Parametric decays in relativistic magnetized electron-positron plasmas with relativistic temperatures

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Introduction

Relativistic electron-positron plasmas have received much attention because they are relevant in several environments, either of astrophysical or laboratory nature. Examples of this are accretion disks, models of early universe, ultra-intense lasers, laboratory and tokamak plasmas, pulsar magnetospheres or hypothetical quark stars. Several effects in these plasmas relate to wave propagation, such as the proposed pulsar radio emission processes, bulk acceleration of relativistic jets, quasar relativistic jets, or electron-positron pair annihilation into one-photon in the presence of a strong magnetic field.

In several of the environments mentioned above, relativistic effects and temperature play an important role, thus it is fundamental to understand wave propagation modes in relativistic plasmas with temperature. Recently, a finite amplitude nonlinear solution for relativistic electron-positron plasmas has been found for relativistic temperatures [1], using an approach based on the magnetofluid field unification formalism of Ref. [2].

In this work we will consider the parametric perturbations of finite amplitude circularly polarized electromagnetic waves in a relativistic electron-positron thermal plasma, which was solved in Ref. [1]. Although simple, this analysis will allow us to study in detail the effect of relativistic temperatures on wave propagation, and its decay, in relativistic hot plasmas.

Exact Solution

The relativistic plasma, for each species j (e for electrons and p for positrons), obeys the fluid equation

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_j\cdot\nabla\right)\left(f_j\gamma_j\mathbf{v}_j\right)=\frac{q_j}{m_j}\left(\mathbf{E}+\frac{1}{c}\mathbf{v}_j\mathbf{B}\right)-\frac{1}{m_jn_j}\nabla p_j\,, \tag{1}$$

where n_j is the density in the laboratory frame, γ_j is the relativistic factor, and f_j is a relativistic thermal factor which is related to the enthalpy density and depends on the thermodynamical properties of the plasma. For instance, if the system follows a Maxwell-Jüttner equilibrium

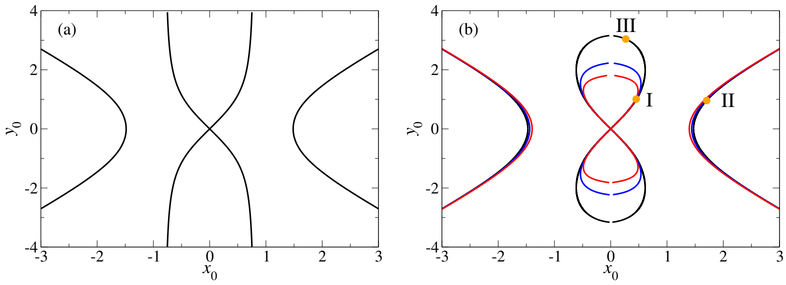

Figure 1: Dispersion relation of the pump wave, Eq. (3). Normalized wave number $y_0 = k_0 c/\Omega_c$ vs. normalized frequency $x_0 = \omega_0/\Omega_c$ for $\omega_p/\Omega_c = 1$, $1/\mu = 0.01$. (a) $\alpha = 0$. (b) $\alpha = 0.1$, $\alpha = 0.2$, $\alpha = 0.3$.

distribution, then $f_j = f(\mu_j) = K_3(\mu_j)/K_2(\mu_j)$, where $\mu_j = m_j c^2/k_B T_j$; K_2 and K_3 are the modified Bessel functions of order 2 and 3, respectively; and k_B is the Boltzmann constant.

As shown in [1] we can find an exact nonlinear transverse solution to these equations, which will be our zeroth order quantities in our approach, with the EM fields written as

$$\begin{aligned}\mathbf{E}_0(z,t) &= E_0\left[\sin(k_0 z - \omega_0 t)\hat{x} - \cos(k_0 z - \omega_0 t)\hat{y}\right] ,\\ \mathbf{B}_0(z,t) &= B_0\left[\cos(k_0 z - \omega_0 t)\hat{x} + \sin(k_0 z - \omega_0 t)\hat{y}\right] + B_{0z}\hat{z} .\end{aligned}$$

We represent the transverse quantities as $D_\perp = D_x + iD_y = De^{i(k_0 z - \omega_0 t)}$. Thus, from Eq. (1) we find the exact transverse velocity for each fluid as [1]

$$v_{0j} = \left(\frac{\omega_0}{f_{0j}\gamma_{0j}\omega_0 - \Omega_{cj}}\right)\frac{q_j B_0}{mck_0} , \qquad \alpha = \frac{e|B_0|}{mc^2 k_0} = \frac{e|A_0|}{mc} , \tag{2}$$

where $\Omega_{cp} = -\Omega_{ce} = \Omega_c = eB_{0z}/mc$ is the positron gyrofrequency and $\gamma_{0j} = (1 - v_{0j}^2/c^2)^{-1/2}$ is the relativistic factor.

The dispersion relation for circularly polarized EM wave to zeroth order, is given by [1]

$$\omega_0^2 - c^2 k_0^2 = \sum_j \omega_p^2 \left(\frac{\omega_0}{f_{0j}\gamma_{0j}\omega_0 - \Omega_{cj}}\right) , \tag{3}$$

where ω_p is the plasma frequency in the laboratory frame.

Parametric decays

Now, considering the finite amplitude transverse circularly polarized wave propagating in our electron-positron plasma system, with the dispersion relation (3), we introduce perturbations (represented by the δ symbol in front of the variables) for every quantity in the dynamical equation (1):

$$f_0\frac{\partial}{\partial t}\left(\gamma_{0j}\delta\mathbf{v}+\delta\gamma_j\mathbf{v}_{0j}\right)+f_0\left(\delta\mathbf{v}_j\cdot\nabla\right)\left(\gamma_{0j}\mathbf{v}_{0j}\right)+\gamma_{0j}\frac{\partial}{\partial t}\left(\mathbf{v}_{0j}\delta f_j\right)$$
$$=\frac{q_j}{m}\left(\delta\mathbf{E}+\frac{1}{c}\mathbf{v}_{0j}\delta\mathbf{B}\right)+\frac{q_j}{mc}\delta\mathbf{v}_j\mathbf{B}_0-\frac{1}{mn_0}\nabla\delta p_j\,, \tag{4}$$

where $\mathbf{v}_{0j}$ is the zeroth order transverse velocity, given by (2); n_{0j} is the zeroth order density in the laboratory frame; and f_0 is the unperturbed part of the function f_j.

We separate Eq. (4) in a longitudinal and transverse part and we assume that every longitudinal and transverse perturbation has the form

$$\delta D_z=\mathrm{Re}\left[\tilde{D}e^{i(kz-\omega t)}\right]=\frac{1}{2}\left(\tilde{D}e^{i(kz-\omega t)}+\tilde{D}^*e^{-i(k^*z-\omega^*t)}\right)\,, \tag{5}$$

$$\delta D=d_+e^{i(k_+z-\omega_+t)}+d_-e^{i(k_-z-\omega_-t)}\,, \tag{6}$$

respectively, where $k_+=k_0+k$, $k_-=k_0-k^*$, $\omega_+=\omega_0+\omega$ and $\omega_-=\omega_0-\omega^*$. Now we get a system of equations for the quantities $\mathbf{x}=(v_{+e},v_{+p},v^*_{-e},v^*_{-p},\tilde{v}_e,\tilde{v}_p,b_+,b^*_-)$. The dispersion relation can be found through the determinant of the set formed by these equations, $A\mathbf{x}=0$, so the dispersion relation will be,

$$F(k,\omega)=\det(A)=0\,. \tag{7}$$

We study the dispersion relation (7) for various pump waves. As we see in Fig. 1, we have three cases: Case I: the pump wave is on the Alfvén branch. Case II: the pump wave is on the electromagnetic branch. Case III: the pump wave is in the anomalous dispersion zone on the Alfvén branch. For each case, we choose $y_0=1$, $y_0=1$, and $y_0=4.632$, respectively.

In Fig. 2, we shown the dispersion relation (7) for the case I and II. We can see from Fig. 2 that there are several possible crossings between solutions of the dispersion relation. At these crossings, complex solutions can appear when $\alpha\neq0$. Since the polynomial being solved has real coefficients, these solutions always occur as complex conjugate pairs, thus one of them has a positive imaginary frequency. Therefore, the disappearance of real solutions when $\alpha\neq0$ implies the presence of unstable waves, corresponding to the parametric decays of the pump wave. When we turn the pump wave on by considering $\alpha\neq0$. In Fig. 2, $\alpha=0.1$, we notice that some crossings become gaps. This means that at these crossings we have complex solutions whose real parts are indicated as dotted lines, while the real solutions correspond to the continuous lines. Hence, we now have instabilities, indicating wave coupling.

Now in Fig. 3 we show the results for case III (anomalous dispersion, $d\omega/dk<0$ for $\omega>0$). It is interesting to note the instability that occurs for large y values, close to $x\approx0$, due to the (p_-,p_+) coupling, which can be considered as an electromagnetic modulational instability.

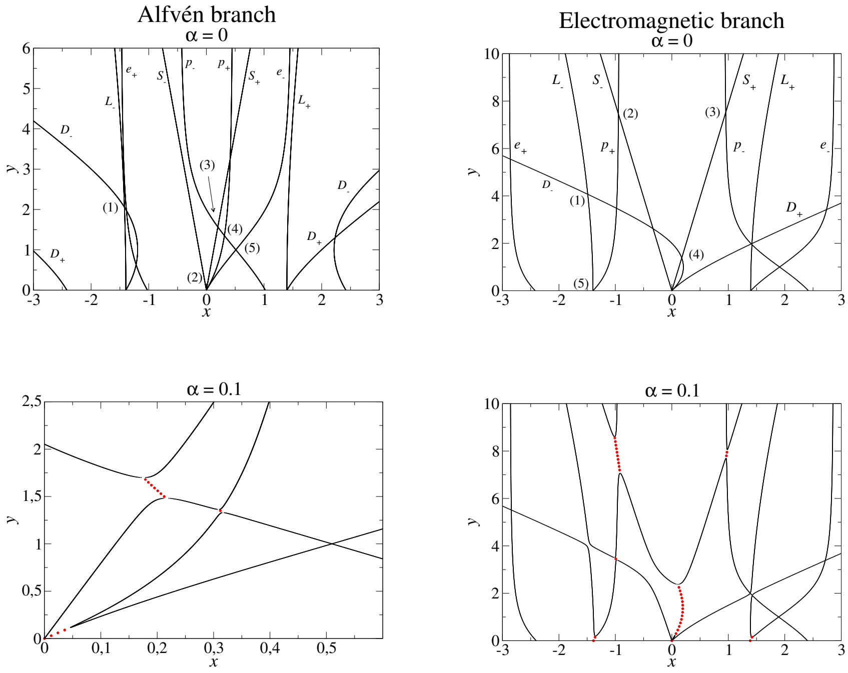

Figure 2: Solution of the dispersion relation Eq. (7). Normalized wave number $y = kc/\Omega_c$ vs. normalized frequency $x = \omega/\Omega_c$ for $y_0 = 1$, $\omega_p/\Omega_c = 1$, $1/\mu = 0.01$. The dotted lines correspond to complex conjugate pair solutions.

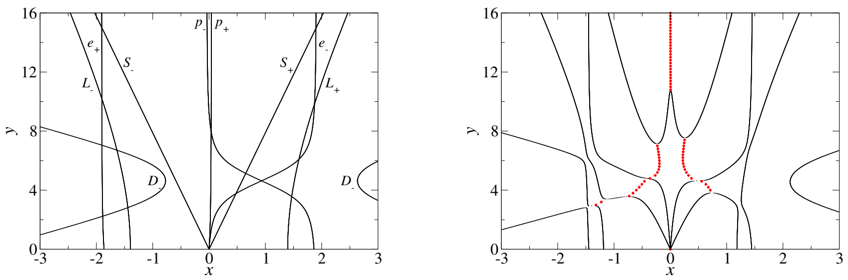

Figure 3: Dispersion relation Eq. (7). Normalized wave number vs. normalized frequency for $y_0 = 4.632$, $\omega_p/\Omega_c = 1$, $1/\mu = 0.01$. Left: $\alpha = 0$. Right: $\alpha = 0.04$. Dotted lines represent the real part of the complex solution.

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