PARAMETRIC ACOUSTIC ARRAYS - A STATE OF THE ART REVIEW

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SUMMARY

Following a brief introduction to the concept of parametric acoustic interactions, the basic properties of Parametric Transmitting and Receiving Arrays are considered in the light of conceptual advances resulting from experimental and theoretical investigations that have taken place since Westervelt's (ref. 1) landmark paper in 1963.

INTRODUCTION

It is interesting to observe that the concept of a Parametric Acoustic Array which was first introduced by Westervelt (ref. 1) in 1963 can be viewed retrospectively as the inevitable consequence of his earlier investigations (ref. 2) of the scattering of sound by sound. Adopting this perspective as a framework for discussion, we begin by considering the propagation of isentropic finite-amplitude acoustical disturbances (i.e., waves of maximum Mach Number $\varepsilon_o < 0.1$) in an unbounded dispersionless, thermo-viscous fluid at rest. Such disturbances, as shown by Westervelt (ref. 3) are governed by a second-order nonlinear wave equation which can be derived from Lighthill's (ref. 4) 'acoustic analog equation' [i.e., a cleverly rearranged form of the Navier-Stokes (ref. 5) equations]. The excess pressure p' induced in the fluid by a finite-amplitude disturbance of initial peak pressure p_o is thus described by the equation,

$$\Box^2 P = -\frac{1}{2}\beta\varepsilon_o (P^2)_{tt} \qquad P(\underline{r},t) = p'/p_o \qquad \varepsilon_o = p_o/\rho_o c_o^2 \qquad (1)$$

where the coefficient of nonlinearity of the fluid (ref. 6) β has a value of ~3.5 in water at 20°C and atmospheric pressure. Taking the Fourier transform of eq. (1) gives,

$$(\nabla^2 + k^2)\tilde{P}_\omega = \frac{1}{2}\beta\varepsilon_o k^2 F_\omega\{P^2\} \qquad \tilde{P}_\omega(\underline{r}) = F_\omega\{P(\underline{r},t)\} \qquad k = \omega/c_o \qquad (2)$$

where the effect of viscous absorption can be included by treating k as a complex wavenumber.

If two finite-amplitude plane waves of initial peak amplitudes p_{o1}, p_{o2} and angular frequency-wavenumber pairs $(\omega_1,\underline{k}_1)$, $(\omega_2,\underline{k}_2)$, termed 'primary'

waves interact weakly (i.e., without incurring significant distortion) their combined field is obtained to a first-approximation via linear superposition giving,

$$P(r,t) = \mathrm{Re}\{P_{o1} \exp(j\omega_1 t - j\underline{k}_1 \cdot \underline{r}) + P_{o2} \exp(j\omega_2 t - j\underline{k}_2 \cdot \underline{r})\} + O(\varepsilon_o^{\,2}) \tag{3}$$

The right-hand-side of eq. (2) thus consists of forcing functions at the second harmonic and combination frequencies so that as in the case of a linear harmonic oscillator its response to any one of these applied forces remains small unless their frequencies coincide with characteristic frequencies of the homogeneous equation. For weakly interacting primary waves this occurs at the combination frequencies whenever the following 'resonance' conditions are satisfied (ref. 7):

$$\omega_1 \pm \omega_2 = \omega_{\pm} \qquad \underline{k}_1 \pm \underline{k}_2 = \underline{k}_{\pm} \tag{4}$$

Since the second of these conditions can be reexpressed for interaction in a dispersionless fluid (i.e., $\omega_1/k_1 = \omega_2/k_2 = \omega_{\pm}/k_{\pm} = c_o$) as,

$$\omega_1^{\,2} + \omega_2^{\,2} \pm 2\omega_1\omega_2 \cos\theta = \omega_{\pm}^{\,2}$$

where θ is the angle of intersection between the wave normals, it follows from the first condition that $\theta = 0$ is the only angle of intersection for which eq. (4) can be satisfied. As Westervelt (ref. 2) concluded therefore, two perfectly collimated overlapping finite-amplitude plane waves can only interact 'resonantly' when their wave vectors $\underline{k}_1$ and $\underline{k}_2$ are aligned in the same direction. On the other hand, it should be noted as Rudenko, et. al. (ref. 8) have shown that 'resonance' occurs at non-zero intersection angles in dispersive fluids.

In the case of 'non resonant' or 'asynchronous' interactions the combination tones are subject to spatial oscillations which inhibit their effective amplification. Alternatively, 'resonant' or 'synchronous' interactions result in continuous energy transfer from the primary waves to the nonlinearly generated 'secondary' waves (i.e., combination tones, etc.). If the initial amplitudes of the 'secondary' waves are zero they will thus grow linearly with range at the expense of the primary waves until the latter, and hence the amplitudes of the 'forcing functions' on the right-hand-side of eq. (2), are sufficiently diminished by this type of "finite-amplitude absorption" and by conventional 'linear' losses such as viscous absorption and spherical spreading. At distances from the source of the disturbance where the primary waves are no longer of

finite-amplitude, nonlinear interaction ceases, and the secondary waves formed in the "interaction zone" eventually decay at rates determined by their viscous attenuation coefficients and by spherical spreading losses. The range at which this occurs defines the 'far-field' of the secondary waves which is generally much greater than that of the primary waves. The interaction zone can thus be viewed as an extension of the source itself, the generation of secondary waves within it resulting from the establishment of volume distributed "virtual sources" created by the primary fields which formed as envisaged by Westervelt (ref. 1), a "virtual acoustic array". Moreover, the term 'parametric' which Westervelt (ref. 1) used to describe such arrays was chosen, by analogy with the concept of electrical parametric amplification, to convey the idea that their performance is dependent on parameters of the medium (i.e., β, ρ_o, c_o, attenuation characteristics, etc.) and of the source distribution (i.e., primary wave amplitudes, frequencies, and aperture dimensions). Since the spatial directivity of the secondary waves is in most instances equivalent to that of the primary waves, highly directive low frequency "parametric transmitting arrays" can thus be formed by bifrequency projectors simultaneously radiating highly directive primary waves of nearly equal frequencies to generate a low difference-frequency signal via nonlinear interaction in the medium. The converse task of directive low frequency reception, can likewise be accomplished by means of "parametric receiving arrays".

PARAMETRIC TRANSMITTING ARRAYS

When the primary waves are radiated by a plane piston projector of area A_o, they propagate as essentially collimated plane waves within their mean Rayleigh distance $r_o = A_o/\lambda_o$, λ_o being the mean primary wavelength, and as directive spherical waves beyond this range. If α_o is the mean primary wave attenuation coefficient, then $2\alpha_o r_o$ represents the total 'linear' loss incurred by the primary waves within r_o. Consequently, when $2\alpha_o r_o$ is such that the primary wave amplitudes are reduced to small-signal levels within r_o (i.e., $2\alpha_o r_o >> 1$), a plane wave primary interaction of the type considered by Westervelt (ref. 1) occurs in the fluid. This type of parametric interaction, which is described as 'absorption-limited', results in the virtual sources being phased in such a manner that they form a "virtual-end-fire array whose 'far-field' spectrum contains only the difference-frequency (and possibly some of its harmonics). In most instances the latter signal overrides the primary waves and upper sideband components to survive in the far-field (i) because it has been amplified throughout the interaction zone and (ii) because of its significantly lower rate of viscous absorption. The 'far-field' pressure of an axially symmetric 'absorption-limited' parametric array obtained from eq. (2) thus becomes (refs. 1 and 9),

$$\tilde{p}_{\omega_-}(r,\theta) = D_-(\theta) \left(\frac{\omega_-^2 \beta p_{o1} p_{o2} A_o}{4\pi \rho_o c_o^4} \right) \left(\frac{1}{\alpha_T} \right) \frac{e^{-\alpha_- r - jk_- r}}{r} \qquad 2\alpha_o r_o >> 1 \qquad (5)$$

where $\alpha_T = \alpha_1 + \alpha_2 - \alpha_- \simeq 2\alpha_o$ is the effective length of the virtual-end-fire-array and its directivity-function $D_-(\theta)$ is given by,

$$|D_-(\theta)| = \frac{|D_B(\theta)|}{\sqrt{1 + (2\alpha_T/k_-)^2 \sin^4(\theta/2)}} \qquad 2\alpha_o r_o >> 1 \qquad (6)$$

$D_B(\theta)$ being the far-field directivity function of the radiator at the difference-frequency - a necessary modification of Westervelt's (ref. 1) solution for $k_- a > 1$, introduced by Naze and Tjotta (ref. 9), where $2a$ is the characteristic dimension of the aperture. If $k_- a < 1$ then $D_B(\theta) \simeq 1$ over the angular domain of interest, so that in this instance the directivity function defined by eq. (6) assumes the form originally derived by Westervelt (ref. 1). This directivity function has no sidelobes, a most attractive feature of 'absorption-limited' parametric arrays, which has been confirmed experimentally by Bellin and Beyer (ref. 10), Berktay (ref. 11), Zverev and Kalachev (ref. 12), and by Muir and Blue (ref. 13). Using a 25 cm^2 square projector simultaneously radiating primary waves of frequencies 1.124 MHz and 0.981 MHz at finite-amplitudes in fresh water, the latter (ref. 13) showed that the far-field directivity function of the 143 kHz difference-frequency signal was in very good agreement with that predicted by eq. (6), thus demonstrating that in this instance the parametric array was capable of achieving the same directivity as a conventional source operating at 143 kHz, but with an aperture of characteristic dimension approximately eight times smaller.

If the near-field primary wave absorption loss $2\alpha_o r_o$ is very small (i.e., $2\alpha_o r_o << 1$), significant nonlinear interaction occurs beyond r_o where the primary fields propagate as directive spherical waves. A parametric array formed by this type of interaction is termed 'diffraction-limited' because the virtual-end-fire array which now extends beyond r_o is effectively truncated by spherical spreading losses at a distance $r_o' = r_o(\omega_o/\omega_-)$ where the half-power beamwidth of the virtual-end-fire-array begins to asymptotically approach that of the mean primary wave directivity function. Lauvstad and Tjotta (ref. 14), Cary (ref. 15), Fenlon (refs. 15 and 16), and Muir and Willette (ref. 17) have investigated the properties of 'diffraction-limited' parametric arrays, whose 'far-field' difference-frequency pressure for axially symmetric primary waves is given by eq. (2) as,

$$\tilde{p}_{\omega_-}(r,\theta) = -j\, D_-(\theta) \left(\frac{\omega_-^2\, \beta\, p_{o1} p_{o2}\, A_o}{4\pi\, \rho_o c_o^4} \right) \left(r_o' \ln \frac{1}{\alpha_T r_o'} \right) \frac{e^{-\alpha_- r - jk_- r}}{r}$$

$$2\alpha_o r_o << 1 \qquad (7)$$

the effective array length $r_o' \ln \frac{1}{\alpha_T r_o'}$ in this instance being considerably less than the 'absorption-limited' length $1/\alpha_T$. Moreover, as shown by

Fenlon (ref. 18) and Lockwood (ref. 19) the 'far-field' difference-frequency directivity function $D_-(\theta)$ for an axially symmetric diffraction-limited array is given by,

$$D_-(\theta) = D_1(\theta)\, D_2(\theta) \qquad 2\alpha_o r_o << 1 \tag{8}$$

where $D_i(\theta)$, $(i = 1,2)$ are the far-field primary wave directivity functions.

Combining the asymptotic solutions defined by eqs. (5)-(8), Fenlon (ref. 20), Berktay and Leahy (ref. 21), and (although not explicit in their analysis) Mellen and Moffett (ref. 22) have shown that the difference-frequency pressure in the 'far-field' of an axially symmetric parametric array can be expressed for all values of $2\alpha_o r_o$ as,

$$\left|\tilde{p}_{\omega_-}(r,\theta)\right| = D_-(\theta) \left(\frac{\omega_-}{\omega_o}\right)\left(\frac{\beta\, k_o\, p_{o1} p_{o2}\, r_o^2}{2\rho_o c_o^2}\right) R_L \frac{e^{-\alpha_- r}}{r} \tag{9}$$

where

$$R_L \to \frac{\omega_-/\omega_o}{\alpha_T r_o} \qquad \alpha_T r_o' >> 1 \tag{9a}$$

$$\to \ln \frac{1}{\alpha_T r_o'} \qquad \alpha_T r_o' << 1 \tag{9b}$$

$R_L = r_L/r_o'$ being the effective length of the parametric array r_L , normalized with respect to $r_o' = r_o(\omega_o/\omega_-)$. The dependence of R_L on $\alpha_T r_o'$ obtained from refs. 19 and 20 is shown in figure 1. Again, the general form of the difference-frequency directivity function $D_-(\theta)$ is obtained by convolving eqs. (6) and (8), as shown implicitly by Lauvstad and Tjotta (ref. 13) and explicitly by Blue (unpublished report). It should be noted that Berktay and Leahy (ref. 21) have evaluated the convolution integral numerically to obtain $D_-(\theta,\phi)$ for both axially symmetric and asymmetric 'diffraction-limited' arrays, the computed directivity functions being in excellent agreement with experimental results.

Returning to eq. (9) it is convenient to reexpress it in terms of the equivalent <u>peak</u> primary wave and difference-frequency source levels at 1m giving

$$\hat{SL}_- = \hat{SL}_1 + \hat{SL}_2 + 20 \log_{10}(\omega_-/2\pi \times 1\ \text{kHz}) + 20 \log_{10} R_L - 290\ \text{dB re 1 μPa at 1m in } \underline{\text{water}} \tag{10}$$

Since the dependence of R_L on $\alpha_T r_o'$ depicted in fig. 1 has been confirmed

experimentally (refs. 23 and 24) over the range $10^{-5} \leq \alpha_T r'_o \leq 10$, it follows that eq. (10) can be applied over the entire range of sonar frequencies provided that the combined peak primary wave pressure does not excede the shock threshold (i.e. the amplitude at which the primary waves become so distorted due to repeated self interaction that shock formation occurs within the interaction zone). Denoting the critical peak source level corresponding to the shock threshold as $\hat{SL}_{oc}$ it can be shown (ref. 20) that,

$$\hat{SL}_{oc} = 20 \log_{10} \sigma_{oc} - 20 \log_{10}(\omega_o/2\pi \times 1 \text{ kHz}) + 287 \text{ dB re 1 } \mu\text{Pa at 1m in water} \qquad (11)$$

where the parameter σ_{oc} is given as a function of $\alpha_o r_o$ in fig. 2 for a plane piston projector. It can also be shown that the half-power beamwidth $2\theta_-$ of the difference-frequency directivity function obtained from the convolution integral (refs. 13 and 21) is given to a good approximation by the expressions,

$$2\theta_- \simeq \frac{2\theta_o}{\sqrt{2}} \left\{ 1 + \sqrt{\alpha_T r'_o} \right\} \qquad 2\theta_o = \frac{0.88\,\lambda_o}{d} \qquad \text{for a square piston of side length } d \qquad (12a)$$

$$\simeq \frac{2\theta_o}{\sqrt{2}} \left\{ 1 + \frac{4}{\pi} \sqrt{\alpha_T r'_o} \right\} \qquad 2\theta_o = \frac{\lambda_o}{d} \qquad \text{for a circular piston of diameter } d\,. \qquad (12b)$$

Several examples illustrating the application of eqs. (10)-(12) to experiments reported in the literature are included in Tables 1a and 1b, the "frequency response index" $\bar{n}$ which appears in Table 1b being defined as,

$$\bar{n} = 1 + \frac{\ln\{R_L(\alpha_T r_o)/R_L(\alpha_T r'_o)\}}{\ln(\omega_o/\omega_-)} \qquad 1 \leq n \leq 2 \qquad (13)$$

where $R_L(\alpha_T r'_o)$ and $R_L(\alpha_T r_o)$ are both defined by the characteristic in fig. 1. It should be noted that from eq. (13), $\bar{n} \to 2$ for 'absorption-limited' arrays (i.e., $2\alpha_o r_o \gg 1$) and likewise $\bar{n} \to 1$ for 'diffraction-limited' arrays (i.e., $2\alpha_o r_o \ll 1$), as required. The difference-frequency pressure distribution in the 'near-field' of 'absorption-limited' parametric transmitting arrays has been analyzed by Berktay (ref. 25), Hobaek and Vestrheim (ref. 26) and by Novikov et. al. (ref. 27). A 'near-field' solution for 'diffraction-limited' arrays has also been obtained by Rolleigh (ref. 28) although it can be shown that this approximation is only valid for $10^{-2} \leq \alpha_T r'_o \leq 1$. A more comprehensive 'near-field' which include both 'absorption-limited' and 'diffraction limited' interactions has recently been derived by Mellen (ref. 29). However, this approximation has not as yet been sufficiently tested to confirm its applicability over a wide range of the parameter $\alpha_T r'_o$.

More complex parametric interactions between spatially separated primary sources have been treated analytically by Lauvstad (ref. 30) and by Cary and Fenlon (ref. 31).

An 'absorption-limited' parametric transmitting array was first formed in air by Bellin and Beyer (ref. 10) but the formation of 'diffraction-limited' arrays in air was only recently accomplished by Bennett and Blackstock (ref. 32) and independently by Muir (ref. 33). The latter, who made use of a small bifrequency transducer (i.e., operating simultaneously at 15.5 kHz and 16.5 kHz) located at the Newtonian focus of a 55.9 cm diameter parabolic reflector to form the primary waves, concluded from the success of his experiment that the advent of directional parametric megaphones is virtually assured.

Muir (ref. 33) also formed and successfully steered over a 36° sector a 21 kHz difference-frequency signal resulting from the interaction of primary waves (i.e., 185 kHz and 206 kHz) simultaneously radiated by small bifrequency transducers located on the focal surface of a 43 cm diameter solid polystyrene plastic refracting lens in water. Widener and Rolleigh (ref. 34) have subsequently shown that the difference-frequency pressure and directivity are not adversely affected by mechanically steered primary waves if the frequency of rotation is small compared to the difference-frequency.

In another recent experiment Ryder, Rogers, and Jarzynski (ref. 35) generated difference-frequencies of 10 kHz - 20 kHz via an 'absorption-limited' parametric transmitting array formed by primary waves of mean frequency 1.4 MHz propagating in a 16.5 cm diameter, 23 cm long silicone rubber cylinder immersed in water, the primary waves being radiated by 2 cm diameter circular piston centered at the back end of the cylinder. Although the axial field dependence of the difference-frequency signals was found to be in good agreement with eq. (5) when $1/\alpha_T$ was replaced by a 'slow-waveguide-antenna-absorption-distance-parameter', the 'far-field' difference-frequency directivity functions were much more directive than those predicted by eq. (6). However, despite the fact that the coefficient of nonlinearity in silicone rubber exceeds that of water by a factor of ~1.4 whilst its sound velocity is ~1.5 less than that of water, parametric arrays are formed less efficiently in this material because of its significantly greater rate of absorption per wavelength.

Attempts to address the problem of defining the maximum realizable conversion efficiency of parametric transmitting arrays have been made by Mellen and Moffett (ref. 22) and by Fenlon (ref. 36) via saturated parametric array models. Differences between these models at very high primary wave amplitudes however, have not yet been resolved experimentally.

Following Muir and Blue's (ref. 37) demonstration of the broadband (low Q) nature of parametric transmitting arrays, resulting from the transfer of primary wave bandwidths to the difference-frequency signal, it was evident that pulse compression techniques could be used, as in the case of peak-power-limited radars, to offset the poor conversion efficiency of these arrays.

Furthermore, when it was realized that the process of simultaneously radiating finite-amplitude tones of angular frequencies ω_1 and ω_2 each of initial amplitude P_o is equivalent to radiating a sinusoidal finite-amplitude carrier wave of angular frequency $\omega_o = (\omega_1 + \omega_2)/2$ and peak amplitude $2P_o$, modulated by a cosine envelope function of angular frequency $\Omega = (\omega_1 - \omega_2)/2$, it became obvious that parametric amplification is simply the converse of 'pulse demodulation' - a concept introduced by Berktay (ref. 11) and confirmed experimentally by Moffett, Westervelt, and Beyer (ref. 38) to explain the enhanced demodulation of a narrow-band-modulated finite-amplitude carrier resulting from propagation in a fluid (i.e., in addition to demodulation caused by viscous absorption) in terms of energy transferred by the carrier to its squared envelope frequency components. These components, being of lower frequency than the carrier survive the latter in the 'far-field' having been endowed with spatial directivities and bandwidths closely related to those of the carrier via angular and frequency convolution of the time waveform squared in the interaction zone. Eller (refs. 39 and 40) who investigated biased cosine modulation (i.e., a.m. with carrier) and narrow-band N-spectral line modulation showed, independently of Merklinger's (ref. 41) analysis of rectangular envelope modulation, that in principle, a maximum gain of 6 dB in conversion efficiency relative to that afforded by cosine modulation of angular frequency $\Omega/2$ could be realized for the same average carrier power by a periodic impulse function of repetition frequency Ω. In practice, however, since this form of modulation cannot be implemented by conventional band-limited, peak-power-limited acoustic sources, Merklinger (ref. 41) suggested the alternative of using a periodic rectangular envelope with a 25% 'mark-space-ratio' which results in a 5.1 dB gain in conversion efficiency for the same average power as a cosine modulated wave, provided that the source has sufficient bandwidth to form the rectangular envelope, and can at the same time sustain a 50% increase in peak pressure. On the other hand, if the source is peak-power-limited but not band-limited, a gain in conversion efficiency of 2.1 dB can still be realized for the same average power as a cosine modulated carrier, via periodic square wave modulation (i.e, rectangular modulation with a 50% mark-space-ratio) without incurring any increase in peak power. In general therefore, rectangular modulation is a very advantageous means of launching a parametric array, particularly as it can readily be implemented via switching amplifiers.

More recently, a procedure for optimizing the performance of parametric transmitting arrays by spectral design of the modulating envelope has been outlined in a preliminary study by Clynch (ref. 42).

PARAMETRIC RECEIVING ARRAYS

Parametric Receiving Arrays are formed in a fluid by projecting a finite-amplitude 'pump wave' of angular frequency ω_o into the medium to serve as a 'carrier' wave for a weak incoming signal of angular frequency ω_s, where in general $\omega_o/\omega_s \gg 1$. Since the pump wave is sufficiently intense to make the compressibility of the fluid amplitude dependent, the presence of any other wave,

such as the spatial component of a weak signal traveling along the pump axis, will result in a combined pressure field which is effectively squared by the inherent nonlinearity of the medium. The nonlinear interaction thus gives rise to sinusoidal modulation of the pump wave by the spatial component of the signal along its axis which in turn produces an intermodulation spectrum, the "sum" and "difference" components of angular frequencies $\omega_o \pm \omega_s$ being of greatest interest. For an efficient nonlinear interaction the 'resonance conditions' require that the spatial component of the signal along the pump axis be propagating in the same direction as the pump wave. On account of the fact that $\omega_o/\omega_s >> 1$ these sidebands are in close spectral proximity to the pump frequency, but unlike the latter, their directivity is equivalent to that of a virtual-end-fire line array of length L/λ_s (in wavelengths of the signal frequency), where L is the distance from the pump projector along its axis at which a receiving hydrophone resonant at $\omega_o + \omega_s$ or $\omega_o - \omega_s$ is located. Upon reception the "up-converted" signal is fed to a low pass filter to remove the pump frequency and recover the signal of frequency ω_s.

Although implicit in Westervelt's (ref. 2) work, the process of Parametric Reception was identified and made explicit by the extensive theoretical and experimental investigations of Berktay (ref. 43) who in cooperation with Al-Temimi (refs. 44, 45) and Shooter (ref. 46) considered the practical implications of the up-conversion process. Subsequent experimental work by Barnard et. al. (ref. 47) and by Berktay and Muir (ref. 48) has been directed to long wavelength up-conversion in fresh water lakes and to the consideration of arrays of parametric receivers, respectively, thus involving significant practical extensions of the original scaled laboratory experiments. Further theoretical extensions by Rogers et. al. (ref. 49) and by Truchard (ref. 50) have also been made to provide a more precise description of the pump fields radiated by practical sources and the resulting effect of such refinements upon the analytical form of solutions for the up-converted fields. More recently Goldsberry (ref. 51) and McDonough (ref. 52) have derived optimum operating conditions for parametric receiving arrays from systems analyses based on Berktay and Al-Temimi's analytical model (ref. 45) for a spherically spreading pump wave. It should be noted however, that Goldsberry's (ref. 51) analysis which attempts to include the effect of noise is much more realistic than that of McDonough (ref. 52) who neglected to include this vital effect. With the exception of a preliminary study by Bartram (ref. 53), no systematic analysis had been made prior to Fenlon and Kesner's analysis (ref. 54) of the effect of finite-amplitude absorption on the performance of parametric receivers, which although insignificant at low pump amplitudes, ultimately determines the maximum achievable efficiency of these arrays when the pump wave becomes saturated.

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Table 1a

Experiment	f_1 kHz	f_2 kHz	f_o kHz	f_- kHz	f_o/f_-	A_o m^2	λ_o m	r_o m	α_o np/m	$\alpha_o r_o$ np	$\alpha_T r'_o$ np	σ_{oc}	R_L
Muir & Willette(ref. 17)	482	418	450	64	7.03	4.56×10^{-3}	3.33×10^{-3}	1.37	0.006	8.2×10^{-3}	0.116	0.3	1.9
Eller (ref. 39)	1460	1410	1435	50	28.7	3.14×10^{-3}	1.03×10^{-3}	0.304	0.04	1.2×10^{-2}	0.69	0.4	0.76

Table 1b

Experiment	$\hat{SL}_1$ dB µPam	$\hat{SL}_2$ dB µPam	$\hat{SL}_o$ dB µPam	$\hat{SL}_{oc}$ dB µPam	$\hat{SL}_-$ dB re 1µPam		θ_- degrees		$\bar{n}$	r_L m
					predicted via eq(10)	measured	predicted via eq(12)	measured		
Muir & Willette(ref. 17)	207.2	207.2	213.2	223.5	166	167	2.47	2.5	1.36	18.3
Eller (ref. 39)	206.7	206.7	212.7	215.9	155	154	4.32	4.5	1.44	6.5

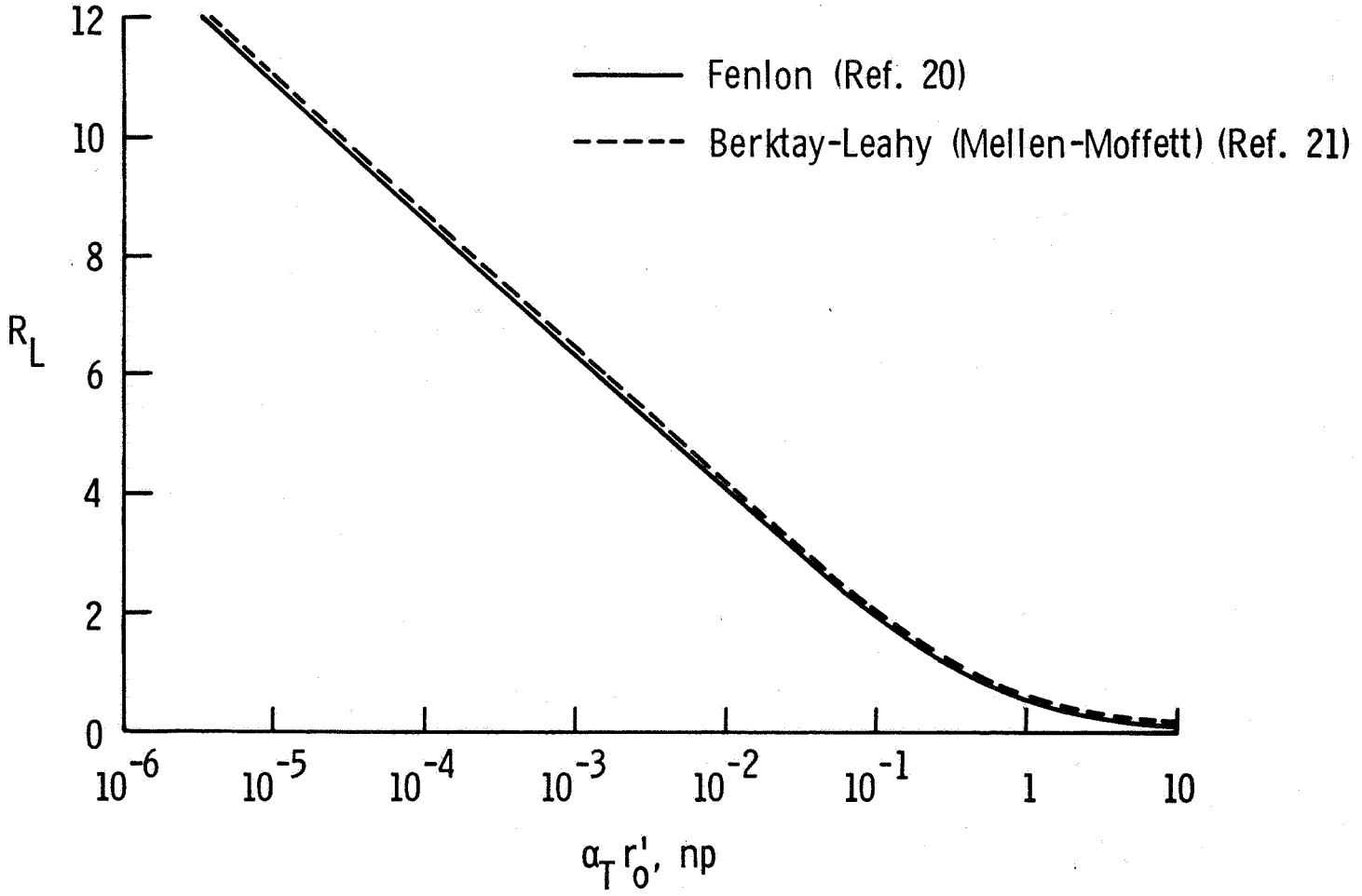

Figure 1.- Effective parametric array length characteristic.

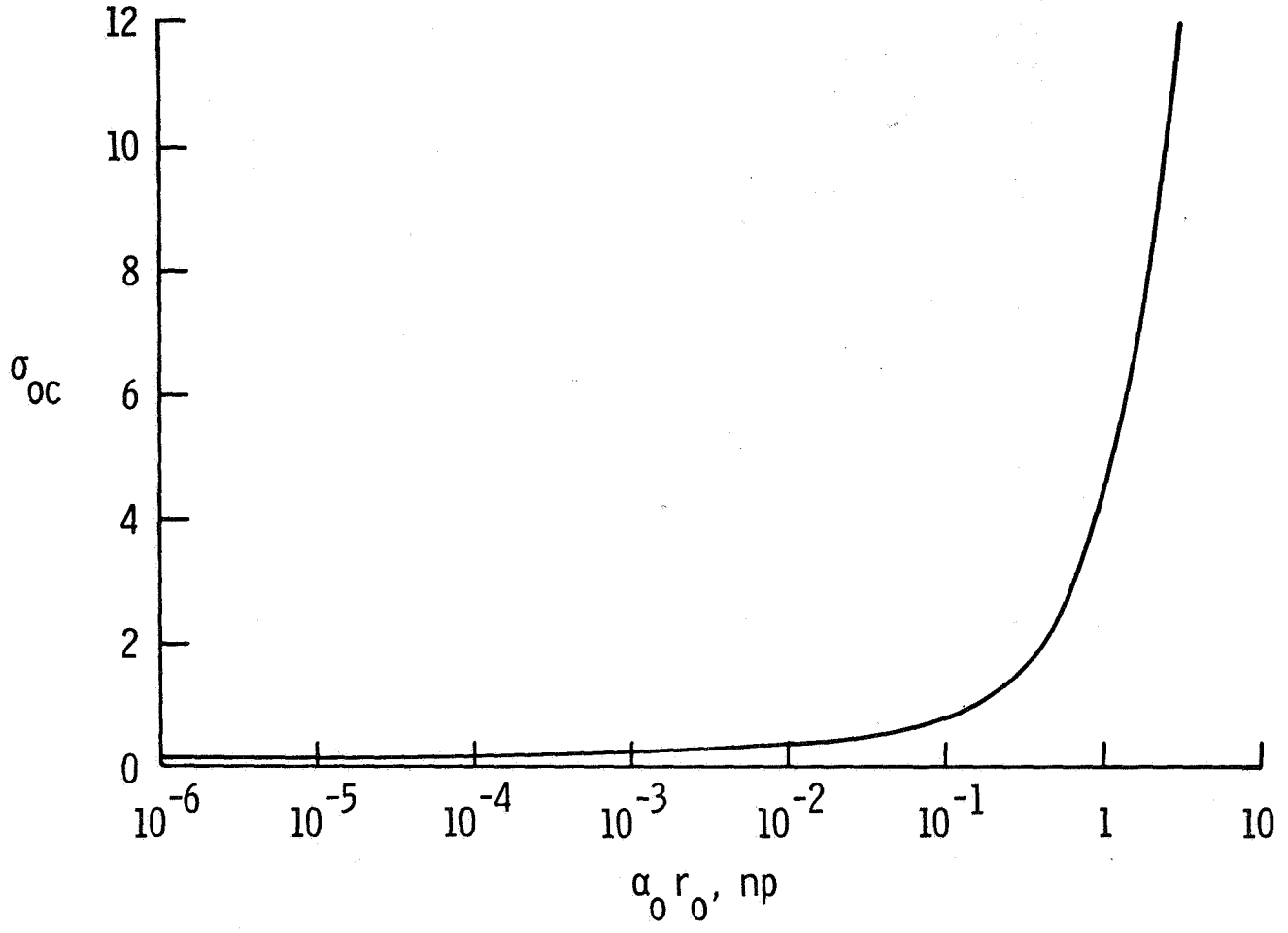

Figure 2.- Shock threshold characteristic.