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Parametric Analysis of the Car Body Suspended Equipment for Railway Vehicles Vibration Reduction

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ABSTRACT A generalized railway vehicle model is built to study the coupled vibrations between a flexible car body (CB) and the equipment suspended beneath it by taking the complex suspension characteristics of the equipment into consideration. Associated theories for the determination of the suspension parameters for CB suspended equipment are summarized systematically, including the vibration isolation methods and the dynamic vibration absorber (DVA) theories for both undamped and damped cases. The equipment is then grouped into four categories by weight, location, and the excitation that it is subjected to. A general principle for the determination of the suspension parameters is proposed for better vibration isolation effects concerning bogie hunting, the elastic modes of the CB and its excitations. Analysis based on DVA theory shows that 1) siting equipment heavier than 2 t near the CB center can reduce the structural vibrations of the CB considerably, 2) siting medium-weight equipment near one end of the CB is good for absorbing the vibrations resulting from bogie hunting, and 3) a rigid connection is suggested for light equipment. Furthermore, on-track field tests of a high speed train confirmed that the heavy equipment vibrated violently upon absorbing some of the CB vibration energy resulting from bogie hunting. However, solid proof of the vertical bending of the CB was not obtained yet because it was sufficiently damped and is not the first natural mode of a modern aluminum-alloy CB. Further research on the lateral flexibility of the CB and its coupled vibrations with the heavy equipment mounted near its center are in need.

INDEX TERMS Car body, flexibility, railway vehicle, suspended equipment, suspension parameters.

I. INTRODUCTION

Lightening design of a car body (CB) is a tendency for high speed trains, however, it often leads to a lower structural stiffness and therefore lower Eigen frequencies, which can fall in the 6-12 Hz range, to which the human body is particularly sensitive. This facilitates easy excitation of CB structural vibrations which would reduce ride comfort significantly. Similarly, the CB structural stiffness is inversely proportional to the distance between the bogie centers. With higher operation speeds and the occurrence of abnormal large equivalent conicity of wheelset on certain track segments, the bogie hunting frequency arises more on high speed vehicles than on traditional railway vehicles. Indeed, the frequency of the bogie hunting motion is so high that it is close to the frequency of the first natural mode of CB. Thus, resonance occurs worsening the vibrations of the floor, seats and accessory components mounted inside the car body. This resonance detriment severely the ride comfort of high speed trains. Therefore, it is necessary to consider the modal vibration of CB and the complex structure of the floor and seats when analyzing the ride comfort. The factors involved in the ride comfort analysis in turn provide solutions to improving the ride quality for high speed trains.

Furthermore, electric multiple units (EMUs), urban and metro vehicles are equipped with functional devices. These devices include the traction transformer, traction converter, braking unit, cooling unit, air compressor and waste collection unit, as shown in Figure 1. Some of them weigh tons and are subjected to vibrational excitations, such as cooling

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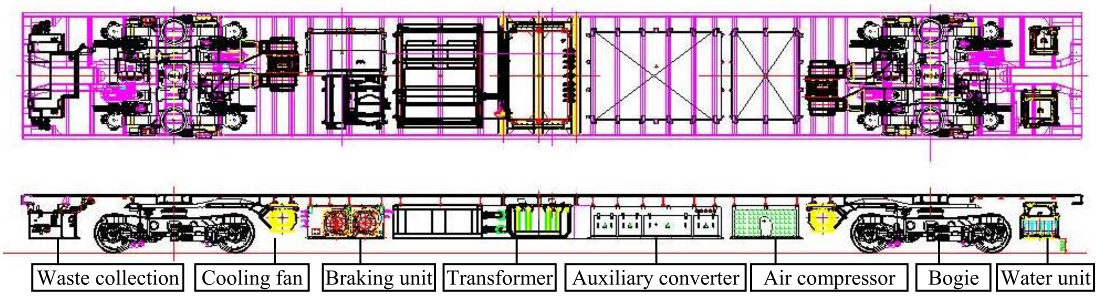

FIGURE 1. Arrangement of car body (CB) and CB suspended equipment (CBSE) of a high speed train.

fans and mechanical switches. Such devices are connected to the CB chassis by elastic suspension elements (i.e. rubber components) to avoid transmitting excitation and noise to the CB, which would reduce the ride comfort considerably. Modal matching between the heavy equipment and the CB is required because vibration of the equipment affects that of the CB considerably. This is because the modal mass of the whole system including the CB and the equipment suspended beneath it (hereinafter referred to as the CB suspended equipment (CBSE)) changes with the suspension parameters of equipment. Furthermore, regarding the vibration characteristics of this coupled system, the equipment vibrations should be limited to ensure satisfactory ride comfort and acceptable force and stress environments for the suspensions.

It is observed that the hunting frequency of the bogies approaches that of the diamond mode of the CB when the vehicle travels on certain track segments during operation, thus causing resonance and reducing the ride comfort severely [1]–[3]. Besides the optimization of the wheel-rail interaction to decrease the frequency and magnitude of bogie hunting, the frequency of the first natural mode of the CB must also be increased to avoid resonance. It is known that attaching an active control and damping layer on the CB surface can suppress vertical bending of the CB, but the response of the CB depends strongly on the output force of the actuator and the damping coefficient of the damping layer, as well as where these output forces is located on the CB [4]–[8]. However, such active layers are generally not used by the rail industry because of their high cost and low reliability. On the other hand, mounting a dynamic vibration absorber (DVA) under the CB can reduce both the rigid and elastic vibrations of the CB. The modal vibration can be reduced effectively by a CBSE with reasonably optimized suspension parameters, which works as a DVA. Both numerical analyses and field tests on rig and on track verified that [9]–[11]. However, the vibration reduction measured in the field is less than that predicted by numerical simulations and lab tests because of restricted installation space and the limited mass and actual volume of the CBSE in operation. Consequently, CB modal vibration still arises on track even when using vibration isolation and optimized suspension parameters based on DVA theory [13], [14].

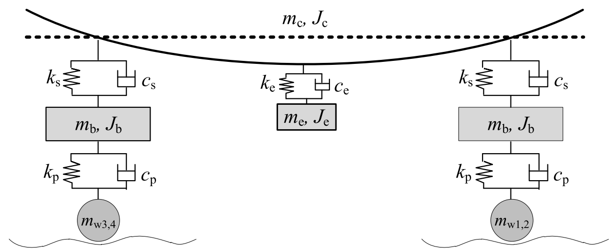

FIGURE 2. Generalized model of vehicle with flexible CB and CBSE.

At present, there are no generalized principles for designing the suspension parameters of the CBSE for a railway vehicle. Further consideration is required to determine the objective of vibration reduction, namely either the CB or the CBSE. Meanwhile, there has been no comprehensive study of how to classify the CBSE and the corresponding parametric design methods. In the present investigation, the coupled vibrations between a flexible CB and its CBSE are studied by employing multi-body dynamics of a railway vehicle system, and the CBSE suspension parameters are analyzed. The CBSEs are grouped into different categories according to factors such as the mounting location, excitation and mass. Moreover, the design principles of each category are discussed and the corresponding theory is introduced. The design goals and principles of the suspension parameters are then proposed concerning the bogie hunting characteristics in field. Finally, the suspension parameters for the heaviest CBSE are verified through on-track tests of a high speed train at a speed of 250 km/h.

II. GENERALISED VEHICLE MODEL OF FLEXIBLE CAR BODY WITH EQUIPMENT SUSPENDED BENEATH IT

The horizontal bending and diamond mode shapes of the CB of a high speed train can be much more dominant than the vertical bending, consequently all those modes should be considered when modelling the railway vehicle system. Since the physics is pretty much the same for modelling both the vertical and lateral modes, a model generally used to analyze the vertical vibrations is introduced, in which only vertical modes (i.e. bounce, pitch and bending modes) are included. As depicted in Figure 2, the model is a rigid-flexible coupling system comprising of (i) the CBSE (ii) a flexible beam as the CB and (iii) rigid bodies, i.e., the two bogie frames and four wheelsets. The CB is modelled as a simple uniform Euler Bernoulli beam that is supported by the bogie frames via the secondary suspension. The equipment is treated as a two-degree-of-freedom (2DOF) rigid body that is fastened to the CB via an elastic or rigid suspension. The bogie frame is treated as a 2DOF rigid body that is supported by the wheelsets via the primary suspension. The wheelsets are supported closely by the rails, which means that no wheel jump happens and the vertical movements of the wheelsets are exactly the same as the track irregularities. This hypothesis assumes that the track is rigid; although this deviates somewhat from reality, it does not affect the conclusions of the present study because the natural frequencies of the rail are much higher than that of the lowest CB mode.

Travelling at a constant speed v to the right along the track centerline (x axis) with a downward direction for the positive z axis, the origin of the coordinate system is located at the geometrical center of each body. The differential equation of each body is written according to the load acting on that body. When the CB is considered as a rigid body, its bounce and pitch movement can be represented by z_c and θ_c, z_{bi} and θ_{bi} $(i = 1, 2)$ describe the movements of bounce and pitch of bogie i, z_{wj} $(j = 1-4)$ represents the movement of bounce in wheelset j and z_{ej} and θ_{ej} (j = 1-N, N represents the total number of CBSE mounted on the CB) describe the movements of bounce and pitch of CBSE j. The total number of degrees of freedom of all rigid bodies is $10 + 2N$, becoming $10 + 2N + m$ if m natural modes of the CB are included for the whole system. Define the distance between bogie centers as L_b and the distance between wheelsets as L_w. Other unmentioned symbols are defined as shown in Figure 2.

A. KINEMATICS OF BOGIE

The equations of motion for the bounce and pitch of the two bogies are

$$
\begin{aligned}
m_b\ddot{z}_{b1} &= -c_p\left(2\dot{z}_{b1} - \dot{z}_{w1} - \dot{z}_{w2}\right) - k_p\left(2z_{b1} - z_{w1} - z_{w2}\right) \\
&\quad -c_s\left[\dot{z}_{b1} - \dot{z}\left(x_1, t\right)\right] - k_s\left[z_{b1} - z\left(x_1, t\right)\right], \qquad (1)\\
J_b\ddot{\theta}_{b1} &= -c_pL_w\left(2\,L_w\dot{\theta}_{b1} - \dot{z}_{w1} + \dot{z}_{w2}\right) \\
&\quad -k_pL_w\left(2\,L_w\theta_{b1} - z_{w1} + z_{w2}\right), \qquad (2)\\
m_b\ddot{z}_{b2} &= -c_p\left(2\dot{z}_{b2} - \dot{z}_{w3} - \dot{z}_{w4}\right) - k_p\left(2z_{b2} - z_{w3} - z_{w4}\right) \\
&\quad -c_s\left[\dot{z}_{b2} - \dot{z}\left(x_2, t\right)\right] - k_s\left[z_{b2} - z\left(x_2, t\right)\right], \qquad (3)\\
J_b\ddot{\theta}_{b2} &= -c_pL_w\left(2\,L_w\dot{\theta}_{b2} - \dot{z}_{w3} + \dot{z}_{w4}\right) \\
&\quad -k_pL_w\left(2\,L_w\theta_{b2} - z_{w3} + z_{w4}\right), \qquad (4)
\end{aligned}
$$

where $z(x_1, t)$, $z(x_2, t)$ represent the total vertical displacement of the CB above the center of the front bogie and rear bogie respectively, including both the rigid displacement and flexible vibrations. m_b is the mass of bogie, while J_b represents the moment of inertia related to the pitching motion of bogie and k_s and c_s are the secondary-suspension stiffness and damping, respectively. The equations of motion for the bounce of the four wheelsets are

$$
\begin{aligned}
m_w\ddot{z}_{w1} &= -c_p\left(\dot{z}_{w1} - \dot{z}_{b1} - L_w\dot{\theta}_{b1}\right) \\
&\quad - k_p\left(z_{w1} - z_{b1} - L_w\theta_{b1}\right) - 2p_1, \qquad (5)\\
m_w\ddot{z}_{w2} &= -c_p\left(\dot{z}_{w2} - \dot{z}_{b1} + L_w\dot{\theta}_{b1}\right) \\
&\quad - k_p\left(z_{w2} - z_{b1} + L_w\theta_{b1}\right) - 2p_2 \qquad (6)\\
m_w\ddot{z}_{w3} &= -c_p\left(\dot{z}_{w3} - \dot{z}_{b2} - L_w\dot{\theta}_{b2}\right) \\
&\quad - k_p\left(z_{w3} - z_{b2} - L_w\theta_{b2}\right) - 2p_3, \qquad (7)\\
m_w\ddot{z}_{w4} &= -c_p\left(\dot{z}_{w4} - \dot{z}_{b2} + L_w\dot{\theta}_{b2}\right) \\
&\quad - k_p\left(z_{w4} - z_{b2} + L_w\theta_{b2}\right) - 2p_4, \qquad (8)
\end{aligned}
$$

where m_w is the mass of a wheelset and k_p and c_p are the primary suspension stiffness and damping, respectively. The term p_j is half the contact force of the wheel-rail interaction on wheelset j and is derived using nonlinear Hertz elastic contact theory [15] as

$$
p_j = \begin{cases} \left[\frac{1}{G}\left(z_{wj} - z_{tj}\right)\right]^{3/2} & \left(z_{wj} - z_{tj}\right) > 0 \\ 0 & \left(z_{wj} - z_{tj}\right) \le 0, \end{cases} \qquad (9)
$$

where z_tj is the track irregularity and z_wj is the bouncing displacement of wheelset j. The constant G $(= 3.86R^{-0.115} \times 10^{-8}$ m/N$^{2/3})$ arises from the wheel-rail interaction. Equation 9 shows that the wheel-rail contact force is determined by the wheelset displacement and the track irregularity. The irregularities associated with wheelsets 2-4 are represented as functions of that associated with wheelset 1:

$$
\begin{cases} z_{t2} = z_{t1}\left(t - 2L_w/v\right) \\ z_{t3} = z_{t1}\left(t - 2L_b/v\right) \\ z_{t4} = z_{t1}\left[t - 2\left(L_b + L_w\right)/v\right]. \end{cases} \qquad (10)
$$

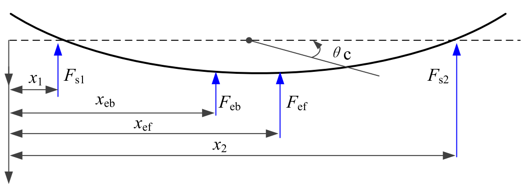

FIGURE 3. Forces acting on flexible CB with one piece of CBSE.

B. KINEMATICS OF CAR BODY WITH ONE PIECE OF SUSPENDED EQUIPMENT

In Figure 3, the CB is represented as a simple uniform Euler Bernoulli beam with free-free ends, constant cross section and uniformly distributed mass. The vertical displacement of the CB at location x and time t is denoted by $z(x, t)$. The forces applied on the CB by the air springs are denoted as F_{s1} and F_{s2}, for which the front and rear bogies are located at x_1 and x_2, respectively from the left-hand end of the CB. The forces applied by the CBSE are F_{eb} and F_{ef} through two connections (i.e. front and back) at locations x_{eb} and x_{ef}, respectively, from the left-hand end of the CB.

The differential equation of motion for the flexible CB is

$$EI\frac{\partial^4 z(x,t)}{\partial x^4}+\mu I\frac{\partial^5 z(x,t)}{\partial t\partial x^4}+\rho A\frac{\partial^2 z(x,t)}{\partial t^2}=\sum_{j=1}^{2}F_{sj}\delta\left(x-x_j\right)+F_{ef,b}\delta\left(x-x_{ef,b}\right), \quad (11)$$

where $z(x, t)$ is the vertical displacement of the CB, t is time, x is the distance along the beam, EI, μI and ρA are the bending rigidity, mass per unit length and internal coefficient of the beam, respectively, $\delta(x)$ is the Dirac delta function, F_{sj} is the force acted at points x_1 and x_2 by the secondary suspension and $F_{ef,b}$ is the force acted at point $x_{ef,b}$ by the CBSE suspension. F_{sj} and $F_{ef,b}$ are expressed as

$$F_{sj}=-k_s\left[z(x_j,t)-z_{bj}\right]-c_s\left[\dot{z}(x_j,t)-\dot{z}_{bj}\right],\quad j=1,2, \quad (12)$$

$$F_{ef,b}=-k_e\left[z\left(x_{ef,b},t\right)-z_e\pm\theta_e l_{ef,b}\right]-c_e\left[\dot{z}\left(x_{ef,b},t\right)-\dot{z}_e\pm\dot{\theta}_e l_{ef,b}\right], \quad (13)$$

where k_e and c_e are the suspension stiffness and damping of the CBSE, respectively. Equation (11) can be solved using separation of variables. Terms $Y_i(x)$ and $q_i(t)$ denote the Eigen function of the beam and the modal coordinate of its i^{th} mode, respectively, and m is the number of modes considered in the CB flexibility.

The CB displacement at an arbitrary location can be written as

$$z(x,t)=z_c+(x-L/2)\,\theta_c+\sum_{i=1}^{m}Y_i(x)q_i, \quad (14)$$

where L is the CB length, z_c is the CB bouncing motion with unit Eigen function, θ_c is the CB pitching motion with the Eigen function $x - L/2$, and the Eigen function of the beam's flexible bending mode is

$$Y_i(x)=ch\beta_i x+\cos\beta_i x-\frac{ch\lambda_i-\cos\lambda_i}{sh\lambda_i-\sin\lambda_i}\left(sh\beta_i x+\sin\beta_i x\right), \quad (15)$$

where $ch\lambda_i cos\lambda_i = 0$, $\beta_i = \lambda_i/L$ and $\lambda_i \approx (2i + 1)/2$, $i = 1, 2, \ldots m$. The CB equations of motion are obtained by substituting Equation (14) into Equation (11) and integrating along the beam length while acknowledging the orthogonality of the Eigen functions and the performance of the Dirac delta function. The CB equations of motion are

$$\ddot{q}_i+2\xi_i\omega_i\dot{q}_i+\omega_i^2 q_i=\sum_{j=1}^{2}\frac{Y_i(x_j)}{m_c}F_{sj}+\frac{Y_i\left(x_{ef,b}\right)}{m_c}F_{ef,b}, \quad (16)$$

$$m_c\ddot{z}_c=\sum_{j=1}^{2}F_{sj}+F_{ef,b}, \quad (17)$$

$$J_c\ddot{\theta}_c=\sum_{j=1}^{2}F_{sj}(x-L/2)+F_{eb,f}\left(x_{eb,f}-x_e\right), \quad (18)$$

where m_c and J_c are the mass and moment of inertia, respectively, related to the CB pitching motion, ω_i and ξ_i are the angular frequency and damping ratio, respectively, of the i^{th}-order CB bending mode and are expressed via $\frac{EI\beta_i^4}{\rho A}=\omega_i^2$ and $\frac{\mu I\beta_i^4}{\rho A}=2\xi_i\omega_i$, respectively. Similarly, the CBSE bounce and pitch are written as

$$m_e\ddot{z}_e=-F_{eb,f}, \quad (19)$$

$$J_e\ddot{\theta}_e=-F_{eb,f}\left(x_{eb,f}-x_e\right), \quad (20)$$

where m_e and J_e are the mass and moment of inertia, respectively, related to the CBSE pitching motion.

C. KINEMATICS OF CAR BODY WITH MULTIPLE PIECES OF SUSPENDED EQUIPMENT

Figure 4 shows the model of a CB with more than one piece of CBSE. The CB equations of motion in this case are similar to Equations (16)-(18). The force acting on the CB from the CBSE suspension is derived for each piece of CBSE using Equation (13) and the specific position of action on the CB. Similarly, the equation of motion for each piece of CBSE is derived with reference to Equations (19) and (20).

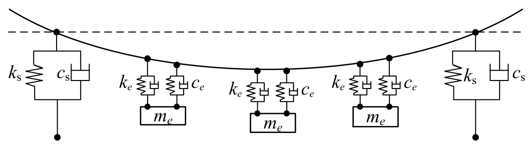

FIGURE 4. Model of flexible CB with multiple pieces of CBSE.

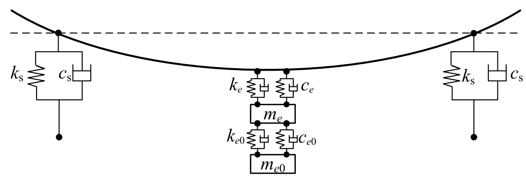

FIGURE 5. Flexible CB with one piece of CBSE using a two-stage suspension.

D. TWO-STAGE SUSPENSION SYSTEM FOR SUSPENDED EQUIPMENT

A two-stage suspension may also be used for the CBSE to avoid its excitation being transmitted to the CB, as shown in Figure 5.

In this case, the kinematic analysis is basically the same as the cases mentioned above while taking the suspended equipment as two rigid bodies, namely its frame and the equipment itself. As shown in Figure 5, m_e is the mass of the frame and m_{e0} is the mass of the functional equipment. Again, their equations of motion are obtained with reference to Equations (19) and (20). If the two-stage suspension is used for multiple pieces of CBSE, the associated equations of motion are derived by the combination of the equations of motion for the multi pieces of CBSE and the two stage suspension.

III. THEORETICAL BASIS OF VIBRATION ANALYSIS

The total mass of CBSEs can be as much as 10 t, while the rest of the CB of a high speed vehicle has a mass of approximately 30 t. This implies that the CBSE vibration can affect the CB considerably, leading to a flexible-rigid coupled vibration. A rigid connection between the CB and CBSE would definitely ensure reliability, whereas an elastic suspension based on DVA and vibration isolation helps to reduce CB/CBSE vibrations if the suspension parameters are determined carefully.

A. VIBRATION ISOLATION THEORY

The vibration mechanism of a one-degree-of-freedom (1DOF) system is quite clear and can be applied easily. While it does not account for the influence of vibration on the base, and it is inapplicable if the vibration of base is influenced significantly by the vibration of the isolated object. However, if the CBSE mass is so small that it does not affect the CB vibration, the vibration isolation theory of a 1DOF system can be used to determine the suspension parameters. In this section, two cases of excitation source are discussed, namely the base (CB) and the object (CBSE) itself.

1) EXCITATION CARRIED BY BASE

When discussing the vibrations transferred from the CB to the CBSE, the CB is regarded as the base and the CBSE is considered as the object. Based on the transition law for a 1DOF system, the acceleration transmission ratio is exactly the same as that of displacement or force, and is given as $T_g = \sqrt{(1+4\xi^2 g^2)/\left((1-g^2)^2+4\xi^2 g^2\right)}$, where $g = \omega/\omega_n$, ω and ω_n are the excitation frequency of the base and the suspension frequency of the isolated object, respectively, and ξ is the damping ratio [16]. The vibration transmission law in this case is widely known: the suspension frequency should be less than $1/\sqrt{2}$ of the base excitation frequency to ensure vibration reduction for the isolated object. It is known that it is generally less than 2.5 Hz for the rigid modes of the CB of a high speed train, meaning that the CBSE suspension frequency should be less than 1.8 Hz according to the vibration transmission law. Meanwhile, the lowest flexible-vibration mode of a CB often has a natural frequency higher than 6 Hz, meaning that the CBSE suspension frequency should be less than 4.2 Hz to isolate the excitation caused by the bending, diamond or torsion modes of the CB. Additionally, the damping ratio strongly affects the resonance magnitude when the excitation frequency is close to the natural frequency of the object. A large damping ratio reduces the resonance magnitude but leads to an undesirable isolation from the high-frequency excitation from the base. If the object experiences broad-spectrum excitation, a compromise in the damping ratio is needed to keep the low-frequency and high-frequency responses within a given limit simultaneously.

2) EXCITATION CARRIED BY OBJECT

When the object (CBSE) carries the excitation, the excitation should be isolated to avoid deteriorating the vibration of the base (CB). The transfer ratio is the same as that when the excitation is carried by the base. To reduce the CB vibration, the CBSE suspension frequency should again be less than $1/\sqrt{2}$ of the excitation frequency. Given that the excitation frequency usually exceeds 20 Hz (e.g. from the unbalanced rotation of a cooling fan), the CBSE suspension frequency should be less than $20/\sqrt{2}$ Hz, which equals 14 Hz approximately.

B. UNDAMPED DVA THEORY

The hunting frequency of the bogies is high enough to approach the lowest natural frequency of the CB, thereby causing resonance in the CB structural vibrations. Based on undamped DVA theory, the CBSE suspension frequency can be set close to the frequency of the bogie hunting movement, thus the hunting excitation transmitted to the CB can be furtherly transmitted to the CBSE. A numerical study using the model in Section II (comprising a flexible CB and one piece of CBSE) is performed to state how undamped DVA theory can be applied. In this examination, the CB mass (without CBSE) is 28.8 t and the CBSE mass is 2.6 t. The damping ratio of the secondary suspension of the vehicle remains constant at 20%, and that of the CBSE suspension is 1% to stimulate a vibration absorber with pretty low damping or even without damping. The frequency of the bogie hunting motion is taken as 7 Hz, and this excitation acts directly on the secondary suspension of the vehicle [14]. As shown in Figure 6, the CB acceleration amplitude is 0.23 m/s^2 with a CBSE suspension frequency of 11 Hz, dropping to 0.05 m/s^2 with a CBSE suspension frequency of 7 Hz (i.e. the excitation frequency). It is concluded that the CB vibration excited by bogie hunting can be reduced effectively when the CBSE suspension frequency is the same as the excitation frequency in the undamped case. In that case, undamped DVA theory can be used to improve the ride comfort of railway vehicles undergoing bogie hunting.

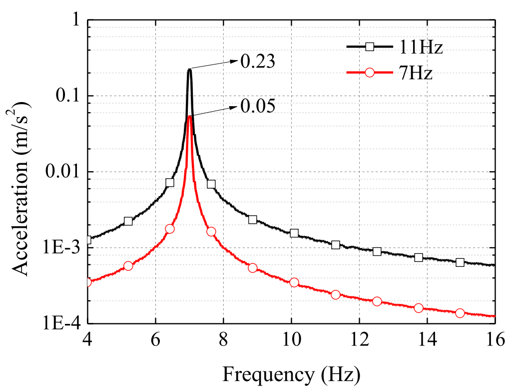

FIGURE 6. Frequency spectrum of CB acceleration.

C. DAMPED DVA THEORY

Rubber elements are commonly used to connect the CBSE to the CB chassis and provide a damping ratio of approximately 25%, meaning that damped DVA theory must be used to analyze the CBSE suspension parameters. The optimized CBSE suspension frequency can be obtained straightforwardly through general DVA analysis. The optimized frequency ratio between the CBSE suspension frequency and the CB natural frequency is written as $f_{opt} = \frac{1}{1+\mu Y^2(x)}$, where μ is the CBSE-to-CB mass ratio and $Y(x)$ is the Eigen function at a specific CB location x as given by Equation (15) [16], [17]. Assuming that the first CB bending mode has a frequency of 12 Hz [10], [11], for a CBSE mass of 4 t mounted at the CB center, the CBSE suspension frequency is 9.8 Hz. Similar to the optimized suspension frequency for a DVA, its optimized damping ratio also has a closed form as stated in the literature [16].

Numerical analysis using the model comprising a flexible CB and one piece of CBSE is performed to study how the CBSE suspension frequency affects the vibration of others. The stiffness of the rubber elements determines the CBSE suspension frequency f by the relationship $f = \sqrt{k/m}/(2\pi)$, where k is the stiffness of the rubber and m is the CBSE mass. In the frequency response analysis, the steady-state oscillation amplitude is studied as a function of the excitation frequency at a certain position. Herein, interest is confined to the CB response to excitations applied to the wheelsets. To excite the CB bending vibration fully, a harmonic excitation is imposed in phase on the four wheelsets. The CBSE mass is taken as 4 t and the CBSE is mounted at the CB center.

Figure 7 shows the frequency response at the CB center and at the CBSE when considering either a rigid CB-CBSE connection or an elastic connection with a suspended frequency of 8-10 Hz. The magnification is defined as the amplitude ratio of vibration of CB/CBSE with respect to that of the base excitation. With the CBSE connected rigidly to the CB center, a single peak with a frequency around 9 Hz is obtained and the frequency and its amplitude is dependent on the CBSE weight. In this case, the two bodies make one coupled body that vibrates in phase. However, when elastic suspension is applied, the single peak is replaced by two peaks with notably lower amplitude; the frequency of one peak is lower than that in the rigid-connection case, while the frequency of the other is higher. The amplitudes of these two peaks are determined by the CBSE suspension frequency, which is optimized to make the two peaks as low as possible; by contrast, the CBSE experiences its severest vibration if the structural vibration on CB is minimized. Thus the vibration energy is absorbed by the CBSE, which fulfil the purpose of DVA. This means that an elastic CBSE suspension can effectively reduce the CB vibration compared with the rigid connection. Further numerical simulations that consider the CBSE as a DVA can be found in previous work [11], [18].

IV. DESIGN OF SUSPENSION PARAMETERS FOR EQUIPMENT SUSPENDED BENEATH CAR BODY

The CBSEs can be classified according to its (i) mass, (ii) location on the CB and (iii) vibration characteristics, whereupon general principles for designing the suspension parameters can be summarized concerning the corresponding

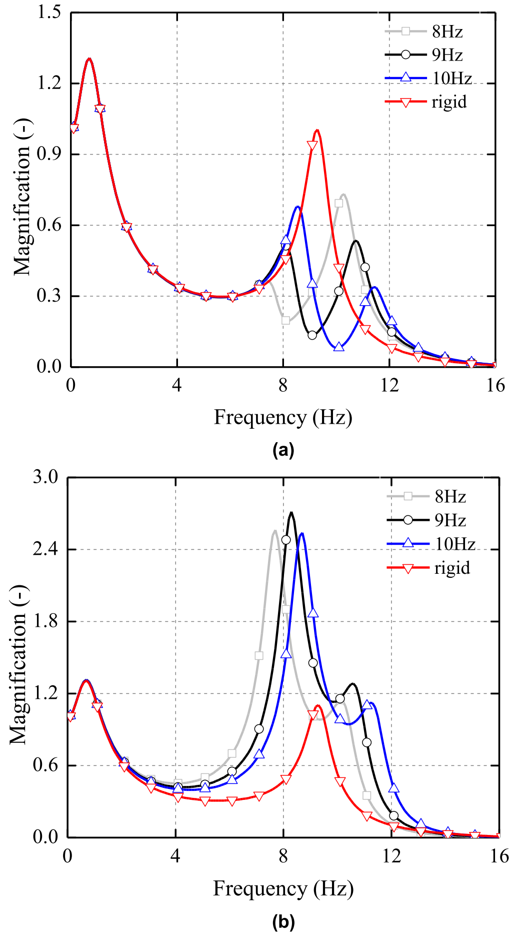

FIGURE 7. Frequency response of displacement with respect to excitation: (a) CB and (b) CBSE.

theories discussed in Section III. As summarized in Table 1, the CBSE of a high speed train is categorized into four types.

Regarding the equipment types listed in Table 1, the corresponding principles in the design of the suspension parameters are summarized in Table 2 based on the theoretical analysis presented in Section III. Because the CBSE suspension frequency determines how this coupled system vibrates, it is used as a reference when introducing the CBSE design principles. The CBSE mass and the excitation performance on CBSE are clearly the two factors considered when choosing which theoretical basis to apply. Regarding the CB structural vibration and the excitations due to bogie hunting and the CBSE, various theoretical bases are used to design the suspension parameters of various types of equipment. Besides the principal focus on the CBSE suspension frequency, other aspects must be considered. The dynamic stiffness of the rubber elements is restricted by their geometry, size and material properties, and may deviate from the required value listed in Table 2. Moreover, coupled vibration between two adjacent pieces of equipment mounted on a CB should be avoided. It is also undesirable if a piece of equipment vibrates violently both vertically and laterally simultaneously. Furthermore, because the static deflection

TABLE 1. Classifications of CBSE.

Type	Characteristics	Instances
A	Electromagnetic excitation at 50 Hz or 100 Hz at the same or higher level than its rigid vibration	Traction transformer, traction converter, auxiliary converter
A	Rotating/switching equipment with high-frequency dynamic imbalance in mass or impacts ≥1 g	Cooling unit, air compressor and waste pumping unit, air conditioner
B	Heavy equipment (≥2 t) without excitation, located near CB center	Traction transformers and converters
C	Medium equipment (≥1 t) without excitation, located near CB end or bogie	Battery storage
D	Light equipment (≤1 t) without excitation	Brake unit, water tank and waste storage

TABLE 2. Design principles for CBSE suspension.

Type	Theory basis	Design principle
A	Vibration isolation	Suspension should be soft enough to isolate the excitation on equipment transmitted to the CB, approximately 7 Hz in case of excitation at 20 Hz because 20/√2≥7
B	Damped DVA	Suspension frequency should be within 7-9 Hz as determined by damped DVA theory to restrain the structural vibration of CB
C	Undamped DVA	Suspension frequency should be the same as the potential hunting frequency of bogie, such as 5-8 Hz
D	Rigid connection	Suspension should be stiff enough, or a rigid fasten can be employed to ensure connection reliability

of the suspension components is restricted by space limitations, the rubber stiffness is adjusted slightly from the design value to allow the CB and CBSE to be assembled correctly. In addition, manufacturing errors and the aging of rubber also change its stiffness. The suspension frequency should always be within the design range even considering all the aforementioned matters. Additionally, electromagnetic oscillation is quite common for type-A equipment, and its frequency is very high comparted to the rigid movements of the CBSE. The damping of rubber can isolate high-frequency vibrations effectively, therefore numerical analyses and rig tests should be conducted to achieve the most satisfactory damping coefficient of the rubber components.

V. FIELD TESTS ON COUPLED VIBRATION OF CAR BODY AND SUSPENDED EQUIPMENT

The vibration of a CB and its CBSE were measured by field tests on a high speed train running at a speed of 250 km/h. The test was conducted on the Ha'erbin-Dalian high speed line in northeast China with the associated equipment and elastic suspensions as shown in Figure 8. In these tests, the wheel profile was measured at different travelling mileages and the associated vehicle acceleration was recorded. The accelerations of the CB and CBSE were analyzed separately along the vertical and lateral axes. Accelerometers with a maximum response amplitude of 2 g and a frequency of 1 kHz were mounted on both the CB and the CBSE around the rubber element. For the on-board data acquisition system, the sampling rate was set as 2 kHz for the sensors on the bogie frame and 5 kHz for those on the wheelset.

The equipment illustrated in Figure 8 is the traction and auxiliary converter. It weighs 3,155 kg and is suspended on the CB chassis by seven pairs of elastic mounts. The vertical stiffness of each connection is 412 N/mm, thus the vertical

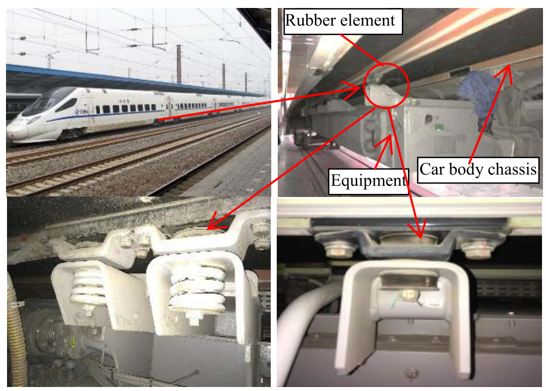

FIGURE 8. Field test of equipment vibration on electric multiple unit (EMU).

suspension frequency of the equipment can be calculated roughly as $f = \sqrt{14 \times 0.412 \times 10^6/3155}/(2\pi) = 6.8$ Hz; the lateral suspension frequency is close to the vertical suspension frequency. The distance between the bogie centers is 19 m, while that between the wheelsets is 2.7 m. The lowest natural mode of the CB is diamond mode in the middle region. The frequency of this natural mode for different types of CB of a high speed train varies within 7-9 Hz and is the lowest mode [17].

A. ANALYSIS OF LATERAL VIBRATION

At a speed of 250 km/h, the lateral acceleration of the frame end of the bogie is shown in Figure 9 with the original signal at a sampling rate of 2 kHz and the band-pass filtered signal in 0.5-15 Hz plotted. This indicates that the bogie experiences a hunting frequency of 7.6 Hz with an amplitude of around 1 g. Consequently, the associated lateral and unfiltered accelera-

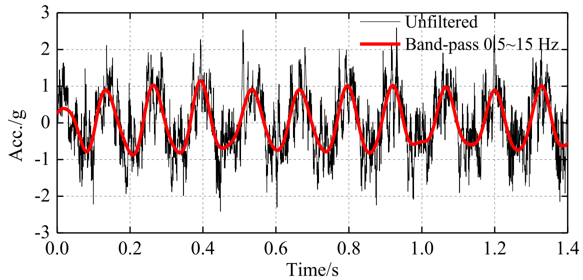

FIGURE 9. Lateral acceleration of frame end of bogie when hunting.

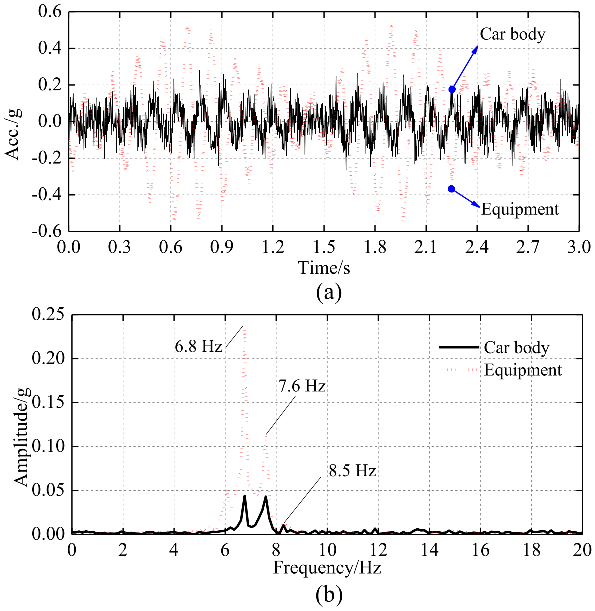

FIGURE 10. Lateral acceleration of CB and CBSE when bogie hunting: (a) time histories; (b) frequency spectra.

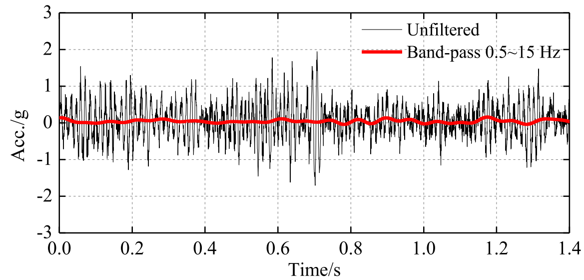

FIGURE 11. Lateral acceleration on the frame end of bogie.

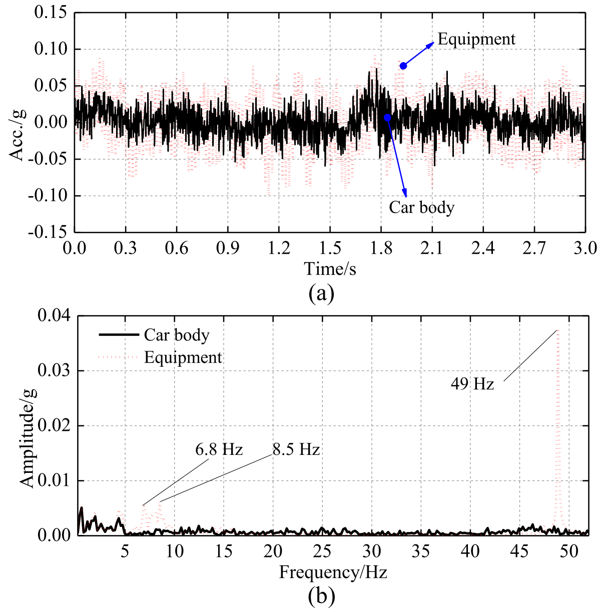

FIGURE 12. Lateral accelerations of CB and CBSE: (a) time histories; (b) frequency spectra.

tions of the CB and CBSE are shown in Figure 10. This shows that the CBSE acceleration has an amplitude of 0.5 g, which is more than twice that of the CB. Similar to the harmonic vibration of the bogie, the CB signals also experience harmonics. In addition, the CB and the traction converter vibrate oppositely in phase. The corresponding frequency spectrum shows the dominant vibration frequencies to be 6.8 Hz, 7.6 Hz and 8.5 Hz, which are close to the suspension frequency of the traction converter, the frequency of bogie hunting and the natural frequency of the CB diamond mode, respectively. Additionally, the frequency spectrum also indicates that the CBSE acceleration amplitude around the dominant frequency 6.8 Hz is 5-6 times that of the CB, in agreement with Figure 6. When the bogie experiences hunting, the equipment vibrates violently by absorbing some of the vibration from the CB, which is transmitted from the bogie. As shown in Figure 6 in Section III, the bogie hunting motion would reduce the ride comfort of the vehicle appreciably if the CBSE suspension frequency was not as close as required to the frequency of bogie hunting.

To verify the conclusions drawn from Figures 9 and 10, a comparative analysis is provided in Figures 11 and 12 in which the bogies run normally without hunting. In this case, the filtered acceleration of the bogie frame is less than 0.2 g and has no dominant frequency, while the CB and CBSE accelerations vary very little and have amplitudes that are generally lower than 0.1 g, which is less than 0.5 g in Figure 10(a). The main CBSE vibration frequency is around 49 Hz and results from electromagnetic excitation. This indicates that the high-frequency CBSE excitation is isolated effectively. In addition to the high-frequency electromagnetic vibration of the CB, the vibrations corresponding to the CBSE suspension frequency and the diamond mode of the structural vibration of the CB experience rather limited amplitudes compared to those when the CB vibrates abnormally. Therefore, setting a reasonable suspension parameter for a heavy equipment can reduce considerably the CB vibration resulting from its structural flexible vibration and the bogie hunting, thereby verifying the principle proposed in Table 2.

B. ANALYSIS OF VERTICAL VIBRATION

Figure 13 shows the corresponding vertical accelerations of the CB and CBSE when the bogie experiences hunting, in which the CBSE acceleration is higher than the CB because of the high-frequency vibrations at 13.5 Hz, 20.2 Hz and 49.0 Hz. These high-frequency vibrations, respectively, result

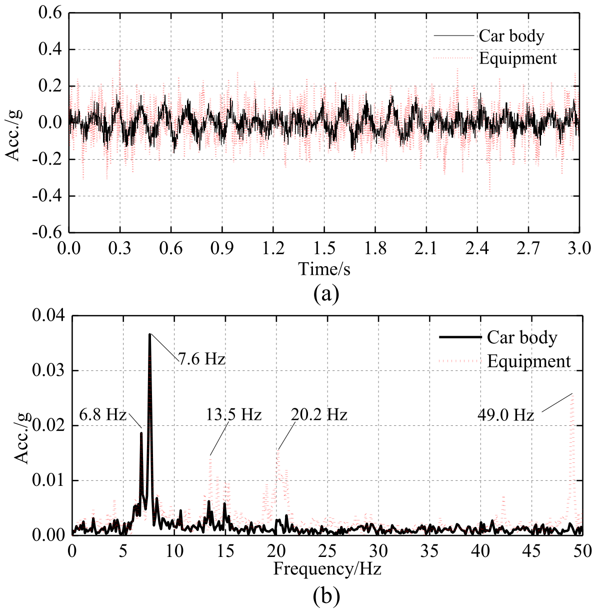

FIGURE 13. **Vertical accelerations of CB and CBSE: (a) time histories; (b) frequency spectra.**

from the potential rolling and pitching movements of the CBSE around its center of gravity and the electromagnetic excitation acting on it. Meanwhile, the CBSE vertical vibration has an amplitude of around 0.3 g, which is much smaller than that of the CBSE lateral vibration and experiences no obvious harmonics. Similar to the lateral acceleration analysis, the dominant peaks from the CBSE suspension frequency and the bogie hunting frequency are still evident in the vertical acceleration of the CB. While the acceleration amplitude corresponds to the two peaks on the CB are nearly same with that on the equipment. This implies that in the vertical direction, there is no obvious vibration absorption between the CBSE and CB. This is because the first vertical bending mode of the CB is sufficiently damped by its structure, thus not excited. Normally a mode is excited because its damping is sufficiently low. Therefore, the corresponding designed suspension parameters of the CBSE show no obvious effect on the vertical bending vibration reduction of the CB. However, the field tests also show that the structural vibration of the CB at high frequencies (6-12 Hz) is much more severe than the rigid movements at low frequencies (0.5-2.5 Hz). Therefore, further potential vibration-reduction techniques are still needed to restrain the flexible vibration of the CB to improve its ride comfort for high speed trains.

VI. CONCLUSIONS

In this paper, a generalized railway vehicle model is built to study the coupled vibrations between a flexible CB and its CBSE. Cases involving one piece of equipment, multiple pieces of equipment and a two-stage suspension are introduced with specific equations of motion for general use. The associated theoretical basis for designing the CBSE suspension parameters is then summarized systematically in terms of vibration-isolation theory and DVA theory. The equipment of high speed trains is categorized and corresponding design principles of the suspension parameters are proposed. The following conclusions are drawn from the results.

The generalized model concerns the CB flexibility and the CBSE vibrations. The effects of the suspension stiffness, damping, mounting-location, mass and its inertia of CBSE on the coupled vibrations between the CB and CBSE can be investigated systematically. The model can then be used for further analysis on more-complex configurations, for instance the center of gravity does not coincide the geometrical center of the equipment, or more than two connections are used. The vibration between two adjacent pieces of equipment can also be analyzed using this generalized model, as can the two-stage vibration-isolation performance for equipment with excitations.

Vibration-isolation theory is discussed for the case in which the base carries the excitation and also for the case in which the object carries the excitation. The focus is on the vibration transmitted from the CB to the CBSE or vice versa, whereupon the corresponding CBSE suspension frequency can be determined. For a railway vehicle, the aim is to isolate the excitation that acts on the CBSE from the CB. An undamped DVA is used to absorb the CB vibration resulting from bogie hunting, for which the CBSE is designed to have the same frequency as that of the bogie hunting. A damped DVA is used to replace the single peak resulting from a CBSE rigid connection by two peaks with notably lower amplitude when an elastic suspension is employed. One peak has a lower frequency than that in the rigid-connection case, and the other one is higher. The amplitude of these two peaks is determined by the CBSE suspension frequency, which is optimized to make the two peaks as low as possible.

The CBSE is grouped into four categories according to its mass, its location and the excitation that it affords. CSBE of type D has a mass of less than 1 t and does not carry any source of excitation, such as the brake unit, water tank and liquid waste storage. Its suspension should be either stiff enough or a rigid connection to ensure connection reliability because it does not help to reduce the CB vibration. CBSE of type C weighs 1-2 t and does not carry any source of excitation either, such as the battery storage located near the CB end or bogie. It should be suspended with the same frequency as that of the bogie hunting, such as 5-8 Hz for high speed trains. For type B, which weighs more than 2 t and is not a source of excitation near the CB center, such as the transformer and converter, damped DVA theory should be used to determine its suspension frequency to reduce the CB structural vibration (e.g. a suspension frequency of 7-9 Hz for the CBSE concerns the lowest natural mode of an aluminum-alloy CB). CBSE of type A carries rotational excitation or mechanical impact. Its suspension should be soft enough to isolate its excitation, which would reduce the ride comfort and increase the noise in the cabin if transmitted to CB (e.g. a suspension frequency of 7 Hz for the CBSE in the case of an excitation whose frequency exceeds 20 Hz on the CBSE).

On-track field tests of a high speed train at a speed of 250 km/h verify that heavy equipment vibrates violently and absorbs some of the CB vibration resulting from bogie hunting. Vertical bending of the CB cannot be excited in operation because it has sufficiently high structural damping and is not the lowest natural mode of the CB. The lowest natural mode for a modern aluminum-alloy CB is the diamond mode in the middle. Therefore, further research on the lateral flexibility of the CB and its coupled vibrations with the heavy equipment mounted near its center are in need.

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