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PARAMETERS IN THE ELECTROWEAK THEORY
II. QUADRATIC DIVERGENCES

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ABSTRACT

One of the major differences between electroweak theory and the better understood quantum electrodynamics is the appearance of quadratic divergences. We discuss this issue using point-splitting regularization. It is shown that these quadratic divergences vanish if a certain mass relation is satisfied. The relation is linear in the squares of all the masses of the theory.

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1. Introduction

Among the various quantum field theories that have been used to describe our physical world, quantum electrodynamics is perhaps the best understood, and the Standard Model of Glashow, Weinberg and Salam [1] the one that has been verified most extensively by high-energy experiments [2]. It is therefore instructive to study the qualitative differences between them.

Two differences are especially striking besides the appearance of the spin-zero Higgs [3] and the Yang-Mills [4] non-Abelian coupling in the Standard Model. First, because of gauge invariance, there is no quadratic divergence in quantum electrodynamics. For the Standard Model, in spite of gauge invariance, there are nevertheless quadratic divergences. Secondly, in quantum electrodynamics, there is only the gauge coupling, without any other interaction. In the Standard Model, there are, besides the gauge couplings, in addition numerous fermion-fermion-Higgs Yukawa couplings that give rise to the masses of the various particles. These two differences are of course closely related to each other.

It is the purpose of this paper to study the quadratic divergences, while the problem of the fermion-fermion-Higgs Yukawa coupling will be addressed in the following paper. All the quadratic divergences appear in connection with the self-energy diagrams, typically those of the electron and the Higgs. Following the preceding paper, Paper I [5], we shall use the method of point-splitting regularization to study the one-loop diagrams. As compared with dimensional regularization [6], point splitting is technically less developed, but without any difficulty with γ_5.

We shall see that the quadratic divergences associated with the electron self energy all stem from the Higgs tadpole diagrams. These will be discussed in section 2. The quadratic divergences vanish if the following mass relation is satisfied,

$$m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) = \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{3}{4}m_H^2. \quad (1.1)$$

This relation was first given by Veltman [7] over 10 years ago. Our derivation is very different from his. Since the point-splitting regularization makes no reference to dimensions of space-time other than four, there is in our derivation no ambiguity related to the dimension of the Dirac matrices or that of Lorentz vectors.

Next, the Higgs self-energy diagrams are discussed in section 3. In this case, the tadpole diagrams enter as a subset of all relevant diagrams, but they are not the only ones. There are also quadratically divergent "bubble" diagrams. It turns out that the quadratically divergent parts of those are simply proportional to the tadpole diagrams.

2. Electron self energy

The various Feynman diagrams contributing to the electron self energy are given in fig. 1. Here, ϕ and ϕ_0 denote the Higgs ghosts, while η^+, η^- and η^Z denote Faddeev-Popov ghosts [8]. Most of these contributions to the electron self energy are divergent, but it is easily seen that only the Higgs tadpole diagrams are actually quadratically divergent. (It should be remembered that no vertex involving the electron field has any momentum factor.) Since we focus on quadratic divergences in the present paper, we consider the Higgs tadpole diagrams in more detail.

The contributions of the various tadpole diagrams are given by

$$T_e = \frac{-1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(-\frac{1}{2}ig\frac{m_e}{m_W}\right)\mathrm{Tr}\frac{i(\not{k}+m_e)}{k^2-m_e^2+i\epsilon}, \tag{2.1}$$

$$T_u = \frac{-3}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(-\frac{1}{2}ig\frac{m_u}{m_W}\right)\mathrm{Tr}\frac{i(\not{k}+m_u)}{k^2-m_u^2+i\epsilon}, \tag{2.2}$$

$$T_d = \frac{-3}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(-\frac{1}{2}ig\frac{m_d}{m_W}\right)\mathrm{Tr}\frac{i(\not{k}+m_d)}{k^2-m_d^2+i\epsilon}, \tag{2.3}$$

$$T_Z = \frac{1}{(2\pi)^4}\frac{1}{2}\int d^4k\, e^{ik\cdot\delta}\left(ig\frac{m_Z^2}{m_W}\right)\frac{i}{k^2-m_Z^2+i\epsilon}\left[-4+(1-\xi)\frac{k^2}{k^2-\xi m_Z^2+i\epsilon}\right], \tag{2.4}$$

$$T_{\phi_0} = \frac{1}{(2\pi)^4}\frac{1}{2}\int d^4k\, e^{ik\cdot\delta}\left(-\frac{1}{2}ig\frac{m_H^2}{m_W}\right)\frac{i}{k^2-\xi m_Z^2+i\epsilon}, \tag{2.5}$$

$$T_W = \frac{1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(igm_W\right)\frac{i}{k^2-m_W^2+i\epsilon}\left[-4+(1-\xi)\frac{k^2}{k^2-\xi m_W^2+i\epsilon}\right], \tag{2.6}$$

$$T_{\phi^+} = \frac{1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(-\frac{1}{2}ig\frac{m_H^2}{m_W}\right)\frac{i}{k^2-\xi m_W^2+i\epsilon}, \tag{2.7}$$

$$T_H = \frac{1}{(2\pi)^4}\frac{1}{2}\int d^4k\, e^{ik\cdot\delta}\left(-\frac{3}{2}ig\frac{m_H^2}{m_W}\right)\frac{i}{k^2-m_H^2+i\epsilon}, \tag{2.8}$$

$$T_{\eta^+} = \frac{-1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(-\tfrac{1}{2}ig\xi m_W\right)\frac{i}{k^2-\xi m_W^2+i\epsilon}, \tag{2.9}$$

$$T_{\eta^-} = \frac{-1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(-\tfrac{1}{2}ig\xi m_W\right)\frac{i}{k^2-\xi m_W^2+i\epsilon}, \tag{2.10}$$

$$T_{\eta^Z} = \frac{-1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(-\frac{1}{2}ig\xi\frac{m_Z^2}{m_W}\right)\frac{i}{k^2-\xi m_Z^2+i\epsilon}, \tag{2.11}$$

where the prefactor $-$ sign is for fermion and Faddeev-Popov ghost loops, 3 is for color, and $1/2$ is a symmetry factor for self-conjugate fields. Furthermore, ξ is a parameter specifying the gauge [9].

The contribution to the electron self energy from these tadpole diagrams is thus proportional to

$$
\begin{aligned}
T &= T_e + T_\mu + T_\tau + T_u + T_d + T_s + T_c + T_t + T_b \\
&\quad + T_Z + T_{\phi_0} + T_W + T_{\phi^+} + T_H + T_{\eta^+} + T_{\eta^-} + T_{\eta^Z} \\
&= \frac{g}{m_W}\frac{1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta} \\
&\quad \times \Bigg\{ -2\Bigg[\frac{m_e^2}{k^2 - m_e^2 + i\epsilon} + \frac{m_\mu^2}{k^2 - m_\mu^2 + i\epsilon} + \frac{m_\tau^2}{k^2 - m_\tau^2 + i\epsilon} + \frac{3m_u^2}{k^2 - m_u^2 + i\epsilon} + \frac{3m_d^2}{k^2 - m_d^2 + i\epsilon} \\
&\quad + \frac{3m_c^2}{k^2 - m_c^2 + i\epsilon} + \frac{3m_s^2}{k^2 - m_s^2 + i\epsilon} + \frac{3m_t^2}{k^2 - m_t^2 + i\epsilon} + \frac{3m_b^2}{k^2 - m_b^2 + i\epsilon} \Bigg] + 3\frac{m_W^2}{k^2 - m_W^2 + i\epsilon} \\
&\quad + \frac{3}{2}\frac{m_Z^2}{k^2 - m_Z^2 + i\epsilon} + \frac{3}{4}\frac{m_H^2}{k^2 - m_H^2 + i\epsilon} + \frac{1}{2}\frac{m_H^2}{k^2 - \xi m_W^2 + i\epsilon} + \frac{1}{4}\frac{m_H^2}{k^2 - \xi m_Z^2 + i\epsilon} \Bigg\}.
\end{aligned}
\tag{2.12}
$$

We note that this is *not* independent of ξ. However, the quadratically divergent part is gauge independent.

All these integrals can be expressed in terms of the $I^{(1)}(\delta)$ defined and evaluated in appendix A of Paper I. We have [5]

$$
\begin{aligned}
I^{(1)}(\delta) &= \int d^4l\, e^{il\cdot\delta}\frac{1}{l^2 - M^2 + i\epsilon} \\
&= -4\pi^2 i M^2 \left\{ \frac{-1}{M^2\delta^2} + \frac{1}{2}\left[\ln\frac{M\delta}{2} + \gamma - \tfrac{1}{2}\right] + i\frac{\pi}{4} + \mathcal{O}(\delta^2) \right\},
\end{aligned}
\tag{2.13}
$$

for δ time-like (and no $i\pi/4$ otherwise). Obviously, the quadratically divergent part of the expression (2.12) is the one corresponding to the $1/\delta^2$ part of $I^{(1)}(\delta)$, which is independent of M^2. Hence, the quadratically divergent part of the electron self energy vanishes when the over-all coefficient of $1/\delta^2$ vanishes. Thus the condition is

$$
m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_b^2 + m_t^2) = \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{3}{4}m_H^2, \tag{2.14}
$$

which is our first mass relation, quoted in the Introduction. We note that this is determined entirely by the tadpole diagrams through the coupling of the Higgs field to the various other fields.

3. Self energy of H

The diagrams that contribute are given in fig. 2. We give below the amplitudes for the diagrams that contain quadratic divergences. From the first set, they are

$$I_e = \frac{-1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(-\frac{1}{2}ig\frac{m_e}{m_W}\right)^2 \mathrm{Tr}\, \frac{i(\not k + \frac{1}{2}\not p + m_e)}{(k+\frac{1}{2}p)^2 - m_e^2 + i\epsilon}\, \frac{i(\not k - \frac{1}{2}\not p + m_e)}{(k-\frac{1}{2}p)^2 - m_e^2 + i\epsilon}, \tag{3.1}$$

$$I_u = \frac{-3}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(-\frac{1}{2}ig\frac{m_u}{m_W}\right)^2 \mathrm{Tr}\, \frac{i(\not k + \frac{1}{2}\not p + m_u)}{(k+\frac{1}{2}p)^2 - m_u^2 + i\epsilon}\, \frac{i(\not k - \frac{1}{2}\not p + m_u)}{(k-\frac{1}{2}p)^2 - m_u^2 + i\epsilon}, \tag{3.2}$$

$$I_d = \frac{-3}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(-\frac{1}{2}ig\frac{m_d}{m_W}\right)^2 \mathrm{Tr}\, \frac{i(\not k + \frac{1}{2}\not p + m_d)}{(k+\frac{1}{2}p)^2 - m_d^2 + i\epsilon}\, \frac{i(\not k - \frac{1}{2}\not p + m_d)}{(k-\frac{1}{2}p)^2 - m_d^2 + i\epsilon}, \tag{3.3}$$

from the second set they are

$$\begin{aligned} I_{Z\phi_0} &= \frac{1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(\frac{g}{2\cos\theta_W}(\tfrac{3}{2}p - k)_\mu\right)\left(-\frac{g}{2\cos\theta_W}(\tfrac{3}{2}p-k)_\nu\right) \\ &\times \frac{i}{(\frac{1}{2}p+k)^2 - m_Z^2 + i\epsilon}\left[-g^{\mu\nu} + \frac{(1-\xi)(\frac{1}{2}p+k)^\mu(\frac{1}{2}p+k)^\nu}{(\frac{1}{2}p+k)^2 - \xi m_Z^2 + i\epsilon}\right]\frac{i}{(\frac{1}{2}p-k)^2 - \xi m_Z^2 + i\epsilon}, \end{aligned} \tag{3.4}$$

$$\begin{aligned} I_{W^-\phi^+} &= I_{W^+\phi^-} \\ &= \frac{1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left[\tfrac{1}{2}ig(\tfrac{3}{2}p-k)_\mu\right]\left[\tfrac{1}{2}ig(\tfrac{3}{2}p-k)_\nu\right] \\ &\times \frac{i}{(\frac{1}{2}p+k)^2 - m_W^2 + i\epsilon}\left[-g^{\mu\nu} + \frac{(1-\xi)(\frac{1}{2}p+k)^\mu(\frac{1}{2}p+k)^\nu}{(\frac{1}{2}p+k)^2 - \xi m_W^2 + i\epsilon}\right]\frac{i}{(\frac{1}{2}p-k)^2 - \xi m_W^2 + i\epsilon}, \end{aligned} \tag{3.5}$$

and from the third set they are

$$I_H = \frac{1}{(2\pi)^4}\frac{1}{2}\int d^4k\, e^{ik\cdot\delta}\left(-i\frac{3}{4}g^2\frac{m_H^2}{m_W^2}\right)\frac{i}{k^2 - m_H^2 + i\epsilon}, \tag{3.6}$$

$$I_Z = \frac{1}{(2\pi)^4}\frac{1}{2}\int d^4k\, e^{ik\cdot\delta}\left(i\frac{1}{2}g^2\frac{m_Z^2}{m_W^2}\right)\frac{i}{k^2 - m_Z^2 + i\epsilon}\left[-4 + \frac{(1-\xi)k^2}{k^2 - \xi m_Z^2 + i\epsilon}\right], \tag{3.7}$$

$$I_W = \frac{1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}(i\tfrac{1}{2}g^2)\frac{i}{k^2 - m_W^2 + i\epsilon}\left[-4 + \frac{(1-\xi)k^2}{k^2 - \xi m_W^2 + i\epsilon}\right], \tag{3.8}$$

$$I_{\phi^+} = \frac{1}{(2\pi)^4}\int d^4k\, e^{ik\cdot\delta}\left(-i\frac{1}{4}g^2\frac{m_H^2}{m_W^2}\right)\frac{i}{k^2 - \xi m_W^2 + i\epsilon}, \tag{3.9}$$

$$I_{\phi_0} = \frac{1}{(2\pi)^4}\frac{1}{2}\int d^4k\, e^{ik\cdot\delta}\left(-i\frac{1}{4}g^2\frac{m_H^2}{m_W^2}\right)\frac{i}{k^2 - \xi m_Z^2 + i\epsilon}. \tag{3.10}$$

There is no quadratically divergent contribution from the fourth set, namely the diagrams involving Faddeev-Popov ghosts.

If we here only extract the leading, quadratically divergent terms, we find

$$I_e \sim -g^2 \frac{m_e^2}{m_W^2} \frac{1}{(2\pi)^4} \int d^4k\, e^{ik\cdot\delta} \frac{k^2}{[(k+\frac{1}{2}p)^2 - m_e^2 + i\epsilon][(k-\frac{1}{2}p)^2 - m_e^2 + i\epsilon]}, \tag{3.11}$$

$$I_u \sim -3g^2 \frac{m_u^2}{m_W^2} \frac{1}{(2\pi)^4} \int d^4k\, e^{ik\cdot\delta} \frac{k^2}{[(k+\frac{1}{2}p)^2 - m_u^2 + i\epsilon][(k-\frac{1}{2}p)^2 - m_u^2 + i\epsilon]}, \tag{3.12}$$

$$I_d \sim -3g^2 \frac{m_d^2}{m_W^2} \frac{1}{(2\pi)^4} \int d^4k\, e^{ik\cdot\delta} \frac{k^2}{[(k+\frac{1}{2}p)^2 - m_d^2 + i\epsilon][(k-\frac{1}{2}p)^2 - m_d^2 + i\epsilon]}, \tag{3.13}$$

$$\begin{aligned} I_{Z\phi_0} \sim & \frac{g^2}{4\cos^2\theta_W} \frac{1}{(2\pi)^4} \int d^4k\, e^{ik\cdot\delta} \frac{1}{(\frac{1}{2}p+k)^2 - m_Z^2 + i\epsilon} \\ & \times \left[-k^2 + \frac{(1-\xi)(k^2)^2}{(\frac{1}{2}p+k)^2 - \xi m_Z^2 + i\epsilon}\right] \frac{1}{(\frac{1}{2}p-k)^2 - \xi m_Z^2 + i\epsilon}, \end{aligned} \tag{3.14}$$

$$\begin{aligned} I_{W^-\phi^+} &= I_{W^+\phi^-} \\ & \sim \frac{g^2}{4} \frac{1}{(2\pi)^4} \int d^4k\, e^{ik\cdot\delta} \frac{1}{(\frac{1}{2}p+k)^2 - m_W^2 + i\epsilon} \\ & \quad \times \left[-k^2 + \frac{(1-\xi)(k^2)^2}{(\frac{1}{2}p+k)^2 - \xi m_W^2 + i\epsilon}\right] \frac{1}{(\frac{1}{2}p-k)^2 - \xi m_W^2 + i\epsilon}, \end{aligned} \tag{3.15}$$

$$I_H = \frac{3g^2}{8m_W^2} \frac{1}{(2\pi)^4} \int d^4k\, e^{ik\cdot\delta} \frac{m_H^2}{k^2 - m_H^2 + i\epsilon}, \tag{3.16}$$

$$I_Z = -\frac{g^2 m_Z^2}{4m_W^2} \frac{1}{(2\pi)^4} \int d^4k\, e^{ik\cdot\delta} \frac{1}{k^2 - m_Z^2 + i\epsilon}\left[-4 + \frac{(1-\xi)k^2}{k^2 - \xi m_Z^2 + i\epsilon}\right], \tag{3.17}$$

$$I_W = -\frac{g^2}{2} \frac{1}{(2\pi)^4} \int d^4k\, e^{ik\cdot\delta} \frac{1}{k^2 - m_W^2 + i\epsilon}\left[-4 + \frac{(1-\xi)k^2}{k^2 - \xi m_W^2 + i\epsilon}\right], \tag{3.18}$$

$$I_{\phi^+} = \frac{g^2 m_H^2}{4m_W^2} \frac{1}{(2\pi)^4} \int d^4k\, e^{ik\cdot\delta} \frac{1}{k^2 - \xi m_W^2 + i\epsilon}, \tag{3.19}$$

$$I_{\phi_0} = \frac{g^2 m_H^2}{8m_W^2} \frac{1}{(2\pi)^4} \int d^4k\, e^{ik\cdot\delta} \frac{1}{k^2 - \xi m_Z^2 + i\epsilon}. \tag{3.20}$$

In writing down these expressions, we have kept all denominators exact, but retained only the relevant parts of the numerators, i. e., those terms that contribute to the quadratic divergences.

All of these integrals can immediately be reduced to integrals of the form $I^{(1)}(\delta)$ discussed in the previous section, whose leading behavior for small δ is proportional to $1/\delta^2$. This is true also for $I_{Z\phi_0}$, $I_{W^-\phi^+}$ and $I_{W^+\phi^-}$, since

$$\left[-k^2 + \frac{(1-\xi)k^2k^2}{(\frac{1}{2}p+k)^2 - \xi m_Z^2 + i\epsilon}\right]_{|k^2|\to\infty} \sim \; -\xi k^2.$$

Thus, the quadratic divergences are proportional to

$$I = \frac{g^2}{m_W^2}\{-[m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_b^2 + m_t^2)]$$
$$- \tfrac{1}{4}m_Z^2\xi - \tfrac{1}{2}m_W^2\xi + \tfrac{3}{8}m_H^2 + \tfrac{1}{4}(3+\xi)m_Z^2 + \tfrac{1}{2}(3+\xi)m_W^2 + \tfrac{1}{4}m_H^2 + \tfrac{1}{8}m_H^2\}, \quad (3.21)$$

where we have introduced also the contributions from the fermions of the second and third generations. We note that the gauge-dependent terms cancel, and that we are left with an expression proportional to that of the Higgs tadpole encountered for the electron self energy in the preceding section.

4. Self energies in dimensional regularization

The mass relation (1.1) cannot be obtained within the framework of dimensional regularization. This will be illustrated in some detail in the present section.

4.1 Electron self energy

In d dimensions, the tadpole diagrams of fig. 1 contribute

$$T_e = \frac{-1}{(2\pi)^d}\int d^dk \left(-\frac{1}{2}ig\frac{m_e}{m_W}\right)\mathrm{Tr}\frac{i(\not k + m_e)}{k^2 - m_e^2 + i\epsilon}, \quad (4.1)$$

$$T_u = \frac{-3}{(2\pi)^d}\int d^dk \left(-\frac{1}{2}ig\frac{m_u}{m_W}\right)\mathrm{Tr}\frac{i(\not k + m_u)}{k^2 - m_u^2 + i\epsilon}, \quad (4.2)$$

$$T_d = \frac{-3}{(2\pi)^d}\int d^dk \left(-\frac{1}{2}ig\frac{m_d}{m_W}\right)\mathrm{Tr}\frac{i(\not k + m_d)}{k^2 - m_d^2 + i\epsilon}, \quad (4.3)$$

$$T_Z = \frac{1}{(2\pi)^d}\frac{1}{2}\int d^dk \left(ig\frac{m_Z^2}{m_W}\right)\frac{i}{k^2 - m_Z^2 + i\epsilon}\left[-d + (1-\xi)\frac{k^2}{k^2 - \xi m_Z^2 + i\epsilon}\right], \quad (4.4)$$

$$T_{\phi_0} = \frac{1}{(2\pi)^d}\frac{1}{2}\int d^dk \left(-\frac{1}{2}ig\frac{m_H^2}{m_W}\right)\frac{i}{k^2 - \xi m_Z^2 + i\epsilon}, \quad (4.5)$$

$$T_W = \frac{1}{(2\pi)^d}\int d^dk \,(igm_W)\,\frac{i}{k^2 - m_W^2 + i\epsilon}\left[-d + (1-\xi)\frac{k^2}{k^2 - \xi m_W^2 + i\epsilon}\right], \quad (4.6)$$

$$T_{\phi^+} = \frac{1}{(2\pi)^d} \int d^d k \left(-\frac{1}{2} ig \frac{m_H^2}{m_W} \right) \frac{i}{k^2 - \xi m_W^2 + i\epsilon}, \tag{4.7}$$

$$T_H = \frac{1}{(2\pi)^d} \frac{1}{2} \int d^d k \left(-\frac{3}{2} ig \frac{m_H^2}{m_W} \right) \frac{i}{k^2 - m_H^2 + i\epsilon}, \tag{4.8}$$

$$T_{\eta^+} = \frac{-1}{(2\pi)^d} \int d^d k \left(-\tfrac{1}{2} ig\xi m_W \right) \frac{i}{k^2 - \xi m_W^2 + i\epsilon}, \tag{4.9}$$

$$T_{\eta^-} = \frac{-1}{(2\pi)^d} \int d^d k \left(-\tfrac{1}{2} ig\xi m_W \right) \frac{i}{k^2 - \xi m_W^2 + i\epsilon}, \tag{4.10}$$

$$T_{\eta^z} = \frac{-1}{(2\pi)^d} \int d^d k \left(-\frac{1}{2} ig\xi \frac{m_Z^2}{m_W} \right) \frac{i}{k^2 - \xi m_Z^2 + i\epsilon}. \tag{4.11}$$

Note the factors of d in the square brackets of (4.4) and (4.6). They arise from the sum over vector indices,

$$g_{\mu\nu} g^{\mu\nu} = d. \tag{4.12}$$

The contribution to the electron self energy from these tadpole diagrams is thus proportional to

$$\begin{aligned}
T &= T_e + T_\mu + T_\tau + T_u + T_d + T_s + T_c + T_t + T_b \\
&\quad + T_Z + T_{\phi_0} + T_W + T_{\phi^+} + T_H + T_{\eta^+} + T_{\eta^-} + T_{\eta^z} \\
&= \frac{g}{m_W} \frac{1}{(2\pi)^d} \int d^d k \\
&\times \Bigg\{ -\frac{1}{2} f(d) \Bigg[\frac{m_e^2}{k^2 - m_e^2 + i\epsilon} + \frac{m_\mu^2}{k^2 - m_\mu^2 + i\epsilon} + \frac{m_\tau^2}{k^2 - m_\tau^2 + i\epsilon} + \frac{3m_u^2}{k^2 - m_u^2 + i\epsilon} + \frac{3m_d^2}{k^2 - m_d^2 + i\epsilon} \\
&+ \frac{3m_c^2}{k^2 - m_c^2 + i\epsilon} + \frac{3m_s^2}{k^2 - m_s^2 + i\epsilon} + \frac{3m_t^2}{k^2 - m_t^2 + i\epsilon} + \frac{3m_b^2}{k^2 - m_b^2 + i\epsilon} \Bigg] \\
&+ (d-1) \frac{m_W^2}{k^2 - m_W^2 + i\epsilon} + \frac{d-1}{2} \frac{m_Z^2}{k^2 - m_Z^2 + i\epsilon} \\
&+ \frac{3}{4} \frac{m_H^2}{k^2 - m_H^2 + i\epsilon} + \frac{1}{2} \frac{m_H^2}{k^2 - \xi m_W^2 + i\epsilon} + \frac{1}{4} \frac{m_H^2}{k^2 - \xi m_Z^2 + i\epsilon} \Bigg\},
\end{aligned} \tag{4.13}$$

where $f(d) = \mathrm{Tr}[1]$ arises from the trace over γ matrices as represented in d dimensions.

In dimensional regularization, the quadratic divergences manifest themselves as pole singularities at $d = 2$. Thus, in order to make them vanish, one has to impose the condition

$$\begin{aligned}
&\tfrac{1}{4} f(d) \left[m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_b^2 + m_t^2) \right] \\
&= \frac{d-1}{2} m_W^2 + \frac{d-1}{4} m_Z^2 + \frac{3}{4} m_H^2,
\end{aligned} \tag{4.14}$$

evaluated at $d = 2$. This condition is different from (1.1) for any choice of $f(2)$.

4.2 Self energy of H

We next turn to the Higgs self energy, using the same notation as was used in Sec. 3. From the first set of diagrams in fig. 2, the contributions are

$$I_e = \frac{-1}{(2\pi)^d}\int d^dk\left(-\frac{1}{2}ig\frac{m_e}{m_W}\right)^2 \mathrm{Tr}\,\frac{i(\not k+\frac{1}{2}\not p+m_e)}{(k+\frac{1}{2}p)^2-m_e^2+i\epsilon}\,\frac{i(\not k-\frac{1}{2}\not p+m_e)}{(k-\frac{1}{2}p)^2-m_e^2+i\epsilon}, \tag{4.15}$$

$$I_u = \frac{-3}{(2\pi)^d}\int d^dk\left(-\frac{1}{2}ig\frac{m_u}{m_W}\right)^2 \mathrm{Tr}\,\frac{i(\not k+\frac{1}{2}\not p+m_u)}{(k+\frac{1}{2}p)^2-m_u^2+i\epsilon}\,\frac{i(\not k-\frac{1}{2}\not p+m_u)}{(k-\frac{1}{2}p)^2-m_u^2+i\epsilon}, \tag{4.16}$$

$$I_d = \frac{-3}{(2\pi)^d}\int d^dk\left(-\frac{1}{2}ig\frac{m_d}{m_W}\right)^2 \mathrm{Tr}\,\frac{i(\not k+\frac{1}{2}\not p+m_d)}{(k+\frac{1}{2}p)^2-m_d^2+i\epsilon}\,\frac{i(\not k-\frac{1}{2}\not p+m_d)}{(k-\frac{1}{2}p)^2-m_d^2+i\epsilon}, \tag{4.17}$$

from the second set they are

$$\begin{aligned} I_{Z\phi_0} &= \frac{1}{(2\pi)^d}\int d^dk\left(\frac{g}{2\cos\theta_W}(\tfrac{3}{2}p-k)_\mu\right)\left(-\frac{g}{2\cos\theta_W}(\tfrac{3}{2}p-k)_\nu\right)\\ &\times\frac{i}{(\frac{1}{2}p+k)^2-m_Z^2+i\epsilon}\left[-g^{\mu\nu}+\frac{(1-\xi)(\frac{1}{2}p+k)^\mu(\frac{1}{2}p+k)^\nu}{(\frac{1}{2}p+k)^2-\xi m_Z^2+i\epsilon}\right]\frac{i}{(\frac{1}{2}p-k)^2-\xi m_Z^2+i\epsilon}, \end{aligned} \tag{4.18}$$

$$\begin{aligned} I_{W^-\phi^+} &= I_{W^+\phi^-}\\ &= \frac{1}{(2\pi)^d}\int d^dk\left[\tfrac{1}{2}ig(\tfrac{3}{2}p-k)_\mu\right]\left[\tfrac{1}{2}ig(\tfrac{3}{2}p-k)_\nu\right]\\ &\times\frac{i}{(\frac{1}{2}p+k)^2-m_W^2+i\epsilon}\left[-g^{\mu\nu}+\frac{(1-\xi)(\frac{1}{2}p+k)^\mu(\frac{1}{2}p+k)^\nu}{(\frac{1}{2}p+k)^2-\xi m_W^2+i\epsilon}\right]\frac{i}{(\frac{1}{2}p-k)^2-\xi m_W^2+i\epsilon}, \end{aligned} \tag{4.19}$$

and from the third set they are

$$I_H = \frac{1}{(2\pi)^d}\frac{1}{2}\int d^dk\left(-i\frac{3}{4}g^2\frac{m_H^2}{m_W^2}\right)\frac{i}{k^2-m_H^2+i\epsilon}, \tag{4.20}$$

$$I_Z = \frac{1}{(2\pi)^d}\frac{1}{2}\int d^dk\left(i\frac{1}{2}g^2\frac{m_Z^2}{m_W^2}\right)\frac{i}{k^2-m_Z^2+i\epsilon}\left[-d+\frac{(1-\xi)k^2}{k^2-\xi m_Z^2+i\epsilon}\right], \tag{4.21}$$

$$I_W = \frac{1}{(2\pi)^d}\int d^dk\,(i\tfrac{1}{2}g^2)\frac{i}{k^2-m_W^2+i\epsilon}\left[-d+\frac{(1-\xi)k^2}{k^2-\xi m_W^2+i\epsilon}\right], \tag{4.22}$$

$$I_{\phi^+} = \frac{1}{(2\pi)^d} \int d^dk \left(-i\frac{1}{4}g^2\frac{m_H^2}{m_W^2}\right) \frac{i}{k^2 - \xi m_W^2 + i\epsilon}, \tag{4.23}$$

$$I_{\phi_0} = \frac{1}{(2\pi)^d}\,\frac{1}{2} \int d^dk \left(-i\frac{1}{4}g^2\frac{m_H^2}{m_W^2}\right) \frac{i}{k^2 - \xi m_Z^2 + i\epsilon}. \tag{4.24}$$

Again, there is no quadratically divergent contribution from the fourth set.

Extracting the leading, quadratically divergent terms, we find

$$I_e \sim -g^2\,\frac{m_e^2}{m_W^2}\,f(d)\,\frac{1}{(2\pi)^d} \int d^dk\, \frac{k^2}{[(k+\frac{1}{2}p)^2 - m_e^2 + i\epsilon][(k-\frac{1}{2}p)^2 - m_e^2 + i\epsilon]}, \tag{4.25}$$

$$I_u \sim -3g^2\,\frac{m_u^2}{m_W^2}\,f(d)\,\frac{1}{(2\pi)^d} \int d^dk\, \frac{k^2}{[(k+\frac{1}{2}p)^2 - m_u^2 + i\epsilon][(k-\frac{1}{2}p)^2 - m_u^2 + i\epsilon]}, \tag{4.26}$$

$$I_d \sim -3g^2\,\frac{m_d^2}{m_W^2}\,f(d)\,\frac{1}{(2\pi)^d} \int d^dk\, \frac{k^2}{[(k+\frac{1}{2}p)^2 - m_d^2 + i\epsilon][(k-\frac{1}{2}p)^2 - m_d^2 + i\epsilon]}, \tag{4.27}$$

$$\begin{aligned} I_{Z\phi_0} \sim{}& \frac{g^2}{4\cos^2\theta_W}\,\frac{1}{(2\pi)^d} \int d^dk\, \frac{1}{(\frac{1}{2}p+k)^2 - m_Z^2 + i\epsilon} \\ &\times \left[-k^2 + \frac{(1-\xi)(k^2)^2}{(\frac{1}{2}p+k)^2 - \xi m_Z^2 + i\epsilon}\right] \frac{1}{(\frac{1}{2}p-k)^2 - \xi m_Z^2 + i\epsilon}, \end{aligned} \tag{4.28}$$

$$\begin{aligned} I_{W^-\phi^+} &= I_{W^+\phi^-} \\ &\sim \frac{g^2}{4}\,\frac{1}{(2\pi)^d} \int d^dk\, \frac{1}{(\frac{1}{2}p+k)^2 - m_W^2 + i\epsilon} \\ &\quad\times \left[-k^2 + \frac{(1-\xi)(k^2)^2}{(\frac{1}{2}p+k)^2 - \xi m_W^2 + i\epsilon}\right] \frac{1}{(\frac{1}{2}p-k)^2 - \xi m_W^2 + i\epsilon}, \end{aligned} \tag{4.29}$$

$$I_H = \frac{3g^2}{8m_W^2}\,\frac{1}{(2\pi)^d} \int d^dk\, \frac{m_H^2}{k^2 - m_H^2 + i\epsilon}, \tag{4.30}$$

$$I_Z = -\frac{g^2 m_Z^2}{4m_W^2}\,\frac{1}{(2\pi)^d} \int d^dk\, \frac{1}{k^2 - m_Z^2 + i\epsilon}\left[-d + \frac{(1-\xi)k^2}{k^2 - \xi m_Z^2 + i\epsilon}\right], \tag{4.31}$$

$$I_W = -\frac{g^2}{2}\,\frac{1}{(2\pi)^d} \int d^dk\, \frac{1}{k^2 - m_W^2 + i\epsilon}\left[-d + \frac{(1-\xi)k^2}{k^2 - \xi m_W^2 + i\epsilon}\right], \tag{4.32}$$

$$I_{\phi^+} = \frac{g^2 m_H^2}{4m_W^2}\,\frac{1}{(2\pi)^d} \int d^dk\, \frac{1}{k^2 - \xi m_W^2 + i\epsilon}, \tag{4.33}$$

$$I_{\phi_0} = \frac{g^2 m_H^2}{8m_W^2}\,\frac{1}{(2\pi)^d} \int d^dk\, \frac{1}{k^2 - \xi m_Z^2 + i\epsilon}. \tag{4.34}$$

Again we collect all integrands, and demand that the over-all integrand vanish. It is proportional to

$$I = \frac{g^2}{m_W^2}\{-\tfrac{1}{4}f(d)[m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_b^2 + m_t^2)]$$
$$- \tfrac{1}{4}m_Z^2\xi - \tfrac{1}{2}m_W^2\xi + \tfrac{3}{8}m_H^2 + \tfrac{1}{4}(d-1+\xi)m_Z^2 + \tfrac{1}{2}(d-1+\xi)m_W^2\xi + \tfrac{1}{4}m_H^2 + \tfrac{1}{8}m_H^2\}, \tag{4.35}$$

which must vanish if the Higgs self energy is to be free of quadratic divergences. This condition is seen to be the same as (4.14).

5. Comparison with the Derivation of Veltman

It is perhaps instructive to compare the present derivation of the mass relation (1.1) with the one given by Veltman over ten years ago [7].

(1) So far as we know, Ferrara, Girardello and Palumbo [10] are the first ones to derive a quadratic mass formula. They consider the soft breaking of a supersymmetric theory, and find that, under very general conditions, the masses satisfy the relation

$$\sum_J (-1)^{2J}(2J+1)m_J^2 = 0. \tag{5.1}$$

Dimensional regularization plays no role in this work. Suppose this mass formula is applied to the Standard Model, then one gets

$$m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_b^2 + m_t^2) = \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{1}{4}m_H^2.$$

Note that this mass formula differs from that of (1.1) only by a factor of 3 in the last term.

(2) Veltman's paper appeared a year later. In this work, he used dimensional regularization and chose throughout the dimension of the Dirac matrices to be 4, independent of the space-time dimension, i.e.,

$$f(d) = 4 \tag{5.2}$$

Thus the result (4.14) with dimensional regularization becomes

$$m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_b^2 + m_t^2)$$
$$= \frac{d-1}{2}m_W^2 + \frac{d-1}{4}m_Z^2 + \frac{3}{4}m_H^2, \tag{5.3}$$

Since this is obtained by setting the residue of the pole at $d = 2$ to zero, there is no choice, within dimensional regularization, but to let d be 2 on the right-hand side of (5.3). However, Veltman argued, using supersymmetry [11], that the d in (5.3) should be 4. This is based on the work of Siegel [12] and Capper, Jones and Van Nieuwenhuizen [13]. However, it is not clear what supersymmetry has to do with the non-supersymmetric Standard Model. Indeed, it is the power of point-splitting regularization that the mass relation (1.1) is obtained directly without reference to any theory outside of the Standard Model.

(3) There is another major difference between dimensional regularization and point-splitting regularization. Consider the case of the electron self-mass. In QED, the electron self-mass has only a logarithmic divergence: a pole at $d = 4$ with dimensional regularization and a $\ln \delta$ term with point-splitting regularization. For the Standard Model, as we have seen, there are quadratic divergences: a pole at $d = 2$ with dimensional regularization and a δ^{-2} term with point-splitting regularization. With dimensional regularization, the pole at $d = 2$ and the pole at $d = 4$ have nothing to do with each other, and hence one cannot be used to cancel the other. This is quite difference with point-splitting regularization: in the Standard Model, it is possible to go further to make the electron self-mass finite. The point here is that, when the point splitting δ is introduced, the various coupling constants must also be allowed to depend on δ; it is only necessary for the deviations in the coupling constants to approach zero as δ approaches zero.

(4) In view of the above points, it is now possible to proceed further. The point (3) implies that it does not make any sense to try to get another relation by setting the coefficient of the $\ln \delta$ term to zero in the electron self-mass, since this is automatically accomplished. The point (2) implies that, although there is only one mass relation, the one expressed by (1.1), that can be gleaned from supersymmetry, there may well be other relations that are present within the Standard Model. Indeed there is, and this second mass relation is to be derived in Paper III.

6. Concluding remarks

Quadratic mass relations apparently first appeared in the context of supersymmetric theories that are softly broken [10]. When the mass relation from supersymmetry is applied to the Standard Model, the result is very similar to (1.1), the difference being a factor of 3 in one of the terms. As the Standard Model [1] became generally accepted around 1980, a number of groups independently revived the original idea of Stückelberg [14] to determine conditions so that the self energies are finite. Some of the early work can be found in ref. [15]. Shortly thereafter, the quadratic mass relation (1.1) for the Standard Model was first given by Veltman [7] who made use of supersymmetry. We note that it cannot be obtained from dimensional regularization. In that approach quadratic divergences appear as poles in the extrapolation of the amplitudes to two dimensions. However, in two dimensions Lorentz indices can only take two values, and therefore expressions like $k^\mu k^\nu$ effectively become replaced by $\frac{1}{2}k^2 g^{\mu\nu}$ instead of $\frac{1}{4}k^2 g^{\mu\nu}$ in four dimensions. Similarly, the trace over Dirac matrices is ambiguous. When point-splitting regularization is used in the way discussed here, these problems do not appear and (1.1) is obtained directly. It is impressive that Veltman, who is one of the originators of dimensional regularization, and has consistently advocated for its use, has abandoned dimensional regularization in this particular case because of its failure to give the correct answer. This problem is also addressed by Capdequi Peyranère, Montero and Moultaka [16].

We are also aware of some more recent work along these lines by Decker and Pestieau [17], and by Lee and Drell [18].

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Figure captions

Fig. 1. One-loop contributions to the electron self-energy.

Fig. 2. One-loop contributions to the Higgs self-energy. The tadpole diagrams are not shown.

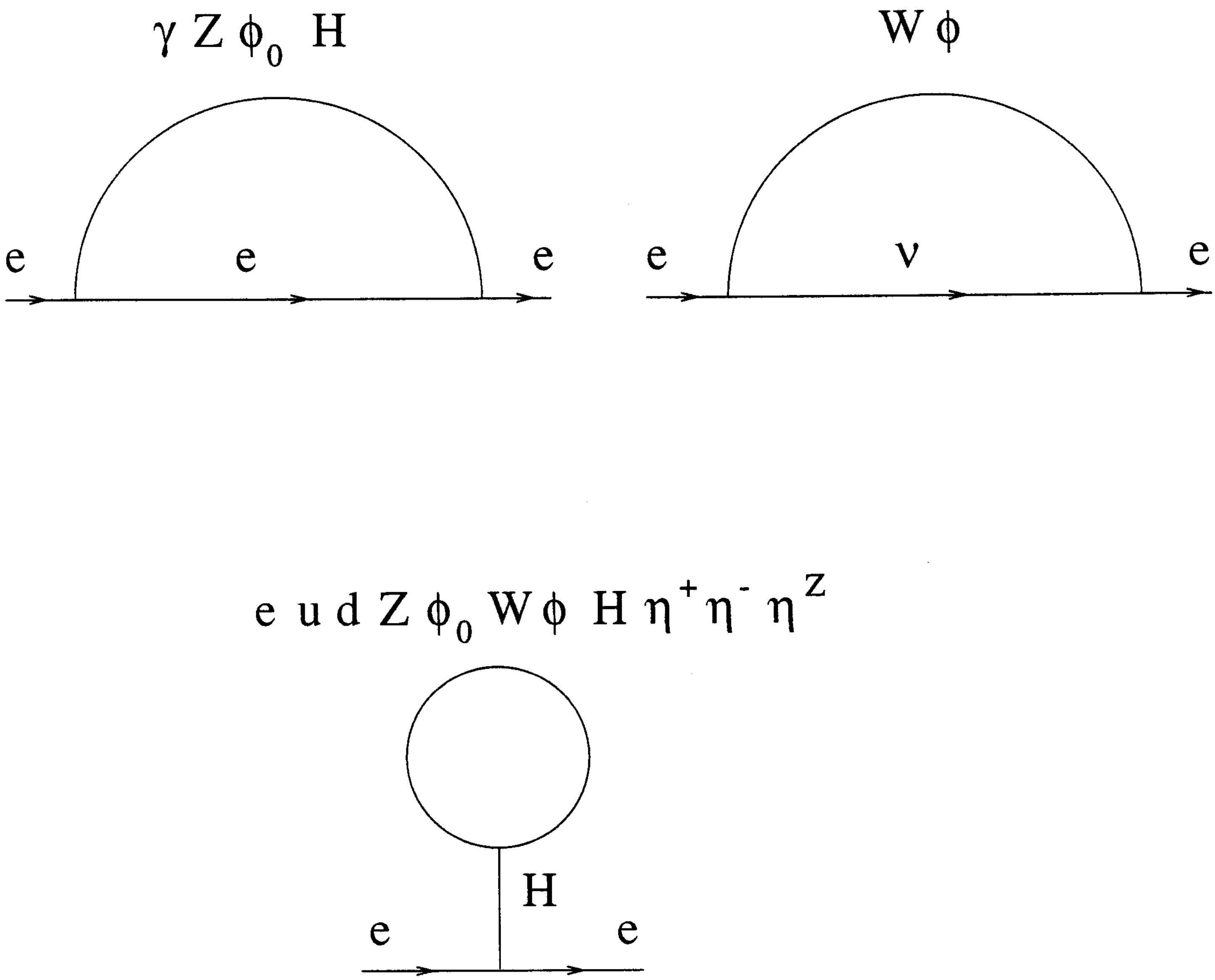

Figure 1

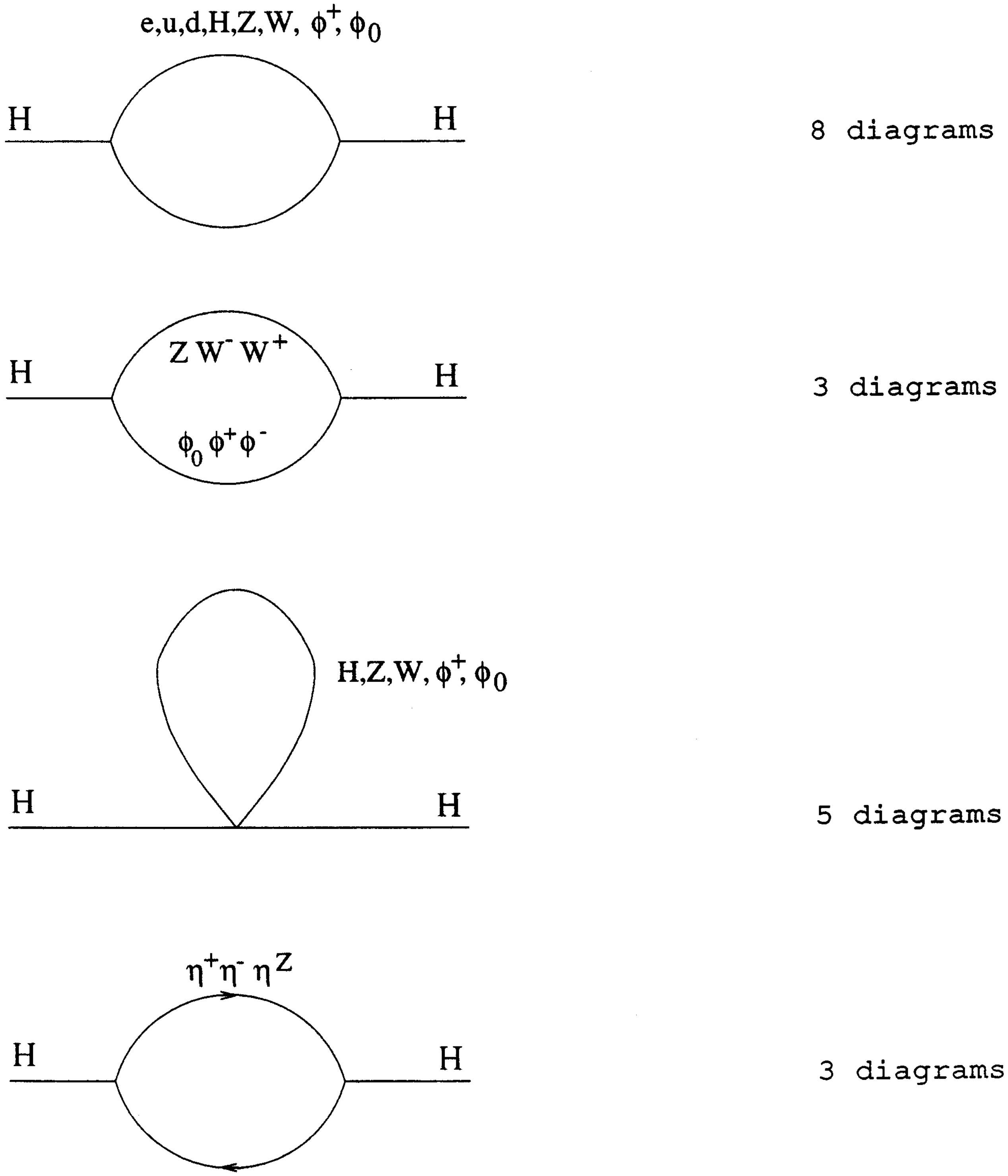
e,u,d,H,Z,W, φ+, φ0
H
H
8 diagrams
Z W- W+
H
H
φ0 φ+ φ-
3 diagrams
H,Z,W, φ+, φ0
H
H
5 diagrams
η+ η- ηZ
H
H
3 diagrams

Figure 2