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PARAMETRIC EXCITATION OF PLASMA OSCILLATIONS IN JOSEPHSON JUNCTIONS

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Résumé. — Des expériences sur un modèle analogique d'une jonction Josephson montrent la présence d'une excitation paramétrique de la résonance de plasma. Les conditions expérimentales sont celles d'une jonction polarisée en courant et excitée par une puissance rf supérieure à un certain seuil qui dépend des pertes du circuit. La radiofréquence est à peu près 2 fois la fréquence de plasma. La génération d'un sous-harmonique d'ordre ½ et de grande amplitude peut être expliquée par les propriétés de stabilité de l'équation différentielle de Mathieu.

Abstract. — Experiments on a Josephson junction analog showed a parametric excitation of the plasma resonance. The experimental conditions were that the junction was dc biased in the supercurrent mode, and that the applied rf-power exceeded a certain threshold value depending on the circuit losses. The frequency of the applied rf was approximately twice the plasma frequency. The observed half harmonic generation with big amplitude may be understood from a discussion of the stability properties of the Mathieu differential equation.

Parametric effects in Josephson junctions at finite voltages have been discussed earlier [1]-[4]. In this communication we report the observation of a parametric excitation of the plasma resonance in connection with a Josephson junction analog. The analog is described elsewhere [5], and the equivalent diagram is shown in figure 1. The conditions for the observation of the parametric excitation were the following : The junction was dc-biased in the supercurrent mode and an rf-current of frequency ω was applied. Under these circumstances it is well known that the Josephson element acts as an inductance which together with the capacitance gives rise to the plasma frequency

$$\omega_p = \sqrt{\frac{2\,eI_0}{\hbar C}\cos\varphi_0}$$

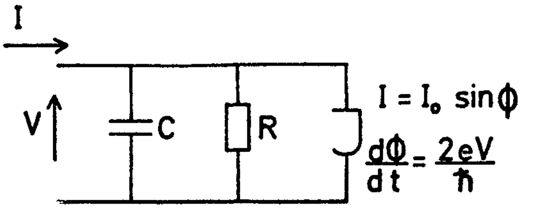

FIG. 1. — The junction model. For the analog $R = 500\ \Omega$, $I_0 = 1$ mA, $C = 100$ nF, and $k = «2\,e/h» = 200$ kHz/V.

where I_0 is the maximum supercurrent and $\sin\varphi_0 = I_{DC}/I_0$. The rf voltage over the junction was measured on an oscilloscope. As could be expected the voltage had a frequency component equal to the applied one and in addition a small content of higher harmonics as long as the rf-amplitude was kept small. As could also be expected a resonance occurred when the applied frequency ω was equal to the plasma frequency ω_p. The new observation, however, was that when $\omega \sim 2\,\omega_p$ half harmonics notably $\sim \omega_p$ suddenly appeared with a large amplitude at a threshold value of the applied rf-power. This is illustrated in figure 2 which shows the rf-voltage just below

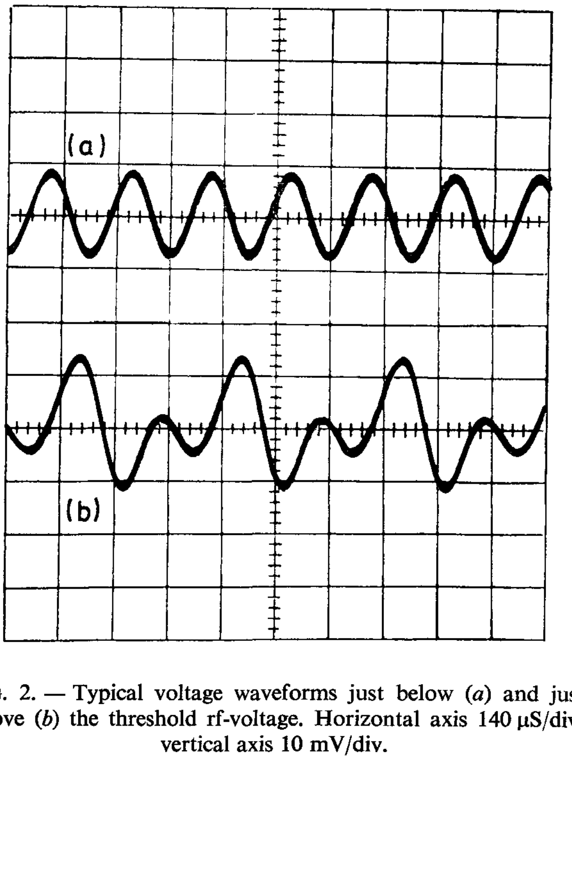

FIG. 2. — Typical voltage waveforms just below (*a*) and just above (*b*) the threshold rf-voltage. Horizontal axis 140 μS/div, vertical axis 10 mV/div.

and just above the threshold when $\omega \sim 2\,\omega_p$. This behaviour is different from the usual harmonic generation which occur gradually with the junction nonlinearity. By investigating this instability further the following observations were made. The threshold rf-voltage was proportional to the circuit losses. At $\omega = 2\,\omega_p$ the threshold condition was $\alpha_c \tan \varphi_0 = 2/Q$ where

$$\alpha_c = \frac{2\,eV_{rf}^{crit}}{\hbar\omega} \quad \text{and} \quad Q = \omega_p RC\,.$$

This is illustrated in figure 3*a*.

For small variations of the frequency in the vicinity of $2\,\omega_p$ the threshold was found to increase as shown in figure 3*b*.

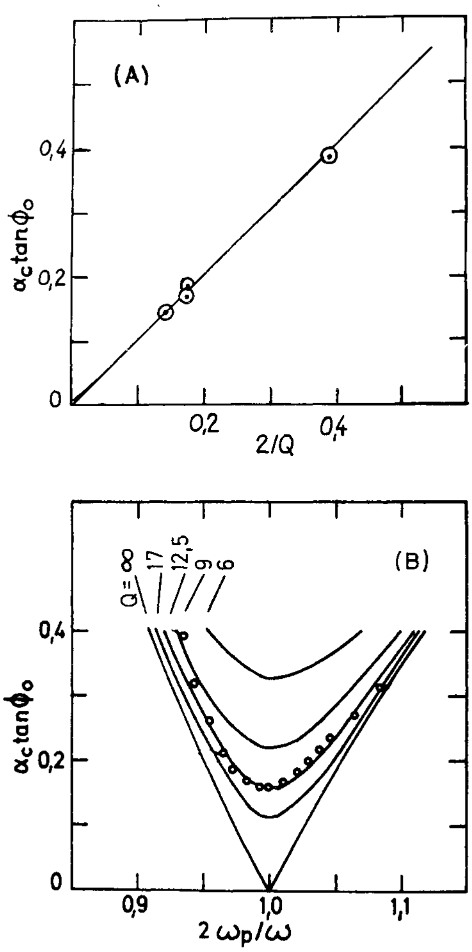

FIG. 3. — (*a*) $\alpha_c \tan \varphi_0$ vs. $2/Q$ at $\omega = 2\,\omega_p$. Straight line : theory. Circles : analog experiments. (*b*) $\alpha_c \tan \varphi_0$ vs. $2\,\omega_p/\omega$. Full curves : Stability curves for the Mathièu equation. Circles : Analog experiments, $\omega_p = 14$ kHz.

Two natural questions are now (1) is it possible to understand this behaviour from a usual junction model ? and (2) can the effect be observed and utilized in a real junction ? The answer to the first question is that it indeed is possible to set up equations which in almost all detail explain the observed instability and the generation of half harmonics. This represents an extension of the usual small signal calculations which can be made for a junction dc biased in the supercurrent mode. The model is the same as shown in figure 1. Expressing now that the total current through the junction is equal to the applied dc and ac currents one obtains the following differential equation

$$I_{DC} + I_{rf} \cos \omega t = \frac{\hbar C}{2\,e}\left(\ddot{\varphi} + \frac{1}{RC}\dot{\varphi} + \frac{2\,eI_0}{\hbar C}\sin \varphi\right). \tag{1}$$

In order to linearize this equation we will try the following *ansatz*

$$\varphi = \varphi_0 + \varphi_1 \cos(\omega t - \theta) + \delta\varphi(t) \tag{2}$$

which expresses that one expects the phase to have a dc component, a component at ω, and something else. Here we need to require that φ_1, $\delta\varphi(t) \ll 1$. By inserting this « ansatz » in eq. (1) and extracting the dc components and the components at ω, expressions are obtained for φ_0 and φ_1. The remaining terms give rise to a differential equation in $\delta\varphi(t)$. To first order in φ_1 the results are

$$\sin \varphi_0 = \frac{I_{DC}}{I_0}$$

$$\varphi_1 = \frac{2\,eI_{rf}}{\hbar C}\,\frac{1}{\sqrt{(\omega_p^2 - \omega^2)^2 + (\omega/RC)^2}}$$

where ω_p again is the plasma frequency. φ_1 may be shown to be equal to the more commonly used quantity $\alpha\left(\varphi_1 = \alpha = \dfrac{2\,eV_{rf}}{\hbar\omega}\right)$.

The remaining terms give rise to the following differential equation in $\delta\varphi(t)$.

$$\delta\ddot{\varphi} + \frac{1}{RC}\delta\dot{\varphi} + {} + \omega_p^2[1 - \varphi_1 \tan \varphi_0 \cos(\omega t - \theta)]\,\delta\varphi = 0\,. \tag{3}$$

On the right hand side of this equation terms which are of order φ_1^2 or higher have been neglected. They are retained in [6]. Eq. (3) has the character of a damped harmonic oscillator with an eigenfrequency which is modulated with the applied frequency around the plasma frequency. The depth of modulation is given by the parameter $\varphi_1 \tan \varphi_0$.

By making the substitution

$$\delta\varphi = e^{-t/2RC}\,\delta\varphi' \tag{4}$$

the differential equation in $\delta\varphi'$ becomes the standard form of the Mathieu equation [7]

$$\frac{d^2\delta\varphi'}{dz^2} + (a - 2\,q \cos 2\,z)\,\delta\varphi' = 0 \tag{5}$$

with parameters a and q given by

$$a = \left(\frac{2\,\omega_p}{\omega}\right)^2 \left[1 - \left(\frac{1}{2\,Q}\right)^2\right]$$

$$2\,q = \left(\frac{2\,\omega_p}{\omega}\right)^2 \alpha \tan \varphi_0$$

where $Q = \omega_p\, RC$. It is well known that the Mathieu equation may have unstable, that is exponentially increasing solutions [7], depending on the magnitude of a and $2\,q$.

The solution for $\delta\varphi'$ which is of interest here is the one where $\omega \sim 2\,\omega_p$ that is $a \sim 1$. The corresponding solutions for $\delta\varphi'$ consists — above a certain value of $2\,q$ — of an exponentially decreasing and an exponentially increasing part both of which are periodic with frequency $\omega/2$. However, only when

$$\frac{2\,q}{a} \sim \alpha \tan \varphi_0 > \frac{2}{Q}$$

will the solution for $\delta\varphi$ itself become exponentially increasing. In that case the rate of increase of $\delta\varphi'$ overcomes the exponential damping due to the circuit losses as described in eq. (4). Once $\delta\varphi$ becomes exponentially increasing the approximation of the smallness of $\delta\varphi$ has broken down, however the derived threshold condition for the parametric half harmonic generation is still valid. The threshold curves around $\omega = 2\,\omega_p$ for various values of Q are shown in figure 3*b* together with the experimental points from the analog measurements. The fitted value of Q can be accounted for by making up the total circuit losses.

A number of other observations have been made including the generation of $\frac{3}{2}$, $\frac{5}{2}$, etc. harmonics and parametric excitations on rf-induced steps. In conclusion — and as a partial answer to the second question we note that for typical tunnel junction parameters, and frequencies around 35 GHz the threshold power for observing the parametric effect may correspond to α's of the order 0.1. In point contact junctions it appears less likely to observe the effect.

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