Parameterized complexity of generalized domination problems on bounded tree-width graphs

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- Usual studied cases of $\exists[\sigma, \varrho]$-**Dominating Set** are **FPT** when parameterized by tree-width;
  $\rightarrow$ Is it always **FPT**?
- We prove $\exists[\sigma, \varrho]$-**Dominating Set** becomes **W**[1]-hard for (many) other cases when parameterized by tree-width.

Parameterized complexity

Definition (FPT)

A problem $\mathcal{P}$ is **FPT** parameterized by k if it can be solved in time $\mathcal{O}\big(f(k) \cdot p(n)\big)$.

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Remark: To prove $\mathcal{P}$ is **W**[1]-hard, take a **W**[1]-hard problem $\mathcal{Q}$ and prove that $\mathcal{Q} \leq_{\text{fpt}} \mathcal{P}$.

Generalized domination

Definition (dominating set)

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- $\exists u \ (u \in D \ \wedge \ \mathrm{adj}(u, v))$.

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- $v \in D \Rightarrow |D \cap N(v)| \geq 0$; or
- $v \notin D \Rightarrow |D \cap N(v)| \geq 1$.

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$D \subseteq V$ is a *dominating set* if, $\forall v \in V$:

- $v \in D \Rightarrow |D \cap N(v)| \in \{0, 1, 2, \ldots\}$; or
- $v \notin D \Rightarrow |D \cap N(v)| \in \{1, 2, 3, \ldots\}$.

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Definition ($[\sigma, \varrho]$-dominating set)

Let $\sigma, \varrho \subseteq \mathbb{N}$. $D \subseteq V$ is a *$[\sigma, \varrho]$-dominating set* if, $\forall v \in V$:

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σ and ϱ fix some constraints on the neighborhood of every vertex:

- σ constraints the neighborhood of vertices which are in D.
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Remark: We usually suppose that $0 \notin \varrho$, as otherwise $D = \emptyset$ would be a trivial $[\sigma, \varrho]$-dominating set.

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Known results

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Theorem [van Rooij, Bodlaender, Rossmanith, 2009]
$\exists[\sigma, \varrho]$-**Dominating Set** can be solved in $\mathcal{O}^*\left(s^{\mathrm{tw}}\right)$ time if σ and ϱ are finite or cofinite, where s is the minimum number of states needed to represents σ and ϱ.

Using Courcelle's theorem:
If σ and ϱ are finite or cofinite, then $\exists[\sigma,\varrho]$-**Dominating Set** is expressible in **$\mathbf{MSOL_2}$**. Hence it is **FPT** when parameterized by tree-width.

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$\exists[\sigma, \varrho]$-**Dominating Set** can be solved in $\mathcal{O}^*\left(s^{\text{tw}}\right)$ time if σ and ϱ are ultimately periodic, where s is (almost) the minimum number of states needed to represents σ and ϱ.

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- Use fast subset convolution to fasten the join operation.

Some **W**[1]-hard cases

Theorem [C., 2010]

If σ contains arbitrary large gaps between two consecutive elements and ϱ is cofinite (and an additional technical constraint on σ), then $\exists[\sigma,\varrho]$-**Dominating Set** is W[1]-hard parameterized by tree-width.

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Given σ and ϱ, we will reduce k-**Capacitated Dominating Set** to $\exists[\sigma, \varrho]$-**Dominating Set**.

k-**Capacitated Dominating Set** is **W**[1]-hard when parameterized by the tree-width of the input graph and the size k of the expected solution.

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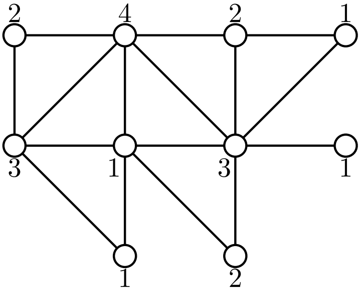
k-**Capacitated Dominating Set**: search $S \subseteq V$ such that $|S| \leq k$.

Ideas of the reduction



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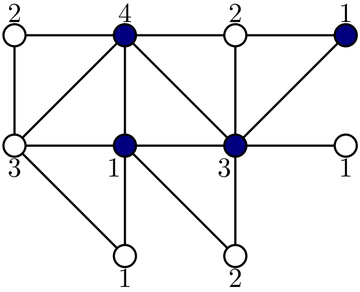

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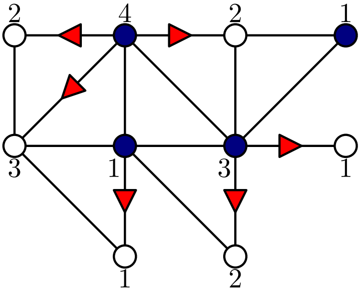

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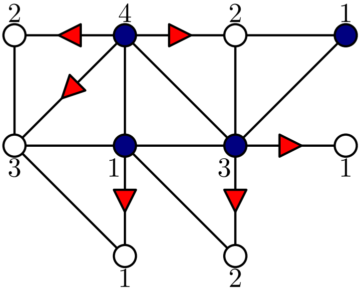

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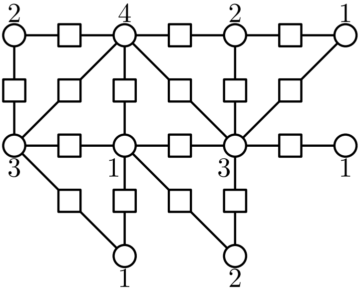

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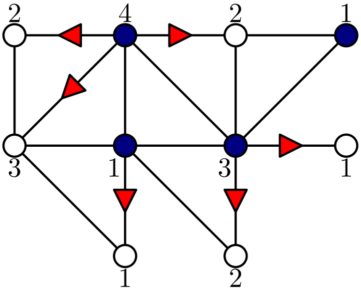

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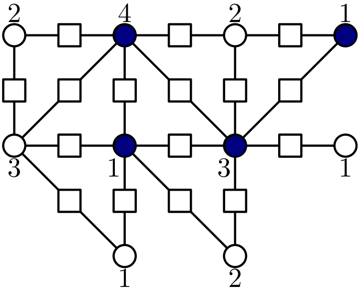

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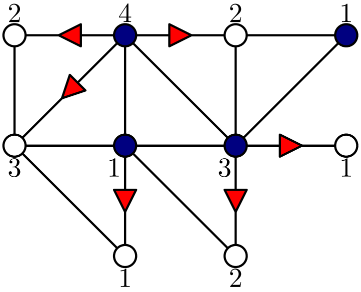

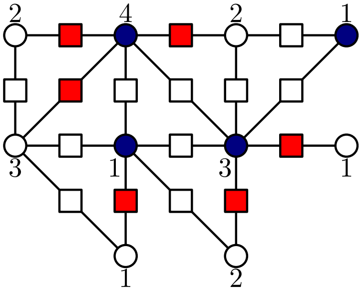

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Technical constraint: We suppose that there exists a polynomial p_σ such that a gap of length t exists at distance $p_\sigma(t)$ in σ.

The reduction

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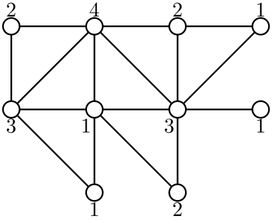

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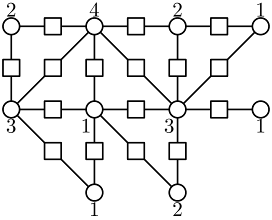

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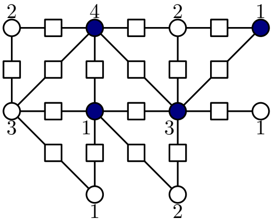

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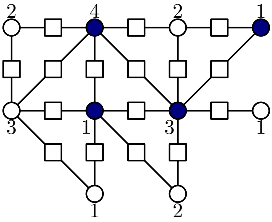

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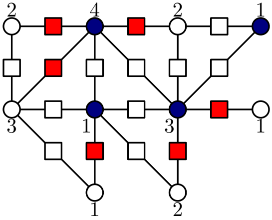

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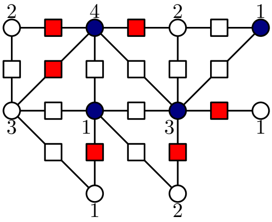

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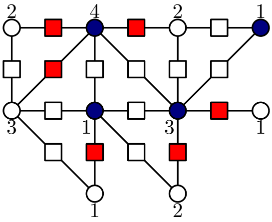

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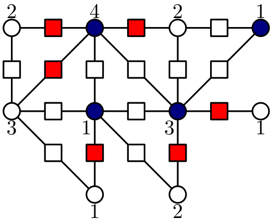

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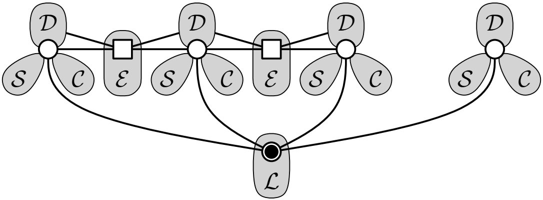

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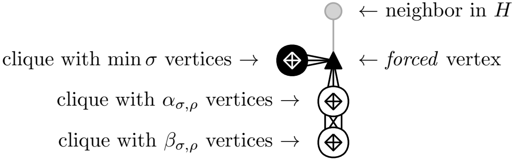

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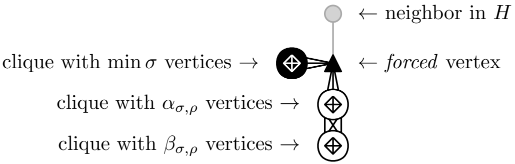

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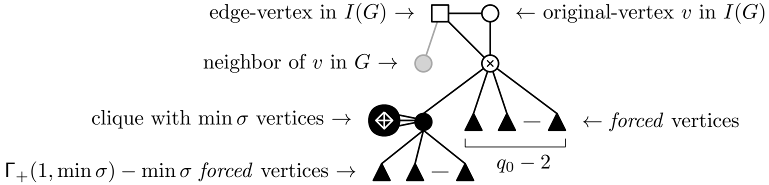
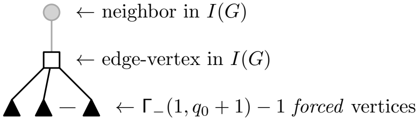


The gadgets

Gadget *edge-selection* ($\mathcal{E}$): encodes the domination function dom.



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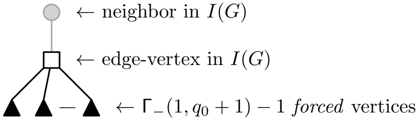
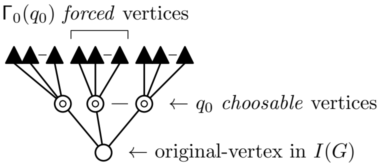
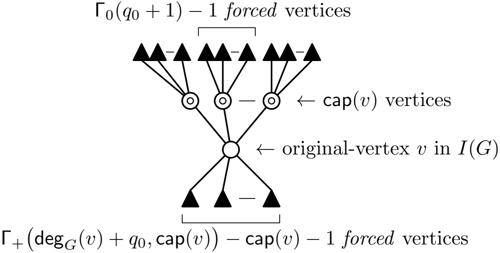


Gadget *satisfiability* ($\mathcal{S}$): allows any non-selected vertex to be satisfied.

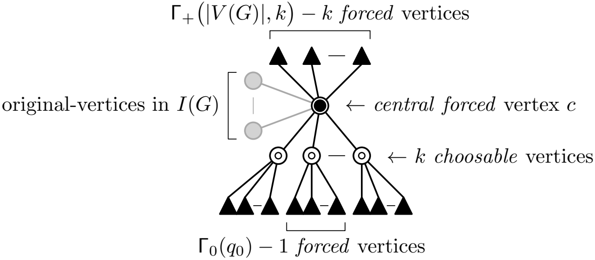


The gadgets

Gadget *capacity* ($\mathcal{C}$): encodes the capacity function cap, and allows any selected vertex to be satisfied.



Gadget *limitation* ($\mathcal{L}$): encodes the parameter k.



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Theorem [C., 2010]

If σ contains arbitrary large gaps between two consecutive elements and ϱ is cofinite (and an additional technical constraint), then $\exists[\sigma, \varrho]$-**Dominating Set** is W[1]-hard parameterized by tree-width.

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