the lenses of the standard Calvo model relative to the microeconomic evidence on price stickiness. For a given slope of the Phillips curve there are, however, three reasons for why the degree of price stickiness can in fact be interpreted as being lower than suggested by the standard Calvo model: *i*) firm-specific capital, *ii*) non-constant price elasticity of demand, and *iii*) aggregation problems of disaggregated price series. Under the standard assumption that aggregation problems are not too severe, it can be shown that our estimated slope of the Phillips curve implies a counterfactually high degree of price stickiness even under firm-specific capital and non-constant price elasticities of demand.[29] In addition, the low slope of the Phillips curve implies that the estimated volatility of the markup shocks is implausibly large, as pointed out by Chari, Kehoe and McGrattan (2008). Our interpretation of these results is that the DSGE model under consideration suffers from misspecification, and that the misspecification problem is mitigated with a parameterisation of the model that is quite implausible in light of the microeconomic evidence. This interpretation of the results is also supported by the findings of Adolfson et al. (2008), who report that the model considered here suffer from misspecification by applying the DSGE-VAR(λ) methodology developed by Del Negro and Schorfheide (2004). Del Negro et al. (2007) also find evidence of misspecification in a closed economy model on US data.

One standard argument (see e.g. Canova and Sala, 2009) why Bayesian methods have be-

[29] For instance, the survey evidence reported by Apel, Friberg and Hallsten (2005) suggest that firm prices in Sweden are reoptimized once per year, whereas the estimated DSGE model suggests that prices are reoptimized one every tenth year. Although the introduction of firm-specific capital and the Kimball aggregator changes the mapping from the slope of the Phillips curve to the implied duration of price contracts, our estimated slope of 0.0005 (i.e., the average slope in the Phillips curves according to the ML estimate) cannot be mapped into the degree of price rigidity reported by Apel et al. (2005) even under the assumption of a non-constant price elasticity of demand and capital being specific to each firm instead of rented from an economy wide market for capital each period.

come so popular recently is that they add curvature to the uninformative log-likelihood function and thus enables successful estimation of DSGE models. Our findings above offer an alternative interpretation why Bayesian methods may have become so popular among macroeconomists: although the likelihood function is very informative about many of the parameters in the model, problems with model misspecification lead to implausible classical ML estimates relative to existing microeconomic evidence. In this environment with model uncertainty and misspecification, Bayesian techniques offer a very natural way to estimate models by allowing the econometricians to examine the performance of the models in a region of the parameter space that can be deemed a priori plausible. The models should then be treated as probability models following the arguments in Sims (2008).

Finally, and importantly, Rubio-Ramirez and Villaverde (2005) compare maximum likelihood estimations of a real business cycle model and argue that estimations based on a non-linear (i.e. second-order) approximation are much more informative about the underlying parameters as opposed to estimations when the underlying DSGE model is log-linearized. Therefore, an interesting extension of the work here would be to examine to what extent the performance of maximum likelihood estimation would be enhanced by working with a second-order as opposed to a log-linearized representation of the model.

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Table 1: Calibrated parameters

Parameter	Description	Calibrated value
β	Households' discount factor	0.999
α	Capital share in production[a]	0.25
η_c	Substitution elasticity between C_t^d and C_t^m	5
μ	Money growth rate (quarterly rate)[a]	1.010445
σ_L	labour supply elasticity	1
δ	Depreciation rate	0.01
λ_w	Wage markup[a]	1.05
ω_i	Share of imported investment goods[a]	0.70
ω_c	Share of imported consumption goods[a]	0.40
τ^y	labour income tax rate[a]	0.30
τ^c	Consumption tax rate[a]	0.24
τ^w	Pay-roll tax rate[a]	0.30
$\rho_{\bar{\pi}}$	Inflation target persistence	0.975
g_r	Government expenditures-output ratio[a]	0.30

[a] Notice that the description of the parameter pertain to it's steady state.

Table 2: Prior and posterior distributions.

Parameter		Prior distribution type	Prior distribution mean	Prior distribution std. dev. / df	Posterior distribution median	Bounds lower	Bounds Upper
Calvo wages	ξ_w	beta	0.750	0.050	0.765	0.001	0.999
Calvo domestic prices	ξ_d	beta	0.750	0.050	0.825	0.001	0.999
Calvo import cons. prices	$\xi_{m,c}$	beta	0.750	0.050	0.900	0.001	0.999
Calvo import inv. prices	$\xi_{m,i}$	beta	0.750	0.050	0.939	0.001	0.999
Calvo export prices	ξ_x	beta	0.750	0.050	0.874	0.001	0.999
Indexation prices	κ_p	beta	0.500	0.150	0.227	0.001	0.999
Indexation wages	κ_w	beta	0.500	0.150	0.323	0.001	0.999
Investment adj. cost	$\tilde{S}''$	normal	7.694	1.500	8.584	0.1	100
Habit formation	b	beta	0.650	0.100	0.679	0.01	0.99
Markup domestic	λ_d	truncnormal	1.200	0.050	1.195	1.001	10
Subst. elasticity invest.	η_i	invgamma	1.500	4	2.715	0.01	20
Subst. elasticity foreign	η_f	invgamma	1.500	4	1.531	0.01	20
Markup imported cons.	$\lambda_{m,c}$	truncnormal	1.200	0.050	1.584	1.001	10
Markup.imported invest.	$\lambda_{m,i}$	truncnormal	1.200	0.050	1.134	1.001	10
Technology growth	μ_z	truncnormal	1.006	0.0005	1.005	1.0001	1.01
Risk premium	$\tilde{\phi}$	invgamma	0.010	2	0.050	0.0001	10
UIP modification	$\tilde{\phi}_s$	beta	0.500	0.15	0.606	0.0001	1
Unit root tech. shock persistence	ρ_{μ_z}	beta	0.850	0.100	0.845	0.0001	0.9999
Stationary tech. shock persistence	ρ_{ε}	beta	0.850	0.100	0.925	0.0001	0.9999
Invest. spec. tech shock persistence	ρ_{Υ}	beta	0.850	0.100	0.694	0.0001	0.9999
Risk premium shock persistence	$\rho_{\tilde{\phi}}$	beta	0.850	0.100	0.684	0.0001	0.9999
Consumption pref. shock persistence	ρ_{ζ_c}	beta	0.850	0.100	0.657	0.0001	0.9999
Labour supply shock persistence	ρ_{ζ_h}	beta	0.850	0.100	0.270	0.0001	0.9999
Asymmetric tech. shock persistence	$\rho_{\tilde{z}^*}$	beta	0.850	0.100	0.964	0.0001	0.9999
Unit root tech. shock std. dev.	σ_z	invgamma	0.200	2	0.133	0.001	10
Stationary tech. shock std. dev.	σ_{ε}	invgamma	0.700	2	0.668	0.001	10
Imp. cons. markup shock std. dev.	$\sigma_{\lambda_{m,c}}$	invgamma	1.000	2	1.126	0.001	400
Imp. invest. markup shock std. dev.	$\sigma_{\lambda_{m,i}}$	invgamma	1.000	2	1.134	0.001	400
Domestic markup shock std. dev.	σ_{λ}	invgamma	1.000	2	0.807	0.001	100
Invest. spec. tech. shock std. dev.	σ_{Υ}	invgamma	0.200	2	0.396	0.001	100
Risk premium shock std. dev.	$\sigma_{\tilde{\phi}}$	invgamma	0.050	2	0.793	0.001	10
Consumption pref. shock std. dev.	σ_{ζ_c}	invgamma	0.200	2	0.263	0.001	5
Labour supply shock std. dev.	σ_{ζ_h}	invgamma	1.000	2	0.386	0.001	15
Asymmetric tech. shock std. dev.	$\sigma_{\tilde{z}^*}$	invgamma	0.400	2	0.188	0.001	2
Export markup shock std. dev.	σ_{λ_x}	invgamma	1.000	2	1.033	0.001	20
Monetary policy shock	$\sigma_{R,}$	invgamma	0.150	2	0.239	0.001	2
Inflation target shock	$\sigma_{\bar{\pi}^c}$	invgamma	0.050	2	0.157	0.001	1.5
Interest rate smoothing	ρ_R	beta	0.800	0.050	0.913	0.001	0.999
Inflation response	r_{π}	truncnormal	1.700	0.100	1.674	1.01	1000
Diff. infl response	$r_{\Delta\pi}$	normal	0.300	0.050	0.098	-0.5	5
Real exch. rate response	r_x	normal	0.000	0.050	-0.016	-5	5
Output response	r_y	normal	0.125	0.050	0.125	-0.5	5
Diff. output response	$r_{\Delta y}$	normal	0.063	0.050	0.178	-0.5	5

*Note: For the inverse gamma distribution the mode and the degrees of freedom are reported. Also, for the parameters $\lambda_d, \eta_i, \eta_f, \lambda_{m,c}, \lambda_{m,i}$ and μ_z the prior distributions are truncated at 1.

Table 3a: Multiple collinearity statistics.

Parameter		Full set of observables posterior median J_2	Full set of observables posterior median $J(T=2)$	Subset of observables posterior median J_2	Subset of observables posterior median $J(T=2)$
Calvo wages	ξ_w	0.986974	0.996896	0.986974	0.999191
Calvo domestic prices	ξ_d	0.846852	0.999620	0.846852	0.999912
Calvo import cons. prices	$\xi_{m,c}$	0.388599	0.999199	0.388599	0.999556
Calvo import inv. prices	$\xi_{m,i}$	0.805859	0.984436	0.805859	0.999767
Calvo export prices	ξ_x	0.665556	0.999734	0.665556	0.999863
Indexation prices	κ	0.890175	0.910726	0.890175	0.989664
Indexation wages	κ_w	0.968640	0.993752	0.968640	0.969346
Investment adj. cost	$\tilde{S}''$	0.777935	0.996781	0.777935	0.999647
Habit formation	b	0.544705	0.994847	0.544705	0.952666
Markup domestic	λ_d	0.614093	0.995530	0.614093	0.999284
Subst. elasticity invest.	η_i	0.914608	0.961628	0.914608	0.997470
Subst. elasticity foreign	η_f	0.731127	0.999773	0.731127	0.999898
Markup imported cons.	$\lambda_{m,c}$	0.612245	0.999495	0.612245	0.995443
Markup.imported invest.	$\lambda_{m,i}$	0.928346	0.994278	0.928346	0.987823
Technology growth	μ_z	0.723811	0.988386	0.723811	0.804138
Risk premium	$\tilde{\phi}$	0.941905	0.995071	0.941905	0.999815
UIP modification	$\tilde{\phi}_s$	0.932155	0.998273	0.932155	0.999777
Unit root tech. persistance	ρ_{μ_z}	0.352182	0.999217	0.352182	0.999351
Stationary tech. persistance	ρ_{ε}	0.530834	0.998080	0.530834	0.999464
Invest. spec. tech. persist.	ρ_{Υ}	0.363288	0.998570	0.363288	0.999297
Risk premium persistence	$\rho_{\tilde{\phi}}$	0.912502	0.999987	0.912502	0.999999
Consumption pref. persist.	ρ_{ζ_c}	0.100695	0.999240	0.100695	0.995582
Labour supply persistance	ρ_{ζ_h}	0.987302	0.999540	0.987302	0.999242
Asymmetric tech. persist.	$\rho_{\tilde{z}*}$	0.019953	0.998235	0.019953	0.999606
Unit root tech. shock	σ_{μ_z}	0.771729	0.998721	0.771729	0.999029
Stationary tech. shock	σ_{ε}	0.379276	0.994601	0.379276	0.996899
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	0.020807	0.996419	0.020807	0.999228
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	0.011689	0.993625	0.011689	0.999759
Domestic markup shock	σ_{λ_d}	0.212869	0.999618	0.212869	0.999900
Invest. spec. tech. shock	σ_{Υ}	0.329197	0.998488	0.329197	0.999337
Risk premium shock	$\sigma_{\tilde{\phi}}$	0.916604	0.999988	0.916604	0.999999
Consumption pref. shock	σ_{ζ_c}	0.200817	0.998114	0.200817	0.992222
Labour supply shock	σ_{ζ_h}	0.009630	0.999331	0.009630	0.998309
Asymmetric tech. shock	$\sigma_{\tilde{z}*}$	0.059400	0.972290	0.059400	0.999762
Export markup shock	σ_{λ_x}	0.037493	0.966417	0.037493	0.999710
Monetary policy shock	σ_R	0.536318	0.998304	0.536318	0.996592
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.793176	0.997999	0.793176	0.999707
Interest rate smoothing	ρ_R	0.988693	0.999460	0.988693	0.999849
Inflation response	r_{π}	0.994991	0.999739	0.994991	0.999817
Diff. infl response	$r_{\Delta\pi}$	0.745399	0.979217	0.745399	0.982905
Real exch. rate response	r_x	0.952986	0.999793	0.952986	0.999904
Output response	r_y	0.984123	0.999952	0.984123	0.999843
Diff. output response	$r_{\Delta y}$	0.924639	0.999870	0.924639	0.999365

Note: The table shows Iskrev's (2008) multiple collinearity statistics w.r.t. the reduced form coefficients in the model only, J_2, and w.r.t. the first and second moments in the data, $J(T=2)$. An value close to one (or -1) indicates that there is very strong degree of collinearity w.r.t. the other deep parameters. Dynare 4.3 has been used to do the calculations.

Table 3b: Rank tests and median condition numbers, smallest singular and tolerance values.

	Full set of observables posterior median		Full set of observables prior sampling		Subset of observables posterior median		Subset of observables prior sampling	
	J_2	$J(T=2)$	J_2	$J(T=2)$	J_2	$J(T=2)$	J_2	$J(T=2)$
Full rank	Yes	Yes	100%	99.94%	Yes	Yes	100%	99.54%
Condition number	5.9634e+004	6.5077e+006	3.4379e+006	6.4944e+008	5.9634e+004	1.0558e+008	3.4379e+006	3.4870e+010
Min singular value	0.9398	0.0039	0.0025	2.1897e-014	0.9398	1.2714e-004	0.0025	3.7298e-012
Tolerance value	2.6674e-008	1.2769e-009	1.7472e-006	1.3076e-006	2.6674e-008	1.4916e-010	1.7472e-006	1.5274e-007

Note: For the columns labelled "prior sampling", the rank of J_2 and $J(T=2)$ have been evaluated at 5,000 draws from the prior distribution in Table 2. For the condition, smallest singular, and tolerance values we report the median from these 5,000 evaluations. Dynare 4.3 has been used to do the calculations.

Table 4: Distribution results from different sample sizes.

Parameter		True values	100 observations: Mean of distribution	100 observations: Median of distribution	100 observations: Std. of distribution	100 observations: Relative MSE	400 observations: Mean of distribution	400 observations: Median of distribution	400 observations: Std. of distribution	400 observations: Relative MSE
Calvo wages	ξ_w	0.77	0.74	0.75	0.13	1.97	0.76	0.76	0.05	1.65
Calvo domestic prices	ξ_d	0.83	0.81	0.82	0.04	1.24	0.82	0.82	0.02	1.10
Calvo import cons. prices	$\xi_{m,c}$	0.90	0.90	0.90	0.02	1.19	0.90	0.90	0.01	1.11
Calvo import inv. prices	$\xi_{m,i}$	0.94	0.94	0.94	0.02	1.28	0.94	0.94	0.01	1.16
Calvo export prices	ξ_x	0.87	0.86	0.86	0.04	1.85	0.87	0.87	0.01	1.31
Indexation prices	κ	0.23	0.22	0.22	0.06	1.35	0.22	0.22	0.03	1.19
Indexation wages	κ_w	0.32	0.32	0.32	0.15	2.05	0.32	0.32	0.07	1.75
Investment adj. cost	$\tilde{S}''$	8.58	8.98	8.08	4.08	2.02	8.65	8.53	1.35	1.38
Habit formation	b	0.68	0.67	0.67	0.07	1.37	0.68	0.68	0.03	1.24
Markup domestic	λ_d	1.20	1.21	1.20	0.14	1.59	1.20	1.19	0.06	1.62
Subst. elasticity invest.	η_i	2.72	2.72	2.71	0.13	1.20	2.71	2.71	0.06	1.04
Subst. elasticity foreign	η_f	1.53	1.59	1.45	0.59	2.61	1.54	1.53	0.14	1.47
Markup imported cons.	$\lambda_{m,c}$	1.58	1.58	1.58	0.01	1.19	1.58	1.58	0.00	1.03
Markup.imported invest.	$\lambda_{m,i}$	1.13	1.14	1.13	0.02	1.19	1.13	1.13	0.01	1.04
Technology growth	μ_z	1.005	1.005	1.005	0.0003	1.05	1.005	1.005	0.0001	1.01
Risk premium	$\tilde{\phi}$	0.05	0.06	0.05	0.02	1.96	0.05	0.05	0.01	1.35
UIP modification	$\tilde{\phi}_s$	0.61	0.61	0.60	0.05	1.78	0.61	0.61	0.02	1.38
Unit root tech. persistance	ρ_{μ_z}	0.85	0.80	0.83	0.14	2.50	0.84	0.85	0.05	1.79
Stationary tech. persistance	ρ_{ε}	0.93	0.89	0.90	0.08	2.25	0.92	0.92	0.02	1.89
Invest. spec. tech. persist.	ρ_{Υ}	0.69	0.65	0.67	0.13	1.99	0.69	0.69	0.05	1.63
Risk premium persistence	$\rho_{\tilde{\phi}}$	0.68	0.65	0.65	0.11	1.73	0.68	0.68	0.04	1.51
Consumption pref. persist.	ρ_{ζ_c}	0.66	0.59	0.61	0.18	2.19	0.64	0.65	0.07	1.82
Labour supply persistance	ρ_{ζ_h}	0.27	0.26	0.26	0.13	1.88	0.27	0.27	0.06	1.55
Asymmetric tech. persist.	$\rho_{\tilde{z}^*}$	0.96	0.73	0.84	0.28	3.15	0.93	0.95	0.11	5.01
Unit root tech. shock	σ_{μ_z}	0.13	0.14	0.14	0.05	1.45	0.13	0.13	0.02	1.43
Stationary tech. shock	σ_{ε}	0.67	0.66	0.65	0.06	1.05	0.67	0.67	0.03	1.01
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	1.13	1.13	1.12	0.11	1.12	1.13	1.13	0.05	1.06
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	1.13	1.14	1.13	0.11	1.08	1.14	1.14	0.05	1.07
Domestic markup shock	σ_{λ_d}	0.81	0.82	0.82	0.08	1.06	0.81	0.81	0.04	1.06
Invest. spec. tech. shock	σ_{Υ}	0.40	0.42	0.41	0.09	1.51	0.40	0.40	0.03	1.39
Risk premium shock	$\sigma_{\tilde{\phi}}$	0.79	0.82	0.80	0.21	1.77	0.80	0.80	0.08	1.44
Consumption pref. shock	σ_{ζ_c}	0.26	0.27	0.27	0.05	1.29	0.27	0.26	0.02	1.14
Labour supply shock	σ_{ζ_h}	0.39	0.39	0.39	0.06	1.48	0.38	0.38	0.03	1.34
Asymmetric tech. shock	$\sigma_{\tilde{z}^*}$	0.19	0.15	0.16	0.06	1.39	0.18	0.19	0.02	1.22
Export markup shock	σ_{λ_x}	1.03	1.13	1.09	0.41	1.95	1.04	1.03	0.11	1.38
Monetary policy shock	σ_R	0.24	0.24	0.23	0.02	1.10	0.24	0.24	0.01	1.05
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.16	0.14	0.14	0.10	2.32	0.16	0.16	0.03	1.54
Interest rate smoothing	ρ_R	0.91	0.91	0.91	0.05	1.74	0.91	0.91	0.02	1.31
Inflation response	r_π	1.67	3.80	1.59	5.08	1.88	2.07	1.66	1.60	2.62
Diff. infl response	$r_{\Delta\pi}$	0.10	0.11	0.10	0.04	1.27	0.10	0.10	0.02	1.16
Real exch. rate response	r_x	-0.02	-0.07	-0.02	0.15	8.28	-0.03	-0.02	0.04	4.07
Output response	r_y	0.13	0.35	0.13	0.63	8.43	0.17	0.13	0.17	4.17
Diff. output response	$r_{\Delta y}$	0.18	0.19	0.18	0.05	1.35	0.18	0.18	0.02	1.20

Note: Out of the 1,500 estimations for the small sample (100 obs.), the results above is based on 1,452 convergent estimations with well behaved inverse Hessians. Out of the 1,500 estimations for the large sample (400 obs.), the results above is based on 1,497 convergent estimations with well behaved inverse Hessians. True parameter values were used as starting values in the estimations. Relative MSE is calculated as; std of distribution (MSE) divided by std error based on (median) inverse Hessian.

Table 5: Distribution results from benchmark model and model with persistent labor supply shocks.

Parameter		True values	Benchmark parameterization: Mean of distribution	Median of distribution	Std. of distribution	Relative MSE	Persistent labor supply shocks: Mean of distribution	Median of distribution	Std. of distribution	Relative MSE
Calvo wages	ξ_w	0.77	0.73	0.74	0.15	2.03	0.79	0.77	0.09	2.08
Calvo domestic prices	ξ_d	0.83	0.81	0.81	0.04	1.24	0.82	0.82	0.02	1.34
Calvo import cons. prices	$\xi_{m,c}$	0.90	0.90	0.90	0.02	1.17	0.90	0.90	0.01	1.55
Calvo import inv. prices	$\xi_{m,i}$	0.94	0.94	0.94	0.02	1.22	0.94	0.94	0.01	1.57
Calvo export prices	ξ_x	0.87	0.86	0.86	0.04	1.68	0.87	0.87	0.02	1.52
Indexation prices	κ	0.23	0.22	0.22	0.06	1.22	0.22	0.22	0.05	1.38
Indexation wages	κ_w	0.32	0.32	0.32	0.15	1.90	0.32	0.32	0.06	1.27
Investment adj. cost	$\tilde{S}''$	8.58	9.10	8.14	4.38	2.05	8.60	8.46	1.88	2.31
Habit formation	b	0.68	0.67	0.67	0.07	1.31	0.68	0.68	0.03	1.43
Markup domestic	λ_d	1.20	1.21	1.20	0.14	1.53	1.20	1.20	0.03	1.33
Subst. elasticity invest.	η_i	2.72	2.72	2.71	0.13	1.21	2.71	2.71	0.05	1.37
Subst. elasticity foreign	η_f	1.53	1.58	1.45	0.58	2.27	1.57	1.50	0.38	1.97
Markup imported cons.	$\lambda_{m,c}$	1.58	1.58	1.58	0.01	1.23	1.58	1.58	0.01	2.61
Markup.imported invest.	$\lambda_{m,i}$	1.13	1.14	1.13	0.02	1.26	1.13	1.13	0.01	1.22
Technology growth	μ_z	1.005	1.005	1.005	0.0003	1.06	1.005	1.005	0.0003	1.05
Risk premium	$\tilde{\phi}$	0.05	0.06	0.05	0.02	1.80	0.05	0.05	0.01	3.85
UIP modification	$\tilde{\phi}_s$	0.61	0.61	0.60	0.06	1.70	0.60	0.60	0.03	2.41
Unit root tech. persistance	ρ_{μ_z}	0.85	0.81	0.84	0.14	2.49	0.81	0.83	0.13	2.20
Stationary tech. persistance	ρ_{ε}	0.93	0.89	0.90	0.08	2.34	0.87	0.88	0.08	2.13
Invest. spec. tech. persist.	ρ_{Υ}	0.69	0.65	0.67	0.13	1.80	0.64	0.65	0.13	1.61
Risk premium persistence	$\rho_{\tilde{\phi}}$	0.68	0.65	0.66	0.11	1.45	0.66	0.65	0.14	1.99
Consumption pref. persist.	ρ_{ζ_c}	0.66	0.60	0.62	0.19	2.02	0.60	0.61	0.17	1.78
Labour supply persistance	ρ_{ζ_h}	0.27/**0.95**	0.27	0.26	0.15	1.79	**0.94**	**0.95**	0.03	3.03
Asymmetric tech. persist.	$\rho_{\tilde{z}^*}$	0.96	0.72	0.82	0.27	4.18	0.73	0.83	0.27	4.12
Unit root tech. shock	σ_{μ_z}	0.13	0.14	0.14	0.05	1.58	0.14	0.14	0.05	1.51
Stationary tech. shock	σ_{ε}	0.67	0.66	0.65	0.14	2.58	0.66	0.65	0.08	1.41
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	1.13	1.12	1.12	0.11	1.10	1.12	1.11	0.12	1.20
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	1.13	1.14	1.14	0.11	1.07	1.13	1.12	0.12	1.25
Domestic markup shock	σ_{λ_d}	0.81	0.82	0.82	0.11	1.39	0.81	0.80	0.09	1.20
Invest. spec. tech. shock	σ_{Υ}	0.40	0.42	0.41	0.09	1.42	0.43	0.42	0.09	1.43
Risk premium shock	$\sigma_{\tilde{\phi}}$	0.79	0.82	0.80	0.21	1.51	0.79	0.79	0.29	2.98
Consumption pref. shock	σ_{ζ_c}	0.26	0.27	0.27	0.07	1.85	0.27	0.27	0.06	1.67
Labour supply shock	σ_{ζ_h}	0.39/**0.13**	0.38	0.38	0.07	1.46	**0.13**	**0.13**	0.06	3.77
Asymmetric tech. shock	$\sigma_{\tilde{z}^*}$	0.19	0.15	0.16	0.06	1.41	0.16	0.16	0.08	1.67
Export markup shock	σ_{λ_x}	1.03	1.13	1.09	0.41	1.80	1.05	1.03	0.26	1.51
Monetary policy shock	σ_R	0.24	0.24	0.24	0.02	1.17	0.24	0.23	0.05	2.50
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.16	0.14	0.14	0.11	2.33	0.10	0.05	0.12	1.96
Interest rate smoothing	ρ_R	0.91	0.91	0.91	0.05	1.65	0.91	0.91	0.04	2.50
Inflation response	r_π	1.67	3.82	1.56	5.22	1.41	2.84	1.55	3.30	4.68
Diff. infl response	$r_{\Delta\pi}$	0.10	0.11	0.10	0.04	1.27	0.10	0.10	0.03	2.07
Real exch. rate response	r_x	-0.02	-0.07	-0.02	0.16	8.53	-0.05	-0.01	0.10	8.96
Output response	r_y	0.13	0.36	0.12	0.67	9.16	0.31	0.11	0.51	11.96
Diff. output response	$r_{\Delta y}$	0.18	0.19	0.18	0.05	1.38	0.18	0.18	0.04	1.75

Note: The results above are based on 1,339 convergent estimations with well behaved inverse Hessians on sample sizes with T=100. Starting values in the estimations were sampled from the prior distribution in Table 2. Relative MSE is calculated as; std of distribution/std error based on (median) inverse Hessian.

Table 6: Maximum Likelihood estimation results on actual data.

Parameter		Bayesian Posterior Distribution			Maximum Likelihood Estimation				
		Median	5%	95%	Point estimate	Hessian based bound 5%	Hessian based bound 95%	Simulated lower bound 5%	Simulated upper bound 95%
Calvo wages	ξ_w	0.765	0.677	0.839	0.830	0.748	0.912	0.576	0.998
Calvo domestic prices	ξ_d	0.825	0.737	0.903	0.949	0.928	0.970	0.886	0.999
Calvo import cons. prices	$\xi_{m,c}$	0.900	0.870	0.926	0.989	0.987	0.991	0.915	0.999
Calvo import inv. prices	$\xi_{m,i}$	0.939	0.922	0.955	0.990	0.990	0.990	0.945	0.999
Calvo export prices	ξ_x	0.874	0.838	0.905	0.987	0.979	0.995	0.852	0.999
Indexation prices	κ_p	0.227	0.135	0.335	0.013	0.010	0.016	0.001	0.184
Indexation wages	κ_w	0.323	0.165	0.515	0.020	0.013	0.027	0.001	0.364
Investment adj. cost	$\tilde{S}''$	8.584	6.510	10.803	22.500	15.297	29.703	4.348	46.295
Habit formation	b	0.679	0.572	0.771	0.871	0.843	0.899	0.602	0.974
Markup domestic	λ_d	1.195	1.117	1.277	1.112	1.099	1.125	1.001	2.846
Subst. elasticity invest.	η_i	2.715	2.280	3.330	1.335	1.093	1.577	1.000	2.567
Subst. elasticity foreign	η_f	1.531	1.349	1.856	2.766	2.246	3.286	1.000	5.152
Markup imported cons.	$\lambda_{m,c}$	1.584	1.529	1.638	2.371	2.152	2.590	1.850	5.387
Markup.imported invest.	$\lambda_{m,i}$	1.134	1.067	1.207	2.315	1.568	3.062	1.167	4.959
Technology growth	μ_z	1.005	1.005	1.005	1.005	1.005	1.005	1.004	1.006
Risk premium	$\tilde{\phi}$	0.050	0.022	0.116	0.228	0.039	0.417	0.002	2.054
UIP modification	$\tilde{\phi}_s$	0.606	0.516	0.728	0.982	0.956	1.008	0.595	1.000
Unit root tech. shock persistence	ρ_{μ_z}	0.845	0.704	0.928	0.906	0.862	0.950	0.432	1.000
Stationary tech. shock persistence	ρ_{ε}	0.925	0.822	0.972	0.994	0.987	1.001	0.699	1.000
Invest. spec. tech shock	ρ_{Υ}	0.694	0.519	0.839	0.319	0.176	0.462	0.000	0.854
Risk premium shock persistence	$\rho_{\tilde{\phi}}$	0.684	0.503	0.855	0.416	0.270	0.562	0.000	0.927
Consumption pref.shock	ρ_{ζ_c}	0.657	0.419	0.848	0.017	-0.143	0.177	0.000	0.649
Labour supply shock persistence	ρ_{ζ_h}	0.270	0.167	0.385	0.025	0.004	0.046	0.000	0.645
Asymmetric tech. shock	$\rho_{\tilde{z}*}$	0.964	0.947	0.978	0.933	0.903	0.963	0.000	1.000
Unit root tech. shock std. dev.	σ_z	0.133	0.098	0.184	0.064	0.039	0.089	0.020	0.227
Stationary tech. shock std. dev.	σ_{ε}	0.668	0.542	0.817	0.664	0.524	0.804	0.453	1.009
Imp. cons. markup shock std. dev.	$\sigma_{\lambda_{m,c}}$	1.126	0.956	1.349	1.285	1.102	1.468	0.968	1.694
Imp. invest. markup shock std.	$\sigma_{\lambda_{m,i}}$	1.134	0.955	1.364	1.726	1.430	2.022	1.128	2.346
Domestic markup shock std. dev.	σ_{λ}	0.807	0.684	0.960	0.823	0.706	0.940	0.575	1.066
Invest. spec. tech. shock std. dev.	σ_{Υ}	0.396	0.294	0.535	0.558	0.443	0.673	0.247	0.968
Risk premium shock std. dev.	$\sigma_{\tilde{\phi}}$	0.793	0.500	1.226	2.021	1.723	2.319	0.657	2.739
Consumption pref. shock std. dev.	σ_{ζ_c}	0.263	0.196	0.348	0.300	0.244	0.356	0.157	0.464
Labour supply shock std. dev.	σ_{ζ_h}	0.386	0.326	0.458	0.344	0.286	0.402	0.171	0.519
Asymmetric tech. shock std. dev.	$\sigma_{\tilde{z}*}$	0.188	0.150	0.240	0.013	0.011	0.015	0.000	0.169
Export markup shock std. dev.	σ_{λ_x}	1.033	0.817	1.282	0.554	0.422	0.686	0.244	2.065
Monetary policy shock	σ_R	0.239	0.207	0.279	0.219	0.189	0.249	0.147	0.293
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.157	0.089	0.247	0.263	0.184	0.342	0.001	0.507
Interest rate smoothing	ρ_R	0.913	0.882	0.938	0.957	0.939	0.975	0.858	0.999
Inflation response	$r_{\pi}(1-\rho_R)$	0.146	0.104	0.197	0.045	0.027	0.063	0.000	0.172
Diff. infl response	$r_{\Delta\pi}$	0.098	0.050	0.152	0.011	-0.022	0.044	-0.079	0.098
Real exch. rate response	$r_x(1-\rho_R)$	-0.001	-0.005	0.002	0.002	0.001	0.003	-0.003	0.009
Output response	$r_y(1-\rho_R)$	0.011	0.005	0.018	-0.001	-0.002	0.000	-0.011	0.039
Diff. output response	$r_{\Delta y}$	0.178	0.118	0.241	0.060	0.012	0.108	-0.116	0.435
Log Likelihood			-2128.6			-2022.2			
Log Marginal Likelihood			-2270.1			---			
Log Posterior			-2173.0			-4351.1			

*Note: The reported parameters for the level of inflation, real exchange and the output gap have been transformed to short-run responses instead of long-run responses as in Tables 2-3. The prior distributions used to obtain the Bayesian posterior median are provided in Table 2. The log likelihood for the Bayesian posterior distribution is computed using the posterior median parameters. The sample period in the estimation is 1980Q1-2004Q4, where the period 1980Q1-1985Q4 is used to compute the unobserved state variables in 1985Q4 and the period 1986Q1-2004Q4 for inference. The ML estimation confidence interval is calculated as: point estimate ± 1.645*std, where std is the standard deviation according to the Hessian.

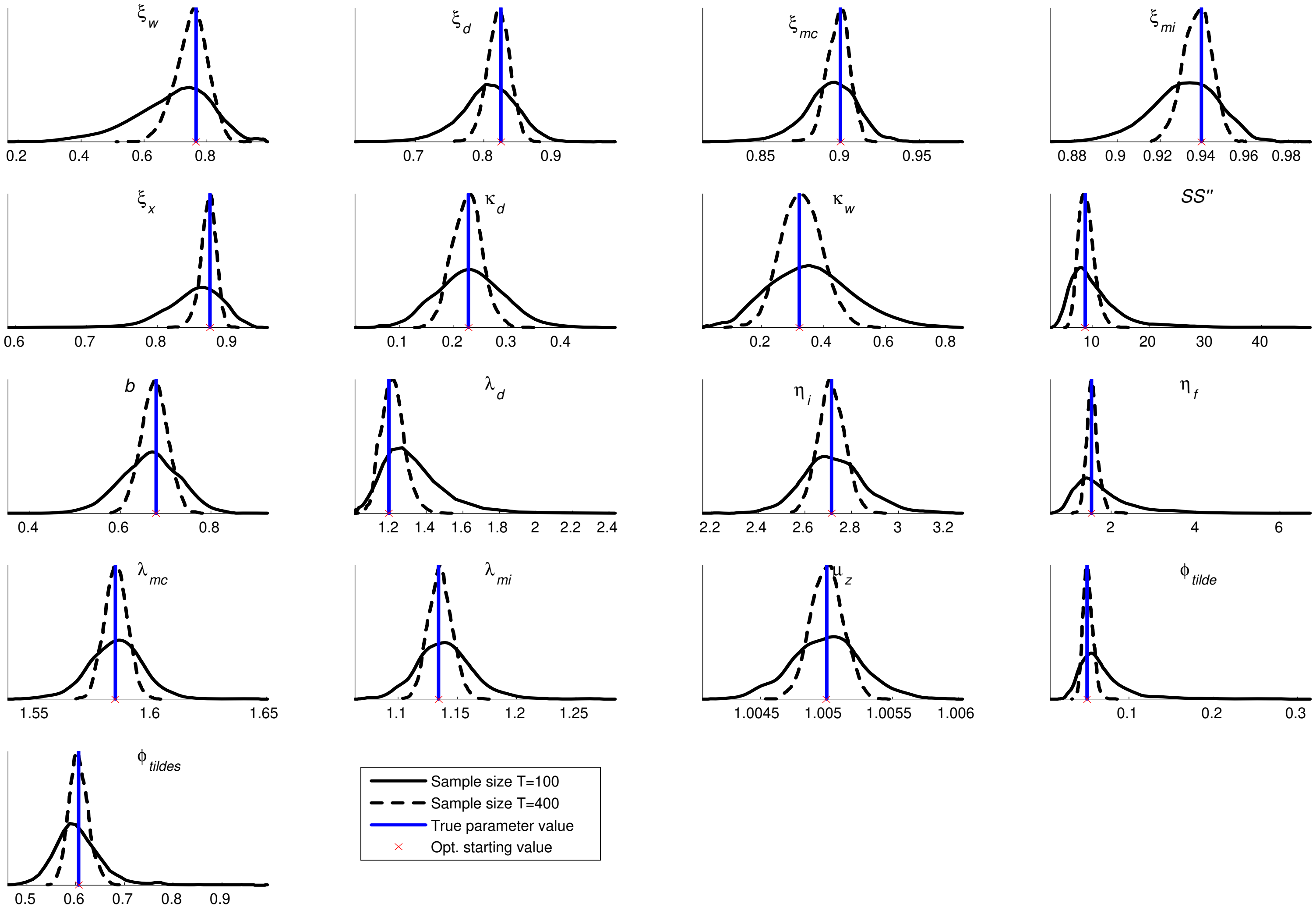

Figure 1a: Kernel density estimates of the small sample distribution for the estimates of the deep model parameters. The solid line show the parameter distribution for $T = 100$, and the dashed line shows the distribution for $T = 400$ observations. The vertical bar shows true parameter value and the cross on the x-axis indicates the starting value in the optimizations.

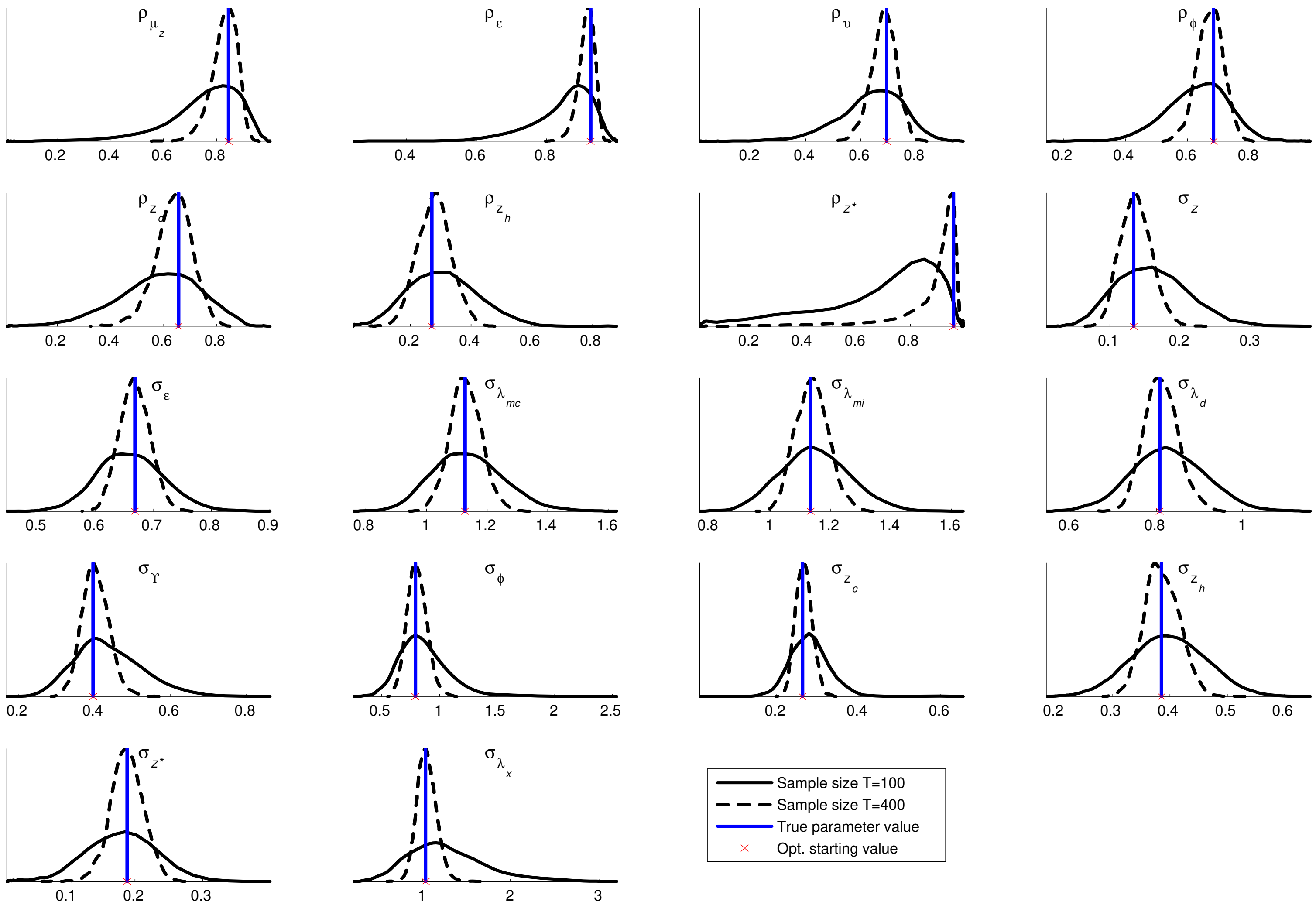

Figure 1b: Kernel density estimates of the small sample distribution for the estimates of the shock parameters. The solid line shows the parameter distribution for $T = 100$, and the dashed line shows the distribution for $T = 400$ observations. The vertical bar shows true parameter value, and the cross on the x-axis indicates the starting value in the optimizations.

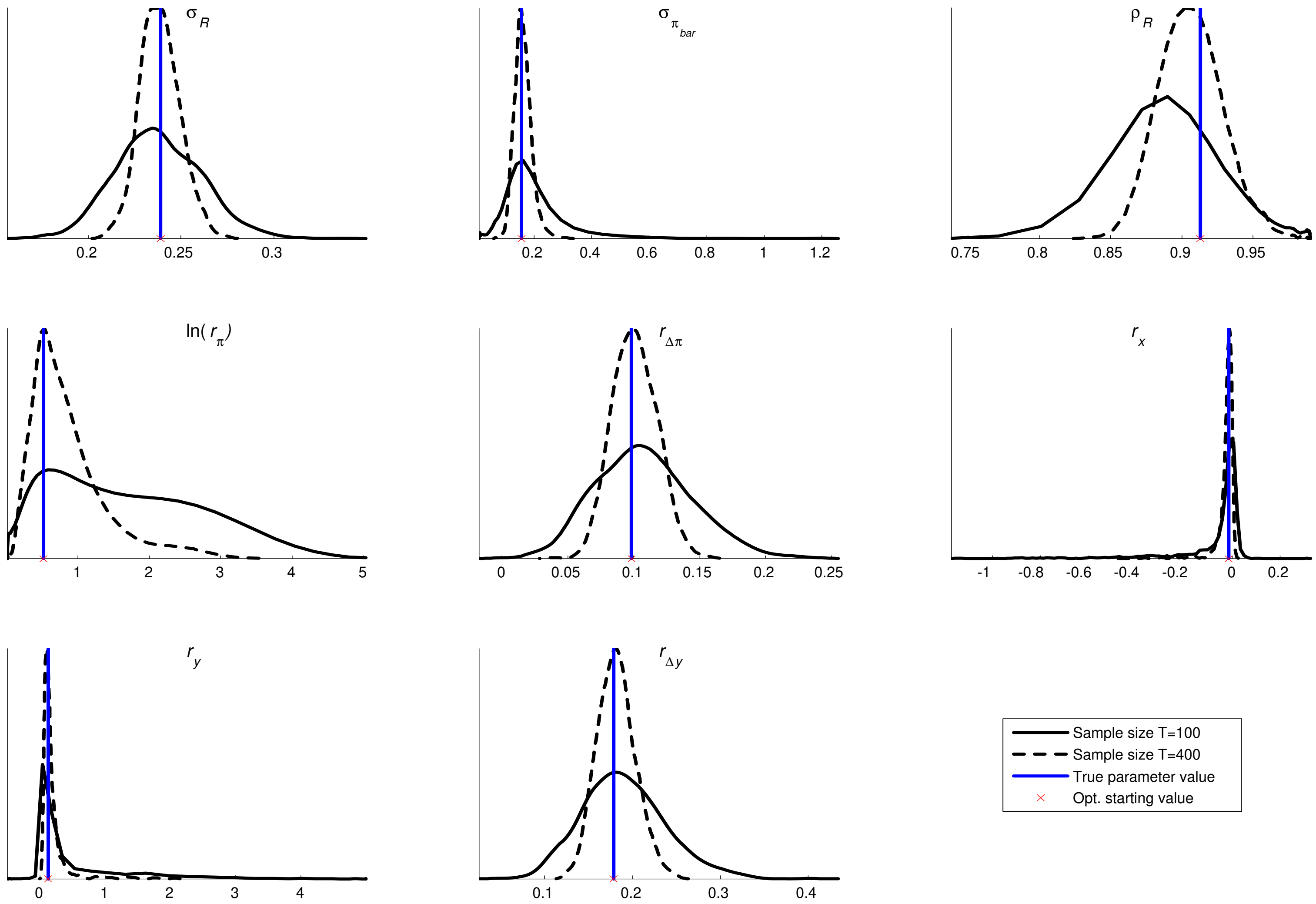

Figure 1c: Kernel density estimates of the small sample distribution for the estimates of the monetary policy parameters. The solid line shows the parameter distribution for $T = 100$, and the dashed line show the distributions for $T = 400$ observations. The vertical bar shows true parameter value, and the cross on the x-axis indicates the starting value in the optimizations.

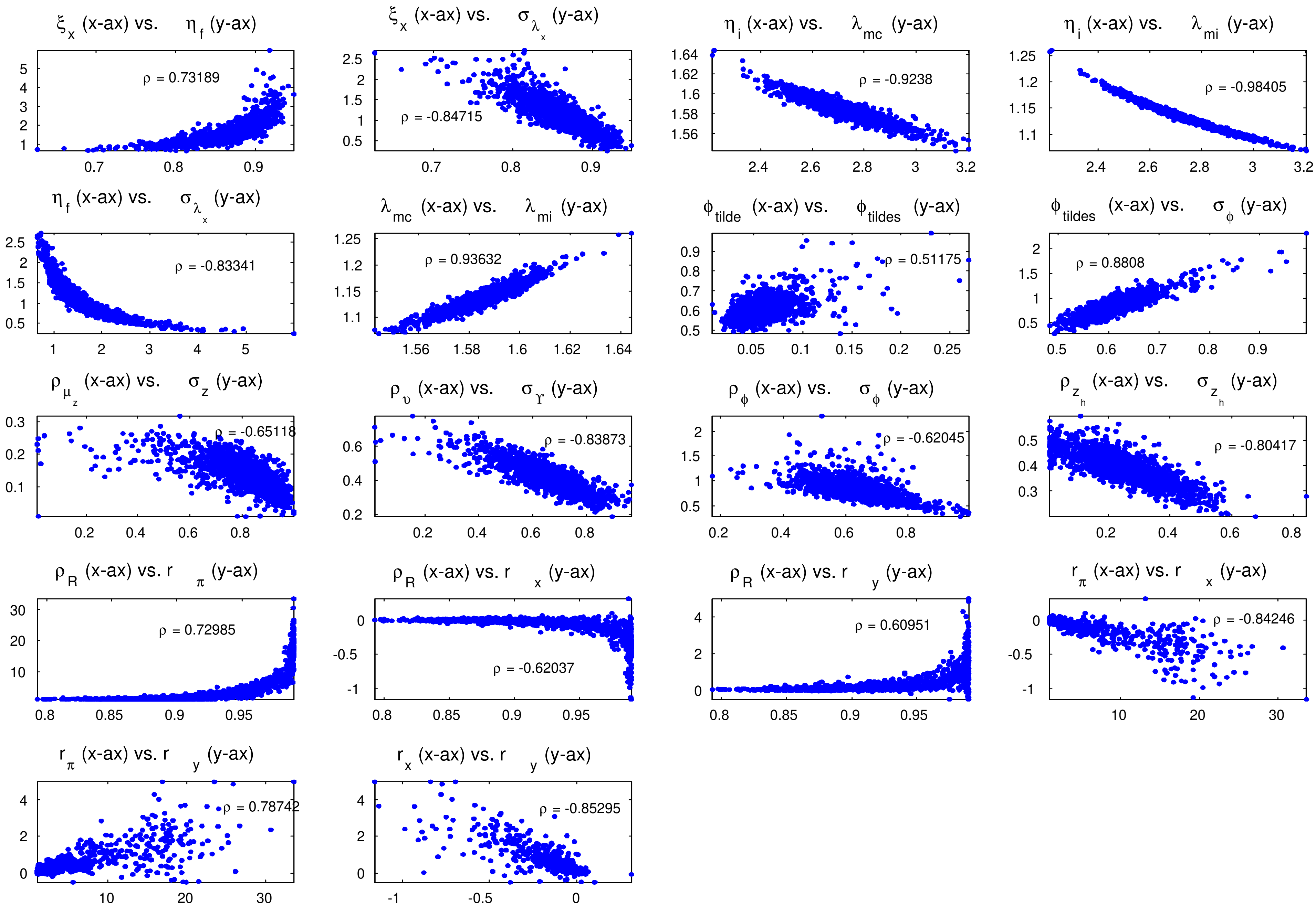

Figure 2: Pairwise estimates for parameters with cross-correlations above 0.5. $T = 100$ observations in each sample, initializing the optimizations with the true parameter values.

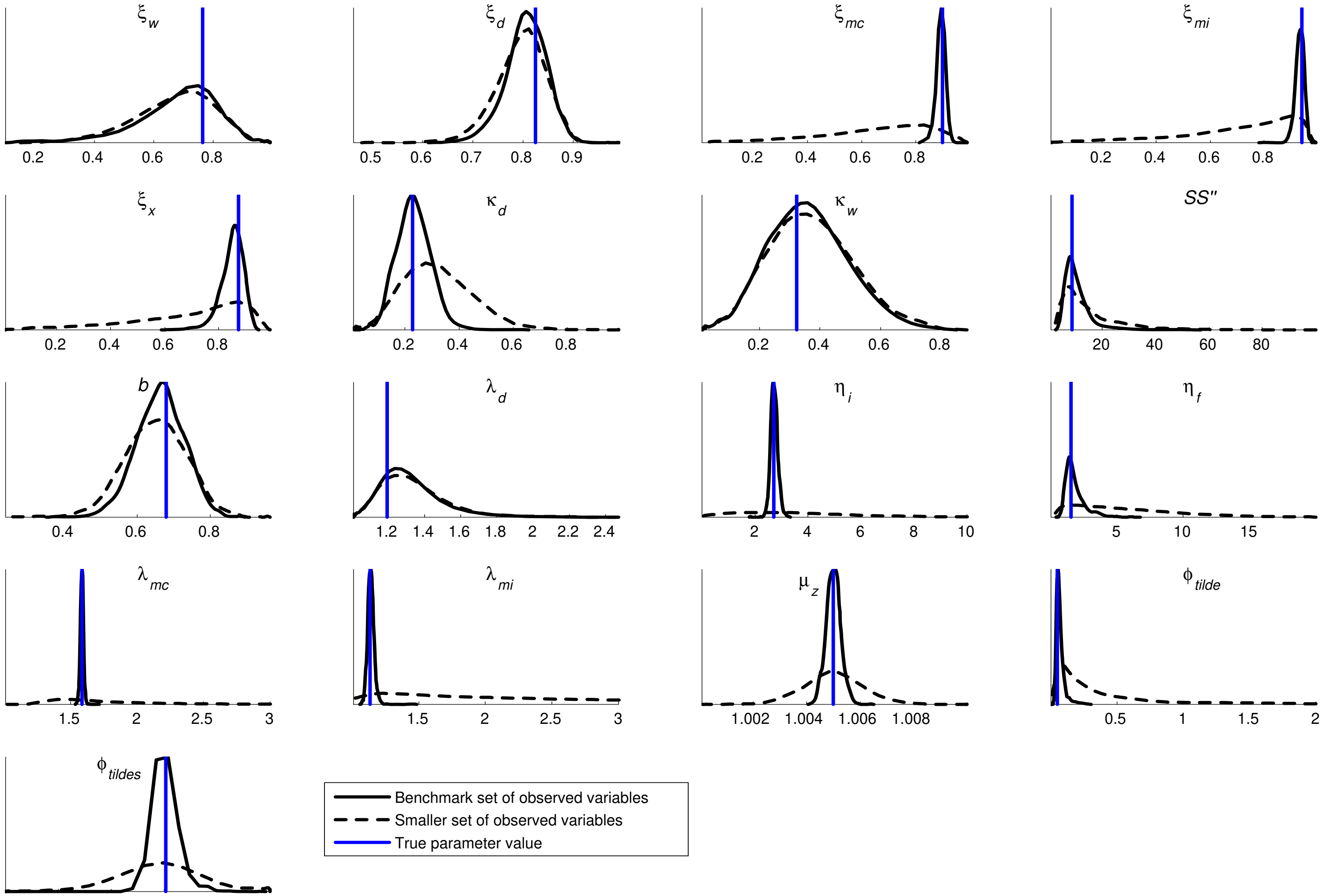

Figure 3a: Kernel density estimates of the small sample distribution for the estimates of the deep model parameters. The solid line shows the parameter distributions when the estimations are based on the full set of observable variables, and the dashed line when the estimations are based on fitting only a subset of variables (i.e., 7 "closed economy" variables). The true parameters are given by the vertical bars. $T = 100$ observations in each of the N artificial samples, and we initialize the estimations by sampling parameters from the prior distributions in Table 2.

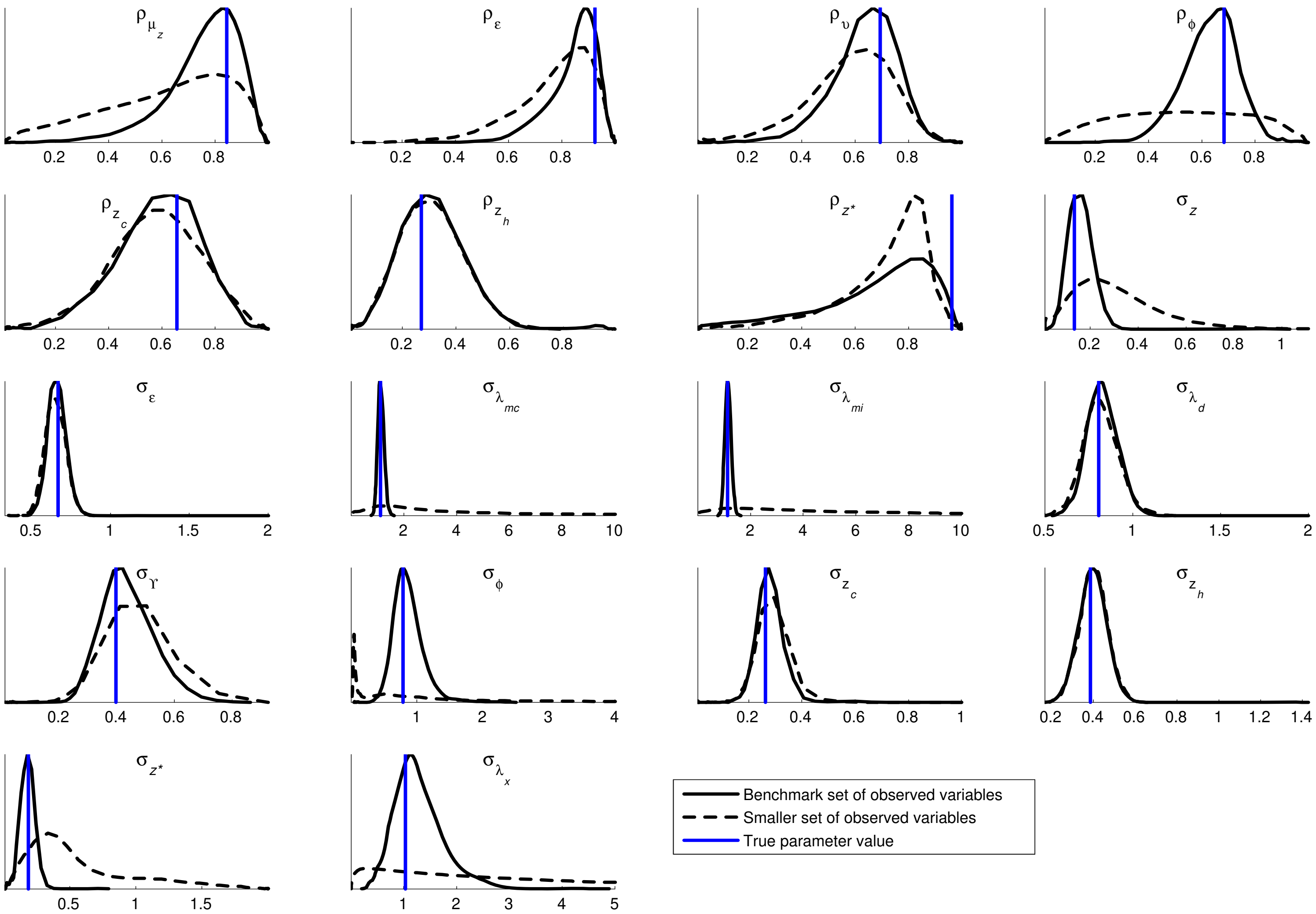

Figure 3b: Kernel density estimates of the small sample distribution for the estimates of the shock parameters. The solid line shows the parameter distribution when the estimations are based on the full set of observable variables, and the dashed line when the estimations are based on fitting only a subset of variables (i.e., 7 "closed economy" variables). The true parameters are given by the vertical bars. $T = 100$ observations in each of the N artificial samples, and we initialize the estimations by sampling parameters from the prior distributions in Table 2.

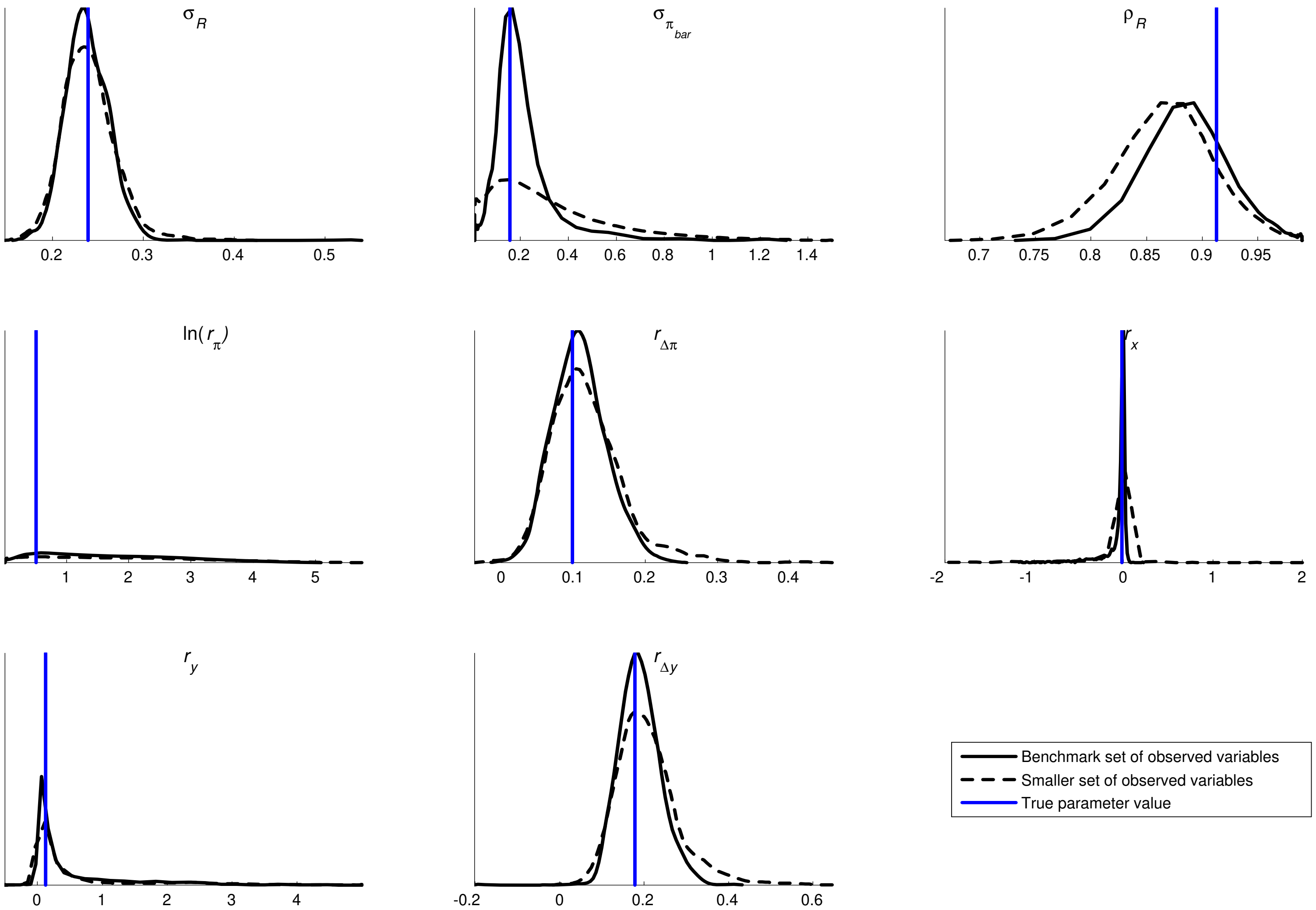

Figure 3c: Kernel density estimates of the small sample distribution for the estimates of the policy rule parameters. The solid line shows the parameter distribution when the estimations are based on the full set of observable variables, and the dashed line when the estimations are based on fitting only a subset of variables (i.e., 7 "closed economy" variables). The true parameters are given by the vertical bars. $T = 100$ observations in each of the N artificial samples, and we initialize the estimations by sampling parameters from the prior distributions in Table 2.

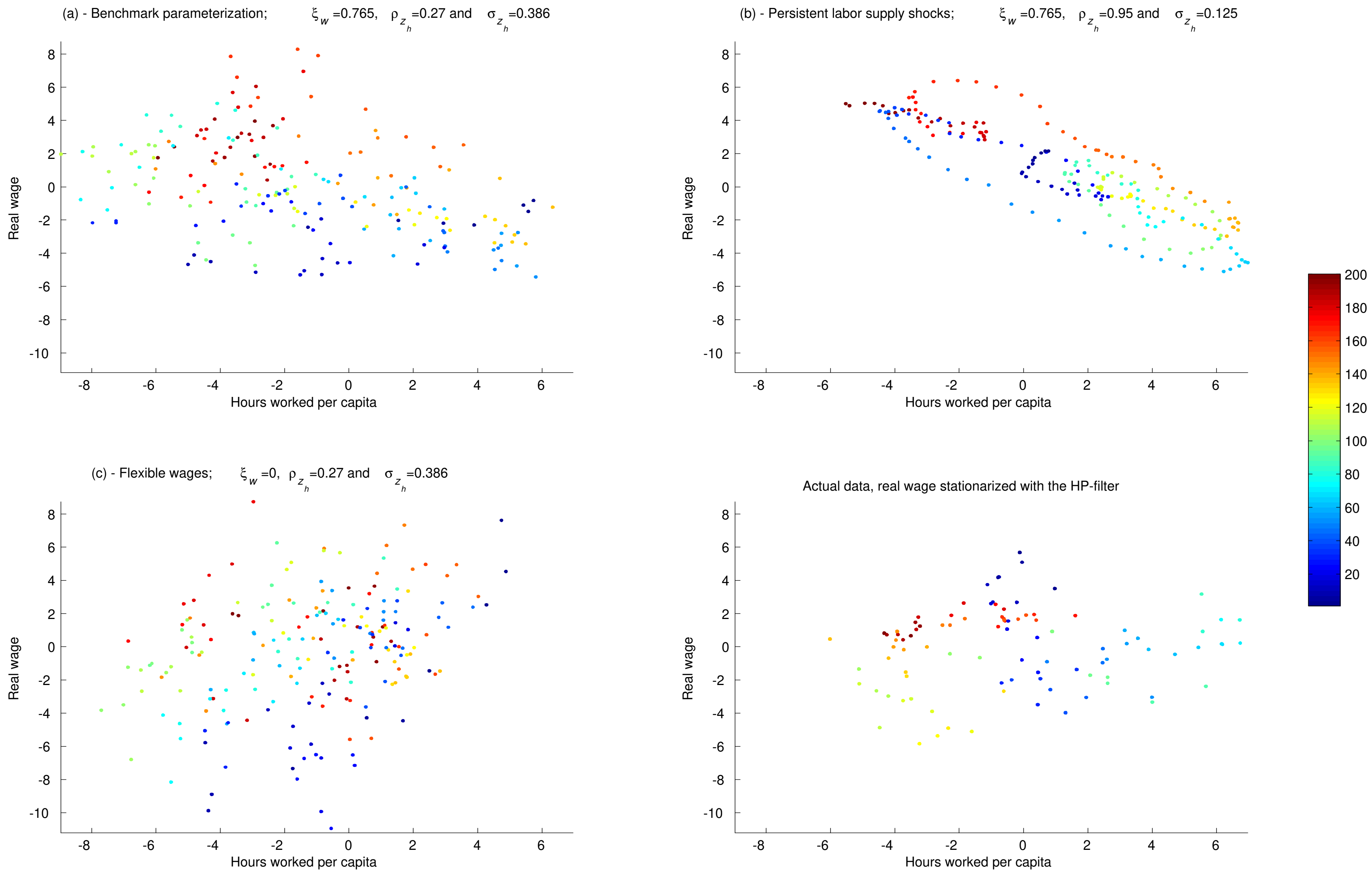

Figure 4: Bivariate real wage and hours worked per capita scatter plots for benchmark (low persistence) and highly persistent labor supply shocks for different degrees of nominal wage stickiness for a random sample of 200 observations. The ordering of the observations $t = 1, 2, ..., 200$ in the sample is indicated by the scale bar on the right hand side of the four panels.

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