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PARAMETERS ESTIMATION OF THE GAMMA DISTRIBUTION IN THE PRESENCE OF OUTLIERS GENERATED FROM GAMMA DISTRIBUTION

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Abstract

The maximum likelihood, moment and mixture of the estimators are derived for samples from the gamma distribution in the presence of outliers generated from gamma distribution. These estimators are compared empirically when all parameters are unknown; their bias and mean squares error are investigated with the help of numerical technique. We have shown that these estimators are asymptotically unbiased. At the end, we conclude that mixture estimators are better than the maximum likelihood and moment estimators.

Key Words: Gamma distribution, Outlier, Moment estimator, Maximum likelihood estimator, Mixture estimator, Newton-Raphson.

1 Introduction

In an experimental situation, many times, an experimenter comes across some observations which are different from the main body of the data and hence are outliers. The problem of outliers is of considerable importance in almost all experimental fields and has revised continued attention in statistical literature. If we ignore the outliers in estimation of parameters, then variance of the estimators will increase and in testing of hypothesis, power will decrease. Hence, we have to consider a different model when outliers are present. For more details, see Hawkins (1980), Miller (1981) and Barnet and Lewis (1994). According to the definition of outliers, we have dependence in the whole data. Also, only we know about the number of outliers but we can not distinguish which of the observation is outlier or no outlier. In other hands, If we select some of the observation from a distribution and the remaining are selected from a different distribution

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then mix them to each other, we have independence in whole of data and it is a mixture model. One should note that in the mixture model, we know about the number and the value of the contamination observation. Also, we can use any observation from any two distributions. But in the outliers problem in actual data the number of outliers is few such as 1,2,3,4.

Consider spread from a point source for example, which might a small plot of plants. During favorable weather conditions, the plants release their pollen and it disperses according to a gamma distribution with distance from the source. However, in less favorable conditions, light, rain or mist, not only are the plants less likely to release pollen, but that which is released still falls with a gamma distribution. Dixit et al. (1996) consider the above example in the context of spread disease amongst plants of viral spores such as barley yellow mosaic dwarf virus (BYMDV). By using the methodology as stated in Dixit et al. (1996), it is possible to estimate the average distance (and hence area) of disease spread in a field from a small patch of infested plants in the presence of some spread caused by insects. Also, Dixit and Nasiri (2001) estimated parameters of the exponential distribution in the presence of outliers generated from uniform distribution. Jabbari Nooghabi et al. (2009) extended their approach to estimate parameters of gamma distribution in the presence of outliers generated from uniform distribution. Further, Jabbari Nooghabi et al. (2010) used some statistics for detecting outliers in gamma distribution.

According to Dixit et al. (1996), we assume that a set of random variables $X_1, X_2, ..., X_n$ represent the distance of an infected sampled plant from a plot of plants inoculated with a virus. Some of the observations are derived from the airborne dispersal of the spores and distributed according to the gamma distribution. The other observations out of n random variables (say k) are present. Because, aphids which are known to be carriers of BYMDV have passed the virus into the plants when the aphids feed on the sap. These k (known) aphids are considered to be gamma distributed with spread scale parameter.

Now, we assume that the random variables $X_1, X_2, ..., X_n$ are such that k of them are distributed with probability density function (pdf)

$$g(x) = \frac{x^{\alpha-1}}{\Gamma(\alpha)\ (\beta\theta)^{\alpha}} \exp\left(-\frac{x}{\beta\theta}\right), \qquad x > 0, \theta > 0, \beta > 0,$$

and the remaining $(n-k)$ random variables are distributed with pdf

$$f(x) = \frac{x^{\alpha-1}}{\Gamma(\alpha)\ \theta^{\alpha}} \exp\left(-\frac{x}{\theta}\right), \qquad x > 0, \theta > 0.$$

The present paper considers the estimation of α, β and θ in the model described above. One should note that $\beta > 0$ and $\beta \neq 1$. Because for $\beta = 1$, the study is reduced to estimation of the parameters of the homogenous case of the Gamma distribution and there is no outliers. In Section 2, we obtain the joint distribution of $X_1, X_2, ..., X_n$ in the presence of k outliers. In Sections 3, 4 and 5, we deal with the method of moment, maximum likelihood and mixture of these two methods (moment and maximum likelihood) to estimate α, β and θ, respectively. We compare empirically the bias and mean square error (MSE) of the estimators in the last section.

2 Joint distribution of $X_1, X_2, ..., X_n$ with k outliers

Let $X_i, i \geq 1$ be a sequence of non negative continuous random variables such that for a given combination $A_1, A_2, ..., A_{n-k}$ of the integers $1, 2, ..., n$, the following conditions hold.
I: The random variables $X_{A_1}, X_{A_2}, ..., X_{A_{n-k}}$, are independent each having the pdf $f(x)$.
II: The remaining random variables are also independent each having the pdf $g(x)$.
III: The two sets of the random variables are also independent.
IV: Further, it is assume that the combinations $A_1, A_2, ..., A_{n-k}$ of the integers $1, 2, ..., n$ are chosen at random with equal probability $[C(n,k)]^{-1}$ for each combinations, where $C(n,k) = \frac{n!}{k!(n-k)!}$.
The joint density of $X_1, X_2, ..., X_n$ is given as

$$f(x_1, x_2, ..., x_n) = \prod_{i=1}^{n} f(x_i) \sum_{A_1, A_2, ..., A_k} \prod_{j=1}^{k} \frac{g(x_{A_j})}{f(x_{A_j})} [C(n,k)]^{-1},$$

where

$$\sum_{A_1, A_2, ..., A_k} = \sum_{A_1=1}^{n-k+1} \sum_{A_2=A_1+1}^{n-k+2} \cdots \sum_{A_k=A_{k-1}+1}^{n}.$$

The above formula is known as the outliers model (refer to Dixit (1989)). By using the pdf of $f(x)$ and $g(x)$ and after some simplification, the joint pdf of $X_1, X_2, ..., X_n$ in presence of k outliers is given by

$$f(x_1, x_2, ..., x_n, \alpha, \beta, \theta) = h_k \prod_{i=1}^{n} x_i^{\alpha-1} \exp\left(-\frac{x_i}{\theta}\right) H(x, \beta, \theta), \tag{1}$$

where

$$h_k = \left[C(n,k)(\Gamma(\alpha))^n \beta^{k\alpha} \theta^{n\alpha}\right]^{-1},$$

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt,$$

and

$$H(x, \beta, \theta) = \sum_{A_1, A_2, ..., A_k} \exp\left(-\frac{1-\beta}{\beta\theta} \sum_{j=1}^{k} x_{A_j}\right).$$

Also, the marginal density of X_i $(i = 1, 2, ..., n)$ can be written as:

$$f(x_i; \alpha, \beta, \theta) = bg(x) + \bar{b} f(x),$$

where $b = \frac{k}{n}$, $\bar{b} = 1 - b$ and $(X_1, X_2, ..., X_n)$ are not independent. For more details see Dixit (1989), Dixit, Ali and Woo (2003), Dixit, Moore and Barnett (1996) and Dixit and Nasiri (2001).

3 Method of moments

To find the moment estimator of the parameters, we should find the sample and population moments. So by comparing these moments, the method of moments estimators are derived. Let $D = \frac{m'_2}{{m'_1}^2}$ and $D_1 = \frac{m'_2}{m'_1}$, where

$$m'_i = \sum_{j=1}^{n} \frac{x_j^i}{n}, \qquad i = 1, 2, 3.$$

If we assume that β is known by using the method of moments, we get

$$D = \frac{(\alpha+1)(b\beta^2+\bar{b})}{\alpha(b\beta+\bar{b})^2}, \tag{2}$$

and

$$D\alpha[b\beta+\bar{b}]^2 = (\alpha+1)(b\beta^2+\bar{b}).$$

So

$$\hat{\alpha} = \frac{b\beta^2+\bar{b}}{D(b\beta+\bar{b})^2-(b\beta^2+\bar{b})}. \tag{3}$$

Also,

$$D_1 = \frac{\theta(\alpha+1)(b\beta^2+\bar{b})}{b\beta+\bar{b}}, \tag{4}$$

and

$$\hat{\theta} = \frac{D_1(b\beta+\bar{b})}{(\alpha+1)(b\beta^2+\bar{b})}. \tag{5}$$

By substituting (3) in (5), we obtain

$$\hat{\theta} = \frac{D_1[D(b\beta+\bar{b})^2-(b\beta^2+\bar{b})]}{D(b\beta^2+\bar{b})(b\beta+\bar{b})}. \tag{6}$$

Now, to estimate β, we should compare the third sample and population moments. So we find the third population moment and get

$$m'_3 = \theta^3(\alpha+2)(\alpha+1)\alpha(b\beta^3+\bar{b}). \tag{7}$$

Substituting (3) and (5) in (7), imply that

$$m'_3 = \frac{{D_1}^3(b\beta^3+\bar{b})[2D(b\beta+\bar{b})^2-(b\beta^2+\bar{b})]}{(b\beta^2+\bar{b})^2(b\beta+\bar{b})D^2}. \tag{8}$$

Therefore, solving the following equation gets the moment estimator (ME) of β as:

$$A_1\beta^5 + A_2\beta^4 + A_3\beta^3 + A_4\beta^2 + A_5\beta + A_6 = 0, \tag{9}$$

where

$$\begin{cases} A_1 = m_3' D^2 b^3 + D_1^3 b^2 - 2D_1^3 D b^3, \\ A_2 = m_3' D^2 b^2 \bar{b} - 4D_1^3 D b^2 \bar{b}, \\ A_3 = 2m_3' D^2 b^2 \bar{b} + D_1^3 b\bar{b} - 2D_1^3 D b\bar{b}^2, \\ A_4 = 2m_3' D^2 b\bar{b}^2 + D_1^3 b\bar{b} - 2D_1^3 D b^2 \bar{b}, \\ A_5 = -4D_1^3 D b\bar{b}^2 + m_3' D^2 b\bar{b}^2, \\ A_6 = m_3' D^2 \bar{b}^3 - 2D_1^3 D\bar{b}^3 + D_1^3 \bar{b}^2. \end{cases} \tag{10}$$

Note: For moment estimator of β, one should note that in either case, we may get more than one feasible solution. In such a situation, estimates can be selected by evaluating the likelihood for each feasible solution and choosing the one that maximizes the likelihood function with respect to β.
Now, it is useful to show that $\hat{\theta}$, $\hat{\alpha}$ and $\hat{\beta}$ are asymptotically unbiased estimators.
Let $W_1 = \sum_{i=1}^n X_i$, $W_2 = \sum_{i=1}^n X_i^2$ and $W_3 = \sum_{i=1}^n X_i^3$, then by using the method of moments

$$D = \frac{nW_2}{W_1^2}, \quad D_1 = \frac{W_2}{W_1} \quad \text{and} \quad \mathrm{W}_3 = \mathrm{nm}_3'.$$

So if β is known, we can write $\hat{\theta}$ as a function of W_1 and W_2

$$\hat{\theta} = f(W_1, W_2). \tag{11}$$

Let $\mathrm{E}(W_1) = \mu$ and $\mathrm{E}(W_2) = \nu$. We expand the function $f(W_1, W_2)$ about the point (μ, ν) using Taylor series

$$f(W_1, W_2) = f(\mu,\nu) + (W_1 - \mu)\frac{\partial f}{\partial W_1}|_{W_1=\mu, W_2=\nu} + (W_2 - \nu)\frac{\partial f}{\partial W_2}|_{W_1=\mu, W_2=\nu} + \dots \,. \tag{12}$$

Therefore from (5), (11) and (12)

$$\mathrm{E}(\hat{\theta}) \simeq f(\mu,\nu) = \frac{\frac{\nu}{\mu}(b\beta + \bar{b})}{(\alpha+1)(b\beta^2 + \bar{b})},$$

where

$$\mu = (b\beta + \bar{b})\alpha\theta \quad \text{and} \quad \nu = (\mathrm{b}\beta^2 + \bar{\mathrm{b}})\theta^2\alpha(\alpha+1).$$

So, we obtain

$$\mathrm{E}(\hat{\theta}) \simeq \theta. \tag{13}$$

Now, assume that $\hat{\alpha}$ is a function of W_1 and W_2 as $\hat{\alpha} = g(W_1, W_2)$. Then by using the expansion of Taylor series of $g(W_1, W_2)$ around (μ, ν), same as (12) and using (3), we find

$$\begin{aligned} \mathrm{E}(\hat{\alpha}) &\simeq \frac{b\beta^2 + \bar{b}}{\frac{\nu}{\mu^2}(b\beta + \bar{b})^2 - (b\beta^2 + \bar{b})} \\ &= \frac{b\beta^2 + \bar{b}}{\frac{(b\beta^2+\bar{b})(\alpha+1)}{\alpha} - (b\beta^2 + \bar{b})} \\ &= \alpha. \end{aligned} \tag{14}$$

Finally to prove $\mathrm{E}(\beta) \simeq \beta$, assuming that $\hat{\beta}$ is a function of W_1, W_2 and W_3 as $\hat{\beta} = h(W_1, W_2, W_3)$ and expand it around (μ, ν, η), where $\eta = \mathrm{E}(W_3)$. So we get

$$\begin{aligned} h(W_1, W_2, W_3) &= h(\mu, \nu, \eta) + (W_1 - \mu)\frac{\partial h}{\partial W_1}|_{W_1=\mu, W_2=\nu, W_3=\eta} + (W_2 - \nu)\frac{\partial h}{\partial W_2}|_{W_1=\mu, W_2=\nu, W_3=\eta} \\ &\quad +(W_3 - \eta)\frac{\partial h}{\partial W_3}|_{W_1=\mu, W_2=\nu, W_3=\eta} + \dots. \end{aligned} \tag{15}$$

Then, $\mathrm{E}(\hat{\beta}) \simeq h(\mu, \nu, \eta)$. To verify the unbiasedness of the estimator, we have

$$\begin{cases} A_1 \simeq \frac{\eta}{n}\left(\frac{n\nu}{\mu^2}\right)^2 b^3 + \left(\frac{\nu}{\mu}\right)^3 b^2 - 2\left(\frac{\nu}{\mu}\right)^3\left(\frac{n\nu}{\mu^2}\right) b^3, \\ A_2 \simeq \frac{\eta}{n}\left(\frac{n\nu}{\mu^2}\right)^2 b^2\bar{b} - 4\left(\frac{\nu}{\mu}\right)^3\left(\frac{n\nu}{\mu^2}\right) b^2\bar{b}, \\ A_3 \simeq 2\frac{\eta}{n}\left(\frac{n\nu}{\mu^2}\right)^2 b^2\bar{b} + \left(\frac{\nu}{\mu}\right)^3 b\bar{b} - 2\left(\frac{\nu}{\mu}\right)^3\left(\frac{n\nu}{\mu^2}\right) b\bar{b}^2, \\ A_4 \simeq 2\frac{\eta}{n}\left(\frac{n\nu}{\mu^2}\right)^2 b\bar{b}^2 + \left(\frac{\nu}{\mu}\right)^3 b\bar{b} - 2\left(\frac{\nu}{\mu}\right)^3\left(\frac{n\nu}{\mu^2}\right) b^2\bar{b}, \\ A_5 \simeq -4\left(\frac{\nu}{\mu}\right)^3\left(\frac{n\nu}{\mu^2}\right) b\bar{b}^2 + \frac{\eta}{n}\left(\frac{n\nu}{\mu^2}\right)^2 b\bar{b}^2, \\ A_6 \simeq \frac{\eta}{n}\left(\frac{n\nu}{\mu^2}\right)^2 \bar{b}^3 - 2\left(\frac{\nu}{\mu}\right)^3\left(\frac{n\nu}{\mu^2}\right)\bar{b}^3 + \left(\frac{\nu}{\mu}\right)^3\bar{b}^2. \end{cases} \tag{16}$$

After substituting (16) in left side of (9) and by using some elementary algebra, we get the equation is equal to 0. This means that the moment estimator of β is asymptotically unbiased. For more details, see Dixit and Nasiri (2001).

4 Maximum likelihood estimator

From (1), the likelihood equation corresponding to $x_1, x_2, ..., x_n$ is

$$\begin{aligned} \mathrm{L} = \mathrm{L}(x_1, x_2, ..., x_n, \alpha, \beta, \theta) &= [\Gamma(\alpha)]^{-n}\beta^{-k\alpha}\theta^{-n\alpha}(C(n,k))^{-1}\prod_{i=1}^{n} x_i^{\alpha-1}\exp\left(-\frac{x_i}{\theta}\right) \\ &\quad \times \sum_{A_1, A_2, \dots, A_k}\exp\left(-\frac{1-\beta}{\beta\theta}\sum_{j=1}^{k} x_{A_j}\right). \end{aligned} \tag{17}$$

Then

$$\frac{\partial \mathrm{L}}{\partial \alpha} \simeq \left\{[\Gamma(\alpha)]^{-n-1}\left[-n\Gamma'(\alpha) - k\Gamma(\alpha)\ln(\beta) - n\Gamma(\alpha)\ln(\theta) + \Gamma(\alpha)\sum_{i=1}^{n}\ln(x_i)\right]\right\} \times \sum_{A_1,A_2,\ldots,A_k} \exp\left\{-\frac{1-\beta}{\beta\theta}\sum_{j=1}^{k} x_{A_j}\right\}, \quad (18)$$

where

$$\Gamma'(\alpha) = \frac{\partial\Gamma(\alpha)}{\partial\alpha}.$$

So to find the maximum likelihood estimator (MLE) of α, we must solve the following equation using numerical methods (Newton-Raphson method is just one among many possible approaches) as

$$-n\Gamma'(\alpha) - k\Gamma(\alpha)\ln(\beta) - n\Gamma(\alpha)\ln(\theta) + \Gamma(\alpha)\sum_{i=1}^{n}\ln(x_i) = 0. \quad (19)$$

If $\beta = 1$, we can rise the MLE of α without outlier as the following:

$$-n\Gamma'(\alpha) - \Gamma(\alpha)\left[n\ln(\theta) - \sum_{i=1}^{n}\ln(x_i)\right] = 0. \quad (20)$$

Also, we have

$$\frac{\partial \mathrm{L}}{\partial \theta} \simeq \theta^{-n\alpha}e^{-\sum_{i=1}^{n} x_i/\theta}[-\frac{n\alpha}{\theta}\sum_{A_1,A_2,\ldots,A_k} \exp\left\{-\frac{1-\beta}{\beta\theta}\sum_{j=1}^{k} x_{A_j}\right\} + \frac{\sum_{i=1}^{n} x_i}{\theta^2}\sum_{A_1,A_2,\ldots,A_k} \exp\left\{-\frac{1-\beta}{\beta\theta}\sum_{j=1}^{k} x_{A_j}\right\} + \frac{1-\beta}{\beta\theta^2}\sum_{A_1,A_2,\ldots,A_k} \exp\left\{-\frac{1-\beta}{\beta\theta}\sum_{j=1}^{k} x_{A_j}\right\}] = 0. \quad (21)$$

Then to obtain the MLE of θ, we must solve the following equation using numerical methods.

$$-\frac{n\alpha}{\theta}\sum_{A_1,A_2,\ldots,A_k} \exp\left\{-\frac{1-\beta}{\beta\theta}\sum_{j=1}^{k} x_{A_j}\right\} + \frac{\sum_{i=1}^{n} x_i}{\theta^2}\sum_{A_1,A_2,\ldots,A_k} \exp\left\{-\frac{1-\beta}{\beta\theta}\sum_{j=1}^{k} x_{A_j}\right\} + \frac{1-\beta}{\beta\theta^2}\sum_{A_1,A_2,\ldots,A_k} \exp\left\{-\frac{1-\beta}{\beta\theta}\sum_{j=1}^{k} x_{A_j}\right\}\sum_{j=1}^{k} x_{A_j} = 0. \quad (22)$$

If we put $\beta = 1$, we obtain $\hat{\theta} = \frac{\sum_{i=1}^{n} x_i}{n\alpha}$ as the MLE of θ without outlier.
Finally, for MLE of β we have

$$\sum_{A_1,A_2,\ldots,A_k} \exp\left\{-\frac{1-\beta}{\beta\theta}\sum_{j=1}^{k} x_{A_j}\right\}\left\{-k\alpha\beta + \frac{1}{\theta}\sum_{j=1}^{k} x_{A_j}\right\} = 0. \quad (23)$$

So, we must solve the above equation to find the MLE of β as well as the equation (22).

5 Mixture of methods of moment and maximum likelihood

Read (1981) proposed the methods which are avoided the difficulty of complicated equations. According to Read's results (1981), we obtain the mixture estimator (MXE) for the two cases as follows.
I) β is known:
In Section 3, we obtained the moment estimator of θ as

$$\hat{\theta} = \frac{D_1[D(b\beta + \bar{b})^2 - (b\beta^2 + \bar{b})]}{D(b\beta^2 + \bar{b})(b\beta + \bar{b})}. \tag{24}$$

Therefore by substituting the above estimator in equation (23) and solving by numerical methods, we can find the MXE of α.
II) β is unknown:
At first, we find the estimator of β according to ME method by solving equation (9). So we substitute this estimator in equation (6) and then solve it to obtain the MXE of θ. Finally, the MXE of α will be obtained by solving the equation that is yielded from replacing the ME of β and θ in equation (23).

6 Numerical Study

In order to get an idea of efficiency between the three types of estimator i.e ME, MLE and MXE in known β and unknown β cases, we have generated a sample of size 5(1)10(5)30 for $k = 1$ and $k = 2$ from the gamma distribution with α=5, β=0.1 and θ=0.5 using R statistical software. For example, for k=1, α=5, β=0.1 and θ=0.5 a sample of size 10 is generated such that a sample of size 9 is taken from $f(x)$ and a sample of size one is taken from $g(x)$. For these observations, we have calculated bias and MSE of the estimators. This process is repeated 1000 times. Further, these 1000 biases and MSEs were divided by 1000. Figures 1 and 2 (a, b, c, d, e and f) show the results based on one thousand independent replication of each experiments for $k = 1$ and $k = 2$, respectively.
In figures 1 and 2 (a, for Bias of $\hat{\alpha}$), one can easily find that the Bias of MXE of $\hat{\alpha}$ in case **II** is less than Bias of the other methods for all n. Figures 1 and 2 (b, for Bias of $\hat{\beta}$) show that the Bias of MXE of $\hat{\beta}$ is between Bias of ME and MLE for all sample sizes. It is clear that in figures 1 and 2 (c, for Bias of $\hat{\theta}$) the Bias of MXE of $\hat{\theta}$ in case **II** is approximately near to the Bias of MXE in case **I** and MLE and also less than the Bias of ME. In addition, in figures 1 and 2 (d, for MSE of $\hat{\alpha}$), it can be obtained that the MSE of MXE of $\hat{\alpha}$ in case **II** is less than the MSE of other methods for all n and it is decreasing when n increases. Figures 1 and 2 (e, for MSE of $\hat{\beta}$) show that the MSE of MXE of $\hat{\beta}$ is less than MSE of MLE and ME for all sample sizes and decreasing respect to n. It is obvious that in figures 1 and 2 (f, for MSE of $\hat{\theta}$) the MSE of MXE of $\hat{\theta}$ in case **II** is less than the MSE of MXE in case **I**, MLE and ME and also is decreasing when n is increasing.

In summary, the graphs show that for the estimation of α, β and θ, the MXE in case **II** (Mixture2, β is unknown) is more efficient than the other estimators. Also in case **I** (Mixture1, β is known), we see that the MXE is better than the MLE and ME. Further, in the two cases the MLE is better than the ME. Finally, we can inference that the MSE of MXE in both cases decreases as n is increased. So it is a consistent estimator.

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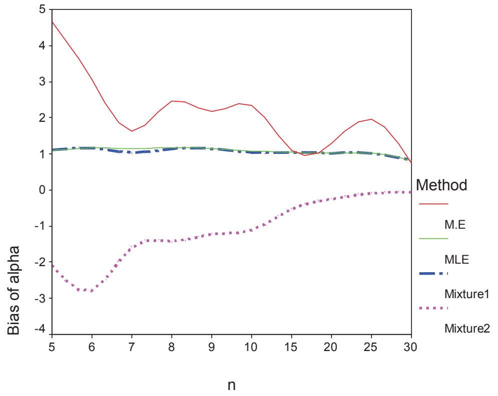
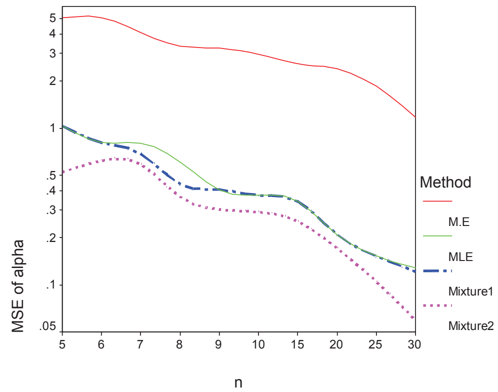
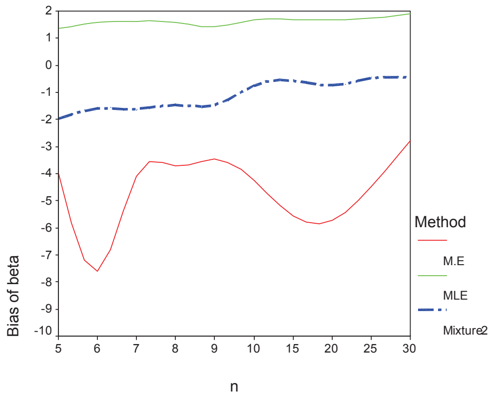


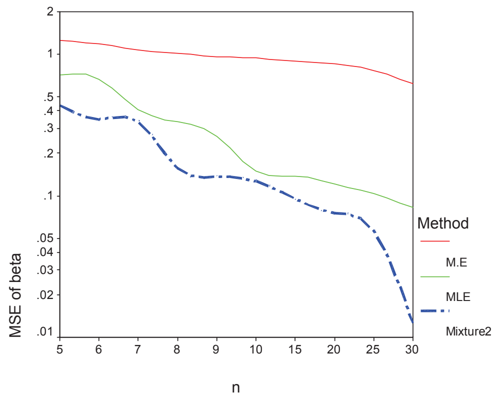
Bias of $\hat{\alpha}$ when k=1, α=5, β=0.1 and θ=0.5



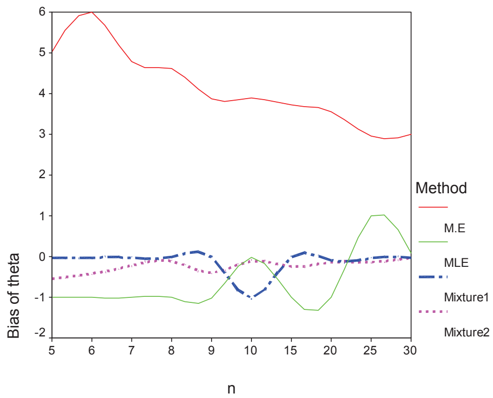
MSE of $\hat{\alpha}$ when k=1, α=5, β=0.1 and θ=0.5



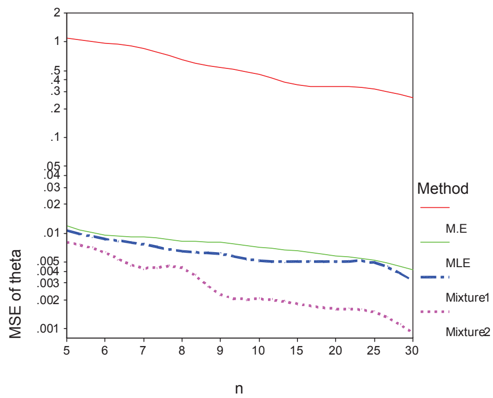
Bias of $\hat{\beta}$ when k=1, α=5, β=0.1 and θ=0.5



MSE of $\hat{\beta}$ when k=1, α=5, β=0.1 and θ=0.5

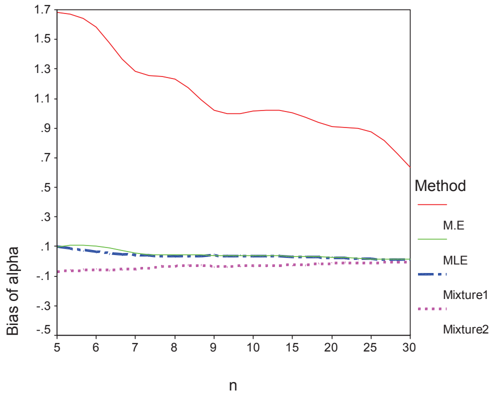


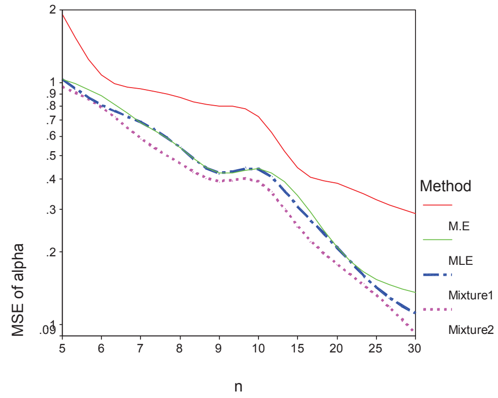
Bias of $\hat{\theta}$ when k=1, α=5, β=0.1 and θ=0.5



MSE of $\hat{\theta}$ when k=1, α=5, β=0.1 and θ=0.5

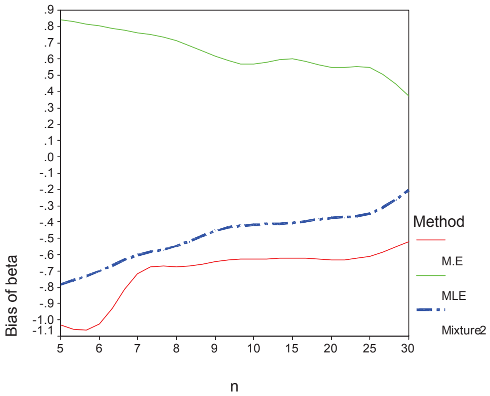
Figure 1. Comparison the bias and MSE of the estimators based on simulation results

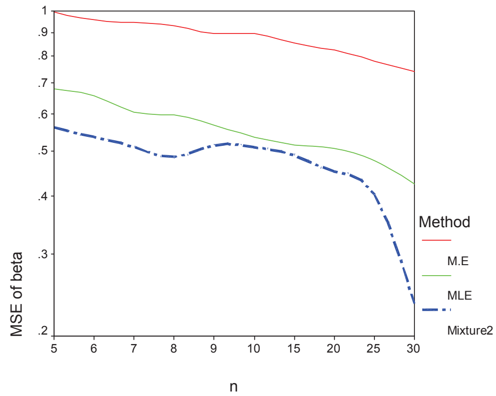




Bias of $\hat{\alpha}$ **when** k**=2,** α**=5,** β**=0.1 and** θ**=0.5**

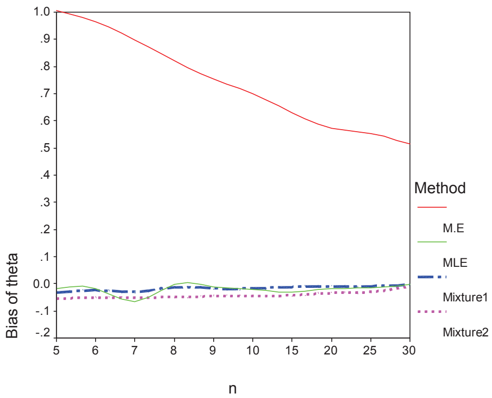
MSE of $\hat{\alpha}$ **when** k**=2,** α**=5,** β**=0.1 and** θ**=0.5**

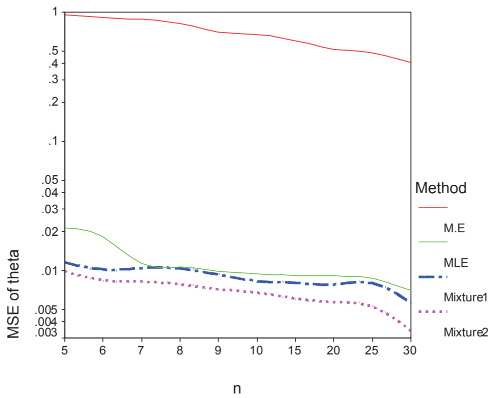




Bias of $\hat{\beta}$ **when** k**=2,** α**=5,** β**=0.1 and** θ**=0.5**

MSE of $\hat{\beta}$ **when** k**=2,** α**=5,** β**=0.1 and** θ**=0.5**





Bias of $\hat{\theta}$ **when** k**=2,** α**=5,** β**=0.1 and** θ**=0.5**

MSE of $\hat{\theta}$ **when** k**=2,** α**=5,** β**=0.1 and** θ**=0.5**

Figure 2. Comparison the bias and MSE of the estimators based on simulation results