Title
PARAMETRIC INSTABILITIES IN TURBULENT, INHOMOGENEOUS PLASMA

Authors
Nicholson, Dwight R.
Kaufman, Allan N.

Publication Date
1974-07-21

Submitted to Physical Review Letters

LBL-3302
Preprint c.

PARAMETRIC INSTABILITIES IN TURBULENT, INHOMOGENEOUS PLASMA

Dwight R. Nicholson and Allan N. Kaufman

July 21, 1974

Prepared for the U. S. Atomic Energy Commission
under Contract W-7405-ENG-48

DISCLAIMER

PARAMETRIC INSTABILITIES IN TURBULENT, INHOMOGENEOUS PLASMA*

Dwight R. Nicholson and Allan N. Kaufman

Department of Physics and Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

July 21, 1974

ABSTRACT

It is known that parametric instabilities involving two coupled modes, with oppositely directed group velocities, saturate convectively if the medium is inhomogeneous. This work considers the modification of that result, when weak long-wave turbulence is present, in addition to the background inhomogeneity. We find that the convective saturation disappears when the turbulence exceeds a certain level, absolute growth occurring instead.

The behavior of parametric instabilities in homogeneous media is now fairly well understood.[1-3] Study of the inhomogeneous situation is progressing rapidly,[4-10] motivated by the problems of laser fusion,[11] heating of magnetically confined plasma, and ionospheric modification.

Most theories of coupled mode effects assume that the pump (or driver), which is responsible for coupling the two modes, has a definite phase relationship to them. In practice, there will be a finite bandwidth[12] in the driver; furthermore, there may be spatial and temporal fluctuations in the background medium which would affect the coupling and propagation of the modes.[13]

When all inhomogeneities are stationary in time, and slowly varying in space with respect to the wavelengths of the three waves, the coupled mode equations can be cast into the form[5,8]

$$\left(\frac{\partial}{\partial t} + V_1 \frac{\partial}{\partial x}\right) a_1(x,t) = \gamma_0\, a_2(x,t) \exp\left[i\int_0^x \kappa(x')dx'\right]$$

$$\left(\frac{\partial}{\partial t} + V_2 \frac{\partial}{\partial x}\right) a_2(x,t) = \gamma_0\, a_1(x,t) \exp\left[-i\int_0^x \kappa(x')dx'\right] \qquad (1)$$

where $a_1(x,t)$ and $a_2(x,t)$ are the slowly varying (in space and time) amplitudes of the waves; V_1 and V_2 are the group velocities of the waves in the absence of coupling; γ_0 is the coupling constant, usually proportional to the amplitude of the third wave, and here assumed constant in space and time; $\kappa(x)$ is the wave number mismatch between the three waves in a WKBJ sense: $\kappa(x) = k_0(x) - k_1(x) - k_2(x)$. The origin $x = 0$ is chosen as the reference point of exact wave number matching, i.e., $\kappa(x = 0) = 0$, in the absence of turbulence. The parameters V_1, V_2, and γ_0 can be combined to form the fundamental length $L_0 \equiv \sqrt{|V_1V_2|}/\gamma_0$. If $V_1V_2 > 0$, L_0 is the amplification length; if $V_1V_2 < 0$, L_0 is the minimum length of a finite system for the existence of absolute instability.

For purposes of this paper, we separate the mismatch function $\kappa(x)$ into a nonrandom part, representing a constant gradient, and a random part with zero mean:

$$\kappa(x) = \kappa' x + \delta\kappa(x) \; ; \qquad \langle \delta\kappa(x) \rangle = 0 \; . \tag{2}$$

The turbulent contribution is characterized by the rms mismatch $\Delta \equiv \left(\left\langle\left(\delta\kappa(x)\right)^2\right\rangle\right)^{\frac{1}{2}}$ and the correlation length L_T.

In the case $V_1V_2 > 0$, there is no possibility of absolute instability,[2] and one can assume a steady state. This case has been considered by Kaw et al.[14] For $\kappa' = 0$, and with a constant source at $x = 0$, the growth length was found to be increased by the presence of turbulence, from L_0 to $(L_0\Delta)^2 L_T$ for $\Delta^2 >> 1/L_TL_0$. For $\kappa' \neq 0$ and no turbulence, we know[5] that the maximum amplification of a temporally constant source at $x = 0$ is $\sim\exp(\pi\lambda)$, where $\lambda \equiv \gamma_0{}^2/V_1V_2\kappa'$. In the turbulent case a similar result was found, but with the amplification length increased.

In this paper we consider the case $V_1V_2 < 0$, which occurs in back-scatter instabilities. If $\kappa' = 0$, and there is no turbulence, an absolute instability exists. On the other hand, for $\kappa' \neq 0$ and in the absence of turbulence, the absolute instability is replaced by saturation of an initial pulse at a value $\exp(\pi\lambda)$. We interpret this saturation, at a fixed point in space, as due to destructive interference by the waves. We expect that the effectiveness of this interference would be reduced, if phase incoherence is introduced by spatial turbulence; and that the saturation could be eliminated, with absolute instability again obtained for a sufficient level of turbulence.

Spatial turbulence is characterized by an amplitude Δ and a correlation length L_T. We take the correlation function to be statistically uniform and Gaussian:

$$\langle \delta\kappa(x)\; \delta\kappa(x') \rangle = \Delta^2 \exp\left[-\frac{1}{2}(x - x')^2/L_T{}^2\right] \; . \tag{3}$$

Consistent with Eq. (3), we represent the random function $\delta\kappa(x)$ as

$$\delta\kappa(x) = (32\pi)^{1/4}\sqrt{\frac{L_T}{L}}\,\Delta \sum_{j=1}^{\infty} e^{-(k_j L_T)^2/4} \sin(k_j x + \alpha_j) \tag{4}$$

where $k_j = 2\pi j/L$; L is a length much larger than L_0, L_T, and the pulse width at all times of interest; α_j is a random phase, with probability uniformly distributed between zero and 2π; and the upper limit of summation is taken to be large,[15] such that $(k_j)_{MAX} L_T >> 1$. For a given realization $\{\alpha_j\}$, and a particular set of parameters, the total mismatch gradient $d\kappa(x)/dx \equiv \kappa' + d\big(\delta\kappa(x)\big)/dx$ is illustrated in Fig. 1a.

Given this model, the coupled-mode equations (1) are integrated numerically to determine the effect of the spatial turbulence on the response of the system to an initial perturbation. The main result of this study is that if Δ exceeds a threshold value (dependent on L_T), the instability no longer saturates at a value $\sim\exp(\pi\lambda)$, but grows exponentially at fixed x for all time, at a growth rate γ_a lower than that for a nonturbulent homogeneous medium. In Fig. 1b we show the temporal development of a typical unstable case with initial conditions $a_1(x,t = 0) = \delta(x)$, $a_2(x,t = 0) = 0$. Fluctuations reminiscent of Ref. 7 are observed, but with a less regular character. The most unstable part of the pulse has the behavior of a temporal normal mode, maintaining its shape while growing exponentially.

In Fig. 1c we show the absolute growth rate γ_a/γ_0 vs Δ/L_0 for $V_1/V_2 = -1$, $\lambda^{-1} \equiv \kappa' L_0^{\,2} = 1$, $L_T/L_0 = 1.27$. The threshold turbulence level is seen to occur at $\Delta/L_0^{-1} \approx 0.1$. The maximum growth rate is $\gamma_a/\gamma_0 \approx 0.70$, which is comparable to the homogeneous growth rate γ_0.

The function $d\kappa(x)/dx$ shown in Fig. 1a corresponds to the threshold case of Fig. 1c. This function is seen to lie in the range $0.80 < L_0^2\, d\kappa(x)/dx < 1.20$. This shows that the coupled mode equations can produce absolute instability even if $d\kappa(x)/dx$ vanishes nowhere in the medium, in contrast to the result of Kaw et al.[14]

In Fig. 1d we show the growth rate γ_a as a function of correlation length L_T, for fixed fluctuation level Δ. For this calculation we use the same realization of the set $\{\alpha_j\}$ in Eq. (4), varying L_T with $\Delta/L_0^{-1} = 0.5$. We see that the absolute growth rate decreases with increasing correlation length.

We have also considered parameters corresponding to Raman backscattering[16] in a laser pellet geometry.[11] Results similar to those above were found.

It should be noted that in this work the turbulent wavelengths are quite long, the shortest being equal to the standard length $L_0 \equiv \sqrt{|V_1 V_2|}/\gamma_0$.

A further point is that for a given value of Δ, the absolute growth rate depends somewhat on the realization of $\{\alpha_j\}$ chosen in Eq. (4). The relative dispersion of the growth rates is of the order of 30-40%.

We interpret our results as follows. The convective saturation of the linearly inhomogeneous coupled mode problem,[5,7] with oppositely directed group velocities, seems to be due to destructive interferences between responses originating at large positive and negative positions. This interpretation is supported by the work of White et al.,[17] who found that replacing the constant pump (γ_0 in Eq. (1)) by a Gaussian in x resulted in absolute instability; i.e., removing the responses at large x removed the destructive interference at $x = 0$. The analogy in our work is that the turbulence upsets the delicately balanced destructive interferences, allowing the system to grow absolutely.

We conclude that the presence of long-wave turbulence tends to destabilize the convective saturation found[5,7] for the coupled mode equations, with oppositely directed group velocities, in an inhomogeneous medium. This destabilization occurs at relatively small turbulence levels; so small that the condition $\partial\kappa(x)/\partial x = 0$ is never satisfied.

ACKNOWLEDGMENTS

For useful discussions we gratefully thank B. Cohen, C. N. Lashmore-Davies, G. Johnston, P. Kaw, Y. C. Lee, C. S. Liu, C. E. Max, M. Mostrom, M. N. Rosenbluth, C. Oberman, G. Smith, and R. White.

FOOTNOTES AND REFERENCES

* Work supported in part by the U. S. Atomic Energy Commission.

1. N. M. Kroll, J. Appl. Phys. 36, 34 (1965).
2. D. L. Bobroff and H. A. Haus, J. Appl. Phys. 38, 390 (1967).
3. E. S. Cassedy and C. R. Evans, J. Appl. Phys. 43, 4452 (1972).
4. F. W. Perkins and J. Flick, Phys. Fluids 14, 2012 (1971).
5. M. N. Rosenbluth, Phys. Rev. Letters 29, 565 (1972).
6. D. F. Dubois and E. Williams, CU #1005 (1973).
7. M. N. Rosenbluth, R. B. White, and C. S. Liu, Phys. Rev. Letters 31, 1190 (1973).

8. A. D. Piliya, Proceedings 10th Conference on Phenomena in Ionized Gases, Oxford (1971).

9. D. W. Forslund, J. M. Kindel, and E. L. Lindman, LA-UR-73-500 (1973).

10. C. Moroli, J. Plasma Physics 10, 165 (1973).

11. J. L. Emmett, J. Nuckolls, and L. Wood, Sci. Am. 230, No. 6, pages 24-37; June, 1974.

12. Finite bandwidth effects have been considered by R. L. Carmen, F. Shimizu, C. S. Wang, and N. Bloembergen, Phys. Rev. A2, 60 (1970); S. E. Bodner, UCRL-74074 (1972); S. Tamor, Phys. Fluids 16, 1169 (1973); E. J. Valeo and C. R. Oberman, Phys. Rev. Letters 30, 1035 (1973); J. J. Thompson, W. Kruer, S. Bodner, and J. De Groot, UCRL-74637 (1973), to be published in Phys. Fluids; J. J. Thompson and J. I. Karush, UCRL-75374 (1973); an important effect is usually found when the bandwidth is greater than the instability growth rate in the zero bandwidth case.

13. The problem of three wave interactions in the presence of random stationary background fluctuations was first considered by V. V. Tamoikin and S. M. Fainshtein, Sov. Phys. JETP 35, 115 (1972). The coupled equations for all three waves were treated, assuming small relative density fluctuations, and it was found that the relaxation oscillations were suppressed by the turbulence.

14. P. Kaw, R. White, D. Pesme, M. Rosenbluth, G. Laval, R. Varma, and R. Huff, IC/73/142 (September 1973; revised version March 1974).

15. Care was taken that all turbulent wavelengths be large with respect to the numerical grid spacing. When this was true, the results were insensitive to the numerical grid spacing.

16. J. Drake, P. Kaw, Y. C. Lee, G. Schmidt, C. S. Liu, and M. N. Rosenbluth, UCLA-PPG-158 (July 1973).

17. R. White, P. Kaw, D. Pesme, M. N. Rosenbluth, G. Laval, R. Huff, and R. Varma, Nuc. Fusion 14, 45 (1974).

FIGURE CAPTIONS

Fig. 1. $V_2/V_1 = -1$, $\lambda^{-1} \equiv \kappa' L_0^2 = 1$ $(L/L_0 = 400)$. A particular realization of the set $\{\alpha_j\}$ is used.

(a) The function $L_0^2 d\kappa(x)/dx$ vs x/L_0 at the threshold value $\Delta/L_0^{-1} \approx 0.1$ in Fig. 1c. $L_T/L_0 = 1.27$.

(b) The temporal evolution of $|a_2(x,t)|$ vs x/L_0 for the initial conditions $a_1(x,t = 0) = \delta(x)$, $a_2(x,t = 0) = 0$. $\Delta/L_0^{-1} = 0.5$, $L_T/L_0 = 1.27$.

(c) The absolute growth rate γ_a/γ_0 vs the RMS mismatch function Δ/L_0^{-1}.

(d) The absolute growth rate γ_a/γ_0 vs the correlation length L_T/L_0.

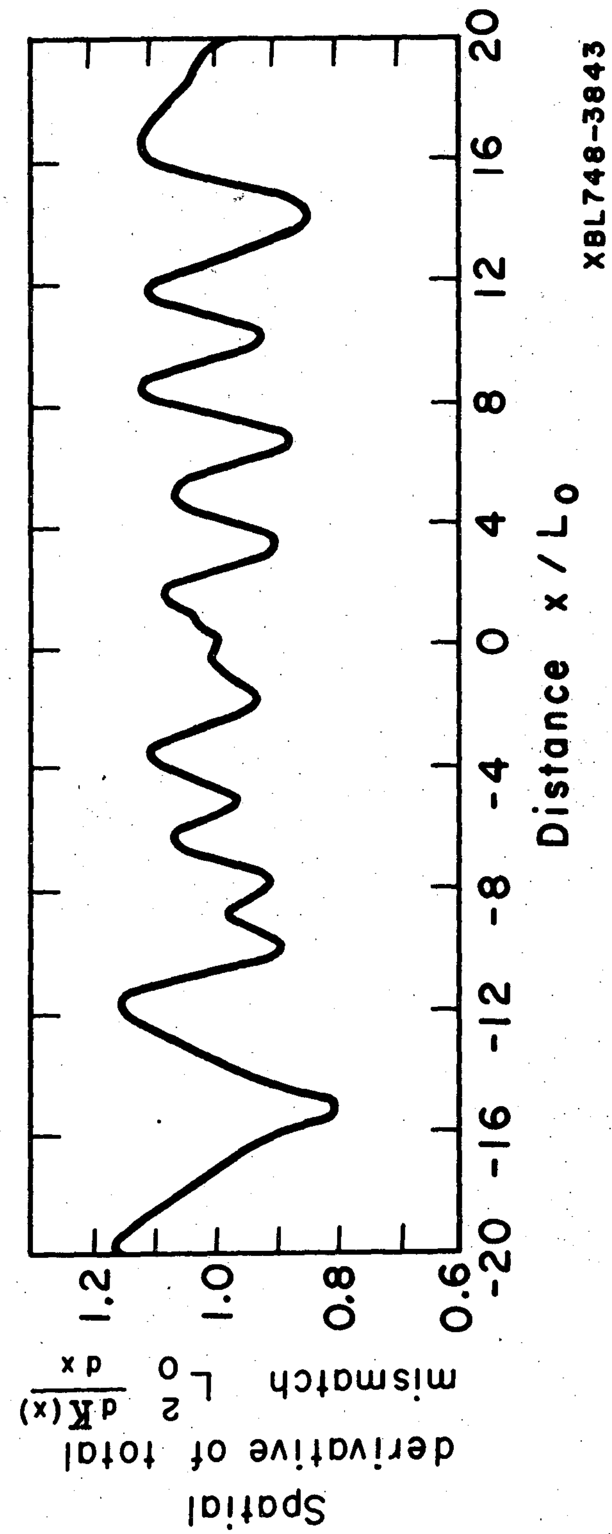
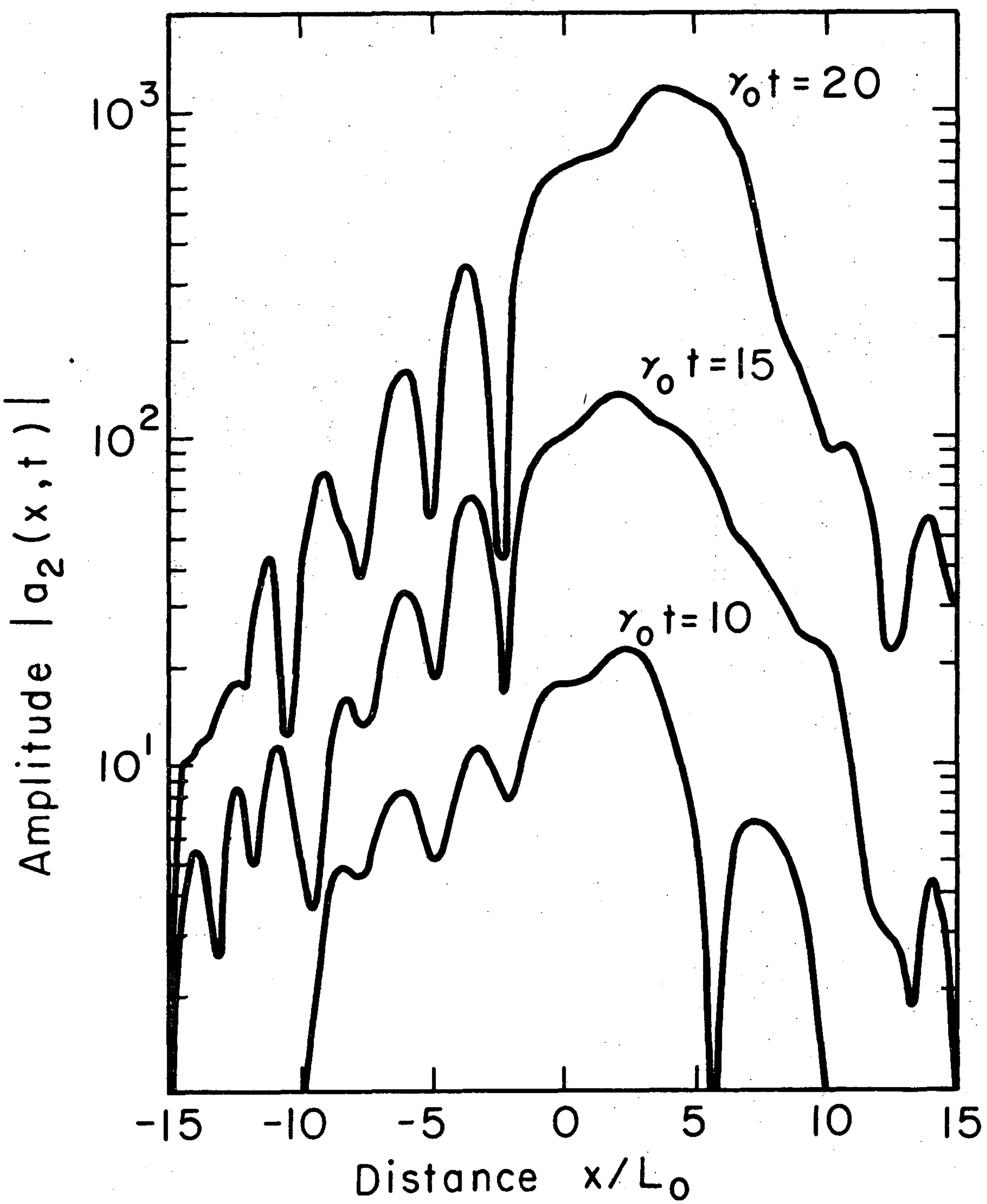

Spatial derivative of total mismatch
$L_0^2 \frac{dK(x)}{dx}$
1.2
1.0
0.8
0.6
-20
-16
-12
-8
-4
0
4
8
12
16
20
Distance x / L_o
XBL748-3843

Fig. 1a.



XBL748-3841

Fig. 1b.

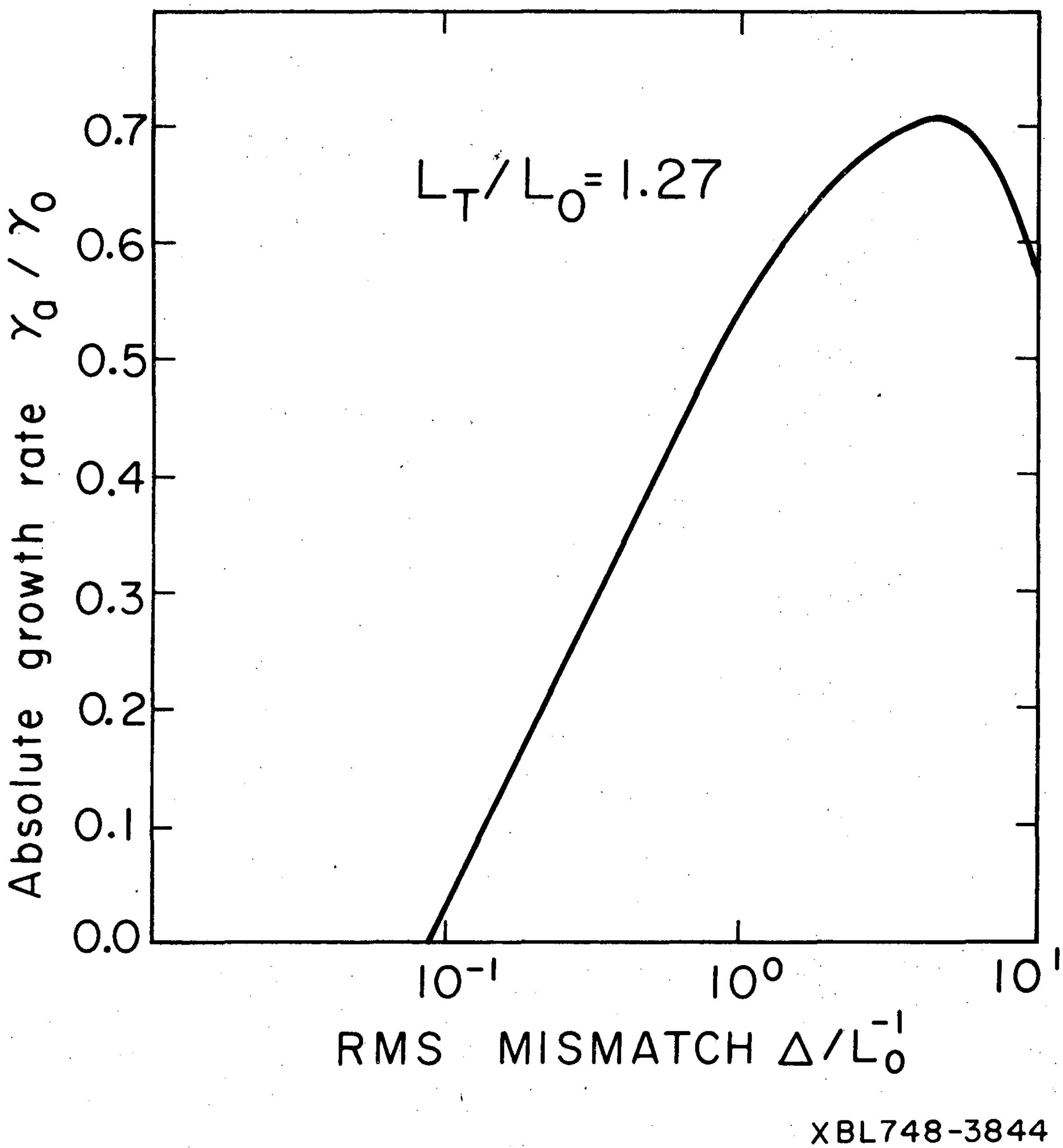
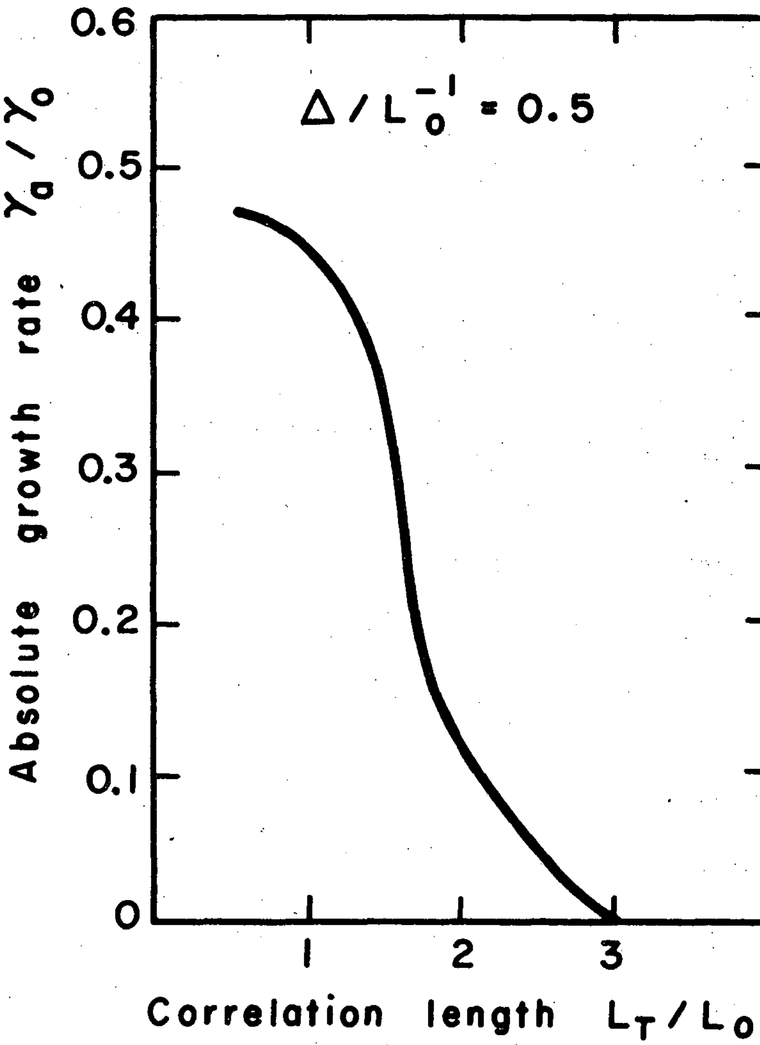

Absolute growth rate γ_a / γ_0
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$L_T / L_0 = 1.27$
10^{-1}
10^0
10^1
RMS MISMATCH Δ / L_0^{-1}
XBL748-3844

Fig. 1c.



XBL 748-3842

Fig. 1d.

0 0 0 0 4 2 0 3 7 1 8

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720