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Parametric excitation of a SiN membrane via piezoelectricity

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We develop a stoichiometric silicon nitride (SiN) membrane-based electromechanical system, in which the spring constant of the mechanical resonator can be dynamically controlled via piezoelectric actuation. The degenerate parametric amplifier is studied in this configuration. We observe the splitting of mechanical mode in the response spectra of a phase-sensitive parametric amplifier. In addition, we demonstrate that the quality factor Q of the membrane oscillator can be significantly enhanced by more than two orders of magnitude due to the coherent amplification, reaching an effective Q factor of $\sim 3 \times 10^8$ at room temperature. The nonlinear effect on the parametric amplification is also investigated, as well as the thermomechanical noise squeezing. This system offers the possibility to integrate electrical, optical and mechanical degrees of freedom without compromising the exceptional material properties of SiN membranes, and can be a useful platform for studying cavity optoelectromechanics.
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Recent years have witnessed great progress in the fabrication and manipulation of micro-/nanomechanical resonators. Micro-/nanomechanical resonators can be utilized to implement precision measurements and ultrasensitive sensors as well as fundamental studies on the boundary between quantum and classical physics.[1,2] Tremendous accomplishments have been achieved in this field, including ground-state cooling of mechanical resonators,[3–6] quantum squeezing of mechanical modes,[7–9] generation of squeezed light,[10–12] and detection of mechanical motion with an imprecision down to the standard quantum limit.[13–16]

Among various mechanical oscillators, silicon nitride (SiN) membranes have attracted a lot of interest due to their exceptional material properties, e.g. very high mechanical Q factors and ultralow optical absorption for near infrared light. These features enable the SiN membrane as an excellent candidate for studying optomechanics. Recently, SiN membranes with intrinsic mechanical Q factors as large as 10^8 have been achieved at room temperature.[17–20] By placing such a flexible SiN membrane inside a high-finesse optical cavity, a cavity optomechanical device with the integration of ultrahigh Q mechanical oscillator and high-finesse optical resonator can be implemented.[21–27]

In the meantime, electrically controllable optomechanical systems exhibit striking advantages and have been a new field of interest recently.[28–35] The electrical actuation can usually be much stronger than the photonic coupling via radiation pressure, which provides the opportunity of more efficient control and the access to nonlinear regime. Typical optoelectromechanical systems utilize the dielectricity or piezoelectricity of the mechanical oscillator itself, such as silica microtoroids,[28] aluminum nitride (AlN) and gallium arsenide (GaAs) optomechanical crystals,[30,34] and AlN microwheel cavities.[31,32] The mechanical Q factors in most of these optoelectromechanical systems require to be further improved in order to obtain longer phonon lifetimes.

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In this work, we develop a SiN membrane-based electromechanical system, and investigate the performance of this system by studying degenerate parametric amplification. Parametric resonance is of interest to many fields of physics.[36] Optical parametric amplifier has witnessed its great success in quantum optics, nonlinear optics, and laser physics. As a counterpart, mechanical parametric effect has attracted significant attention with the development of fabrication techniques recently.[37–48] By directly attaching the substrate of the membrane to a ring piezoelectric actuator, not only can we actuate the membrane oscillation electrically, but more importantly, the stress and thereby the spring constant of the membrane oscillator can also be dynamically modulated via piezoelectric actuation. Such a simple configuration, which does not require complicated nano/micro-fabrication techniques with all the components commercially available, offers several advantages compared to other systems: (i) The spring constant of a high Q SiN membrane oscillator can be electrically controlled without reducing its excellent performance. In principle, the same way can be applied on the ultrahigh Q membranes.[17–20] (ii) Such an electrical channel can also be used within a feedback loop.[49–53] Incorporation of optomechanical interaction and quantum limited detection would offer a useful platform to study cavity optoelectromechanics. (iii) The piezoelectric way is more straightforward than the reservoir engineering method, without requiring appropriate geometric and material design.[54]

In the experiment we use a 50 nm-thick by 500-μm-square window of stoichiometric low-pressure chemical vapor deposition SiN membrane supported by a 5×5 mm^2 size and 200 μm-thick silicon frame from NORCADA Inc. The resonant frequencies of the membrane's vibrational modes are $f_{ij} = \sqrt{\sigma(i^2+j^2)/4\rho l^2}$, where σ ~0.9 GPa is the tensile stress, $\rho \sim 2.7\ g/cm^3$ is the mass density, l is the side length of the square membrane, and i, j are the positive integer mode indices. Typically, such a membrane has large mechanical Q factors in the range of 10^5–10^6 and vibrational frequencies in hundreds kHz to MHz range.[55–60]

The membrane frame is directly glued to the ring piezo actuator at three corners, as shown in Fig. 1(a). As the piezo actuator expands in the thickness mode, it will contact the radial mode, which leads to the deformation of the frame (substrate). The SiN membrane can be understood as a drum with an equilibrium length l_0 stretched to the length l of the frame.[56] The stress is proportional to the difference between l_0 and l. The deformation of the frame, i.e. either squeezed or stretched, results in the changes of stress and the spring constant of the membrane. Figure 1(b) presents the frequency dependence of (3, 3) mode as a function of the voltage applied on the ring piezo actuator. The red dots are the experimental data and the blue line is the linear fit, which indicates that the frequency sensitivity for (3,3) mode is -100Hz/V. This relation holds for a wide dynamic range and hence it can be an additional ingredient to control the property of membrane besides the optomechanical interaction when the membrane is applied for an optomechanical system. The frequency

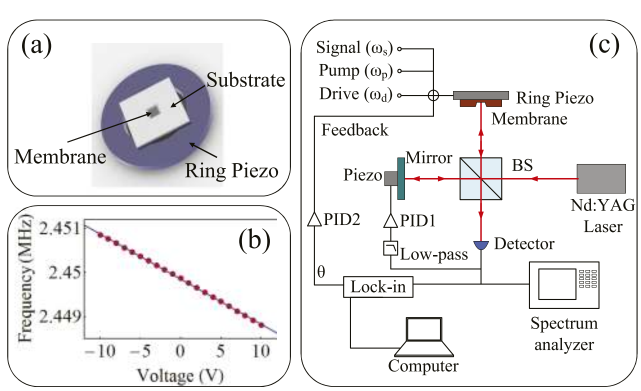

FIG. 1. (a) The image of the SiN membrane attached to the ring piezo actuator. (b) The frequency shift of (3,3) mode as a function of the voltage applied on the ring piezo actuator. (c) The schematic of the experimental system. The signal/pump fields are for the parametric amplification process, and the drive/feedback fields are for the stabilization of the vibrational eigenmodes. Beam splitter (BS); proportional-integral-derivative controller (PID); the phase channel of the lock-in amplifier (θ).

dependence for several other modes is also investigated. We find that the induced frequency fluctuations of different vibrational modes under such conditions are highly correlated. This feature is utilized to actively stabilize the vibrational frequencies of membrane in the experiment, which has a faster response compared to the photothermal feedback.[54,61]

The experimental schematic is shown in Fig. 1(c). The membrane is placed in a vacuum chamber which is ion-pumped to $\sim 10^{-8}$ torr. The electric fields applied on the ring piezo actuator include the signal/pump fields for the parametric amplifier, and the drive/feedback fields for stabilizing the vibrational eigenmodes. The mechanical oscillations are transduced optically by a Michelson interferometer with displacement sensitivity better than 10 fm/Hz$^{1/2}$, which is stabilized with a second piezo actuator, as shown in Fig. 1(c). It is worth noting that the displacement sensitivity can be significantly enhanced by placing the current membrane system inside a high-finesse optical cavity without any modification.

The equation of motion for a parametrically excited mechanical oscillator is[36]

$$\ddot{x} + \frac{\omega_0}{Q}\dot{x} + \frac{k_0 + k_p \cos 2(\omega_0 + \Delta_p)t}{m}x = \frac{F_0}{m}\cos[(\omega_0 + \Delta_s)t + \phi], \tag{1}$$

where $x(t)$ is the membrane displacement, m is the effective mass, ω_0 is the eigenfrequency of the mechanical oscillator, Q is the quality factor, $k_0 = m\omega_0^2$ is the spring constant, $k_p(F_0)$ is the amplitude of the pump (signal) field, $\Delta_p(\Delta_s)$ is the frequency detuning of the pump (signal) field, and ϕ denotes for the phase of the signal relative to the pump field.

When the system is in the limit of weak damping and small oscillation, the secular perturbation theory[62] can be used to solve Eq. (1). We consider the parametrically pumped mechanical response spectrum of the membrane oscillator under the degenerate case, where the pump frequency is always tuned to be twice the signal frequency, i.e. $\Delta_p = \Delta_s = \Delta$. Then the response amplitude is given by[62]

$$a = -\frac{2\Delta + i + (k_p/k_{th})e^{-2i\phi}}{4\Delta^2 + 1 - (k_p/k_{th})^2}. \tag{2}$$

Here $k_{th} = 2k_0/Q$ is the instability threshold of the parametric amplification, and a constant response factor is omitted in Eq. (2).

As one can see in Eq. (2), the parametrically pumped mechanical response spectrum is sensitive to the relative phase between the pump and signal fields. Figure 2(a) shows a Lorentzian profile of the

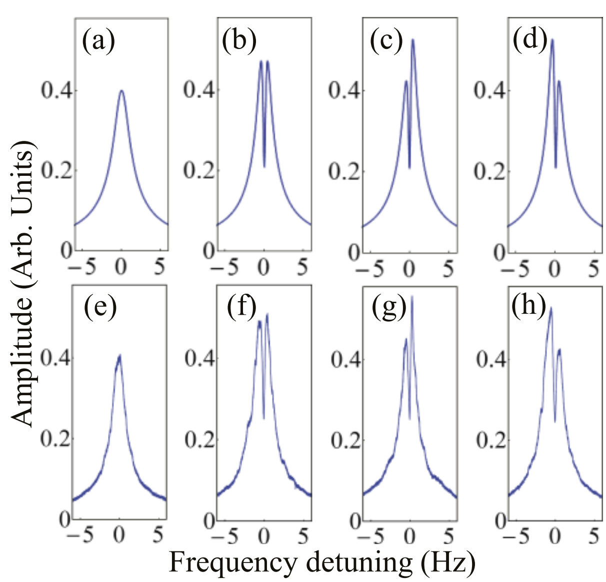

FIG. 2. The mechanical response spectra of the membrane at the parametric excitation. Theoretical simulations for (a) without parametric pump; (b) $\phi = 45°$; (c) $\phi = 45° - 1.5°$; (d) $\phi = 45° + 1.5°$. $k_p/k_{th} = 0.9$ for all the cases in (a-d). (e-h) Experimental measurements for the cases corresponding to (a-d), respectively.

signal field when the parametric pump field is absent. When the signal field has a phase $\pi/4$ relative to the pump, i.e. $\phi = 45°$, there is a symmetric splitting in the mechanical response spectrum,[40,41] as shown in Fig. 2(b). The origin of splitting in the spectrum is the destructive interference in cooperation with the dissipation of the membrane oscillator. When the signal and pump fields are not exactly out of phase ($\phi = 45°$), the splitting of mechanical mode becomes asymmetric, as illustrated in Figs. 2(c) and 2(d), in which the relative phase is 1.5 degrees deviated from $\pi/4$, i.e., $\phi = 45° \pm 1.5°$. Figures 2(e–h) present the experimental measurements corresponding to the cases shown in Figs. 2(a–d), respectively, which finds excellent agreement with the theoretical predictions. During the measurement, the signal and pump fields are both applied on the ring piezo actuator and frequency scan with a fixed phase. This is quite different compared to the situation when the signal frequency scans with the pump frequency fixed, since then the phase difference between the two fields is not a constant.[62] The parametric pump strength is below threshold with $k_p/k_{th} = 0.9$. The signal amplitude is not important for this measurement as long as it is not large enough to consider nonlinearity. Such a spectroscopic signature of parametric amplification in the out of phase situation has been observed in mechanical systems,[40,41] as well as in an optical parametric amplifier.[63] In analogy to the quantum optical systems, the phase-sensitive mechanical parametric amplifier can be useful for applications such as phase-sensitive switching,[64,65] information storage,[66] and logic operations.[67]

When the signal field resonates with the eigenmode of the membrane oscillator, and the pump field is perfectly tuned to be twice the signal field, i.e., $\Delta = 0$, the gain of the parametric amplifier is[62]

$$G = \frac{|x|_{pump\,on}}{|x|_{pump\,off}} = [\frac{\cos^2(\phi + \pi/4)}{(1 - k_p/k_{th})^2} + \frac{\sin^2(\phi + \pi/4)}{(1 + k_p/k_{th})^2}]^{1/2}. \tag{3}$$

Figure 3 represents the amplitude gain as a function of the pump amplitude relative to the pump threshold. The black diamonds represent the deamplification case. The sine term in Eq. (3) dominates when $\phi = \pi/4$, and this leads to the suppression of the signal field, which is limited to -3 dB near the instability threshold.[37] On the contrary, the cosine term remains if $\phi = -\pi/4$, which results in the amplification of the signal field. Previously, we do not consider the nonlinearity, therefore, as the pump amplitude approaches to the threshold, the parametric gain increases to infinity if $\phi = -\pi/4$. However, the mechanical nonlinearities always exist in practical, even comparatively small nonlinearity will limit the gain of amplifier.[44,45] The red squares, blue dots, and green triangles respectively represent the parametric amplification for different signal amplitudes, and they are saturated at different gain levels, as one can see in Fig. 3. The maximum gain obtained below the threshold is ~10 instead of infinity. Although the nonlinearity degrades the performance of parametric amplifier, it provides the opportunity that the nonlinear parametric amplifier can be operated above the linear instability threshold, which can be utilized to study non-equilibrium dynamics.[48]

Now we study the enhanced mechanical Q factor with parametric excitation.[38,42,43,45] As the parametric pump strength increases, the gain goes up and the resonance peak becomes narrower, therefore,

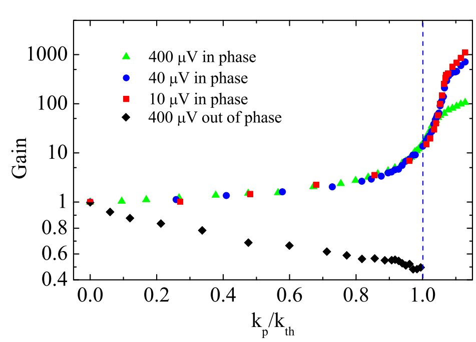

FIG. 3. Parametric amplification/deamplification as a function of the pump amplitude relative to the pump threshold. The black diamonds show the deamplification case, i.e. $\phi = \pi/4$. The red squares, blue dots, and green triangles illustrate the parametric amplification ($\phi = -\pi/4$) for different signal amplitudes respectively. The nonlinearity limits the amplifier gain. The blue dashed line indicates the instability threshold, which is determined by the onset of the self-oscillation.

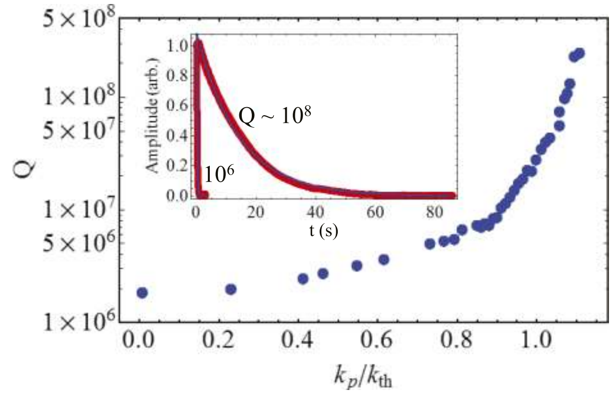

FIG. 4. The enhanced mechanical Q factor with parametric excitation as a function of the pump amplitude relative to the threshold. The inset is the mechanical ringdown response for the membrane without ($Q \sim 10^6$) and with parametric resonant excitation ($Q \sim 10^8$). The red thick curves are the experimental data and the blue thin curves are the exponential fits.

the effective quality factor Q is enhanced, which is measured by the mechanical ringdown technique. To perform the ringdown measurement, we first set $\Delta_p = \Delta_s = 0$, after maximizing the amplitude by tuning the relative phase between the signal and pump fields, we turn off the signal field and measure the decay of the oscillation. The signal field is chosen to be $\sim 10 - 100$ times larger than the thermal amplitude of the membrane. Figure 4 shows the dependence of Q factor for different pump amplitudes. The enhancement of Q factor more than two orders of magnitude has been observed due to the coherent amplification, reaching a Q factor of $\sim 3 \times 10^8$ at room temperature. The inset shows the mechanical ringdown measurements correspond to the cases for the intrinsic and parametrically enhanced quality factors, which are 10^6 and 10^8 respectively. In principle, a higher Q factor is possible to achieve by choosing a vibration mode with a larger intrinsic Q factor.[17] The highest Q factor obtained in the current system is operated slightly above the threshold. The mechanical nonlinearities, e.g. the cubic term, give a smooth response as the pump amplitude goes across the threshold and prevent the collapse of linewidth, since the effective linear damping becomes negative in the instability regime if the non-linearity is ignored.[62] When the parametric pump is above the instability threshold, many interesting nonlinear dynamic processes can emerge, which will be our future studies. It is worth mentioning that the linewidth narrowing or enhancement of Q factor, which is typically related to the amplification of oscillation amplitude, has also been observed in various mechanical systems via other types of excitation, such as optical pumping,[68–70] electrical feedback amplification,[71] and optical band-gap excitation.[72]

A further interesting development related to the parametric amplification is to create a mechanical thermal squeezed state.[37,47,54] The thermal motion of the membrane oscillator can be decomposed into two quadratures $x(t) = X_1(t)\cos\omega_0 t + X_2(t)\sin\omega_0 t$ in a frame rotating at the mechanical resonance frequency ω_0, where $X_1(t)$ and $X_2(t)$ are stochastic Gaussian noise with variance $<X_1^2>=<X_2^2> = k_B T/m\omega_0^2$, where k_B is the Boltzmann constant and T is the temperature. When the parametric pump field is applied, the quadrature of the motion in phase with the parametric modulation

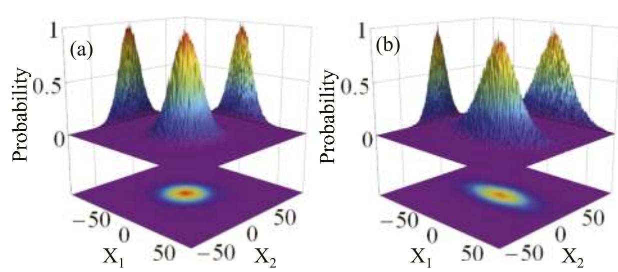

FIG. 5. The probability density functions of the membrane oscillator in the phase space for the thermal (a) and parametrically squeezed (b) cases, respectively.

is amplified and the orthogonal quadrature is deamplified, leading to the parametric squeezing of the thermal mechanical motion in the phase space. $X_1(t)$ and $X_2(t)$ are measured by the two-phase lock-in amplifier (Zurich Instruments HF2LI). The probability density functions of the membrane oscillator in the phase space for the thermal and parametrically squeezed cases are shown in Figs. 5(a) and 5(b), respectively. The thermalmechanical noise reduction of -2.1 dB has been obtained in our system, which is limited to -3 dB since the achievable deamplification with parametric excitation is bounded to 1/2.

In conclusion, we have demonstrated the degenerate parametric amplification in a SiN membrane through piezoelectric driving. The results presented in this letter provide a simple and straightforward way to modulate the spring constant of a SiN membrane with a high intrinsic mechanical Q, which can offer a versatile platform for studying the optoelectromechanics by combining the optomechanical interaction and quantum limited measurements. The implemented system is an ideal nonlinear oscillator with multiple controllable systematic parameters, which allows to study many interesting physical phenomena, such as symmetry breaking,[48] phase transitions,[73] topological operations,[74] and Efimovian dynamics.[75] This might potentially open the avenue to applications in fundamental physics, ultrasensitive measurements, and quantum feedback controls.

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