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Parameters of Butterworth, Tschebyscheff, and Elliptic Prototype Reference Transfer Functions in Discrete-Time Frequency Transformations

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Abstract- Recently the authors proposed a direct discrete-time frequency transformation technique whereby a normalized lowpass prototype reference transfer function can be transformed into a denormalized lowpass, highpass, bandpass, or bandstop transfer function with specified passband as well as stopband frequency edges. This resulted in the development of a general constraint relationship between the passband and stopband edge frequencies of the lowpass prototype reference transfer function. In this paper, the constraint relationship thus obtained is exploited and applied to the determination of the discrete-time prototype reference transfer function in the special cases where the final denormalized discrete-time transfer functions are required to exhibit Butterworth, Tschebyscheff, or elliptic loss-frequency characteristics.

I. INTRODUCTION

There exist three distinct steps involved in the hitherto approach to the derivation of denormalized lowpass (LP), highpass (HP), bandpass (BP), and bandstop (BS) discrete-time transfer functions. These steps are [1]:

Step 1: By employing the classical analog transfer function approximation techniques, a suitable normalized lowpass continuous-time prototype reference transfer function $H_{NC}(s)$ is derived, where s represents the normalized complex frequency variable in the continuous domain.

Step 2: Through the application of the conventional analog frequency transformation techniques, the continuous-time prototype reference transfer function $H_{NC}(s)$ is transformed into a denormalized LP, HP, BP, or BS continuous-time transfer function $H_{XC}(\bar{s})$, where $\bar{s}$ represents the denormalized complex frequency variable in the continuous domain.

Step 3: The transfer function $H_{XC}(\bar{s})$ is transformed into a corresponding denormalized discrete-time transfer function $H_{XD}(\bar{z})$ through the application of the bilinear $\bar{s}$-to-$\bar{z}$ frequency transformation in accordance with

$$H_{XD}(\bar{z}) = H_{XC}(\bar{s})\Big|_{\bar{s} = \frac{\bar{z}-1}{\bar{z}+1}}, \tag{1}$$

where $\bar{z}$ represents the denormalized complex frequency variable in the discrete domain.

An alternative approach to the derivation of denormalized discrete-time transfer functions $H_{XD}(\bar{z})$ is presented in this paper. The proposed approach is based on performing the necessary LP-to-LP, LP-to-HP, LP-to-BP, and LP-to-BS frequency transformations in the discrete domain. Specifically, this approach proceeds in the following three steps:

Step 1: A suitable normalized lowpass continuous-time prototype reference transfer function $H_{NC}(s)$ is derived as in the above approach.

Step 2: The continuous-time prototype reference transfer function $H_{NC}(s)$ is transformed into a corresponding discrete-time prototype reference transfer function $H_{ND}(z)$ through the application of the bilinear s-to-z frequency transformation in accordance with

$$H_{ND}(z) = H_{NC}(s)\Big|_{s = \frac{z-1}{z+1}}, \tag{2}$$

where z represents the normalized complex frequency variable in the discrete domain.

Step 3: Through the application of the discrete-time frequency transformation technique in [2], the prototype reference transfer function $H_{ND}(z)$ is transformed into a denormalized LP, HP, BP, or BS discrete-time transfer function $H_{XC}(\bar{z})$.

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In Section II a review is given of the general constraint relationship that exists between the passband and stopband frequency edges associated with the discrete-time prototype reference transfer function $H_{ND}(z)$. In Section III, it is shown that a similar constraint relationship exists between the passband and stopband frequency edges of the corresponding continuous-time prototype reference transfer function $H_{NC}(s)$. The latter constraint relationship is exploited in Section IV to arrive at denormalized LP, HP, BP, and BS discrete-time transfer functions $H_{XD}(\bar{z})$ having Butterworth, Tschebyscheff, and elliptic loss-frequency characteristics. The parameters of particular interest include the required order for the transfer function $H_{ND}(z)$ as well as the required passband and stopband frequency edges for its loss-frequency characteristic.

II. CONSTRAINTS ON THE PASSBAND AND STOPBAND FREQUENCY EDGES OF DISCRETE-TIME PROTOTYPE REFERENCE TRANSFER FUNCTION

Let $H_{ND}(z)$ represent a normalized lowpass discrete-time prototype reference transfer function characterized by means of a logarithmic loss-frequency

$$A_{ND}(\Omega) = 10\log\frac{1}{|H_{ND}(e^{j\Omega T})|^2}, \tag{3}$$

satisfying general specifications of the form

$$0 \le A_{ND}(\Omega) \le A_p \quad \text{for} \quad 0 \le |\Omega| \le \Omega_p,$$

$$A_{ND}(\Omega) \ge A_a \quad \text{for} \quad \Omega_a \le |\Omega| \le \pi/T. \tag{4}$$

where Ω represents the z-domain real frequency variable, and T represents the sampling period. In these relationships, Ω_p represents the passband frequency edge and A_p the maximum passband ripple, while Ω_a represents the stopband frequency edge and A_a the minimum passband loss associated with $H_{ND}(z)$.

The discrete-time prototype reference transfer function $H_{ND}(z)$ can be transformed into a denormalized LP, HP, BP, or BS transfer function $H_{XD}(\bar{z})$ by using a transformation of the form [2]

$$H_{XD}(\bar{z}) = H_{ND}(z)\Big|_{z = f_{XD}(\bar{z})}. \tag{5}$$

In the case of a LP-to-LP frequency transformation, the loss-frequency characteristic

$$A_{XD}(\bar{\Omega}) = 10\log\frac{1}{|H_{XD}(e^{j\bar{\Omega}T})|^2}, \tag{6}$$

is required to satisfy general frequency specifications of the form

$$0 \le A_{XD}(\bar{\Omega}) \le A_p \quad \text{for} \quad 0 \le |\bar{\Omega}| \le \bar{\Omega}_p,$$

$$A_{XD}(\bar{\Omega}) \ge A_a \quad \text{for} \quad \bar{\Omega}_a \le |\bar{\Omega}| \le \pi/T, \tag{7}$$

while in the case of a LP-to-HP frequency transformation it is required to satisfy frequency specifications of the form

$$A_{XD}(\bar{\Omega}) \ge A_a \quad \text{for} \quad 0 \le |\bar{\Omega}| \le \bar{\Omega}_a,$$

$$0 \le A_{XD}(\bar{\Omega}) \le A_p \quad \text{for} \quad \bar{\Omega}_p \le |\bar{\Omega}| \le \pi/T, \tag{8}$$

where $\tilde{\Omega}_p$ represents the specified passband edge frequency and $\tilde{\Omega}_a$ represents the specified stopband edge frequency associated with $A_{XD}(\bar{\Omega})$. In the case of a *LP*-to-*BP* frequency transformation, on the other hand, the frequency specifications to be satisfied by the loss-frequency characteristic $A_{XD}(\bar{\Omega})$ are of the general form

$$0 \le A_{XD}(\bar{\Omega}) \le A_p \quad \text{for} \quad \tilde{\Omega}_{pl} \le |\bar{\Omega}| \le \tilde{\Omega}_{pu} ,$$

$$A_{XD}(\bar{\Omega}) \ge A_a \quad \text{for} \quad \begin{cases} 0 \le |\bar{\Omega}| \le \tilde{\Omega}_{al} \\ \tilde{\Omega}_{au} \le |\bar{\Omega}| \le \pi/T \end{cases}, \tag{9}$$

while in the case of a *LP*-to-*BS* frequency transformation the specifications to be satisfied are of the form

$$A_{XD}(\bar{\Omega}) \ge A_a \quad \text{for} \quad \tilde{\Omega}_{al} \le |\bar{\Omega}| \le \tilde{\Omega}_{au} ,$$

$$0 \le A_{XD}(\bar{\Omega}) \le A_p \quad \text{for} \quad \begin{cases} 0 \le |\bar{\Omega}| \le \tilde{\Omega}_{pl} \\ \tilde{\Omega}_{pu} \le |\bar{\Omega}| \le \pi/T \end{cases}, \tag{10}$$

where $\tilde{\Omega}_{pl}$ ($\tilde{\Omega}_{pu}$) represents the specified lower (upper) passband frequency edge, and $\tilde{\Omega}_{al}$ ($\tilde{\Omega}_{au}$) represents the specified lower (upper) stopband frequency edge associated with $A_{XD}(\bar{\Omega})$. It has been shown in [3] that if the passband and stopband edge frequencies Ω_p and Ω_a associated with $H_{XD}(z)$ satisfy the conditions given in Table 1, and if the function $f_{XD}(\bar{z})$ is selected as given in Table 2 [2], then the application of the discrete-time frequency transformation in (5) to the prototype reference transfer function $H_{ND}(z)$ leads to denormalized *LP*, *HP*, *BP*, and *BS* transfer functions $H_{XD}(\bar{z})$ satisfying the specifications in (7), (8), (9), and (10), respectively.

TABLE 1

X	*Constraint*
LP	$\tan(\Omega_a T/2) \le \dfrac{\tan(\Omega_p T/2)}{K_0}$
HP	$\tan(\Omega_p T/2) \ge \dfrac{\tan(\Omega_a T/2)}{K_0}$
BP	$\tan(\Omega_a T/2) \le \begin{cases} \dfrac{\tan(\Omega_p T/2)}{K_1} & \text{if } \alpha \ge \alpha' \\ \dfrac{\tan(\Omega_p T/2)}{K_2} & \text{if } \alpha < \alpha' \end{cases}$
BS	$\tan(\Omega_p T/2) \ge \begin{cases} \dfrac{\tan(\Omega_a T/2)}{K_1} & \text{if } \alpha \ge \alpha' \\ \dfrac{\tan(\Omega_a T/2)}{K_2} & \text{if } \alpha < \alpha' \end{cases}$

$$K_0 = \frac{\tan(\tilde{\Omega}_p T/2)}{\tan(\tilde{\Omega}_a T/2)}$$

$$K_1 = \frac{\sin(\tilde{\Omega}_{al} T)}{\cos(\tilde{\Omega}_{al} T) - \alpha} \tan[(\tilde{\Omega}_{pu} - \tilde{\Omega}_{pl})T/2]$$

$$K_2 = \frac{\sin(\tilde{\Omega}_{au} T)}{\cos(\tilde{\Omega}_{au} T) - \alpha} \tan[(\tilde{\Omega}_{pu} - \tilde{\Omega}_{pl})T/2]$$

$$\alpha' = \frac{\cos[(\tilde{\Omega}_{au} + \tilde{\Omega}_{al})T/2]}{\cos[(\tilde{\Omega}_{au} - \tilde{\Omega}_{al})T/2]}$$

III. Constraints on the Passband and Stopband Frequency Edges of Continuous-Time Prototype Reference Transfer Function

Let $H_{NC}(s)$ represent a normalized lowpass continuous-time prototype reference transfer function, let $H_{ND}(z)$ represent a corresponding discrete-time prototype reference transfer function, and let $H_{NC}(s)$ be related to $H_{ND}(z)$ through the bilinear s-to-z frequency transformation in (2). Then, the logarithmic loss-frequency characteristic

$$A_{NC}(\omega) = 10 \log \frac{1}{|H_{NC}(j\omega)|^2} , \tag{11}$$

associated with $H_{NC}(s)$ is related to the corresponding characteristic $A_{ND}(\Omega)$ associated with $H_{ND}(z)$ in accordance with

$$A_{NC}(\omega) = A_{ND}(\Omega)\Big|_{\omega = \tan\frac{\Omega T}{2}} , \tag{12}$$

where ω represents the normalized real frequency variable in the continuous domain. Consequently, the loss-frequency characteristic $A_{NC}(\omega)$ satisfies general specifications of the form

$$0 \le A_{NC}(\omega) \le A_p \quad \text{for} \quad 0 \le |\omega| \le \omega_p ,$$

$$A_{NC}(\omega) \ge A_a \quad \text{for} \quad \omega_a \le |\omega| < \infty , \tag{13}$$

where

$$\omega_p = \tan\frac{\Omega_p T}{2} \tag{14}$$

represents the passband edge frequency, and

$$\omega_a = \tan\frac{\Omega_a T}{2} \tag{15}$$

represents the stopband edge frequency associated with the continuous-time prototype reference transfer function $H_{NC}(s)$.

From (14) and (15),

$$\frac{\omega_a}{\omega_p} = \frac{\tan\frac{\Omega_a T}{2}}{\tan\frac{\Omega_a T}{2}} . \tag{16}$$

It is important to note that in accordance with the results in Table 1,

TABLE 2

X	$f_X(\bar{z})$	α	β
LP	$\dfrac{\bar{z} - \alpha}{1 - \alpha\bar{z}}$	$\dfrac{\sin[(\Omega_p - \tilde{\Omega}_p)T/2]}{\sin[(\Omega_p + \tilde{\Omega}_p)T/2]}$	–
HP	$-\dfrac{\bar{z} - \alpha}{1 - \alpha\bar{z}}$	$\dfrac{\cos[(\Omega_p - \tilde{\Omega}_p)T/2]}{\cos[(\Omega_p + \tilde{\Omega}_p)T/2]}$	–
BP	$-\dfrac{\bar{z}^2 - \frac{2\alpha\beta}{\beta+1}\bar{z} + \frac{\beta-1}{\beta+1}}{1 - \frac{2\alpha\beta}{\beta+1}\bar{z} + \frac{\beta-1}{\beta+1}\bar{z}^2}$	$\dfrac{\cos[(\tilde{\Omega}_{pu} + \tilde{\Omega}_{pl})T/2]}{\cos[(\tilde{\Omega}_{pu} - \tilde{\Omega}_{pl})T/2]}$	$\dfrac{\tan(\Omega_p T/2)}{\tan[(\tilde{\Omega}_{pu} - \tilde{\Omega}_{pl})T/2]}$
BS	$\dfrac{\bar{z}^2 - \frac{2\alpha}{\beta+1}\bar{z} + \frac{1-\beta}{1+\beta}}{1 - \frac{2\alpha}{\beta+1}\bar{z} + \frac{1-\beta}{1+\beta}\bar{z}^2}$	$\dfrac{\cos[(\tilde{\Omega}_{pu} + \tilde{\Omega}_{pl})T/2]}{\cos[(\tilde{\Omega}_{pu} - \tilde{\Omega}_{pl})T/2]}$	$\dfrac{\tan(\Omega_p T/2)}{\cot[(\tilde{\Omega}_{pu} - \tilde{\Omega}_{pl})T/2]}$

$$\frac{\tan\frac{\Omega_a T}{2}}{\tan\frac{\Omega_p T}{2}} \le \frac{1}{K}, \quad (17)$$

where the parameter K is defined in Table 3 for the cases of denormalized LP, HP, BP, and BS discrete-time transfer functions $H_{XD}(\bar{z})$. Therefore, from (16) and (17)

$$\frac{\omega_a}{\omega_p} \le \frac{1}{K}, \quad (18)$$

which implies that there exists a constraint relationship between ω_p and ω_a.

The constraint relationship in (18) is exploited in the following section to arrive at the parameters of the discrete-time prototype reference transfer function $H_{ND}(z)$ required for denormalization into discrete-time LP, HP, BP, and BS transfer functions $H_{XD}(\bar{z})$ having Butterworth, Tschebyscheff, and elliptic loss-frequency characteristics.

TABLE 3

X	LP	HP	BP	BS
K	K_0	$\frac{1}{K_0}$	K_1 if $\alpha \ge \alpha'$; K_2 if $\alpha < \alpha'$	$\frac{1}{K_1}$ if $\alpha \ge \alpha'$; $\frac{1}{K_2}$ if $\alpha < \alpha'$

IV. Parameters of Discrete-time Prototype Reference Transfer Function

A. Parameters Associated with Denormalized Discrete-Time Butterworth Transfer Functions

The loss-frequency characteristic $A_{NC}(\omega)$ associated with a continuous-time Butterworth prototype reference transfer function $H_{NC}(s)$ is of the form [1]:

$$A_{NC}(\omega) = 10\log[1 + \varepsilon^2 \omega^{2n}], \quad (19)$$

where n represents the order of the transfer function $H_{NC}(s)$, and where

$$\varepsilon^2 = 10^{\frac{A_p}{10}} - 1. \quad (20)$$

In accordance with (19),

$$A_p = 10\log[1 + \varepsilon^2 \omega_p^{2n}], \quad (21)$$

and

$$A_a = 10\log[1 + \varepsilon^2 \omega_a^{2n}], \quad (22)$$

From (21),

$$\omega_p = 1, \quad (23)$$

From (23) and (18),

$$\omega_a \le \frac{1}{K}, \quad (24)$$

which provides a means of selecting a value for ω_a. But, from (22) and (20), one can obtain

$$\omega_a = D^{\frac{1}{2n}}, \quad (25)$$

where

$$D = \frac{10^{\frac{A_a}{10}} - 1}{10^{\frac{A_p}{10}} - 1}. \quad (26)$$

Therefore, in accordance with (25) and (24),

$$n \ge \frac{\log D}{2\log\frac{1}{K}}. \quad (27)$$

Furthermore, from (23) and (14),

$$\Omega_p T = \frac{\pi}{2}, \quad (28)$$

and from (25) and (15),

$$\Omega_a T = 2\tan^{-1} D^{\frac{1}{2n}}. \quad (29)$$

Finally, if the passband and stopband edge frequencies Ω_p and Ω_a have been fixed in accordance with (28) and (29), and if the order n has been fixed in accordance with (27), then (29) can be used to get

$$D = \left(\frac{1 - \cos\Omega_a T}{1 + \cos\Omega_a T}\right)^n. \quad (30)$$

In accordance with (30) and (26), there exist a relationship between the maximum passband ripple A_p and the minimum stopband loss A_a. Therefore, if the maximum passband ripple A_p is kept at its specified value, then, in accordance with (30), the actual minimum stopband loss achieved is given by

$$\hat{A}_a = 10\log[1 + (10^{\frac{A_p}{10}} - 1)\left(\frac{1 - \cos\Omega_a T}{1 + \cos\Omega_a T}\right)^n], \quad (31)$$

where $\hat{A}_a \ge A_a$ and improves on the specified value A_a. Conversely, it is possible to fix A_a at its specified value and determine the actual A_p. In this paper, it is always assumed that A_p is kept fixed at its specified value.

B. Parameters Associated with Denormalized Discrete-Time Tschebyscheff Transfer Functions

The loss-frequency characteristic $A_{NC}(\omega)$ associated with a continuous-time Tschebyscheff prototype reference transfer function $H_{NC}(s)$ is given by [1]:

$$A_{NC}(\omega) = 10\log[1 + \varepsilon^2 T_n^2(\omega)], \quad (32)$$

where

$$T_n(\omega) = \begin{cases} \cos[n\cos^{-1}(\omega)] & \text{for } 0 \le \omega \le 1, \\ \cosh[n\cosh^{-1}(\omega)] & \text{for } 1 \le \omega \le \infty, \end{cases} \quad (33)$$

and where n represents the order of $H_{NC}(s)$. In accordance with (32) and (33), one may write

$$A_p = 10\log[1 + \varepsilon^2 \cosh^2(n\cosh^{-1}\omega_p)], \quad (34)$$

and

$$A_a = 10\log[1 + \varepsilon^2 \cosh^2(n\cosh^{-1}\omega_a)]. \quad (35)$$

Then, (34) leads to

$$\omega_p = 1. \quad (36)$$

From (36), and (18),

$$\omega_a \le \frac{1}{K}, \quad (37)$$

providing a means of fixing a value for ω_a. But, from (35),

$$\omega_a = \cosh[\frac{1}{n}\cosh^{-1}\sqrt{D}]. \quad (38)$$

Therefore, in accordance with (38) and (37),

$$n \ge \frac{\cosh^{-1}\sqrt{D}}{\cosh^{-1}\frac{1}{K}}. \quad (39)$$

Furthermore, from (36) and (14),

$$\Omega_p T = \frac{\pi}{2}, \quad (40)$$

and from (38) and (15),

$$\Omega_a T = 2\tan^{-1}[\cosh(\frac{1}{n}\cosh^{-1}\sqrt{D})] . \quad (41)$$

Finally, if Ω_p and Ω_a have been fixed in accordance with (40) and (41), and if n has been fixed in accordance with (39), then (41) can be used to get

$$D = \frac{1 + T_n(\frac{1 - 3\cos\Omega_a T}{1 + \cos\Omega_a T})}{2} . \quad (42)$$

Therefore, from (42) and (26), if A_p is kept at its specified value, then

$$\hat{A}_a = 10\log[1 + (10^{\frac{A_p}{10}} - 1)\frac{1 + T_n(\frac{1 - 3\cos\Omega_a T}{1 + \cos\Omega_a T})}{2}] . \quad (43)$$

C. Parameters Associated with Denormalized Discrete-Time Elliptic Transfer Functions

The passband and stopband edge frequencies of the loss-frequency characteristic $A_{NC}(\omega)$ associated with the continuous-time elliptic prototype reference transfer function $H_{NC}(s)$ can be selected as:

$$\omega_p = \sqrt{k} , \quad (44)$$

and

$$\omega_a = \frac{1}{\sqrt{k}} , \quad (45)$$

where $0 < k < 1$. Then, the order n can be determined as [1]:

$$n \geq \frac{\log 16D}{\log\frac{1}{q}} , \quad (46)$$

where

$$q = e + 2e^5 + 15e^9 + 150e^{13} + 1707e^{17} + \cdots \quad (47)$$

is a rapidly convergent series in e, and where

$$e = \frac{1}{2}\,\frac{1 - \sqrt{1 - k^2}}{1 + \sqrt{1 + k^2}} . \quad (48)$$

By substituting (44) and (45) into (18), one can get

$$k \geq K , \quad (49)$$

which provides a means of fixing a value for k. Furthermore, from (44) and (14),

$$\Omega_p T = \tan^{-1}\sqrt{k} , \quad (50)$$

and from (45) and (15),

$$\Omega_a T = \tan^{-1}\frac{1}{\sqrt{k}} , \quad (51)$$

Finally, if the passband and stopband edge frequencies Ω_p and Ω_a have been fixed in accordance with (50) and (51), and if the order n has been fixed in accordance with (46), then (46) can be used to get

$$D = \frac{(\frac{1}{q})^n}{16} . \quad (52)$$

Therefore, from (52) and (26), if the maximum passband ripple A_p is fixed at its specified value, then the actual minimum stopband loss achieved is given by

$$\hat{A}_a = 10\log[1 + (10^{\frac{A_a}{10}} - 1)\frac{16}{(\frac{1}{q})^n}] , \quad (53)$$

where $\hat{A}_a \geq A_a$ and improves on the specified value A_a.

The main results concerning the parameters of the discrete-time prototype reference transfer function $H_{ND}(z)$ are summarized in Table 4 for the cases of denormalized LP, HP, BP, and BS discrete-time transfer functions $H_{XD}(\bar{z})$ having Butterworth, Tschebyscheff, and elliptic loss-frequency characteristics.

TABLE 4

X	Case		
	Butterworth	*Tschebyscheff*	*elliptic*
	$n \geq \frac{\log D}{2\log\frac{1}{K}}$	$n \geq \frac{\cosh^{-1}\sqrt{D}}{\cosh^{-1}\frac{1}{K}}$	$n \geq \frac{\log 16D}{\log\frac{1}{q}}$
	$\Omega_p T = \frac{\pi}{2}$ $\Omega_a T = 2\tan^{-1}\frac{1}{\omega_a}$	$\Omega_p T = \frac{\pi}{2}$ $\Omega_a T = 2\tan^{-1}\frac{1}{\omega_a}$	$\Omega_p T = 2\tan^{-1}\frac{1}{\omega_a}$ $\Omega_a T = 2\tan^{-1}\omega_a$
	$\omega_a \leq \frac{1}{K}$	$\omega_a \leq \frac{1}{K}$	$\omega_a \leq \frac{1}{\sqrt{K}}$
LP	$K = K_0$		
HP	$K = \frac{1}{K_0}$		
BP	$K = K_1$ if $\alpha \geq \alpha'$; $K = K_2$ if $\alpha < \alpha'$		
BS	$K = \frac{1}{K_1}$ if $\alpha \geq \alpha'$; $K = \frac{1}{K_2}$ if $\alpha < \alpha'$		

V. Conclusions

Explicit relationships have been presented for the parameters of the normalized lowpass discrete-time prototype reference transfer function suitable for the derivation of denormalized lowpass, highpass, bandpass, and bandstop discrete-time transfer functions having Butterworth, Tschebyscheff, and elliptic loss-frequency characteristics. These parameters include the required order of the discrete-time prototype reference transfer function as well as the required passband and stopband edge frequencies for its loss-frequency characteristic.

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