

Parametric analysis of combined turning and ball burnishing process

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Burnishing is a plastic deformation process, which is becoming more popular as a finishing process. Experimental work based on 2³ factorial design has been carried out on Turn master T-40 lathe to establish the effect of the combined turning and two ball-burnishing parameters on the surface roughness and surface hardness of aluminum specimen. The results have been analyzed by the variance technique and the F-test, showing thereby that the lubricant, force, speed, and feed have significant effects on surface roughness and surface hardness. A pre-machined surface roughness of 0.63-0.75 μm (by turning) can be finished up to 0.11 μm (by burnishing) and improved micro hardness is obtained.

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Burnishing is a cold working finishing process, wherein highly polished and hard balls are pressed against a metallic surface of a flat or cylindrical component. In burnishing, initial asperities are compressed and modified. The deformation caused is a function of load applied. The surface material is progressively compressed, then plasticized as the resultant stresses reach a steady maximum value and finally wiped to a superfine finish. As compared with roller burnishing, ball burnishing is more advantageous for cylindrical component, because the ball can easily move in forward and backward direction along the surface direction or parallel to the axis of the cylindrical component. It will reduce the production time and more accuracy can be maintained by turning and ball burnishing process (simultaneous operation of turning and burnishing). For flat surfaces, the roller burnishing is more suitable compared with ball burnishing.

Wu¹ studied the tool life testing by a statistical approach using response surface methodology (turning only) on a lathe. He considered three independent turning parameters namely, speed, feed and depth of cut and by keeping other parameters constant, surface roughness model equations were obtained. Omprakash *et al.*² studied the influence of important ball burnishing parameters on surface finish, depth of work hardening, microstructure and the fatigue life. The burnishing parameters considered were speed, feed and number of passes. It was found that the surface roughness improved initially with an increase in these parameters. After a certain stage, the

surface finish deteriorated and fatigue life decreased. Lah *et al.*³ performed the experiments by using 3⁴ factorial design on a vertical milling machine to find the effect of ball burnishing parameters on the surface roughness of AISI 1045 specimens. By using the analysis of variance technique and F-test, it was seen that ball material, lubricant, feed and depth of penetration had significant effect on the surface roughness. A pre-machined surface roughness could be finished up to 0.77 μm . Bardie⁴ developed a surface roughness model for gray cast iron (154BHN) using carbide tool under dry condition (turning) and for constant depth of cut. The surface roughness model equations were developed in terms of speed, feed, and nose radius of the cutting tool. These variables were investigated using design of experiments and response surface methodology (RSM). Hssan and Ebied⁵ conducted test on brass material on copying lathe machine. Two burnishing parameters were considered namely, burnishing force and number of the passes, while other burnishing parameters were kept constant. The result was improved fatigue life. Ingole and Bahedwar⁶ studied the effect of lubricants on the surface finish of En8 specimens. Using 2³ factorial design, in terms of surface roughness, model equations were developed. The burnishing parameters considered were speed, feed and force and the other parameters were constant. On En8 component, the best lubricant was found to be SAE-40 among SAE-40, grease and mixed lubricant. Siva Prasad and Kotiveerachari⁷ conducted experiment on roller burnishing on

aluminum specimens (FIC, IS 734-1967). It was observed that there was a significant effect of force on surface finish as compared with speed and feed.

All the above information is pertaining to the separate turning and single ball burnishing process. In this paper, a systematic study of burnishing parameters on surface finish and surface hardness of aluminum specimen is presented by using combined turning and two ball burnishing process (lubrication study). Three burnishing parameters are considered here, namely, burnishing force, speed and feed. The other burnishing parameters are kept constant. A combined turning and two ball-burnishing tool has been specifically designed on Kirloskar Turn master T-40 lathe which is used for burnishing of aluminum test specimens. The time required for this operation is less, due to the combined operation. Due to this tool, cylindricity and circularity of the cylindrical component can be maintained. Due to the combined operation, it gives more accuracy and output. Aluminum material (Cu-3.5, Si-6, Fe-1, Zn-1, Ti-0.22, Mn-0.5, Al-remainder in %) is used for the present studies mainly due to the following reasons:

- Pure aluminum cannot be heat treated properly, however some of the aluminum alloys can be heat treated to improve their mechanical properties up to certain extent.
- For improving the properties of non-ferrous material, the combined turning and two-ball burnishing process is more suitable due to cold working/material deformation.
- Aluminum material cannot be machined properly on conventional and even on CNC machine due to poor machinability.

Combined Turning and Two Ball Burnishing Tool

The combined turning and two ball burnishing tool is shown in Fig. 1. The balls are located inside an interchangeable adapter. Diameter of both the balls is 12.5 mm (surface finish is 0.105 μm and hardness is 65 HRC) and made from steel material. The balls are free to rotate with the movement of the work piece due to frictional engagement between their surfaces. When balls are pressed against the surface of metallic specimen, the adaptor compresses pre-calibrated springs.

The springs are used to reduce the possible sticking effect of the balls and also to measure the applied vertical burnishing force with help of the depth nut. By rotating a depth nut in the clockwise direction, the load is applied on the spring through the steel body.

This tool includes two ball bearings (Bearing no. 628X) and two flat-ended springs having stiffness of 7.5 kg/mm. The two ball-burnishing tool is designed in a simple manner so that it can be mounted easily on the lathe machine.

Factorial design

Factorial design (2^3) used in this work is a composite design, which had been initially proposed by Box^{1,3,4}. There are numerous advantages associated with the use of factorial design in conducting experiments. It is more efficient than the conventional one-factor-at-a-time experiments commonly employed by researchers, and also enables the study of both, the main and interaction effects among the factors. Further, should a parameter (e.g. surface roughness) need to be minimized with respect to the combination factors, factorial design will give a combination near to the minimum (or maximum), whereas the one-factor-at-a-time procedure will not.

2^3 factorial design represents a eight-experiments, where the experimental points are located at the vertices of a cube shown in Fig. 2. Four experiments represent an added centre-point to the cube, repeated four times to estimate the pure error. The complete design consists of 12 experiments divided into two blocks, each block containing six experiments and one combined block is considered (trial nos 1 to 12)^{8,9}. This method classifies and identifies the parameters to three different levels (viz. low, center and high). In this experimentation, twelve tests were carried out at these levels. For each block the model equations for surface roughness and the surface

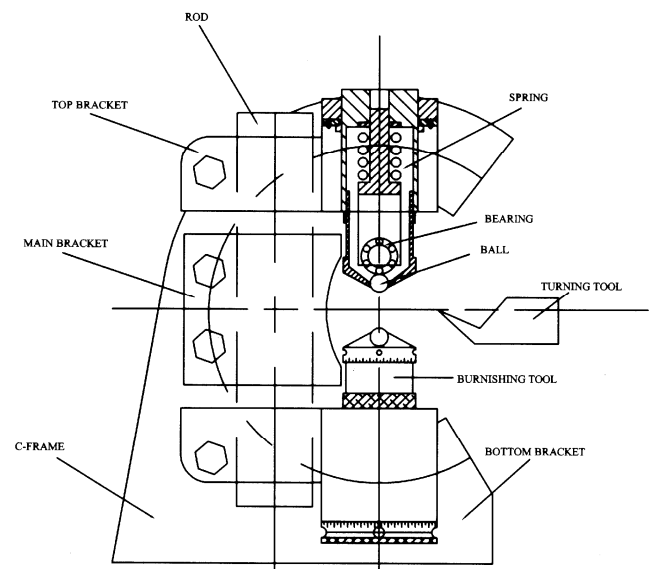


Fig. 1—Combined turning and two-ball burnishing tool

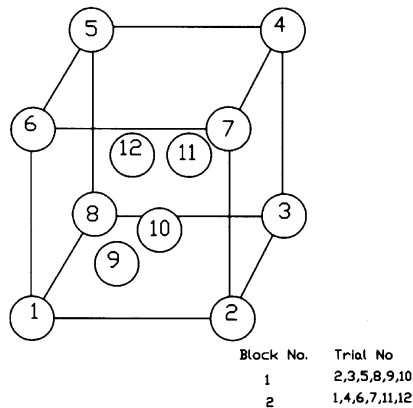


Fig. 2—Composite design

Table 1—Three levels of variables and coding identification

| Level | Force (<i>F</i>) (kgf) | Speed (<i>V</i>) (m/min) | Feed (<i>S</i>) (mm/rev) | Coding | | |
|--------|-----------------------------|-------------------------------|-------------------------------|--------|-------|-------|
| | | | | X_1 | X_2 | X_3 |
| Low | 8 | 20 | 0.01 | -1 | -1 | -1 |
| Centre | 15 | 30 | 0.04 | 0 | 0 | 0 |
| High | 30 | 50 | 0.1 | +1 | +1 | +1 |

hardness are obtained by using the analysis of variance technique, F-test and regression coefficient. (i) First block of six tests (trial nos 2, 3, 5, 8, 9 and 10); (ii) Second block of six tests (trial no. 1, 4, 6, 7, 11 and 12) and (iii) Combined blocks of twelve tests (trial no. 1 to 12).

The trial nos.1 to 8 are at corner points and obtained by varying high and low values of the parameters. Center points (trial nos 9 to 12) are obtained by keeping parameters at center values shown in Fig. 2.

Experimental Procedure

A Kirloskar Turn master T-40 lathe is used for machining and has wide range of parameter settings. Table 1 gives the experimental values of burnishing. The lubricants used are (i) kerosene, (ii) SAE-30 oil, (iii) 5 % graphite by weight in SAE-30 oil and (iv) 10% graphite by weight in SAE-30 oil.

In this work, three burnishing parameters are considered, i.e., speed, feed, force and the other burnishing parameters are considered constant (i. e. ball diameter, nose radius, work piece material and number of passes).

For first, second and combined block, the regression coefficients were calculated by using ANOVA analysis. Highest value of regression coefficient gives best surface roughness model equation. For the first experiment, by using kerosene as a lubricant, second block gave highest value of

regression coefficient. It is the best surface roughness model equation. Similarly by using SAE-30, 5 and 10% graphite powder in SAE-30 oil as a lubricant, first, combined and first blocks were given highest value of the regression coefficient. Second block gave highest value of regression coefficient for surface hardness. One sample of calculation first block readings (Tables 3 and 4) is given in Appendix-A.

The surface roughness was measured on a SURFTEST 221 series 178, Mitutoya (Japan made). The surface roughness was taken perpendicular to the burnishing direction. In this work, the mean average surface roughness (*R_a*) values were measured by taking average of the three readings. The aluminum specimens were turned down to 25 mm diameter and exhibited a surface roughness 0.63-0.75 μm and after burnishing got the surface roughness up to 0.11 μm. The sample micro-hardness was measured on micro-hardness Tester Equipment (Zwick 3212) made by Zwick Ltd, West Germany.

Results and Discussion

By using factorial design, the total 12 experiments are conducted for each lubricant, considering all possible treatment combinations as shown in Table 2. The mathematical surface roughness models are obtained for each lubricant. The model equations are

$$Ra = 0.16758 F^{0.0027} V^{-0.0772} S^{0.0069} \text{ --- Kerosene}$$

$$Ra = 0.2718 F^{-0.1385} V^{0.0865} S^{-0.11145} \text{ --- SAE-30 oil}$$

$$Ra = 1.0718 F^{0.3850} V^{-1.0815} S^{0.1245} \text{ --- 5\% graphite by weight in SAE-30 oil}$$

$$Ra = 2.231 F^{0.2130} V^{-0.8965} S^{-0.3240} \text{ --- 10\% graphite by weight in SAE-30 oil}$$

$$Hv = 108.9 F^{0.5392} V^{-0.579} S^{0.2249} \text{ --- surface -hardness for kerosene as a lubricant}$$

The relationships amongst the burnishing parameters are developed and written above (surface roughness model equations), results are shown in Figs 3-6. Figure 3 shows that when the force increased up to 18 kgf, the surface roughness decreased for kerosene and again when force is increased above 18 kgf, the surface roughness increased. For SAE-30 oil and the mixed lubricants (solid + liquid) 5% and 10% graphite in SAE-30 oil, the surface roughness increased. It is found that the kerosene gives a better surface finish as compared with SAE-30 oil, 5% and 10% graphite by weight in SAE-30 oil. It is evident from Fig 4 that speed has no significant effect on surface roughness.

Table 2—Input parameters and output response

| Trial No | Force (X_1) | Speed (X_2) | Feed (X_3) | Surface Roughness (R_a), μm | | | | Surface hardness in BHN(H_v) for kerosene |
|----------|-----------------|-----------------|----------------|--|------------|-------------------------------------|--------------------------------------|---|
| | | | | Kerosene | SAE-30 oil | 5% Graphite by weight in SAE-30 oil | 10% Graphite by weight in SAE-30 oil | |
| 1 | -1 | -1 | -1 | 0.1146 | 0.2626 | 0.3611 | 0.9453 | 67 |
| 2 | +1 | -1 | -1 | 0.1446 | 0.2187 | 0.4156 | 0.7654 | 69 |
| 3 | -1 | +1 | -1 | 0.1246 | 0.1827 | 0.4342 | 0.7312 | 65 |
| 4 | +1 | +1 | -1 | 0.2367 | 0.2320 | 0.5342 | 0.8569 | 70 |
| 5 | -1 | -1 | +1 | 0.1190 | 0.2031 | 0.6281 | 0.6754 | 64 |
| 6 | +1 | -1 | +1 | 0.1754 | 0.2134 | 0.4278 | 0.8656 | 68 |
| 7 | -1 | +1 | +1 | 0.1145 | 0.1877 | 0.5134 | 0.7468 | 67 |
| 8 | +1 | +1 | +1 | 0.1621 | 0.1563 | 0.7151 | 0.8964 | 71 |
| 9 | 0 | 0 | 0 | 0.1125 | 0.1996 | 0.8196 | 0.5674 | 68 |
| 10 | 0 | 0 | 0 | 0.1385 | 0.1823 | 0.3123 | 0.8754 | 67 |
| 11 | 0 | 0 | 0 | 0.1153 | 0.1223 | 0.21123 | 0.7540 | 68 |
| 12 | 0 | 0 | 0 | 0.1234 | 0.1321 | 0.5212 | 0.8675 | 67 |

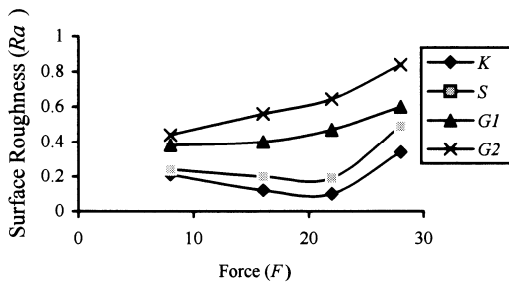


Fig. 3—Effect of force (kgf) on surface roughness (micron)

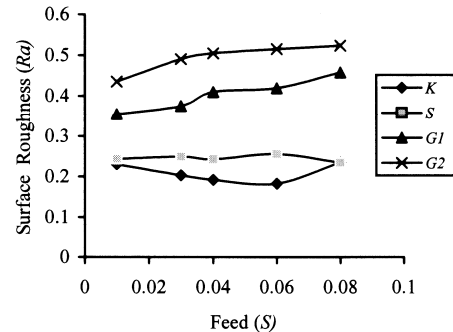


Fig. 5—Effect of feed (mm/rev) on surface roughness (micron)

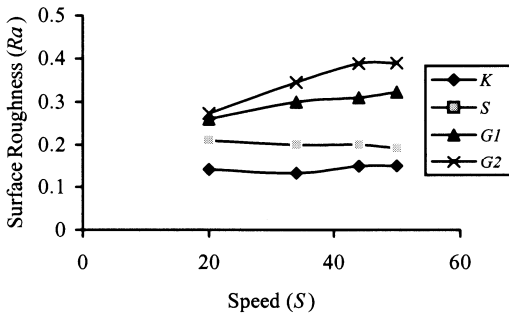


Fig. 4—Effect of speed(m/min) on surface roughness (micron)

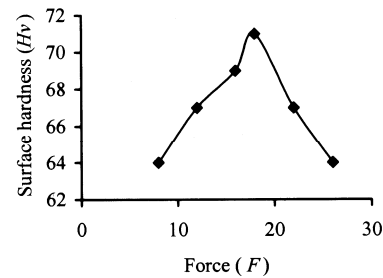


Fig. 6—Effect of force (kgf) on surface hardness (BHN)

It is clear from Fig. 5 that when feed increased the surface finish was increased up to certain extent for kerosene as a lubricant. Fig. 6 shows that as the force increased up to 18 kgf, the surface hardness increased. When the force increased above 18 kgf, the surface hardness decreased. It means that material got deteriorated. The surface hardness varies from outer surface to the centre of the element. Before burnishing hardness of aluminum component was 63 BHN. The best surface finish obtained by kerosene as a lubricant. Only for this lubricant micro hardness is measured, because surface finish obtained by

kerosene is better. For the rough finish, the surface hardness varies unevenly.

Conclusions

- 1 About 600-700% improved surface finish is obtained by combined operation (turning and burnishing simultaneously) on aluminum material (0.63-0.75 μm pre-machined surface can be finished down to 0.11 μm). There is a significant effect of force on surface finish up to 18 kgf.
- 2 At different values of force, speed and feed, the kerosene gave best surface finish than SAE-30 oil,

5% and 10% graphite by weight in SAE-30 oil as a lubricant.

- 3 It is observed that the micro hardness increased with force up to certain extent only.
- 4 The combined turning and burnishing process, it is possible to turn the shafts of low rigidity by balancing the cutting forces. The cutting forces on the component can be balanced with help of two balls and turning tool.
- 5 This process gives more production due to combined operation (simultaneous operation).

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Appendix-A

One sample calculation for first block readings, kerosene as a lubricant

Postulation of a Mathematical Model.

The functional relationship between response (surface roughness) of the burnishing operation and the investigated independent variables can be represented by the following equation

$$Ra = K F^a V^b S^c \quad \dots (1)$$

where Ra is surface roughness (μm), F , V and S are the force (kgf), speed (m/min) and feed (min/rev) respectively. Eq. (1) be written as

$$\ln Ra = \ln K + a \ln F + b \ln V + c \ln S \quad \dots (2)$$

Which may represent the following linear mathematical model

$$\eta = \beta_0 X_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \quad \dots (3)$$

where η is the true response of surface roughness on a logarithmic scale, $X_0 = 1$ (dummy variable), X_1 , X_2 and X_3 are logarithmic transformation of the force, speed and feed, while β_0 , β_1 , β_2 and β_3 are the parameters to be estimated. Eq. (3) can also be written as

$$YI = Y - \varepsilon = b_0 X_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 \quad \dots (4)$$

where YI is the estimated response and Y is the measured surface roughness on a logarithmic scale. ε is the experimental error and the b values are estimates of the β parameters.

Where the b values, that is b_0 , b_1 , b_2 , b_3 are to be estimated by the method of least squares, the basic formula is

$$b = (X^T X)^{-1} X^T Y \quad \dots (6)$$

where the calculation matrix X and the variance matrix $(X^T X)^{-1}$ are shown bellow. Hence, upon determine the b values by using Eq. (6).

The central composite design with 12 experiments which have three levels for each independent variables, as shown in Table 2, and were used to develop the model equation. The independent variables were as follows

$$\begin{aligned} X_1 &= \{(\ln F - \ln 30)/(\ln 30 - \ln 8)\}, \\ X_2 &= \{(\ln V - \ln 50)/(\ln 50 - \ln 20)\}, \\ X_3 &= \{(\ln S - \ln 0.1)/(\ln 0.1 - \ln 0.01)\} \end{aligned} \quad \dots (7)$$

By putting X_1 , X_2 , X_3 in Eq.(6), we will get that equation in F , V and S .

For sample calculation consider the first block reading, (Tables 3 and 4) kerosene as a lubricant.

$$\sum Y = 1.689, \sum X_1 Y = 0.0146, \sum X_2 Y = -0.2832, \sum X_3 Y = 0.0646, \sum Y^2 = 0.44789$$

From Eq. (6)

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----------|---|----|----|----|----|---|----|---|---|---|----|----|----|---|---|---|---|---|---|---|---|---|---|---|---|--|----------|---|--|---|--|---|--|---|--|---|--|----|
| $X =$ | <table style="border-collapse: collapse;"> <tr><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">-1</td><td style="padding: 2px 5px;">-1</td></tr> <tr><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">-1</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">1</td></tr> <tr><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">-1</td><td style="padding: 2px 5px;">-1</td><td style="padding: 2px 5px;">-1</td></tr> <tr><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">1</td></tr> <tr><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">0</td></tr> <tr><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">0</td></tr> </table> | 1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | <table style="border-collapse: collapse;"> <tr><td style="padding: 2px 5px;">Trial No</td><td style="padding: 2px 5px;">2</td></tr> <tr><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">3</td></tr> <tr><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">5</td></tr> <tr><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">8</td></tr> <tr><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">9</td></tr> <tr><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">10</td></tr> </table> | Trial No | 2 | | 3 | | 5 | | 8 | | 9 | | 10 |
| 1 | 1 | -1 | -1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | -1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | -1 | -1 | -1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Trial No | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 8 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 9 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

$$X^T X = \begin{vmatrix} 6 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix}$$

$$(X^T X)^{-1} = \begin{vmatrix} 1/6 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/4 \end{vmatrix}$$

X^T is the transpose of X , $(X^T X)^{-1}$ is the inverse of $(X^T X)$ and Y is the matrix of measured roughness on a logarithmic scale. b_0 , b_1 , b_2 , b_3 can be calculated as follows.

$$b_0 = (\sum Y)/6 = (Y_2 + Y_3 + Y_5 + Y_8 + Y_9 + Y_{10})/6 = 1.689/6 = 0.2815$$

$$b_1 = (\sum YX_1)/4 = (Y_2 - Y_3 + Y_5 - Y_8)/4 = 0.0146/4 = 0.00363$$

$$b_2 = (\sum YX_2)/4 = (-Y_2 + Y_3 - Y_5 + Y_8)/4 = -0.2832/4 = -0.0708$$

$$b_3 = (\sum YX_3)/4 = (-Y_2 - Y_3 + Y_5 + Y_8)/4 = 0.0646/4 = 0.0161$$

By putting X_1 , X_2 , X_3 in terms of F , V , S and b_0 , b_1 , b_2 , b_3 in Eq. (4), Eq. (4) can be written as

Table 3—Analysis of first block reading

| Trial No. | X_0 | X_1 | X_2 | X_3 | Ra | $Y=\ln 10Ra$ | X_1Y | X_2Y | X_3Y |
|-----------|-------|-------|-------|-------|--------|--------------|---------|---------|---------|
| 2 | 1 | +1 | -1 | -1 | 0.1446 | 0.3688 | 0.3688 | -0.3688 | -0.3688 |
| 3 | 1 | -1 | +1 | -1 | 0.1246 | 0.2199 | -0.2199 | 0.2199 | -0.2199 |
| 5 | 1 | -1 | -1 | +1 | 0.1190 | 0.1739 | -0.1739 | -0.1739 | 0.1739 |
| 8 | 1 | +1 | +1 | +1 | 0.1621 | 0.4830 | 0.4830 | 0.4830 | 0.4830 |
| 9 | 1 | 0 | 0 | 0 | 0.1125 | 0.1177 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 1 | 0 | 0 | 0 | 0.1385 | 0.3257 | 0.0000 | 0.0000 | 0.0000 |

Table 4—95% confidence interval, estimated response of the first block reading for kerosene

| Trial No | Y | Y_1 | $Y-Y_1$ | $(Y-Y_1)^2$ | 95% Confidence interval for Y_1 | |
|----------|--------|--------|---------|-------------|-----------------------------------|--------|
| | | | | | Lower | Upper |
| 2 | 0.3688 | 0.3199 | 0.0489 | 0.00239 | -0.3401 | 0.9799 |
| 3 | 0.2199 | 0.5927 | -0.3728 | 0.1389 | -0.0673 | 1.2527 |
| 5 | 0.1739 | 1.0782 | -0.9043 | 0.8177 | 0.4182 | 1.6678 |
| 8 | 0.4830 | 0.4529 | 0.0301 | 0.0009 | -0.2071 | 1.1129 |
| 9 | 0.1177 | 0.6649 | -0.5472 | 0.2994 | 0.0049 | 1.3249 |
| 10 | 0.3257 | 0.6649 | -0.3392 | 0.1150 | 0.0049 | 1.3249 |

$$Y_1 = 0.5163 + 0.0027 \ln F - 0.0772 \ln V + 0.0069 \ln S$$

Taking antilog

$$10Ra = 1.6758 (F^{0.0027} V^{-0.0772} S^{0.0069})$$

$$Ra = 0.16758 F^{0.0027} V^{-0.0772} S^{0.0069}$$

This is the surface roughness model equation of first block, for kerosene as a lubricant. Similarly we can find model equations for surface roughness and surface hardness for all blocks.

Calculation of Regression Coefficient

S^2 is the estimated variance, (degree of freedom =2, degree of freedom means difference between no. of corner trial reading minus central trial reading (dof=4-2=2))

$$S^2 = (\sum S_b) / \text{dof} = 1.3742 / 2 = 0.6871, \text{ where } \sum S_b = \sum (Y - Y_1)^2$$

The covariance matrix b is

$$(X^T * X)^{-1} S^2 = \begin{vmatrix} 1/6 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/4 \end{vmatrix} * S^2$$

$$= (1/6 + 1/4 + 1/4 + 1/4) * S^2$$

$$= (11/12) * S^2$$

Therefore, for two central observations, 95% confidence interval for Y_1 is

$$= Y_1 + t \text{ value } \sqrt{(11/12) S^2}$$

(From F-test and t-table at dof =2 at 0.025 level of significance t value = 4.303)

$$= Y_1 \pm 0.66$$

The experimental error,

$$\epsilon = \sqrt{\sum (Y_m - Y_i)^2 / N}, \text{ where } N = \text{no of trial}$$

$$Y_m = (Y_2 + Y_3 + Y_5 + Y_8 + Y_9 + Y_{10}) / 6 = (\sum Y) / 6 = 1.689 / 6 = 0.2815$$

$$i = 2, 3, 5, 8, 9, 10$$

Regression coefficient (R^2)

$$R^2 = 1 - \frac{\sum S_b}{\sum (Y_i - Y_m)}$$

This is the regression coefficient for first block for kerosene as a lubricant, similarly regression coefficients were found for second and combined block. For the second block, the regression coefficient was the highest value, so it is the best surface roughness of model equation. Similarly, for other lubricants and surface hardness, the model equations were calculated.